A.A.S. **4**, II, 303-337 (2009)

PAPERS FROM ACTUARIAL JOURNALS WORLDWIDE

Single copies of all the papers listed here can be obtained, subject to charge and copyright regulations, from the actuarial profession's libraries. Issues may be borrowed by members. Tel: 0131 240 1311 or 01865 268206/208; libraries@actuaries.org.uk

ASTIN Bulletin

Volume **39** (2), 2009

Alai, D. & Wüthrich, M. V. *Taylor Approximations for Model Uncertainty within the Tweedie Exponential Dispersion Family.* 453-477. The use of generalized linear models (GLM) to estimate claims reserves has become a standard method in insurance. Most frequently, the exponential dispersion family (EDF) is used; see e.g. England, Verrall [2].We study the so-called Tweedie EDF and test the sensitivity of the claims reserves and their mean square error of predictions (MSEP) over this family. Furthermore, we develop second order Taylor approximations for the claims reserves and the MSEPs for members of the Tweedie family that are difficult to obtain in practice, but are close enough to models for which claims reserves and MSEP estimations are easy to determine. As a result of multiple case studies, we find that claims reserves estimation is relatively insensitive to which distribution is chosen amongst the Tweedie family, in contrast to the MSEP, which varies widely.

Belhadi, H., Goulet, V. & Ouellet, T. *On Parameter Estimation in Hierarchical Credibility.* 495-514. One can find in the literature three main sets of estimators for the variance components in the hierarchical credibility model. This paper presents these estimators in a unified notation, studies some of their properties important for numerical evaluation and compares their relative performance by simulation. The paper also demonstrates how function cm of the R package actuar can be used to fit hierarchical models to insurance data.

Biagini, F. & Ulmer, S. *Asymptotics for Operational Risk Quantified with Expected Shortfall.* 735-752. In this paper we estimate operational risk by using the convex risk measure Expected Shortfall (ES) and provide an approximation as the confidence level converges to 100% in the univariate case. Then we extend this approach to the multivariate case, where we represent the dependence structure by using a Lévy copula as in Böcker and Klüppelberg (2006) and Böcker and Klüppelberg, C. (2008). We compare our results to the ones obtained in Böcker and Klüppelberg (2006) and (2008) for Operational VaR and discuss their practical relevance.

Filipovic, D. *Multi-Level Risk Aggregation.* 565-575. In this paper we compare the current Solvency II standard and a genuine bottom-up approach to risk aggregation. This is understood to be essential for developing a deeper insight into the possible differences between the diversification assumptions between the standard approach and internal models.

Hao, X. & Tang, Q. *Asymptotic Ruin Probabilities of the Lévy Insurance Model under Periodic Taxation.* 479-494. Recently, Albrecher and his coauthors have published a series of papers on the ruin probability of the Lévy insurance model under the so-called loss-carry-forward taxation, meaning that taxes are paid at a certain fixed rate immediately when the surplus of the company is at a running maximum. In this paper we assume periodic taxation under which the company pays tax at a fixed rate on its net income during each period. We devote ourselves to deriving explicit asymptotic relations for the ruin probability in the most general Lévy insurance model in which the Lévy measure has a subexponential tail, a convolution-equivalent tail, or an exponential-like tail.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

Ko, B., Russo, R. P. & Shyamalkumar, N. D. *A Note on Nonparametric Estimation of the CTE.* 717-734. The a-level Conditional Tail Expectation (CTE) of a continuous random variable X is defined as its conditional expectation given the event $\{X > qa\}$ where qa represents its a-level quantile. It is well known that the empirical CTE (the average of the $n(1-a)$ largest order statistics in a sample of size n) is a negatively biased estimator of the CTE. This bias vanishes as the sample size increases, but in small samples can be significant. In this article it is shown that an unbiased nonparametric estimator of the CTE does not exist. In addition, the asymptotic behavior of the bias of the empirical CTE is studied, and a closed form expression for its first order term is derived. This expression facilitates the study of the behaviour of the empirical CTE with respect to the underlying distribution, and suggests an alternative (to the bootstrap) approach to bias correction. The performance of the resulting estimator is assessed via simulation.

Kull, A. *Sharing Risk: An Economic Perspective.* 591-613. We revisit the relative retention problem originally introduced by de Finetti using concepts recently developed in risk theory and quantitative risk management. Instead of using the Variance as a risk measure we consider the Expected Shortfall (Tail-Value-at-Risk) and include capital costs and take constraints on risk capital into account. Starting from a risk-based capital allocation, the paper presents an optimization scheme for sharing risk in a multi-risk class environment. Risk sharing takes place between two portfolios and the pricing of risk transfer reflects both portfolio structures. This allows us to shed more light on the question of how optimal risk sharing is characterized in a situation where risk transfer takes place between parties employing similar risk and performance measures. Recent developments in the regulatory domain ('risk-based supervision') pushing for common, insurance industry-wide risk measures underline the importance of this question. The paper includes a simple non-life insurance example illustrating optimal risk transfer in terms of retentions of common reinsurance structures.

Levitan, S. & Thomson, R. *The Application of Expected-Utility Theory to the Choice of Investment Channels in a Defined-Contribution Retirement Fund.* 615-647. This study examines the practical application of a system for the derivation of member utility functions for the purpose of recommending investment-channel choice to members of a defined-contribution retirement fund. The utility functions of post-retirement benefits from members of a defined-contribution fund are elicited. The risk aversion of each member is measured and the results are compared with a standard risk-tolerance assessment method.

Lindset, S. & Persson, S.-A. *Continuous Monitoring: Does Credit Risk Vanish?* 577-589. We present a model for pricing credit risk protection for a limited liability non-life insurance company. The protection is typically provided by a guaranty fund. In the case of continuous monitoring, i.e., where the market values of the company's assets and liabilities are continuously observable, and where the market values of assets and liabilities follow continuous processes, regulators can liquidate the insurance company at the instant the market value of its assets equals the market value of its liabilities, implying that the credit protection is worthless. When jumps are included in the claims process, the protection provided by the guaranty fund has a strictly positive market value. The ability to continuously monitor asset prices with continuous sample paths eliminates economic losses from default. Our analysis suggests that economic losses from default stem from jumps in continuously observed asset prices and/or that asset prices are not continuously observed.

Liu, H. & Verrall, R. *A Bootstrap Estimate of the Predictive Distribution of Outstanding Claims for the Schnieper Model.* 677-689. This paper considers the bootstrapping approach for measuring reserve uncertainty when applying the model of Schnieper for reserves which separate Incurred But Not Reported (IBNR) and Incurred But Not Enough Reserved

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

(IBNER) claims. The Schnieper method has been explored in Liu and Verrall (2009), and the Mean Square Errors of Prediction (MSEP) derived. This paper takes this further by deriving the full predictive distribution, using bootstrapping. Numerical examples are provided and the MSEP from the bootstrapping approach are compared with those obtained analytically.

MacDonald, A. S. & McIvor, K. R. *Modelling Adverse Selection in the Presence of a Common Genetic Disorder: The Breast Cancer Polygene.* 373-402. The cost of adverse selection in the life and critical illness (CI) insurance markets, brought about by restrictions on insurers' use of genetic test information, has been studied for a variety of rare single-gene disorders. Only now do we have a study of a common disorder (breast cancer) that accounts for the risk associated with multiple genes. Such a collection of genes is called a polygene. We take two approaches to modelling the severity of adverse selection which may result from insurers being unable to take account of tests for polygenes as well as major genes. First, we look at several genetic testing scenarios, with a corresponding range of possible insurance-buying behaviours, in a market model for CI insurance. Because a relatively large proportion of the population is exposed to adverse polygenic risk, the costs of adverse selection are potentially much greater than have been associated with rare single genes. Second, we use utility models to map out when adverse selection will appear, and which risk groups will cause it. Levels of risk aversion consistent with some empirical studies do not lead to significant adverse selection in our model, but lower levels of risk aversion could effectively eliminate the market.

Malinovskii, V. K. *Scenario Analysis for a Multi-Period Diffusion Model of Risk.* 649-676. This paper extends and develops the results of a previous paper Malinovskii (2007). Dealing with a simplistic diffusion multi-year model of insurance operations, this paper illustrates the adaptive control approach when the object of control is the balance of solvency and equity. Compared to the previous paper, a new element is the 'scenario of nature', or the incomplete knowledge of future risk, which is quite often the case in insurance. It introduces a new and inevitable randomness in the model and leads to a qualitative difference in its behaviour.

Olivieri, A. & Pitacco, E. *Stochastic Mortality: The Impact on Target Capital.* 541-563. In this paper, we take the point of view of an insurer dealing with life annuities, which aims at building up a (partial) internal model in order to quantify the impact of mortality risks, namely process and longevity risk, in view of taking appropriate risk management actions. We assume that a life table, providing a best-estimate assessment of annuitants' future mortality is available to the insurer; conversely, the insurer has no access to data sets and the methodology underlying the construction of the life table. Nonetheless, the insurer is aware that, in the presence of mortality risks, a stochastic approach is required. The (projected) life table, which provides a deterministic description of future mortality, should then be used as the basic input of a stochastic model.

The model we propose focuses on the annual number of deaths in a given cohort, which we represent allowing for a random mortality rate. To this purpose, we adopt the widely used Poisson model, first assuming a Gamma-distributed random parameter, and second introducing time-dependence in the parameter itself. Further, we define a Bayesian-inferential procedure for updating the parameters to experience in some situations. The setting we define does not demand advanced analytical tools, while allowing for process and longevity risk in a rigorous way. The model is then implemented for capital allocation purposes. We investigate the amount of the required capital for a given life annuity portfolio, based on solvency targets which could be adopted within internal models. The outcomes of such an investigation are compared with the capital required according to some standard rules, in particular those proposed within the Solvency 2 project.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

Rizzo, M. L. *New Goodness-of-Fit Tests for Pareto Distributions.* 691-715. A new approach to goodness-of-fit for Pareto distributions is introduced. Based on Euclidean distances between sample elements, the family of statistics and tests is indexed by an exponent in $(0, 2)$ on Euclidean distance. The corresponding tests are statistically consistent and have excellent performance when applied to heavy-tailed distributions. The exponent can be tailored to the particular Pareto distribution. The goodness-of-fit statistic measures all types of differences between distributions, hence it is also applicable as a minimum distance estimator. Implementation of the test statistics is developed and applied to estimation of the tail index in three well known examples of claims data, and compared with the classical EDF statistics.

Thomas, R. G. *Demand Elasticity, Risk Classification and Loss Coverage.* 403-428. This paper investigates the effects of high or low fair-premium demand elasticity in an insurance market where risk classification is restricted. The effects are represented by the equilibrium premium, and the risk-weighted insurance demand or 'loss coverage'. High fair-premium demand elasticity leads to a collapse in loss coverage, with an equilibrium premium close to the risk of the higher-risk population. Low fair-premium demand elasticity leads to an equilibrium premium close to the risk of the lower-risk population, and high loss coverage — possibly higher than under more complete risk classification. The demand elasticity parameters which are required to generate a collapse in coverage in the model in this paper appear higher than the values for demand elasticity which have been estimated in several empirical studies of various insurance markets. This offers a possible explanation of why some insurance markets appear to operate reasonably well under community rating, without the collapse in coverage which insurance folklore suggests.

Verlaak, R., Hurlimann, W. & Beirlant, J. *Quasi-Likelihood Estimation of Benchmark Rates for Excess of Loss Reinsurance Programs.* 429-452. In this paper a method for determining benchmark rates for the excess of loss reinsurance of a Motor Third Party Liability insurance portfolio will be developed based on observed market rates. The benchmark rates are expressed as a percentage of the expected premium income that is available to cover the whole risk of the portfolio. The rates are assumed to be based on a compound process with a heavy tailed severity, such as Burr or Pareto distributions. In the absence of claim data these assumptions propagate the theoretical benchmark rate component of the regression model. Given the whole set of excess of loss reinsurance rates in a given market, the unknown parameters are estimated within the framework of quasi-likelihood estimation. This framework makes it possible to select a theoretical benchmark rate model and to choose a parsimonious submodel for describing the observed market rates over a 4-years observation period. This method is applied to the Belgian Motor Third Party Liability excess of loss rates observed during the years 2001 till 2004.

Yuen, F. L. & Yang, H. *Option Pricing in a Jump-Diffusion Model with Regime-Switching.* 515-539. Nowadays, the regime switching model has become a popular model in mathematical finance and actuarial science. The market is not complete when the model has regime switching. Thus, pricing the regime switching risk is an important issue. In Naik (1993), a jump diffusion model with two regimes is studied. In this paper, we extend the model of Naik (1993) to a multi-regime case. We present a trinomial tree method to price options in the extended model. Our results show that the trinomial tree method in this paper is an effective method; it is very fast and easy to implement. Compared with the existing methodologies, the proposed method has an obvious advantage when one needs to price exotic options and the number of regime states is large. Various numerical examples are presented to illustrate the ideas and methodologies.

ASTIN papers are available for downloading, at no charge, from http://www.casact.org/library/astin/index.htm although there will be a delay of at least one year from date of publication.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

Members of ASTIN and AFIR receive *ASTIN Bulletin* free of charge. Members of the Faculty of Actuaries and Institute of Actuaries can join ASTIN and AFIR by contacting Georgina Warren (georgina.warren@actuaries.org.uk) at the Institute of Actuaries.

Libraries, insurers, and individuals who are not members of the Faculty or Institute can subscribe to *ASTIN Bulletin* by contacting the publisher, Peeters, Journals Department, Bondgenotenlaan 153, B-3000 Leuven, Belgium. E-mail: peeters@peeters-leuven.be

Australian Actuarial Journal

Volume **15** (2), 2009

Beveridge, C., Denson, N. & Joshi, M. *Comparing Discretisations of the Libor Market Model in the Spot Measure.* 231-253. Various drift approximations for the displaced-diffusion LIBOR market model in the spot measure are compared. The advantages, disadvantages and implementation choices for each of predictor-corrector and the Glasserman–Zhao method are discussed. Numerical tests are carried out and we conclude that the predictor-corrector method is superior.

Madden, R. C. & Pulver, L. R. J. *Aboriginal and Torres Strait Islander Population: More than reported.* 181-208. A realistic estimate of population data and vital statistics such as births and deaths is essential to understanding the history and relative status of any population over time. Estimating the Aboriginal population of Australia has challenged statisticians for well over 100 years. Estimates of the 1788 population are reviewed, as well as estimates from colonial censuses and the early Yearbooks. The wide variation in contemporary estimates is discussed. More recent improvements in data and data quality are examined, and a plausible scenario for the size of the Aboriginal population over time, based on what we know today, is presented.

Siu, T. K. & Yang, H. *Nonparametric Bayesian Credibility.* 209-230. This paper introduces nonparametric Bayesian credibility without imposing stringent parametric assumptions on claim distributions. We suppose that a claim distribution associated with an unknown risk characteristic of a policyholder is an unknown parameter vector with infinite dimension. In this way, we incorporate the uncertainty of the functional form of the claim distribution associated with the unknown risk characteristic in calculating credibility premiums. Using the results of Ferguson (1973), formulas of the Bayesian credibility premiums are obtained. The formula for the Bayesian credibility pure premium is a linear combination of the overall mean and the sample mean of the claims. This is consistent with the result in the classical credibility theory. We perform a simulation study for the nonparametric Bayesian credibility pure premiums and compare them with the corresponding Bühlmann credibility premiums. Estimation results for the credibility premiums using Danish fire insurance loss data are presented.

Available for downloading from:
http://www.actuaries.asn.au/IAA/upload/public/AAJ_Vol15_Iss2_web.pdf

Reproduced with the permission of The Institute of Actuaries of Australia.

Subscription details available from: IAA, Level 7 Challis House, 4 Martin Place, Sydney, NSW 2000, Australia. E-mail: insact@actuaries.asn.au

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

GENEVA RISK AND INSURANCE REVIEW

Volume **34** (2), 2009

BAJTELSMIT, V. & THISTLE, P. *Negligence, Ignorance and the Demand for Liability Insurance.* 105-116. This paper considers whether lack of information regarding risk exposures can lead to a demand for negligence liability insurance. We find that, under the uniform negligence rule, such as the 'reasonable person' standard used to determine negligence in the U.S. and other countries, the value of information is positive and any demand for liability insurance must come from informed individuals. The necessary and sufficient condition is that good risks find it less costly to be negligent and purchase insurance.

CAMPBELL, J. Y. *The Changing Role of Nominal Government Bonds in Asset Allocation.* 89-104. The covariance between nominal bonds and stocks has varied considerably over recent decades and has even switched sign. It has been predominantly positive in periods such as the late 1970s and early 1980s when the economy has experienced supply shocks and the central bank has lacked credibility. It has been predominantly negative in periods such as the 2000s when investors have feared weak aggregate demand and deflation. Nominal bonds are attractive to short-term equity investors when these bonds are negatively correlated with stocks, as has been the case during the 2000s and especially during the downturn of 2007-2008. They are attractive to conservative long-term investors when long-term inflationary expectations are stable, for then these bonds are close substitutes for inflation-indexed bonds that are riskless in the long term.

HAMMITT, J. K., HANINGER, K. & TREICH, N. *Effects of Health and Longevity on Financial Risk Tolerance.* 117-139. We investigate the effects of health and life expectancy on tolerance of financial risk. Using a standard life-cycle model, we find that the effects of health and life expectancy on preferences over lifetime-income risk are theoretically ambiguous. However, risk tolerance is independent of health and life expectancy when utility takes one of the standard (harmonic absolute risk aversion) functional forms or when optimal consumption is constant over time. Our empirical results, using data from a stated-preference survey ($n = 2{,}795$), suggest that financial risk tolerance is positively associated with both health and life expectancy; hence utility is not consistent with standard functional forms.

ZHOU, C. & WU, C. *Optimal Insurance Under the Insurer's VaR Constraint.* 140-154. In this paper, we impose the insurer's Value at Risk (VaR) constraint on Arrow's optimal insurance model. The insured aims to maximize his expected utility of terminal wealth, under the constraint that the insurer wishes to control the VaR of his terminal wealth to be maintained below a prespecified level. It is shown that when the insurer's VaR constraint is binding, the solution to the problem is not linear, but piecewise linear deductible, and the insured's optimal expected utility will increase as the insurer becomes more risk-tolerant. Basak and Shapiro (2001) showed that VaR risk managers often choose larger risk exposures to risky assets. We draw a similar conclusion in this paper. It is shown that when the insured has an exponential utility function, optimal insurance based on VaR constraint causes the insurer to suffer larger losses than optimal insurance without insurer's risk constraint.

Reproduced with the permission of Kluwer Academic Publishers.
Subscription details available from: Kluwer Academic Publishers, PO Box 322, 3300 AH Dordrecht, The Netherlands

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

INSURANCE: MATHEMATICS & ECONOMICS

Volume **45** (1), 2009

BRANGER, N., KRAFT, H. & MEINERDING, C. *What is the impact of stock market contagion on an investor's portfolio choice?* 94-112. Stocks are exposed to the risk of sudden downward jumps. Additionally, a crash in one stock (or index) can increase the risk of crashes in other stocks (or indices). Our paper explicitly takes this contagion risk into account and studies its impact on the portfolio decision of a CRRA investor both in complete and in incomplete market settings. We find that the investor significantly adjusts his portfolio when contagion is more likely to occur. Capturing the time dimension of contagion, i.e. the time span between jumps in two stocks or stock indices, is thus of first-order importance when analyzing portfolio decisions. Investors ignoring contagion completely or accounting for contagion while ignoring its time dimension suffer large and economically significant utility losses. These losses are larger in complete than in incomplete markets, and the investor might be better off if he does not trade derivatives. Furthermore, we emphasize that the risk of contagion has a crucial impact on investors' security demands, since it reduces their ability to diversify their portfolios.

CERQUETI, R., FOSCHI, R. & SPIZZICHINO, F. *A spatial mixed Poisson framework for combination of excess-of-loss and proportional reinsurance contracts.* 59-64. In this paper a purely theoretical reinsurance model is presented, where the reinsurance contract is assumed to be simultaneously of an excess-of-loss and of a proportional type. The stochastic structure of the set of pairs (claim's arrival time, claim's size) is described by a Spatial Mixed Poisson Process. By using an invariance property of the Spatial Mixed Poisson Processes, we estimate the amount that the ceding company obtains in a fixed time interval in force of the reinsurance contract.

CHEUNG, K. C. *Applications of conditional comonotonicity to some optimization problems.* 89-93. In this article, we study two optimization problems. The first is finding the best L1-approximant of a given random vector on some affine subspaces subject to a measurability condition. The second is finding the optimal allocation of policy limits such that the expected retained loss is minimized. Explicit solutions of both problems are constructed by utilizing the notion of conditional comonotonicity.

CHEUNG, K. C. *Upper comonotonicity.* 35-40. In this article, we study a new notion called upper comonotonicity, which is a generalization of the classical notion of comonotonicity. A random vector is upper-comonotonic if its components are moving in the same direction simultaneously when their values are greater than some thresholds. We provide a characterization of this new notion in terms of both the joint distribution function and the underlying copula. The copula characterization allows us to study the coefficient of upper tail dependence as well as the distributional representation of an upper-comonotonic random vector. As an application to financial economics, we show that the several commonly used risk measures, like the Value-at-Risk, the Tail Value-at-Risk, and the expected shortfall, are additive, not only for sum of comonotonic risks, but also for sum of upper-comonotonic risks, provided that the level of probability is greater than a certain threshold.

ELING, M., GATZERT, N. & SCHMEISER, H. *Minimum standards for investment performance: A new perspective on non-life insurer solvency.* 113-122. The aim of this paper is to develop an alternative approach for assessing an insurer's solvency as a proposal for a standard model for Solvency II. Instead of deriving minimum capital requirements — as is done in solvency regulation — our model provides company-specific minimum standards for risk and return of investment performance, given the distribution structure of liabilities and a predefined safety level. The idea behind this approach is that in a situation of weak solvency, an insurer's asset

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

allocation can be adjusted much more easily in the short term than can, for example, claims cost distributions, operating expenses, or equity capital. Hence, instead of using separate models for capital regulation and solvency regulation — as is typically done in most insurance markets — our single model will reduce the complexity and costs for insurers as well as for regulators. In this paper, we first develop the model framework and second test its applicability using data from a German non-life insurer.

Gao, F. & Yan, J. *Sample path large and moderate deviations for risk model with delayed claims.* 74-80. Sample path large and moderate deviation principles for Markov modulated risk models with delayed claims are proved by the exponential martingale method. As applications, asymptotic estimates and exponential bounds of the ruin probability are also studied.

Gao, J. *Optimal investment strategy for annuity contracts under the constant elasticity of variance (CEV) model.* 9-18. This paper focuses on the constant elasticity of variance (CEV) model for studying the optimal investment strategy before and after retirement in a defined contribution pension plan where benefits are paid under the form of annuities; annuities are supposed to be guaranteed during a certain fixed period of time. Using Legendre transform, dual theory and variable change technique, we derive the explicit solutions for the power and exponential utility functions in two different periods (before and after retirement). Each solution contains a modified factor which reflects an investor's decision to hedge the volatility risk. In order to investigate the influence of the modified factor on the optimal strategy, we analyze the property of the modified factor. The results show that the dynamic behavior of the modified factor for the power utility mainly depends on the time and the investor's risk aversion coefficient, whereas it only depends on the time in the exponential case.

Lin, S.-K., Chang, C.-C. & Powers, M. R. *The valuation of contingent capital with catastrophe risks.* 65-73. The Intergovernmental Panel on Climate Change Fourth Assessment Report (2007) indicates that unanticipated catastrophic events could increase with time because of global warming. Therefore, it seems inadequate to assume that arrival process of catastrophic events follows a pure Poisson process adopted by most previous studies (e.g. [Louberge, H., Kellezi, E., Gilli, M., 1999. Using catastrophe-linked securities to diversify insurance risk: A financial analysis of lCAT bonds. *J. Risk Insurance* **22**, 125-146; Lee, J.-P., Yu, M.-T., 2002. Pricing default-risky CAT bonds with moral hazard and basis risk. *J. Risk Insurance* **69**, 25-44; Cox, H., Fairchild, J., Pedersen, H., 2004. Valuation of structured risk management products. *Insurance Math. Econom.* **34**, 259-272; Jaimungal, S., Wang, T., 2006. Catastrophe options with stochastic interest rates and compound Poisson losses. *Insurance Math. Econom.* **38**, 469-483]. In order to overcome this shortcoming, this paper proposes a doubly stochastic Poisson process to model the arrival process for catastrophic events. Furthermore, we generalize the assumption in the last reference mentioned above to define the general loss function presenting that different specific loss would have different impacts on the drop in stock price. Based on modeling the arrival rates for catastrophe risks, the pricing formulas of contingent capital are derived by the Merton measure. Results of empirical experiments of contingent capital prices as well as sensitivity analyses are presented.

Loeffen, R. L. *An optimal dividends problem with transaction costs for spectrally negative Lévy processes.* 41-48. We consider an optimal dividends problem with transaction costs where the reserves are modeled by a spectrally negative Lévy process. We make the connection with the classical de Finetti problem and show in particular that when the Lévy measure has a log-convex density, then an optimal strategy is given by paying out a dividend in such a way that the reserves are reduced to a certain level c1 whenever they are above another level c2. Further we describe a method to numerically find the optimal values of c1 and c2.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

MAURER, R., MITCHELL, O. S. & ROGALLA, R. *Managing contribution and capital market risk in a funded public defined benefit plan: Impact of CVaR cost constraints.* 25-34. Using a Monte Carlo framework, we analyze the risks and rewards of moving from an unfunded defined benefit pension system to a funded plan for German civil servants, allowing for alternative strategic contribution and investment patterns. In the process we integrate a Conditional Value at Risk (CVaR) restriction on overall plan costs into the pension manager's objective of controlling contribution rate volatility. After estimating the contribution rate that would fully fund future benefit promises for current and prospective employees, we identify the optimal contribution and investment strategy that minimizes contribution rate volatility while restricting worst-case plan costs. Finally, we analyze the time path of expected and worst-case contribution rates to assess the chances of reduced contribution rates for current and future generations. Our results show that moving toward a funded public pension system can be beneficial for both civil servants and taxpayers.

NECIR, A. & MERAGHNI, D. *Empirical estimation of the proportional hazard premium for heavy-tailed claim amounts.* 49-58. The asymptotic normality of the sample proportional hazard premium for heavy-tailed claim amounts with infinite variance cannot be obtained by classical results for L-statistics. In this paper, we propose an alternative estimator for this class of premiums and we establish its asymptotic normality.

PLAT, R. *Stochastic portfolio specific mortality and the quantification of mortality basis risk.* 123-132. In the last decade a vast literature on stochastic mortality models has been developed. However, these models are often not directly applicable to insurance portfolios because:

(a) For insurers and pension funds it is more relevant to model mortality rates measured in insured amounts instead of measured in the number of policies.

(b) Often there is not enough insurance portfolio specific mortality data available to fit such stochastic mortality models reliably.

Therefore, in this paper a stochastic model is proposed for portfolio specific mortality experience. Combining this stochastic process with a stochastic country population mortality process leads to stochastic portfolio specific mortality rates, measured in insured amounts. The proposed stochastic process is applied to two insurance portfolios, and the impact on the Value at Risk for longevity risk is quantified. Furthermore, the model can be used to quantify the basis risk that remains when hedging portfolio specific mortality risk with instruments of which the payoff depends on population mortality rates.

SAKKAS, E. & LE, H. *A Markov-modulated model for stocks paying discrete dividends.* 19-24. We extend the model in [Korn, R., Rogers, L. C. G., 2005. Stock paying discrete dividends: modelling and option pricing. *Journal of Derivatives* **13**, 44-49] for (discrete) dividend processes to incorporate the dependence of assets on the market mode or the state of the economy, where the latter is modeled by a hidden finite-state Markov chain. We then derive the resulting dynamics of the stock price and various option-pricing formulae. It turns out that the stock price jumps not only at the time of the dividend payment, but also when the underlying Markov chain jumps.

WEI, L. *Ruin probability in the presence of interest earnings and tax payments.* 133-138. In this paper we investigate the ruin probability in a general risk model driven by a compound Poisson process. We derive a formula for the ruin probability from which the Albrecher–Hipp tax identity follows as a corollary. Then we study, as an important special case, the classical risk model with a constant force of interest and loss-carried-forward tax payments. For this case we derive an exact formula for the ruin probability when the claims are exponential and an explicit asymptotic formula when the claims are subexponential.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

Xie, S. *Continuous-time mean–variance portfolio selection with liability and regime switching.* 148-155. A continuous-time mean–variance model for individual investors with stochastic liability in a Markovian regime switching financial market, is investigated as a generalization of the model of Zhou and Yin [Zhou, X. Y., Yin, G., 2003. Markowitz's mean–variance portfolio selection with regime switching: A continuous-time model, *SIAM J. Control Optim.* **42** (4), 1466-1482]. We assume that the risky stock's price is governed by a Markovian regime-switching geometric Brownian motion, and the liability follows a Markovian regime-switching Brownian motion with drift, respectively. The evolution of appreciation rates, volatility rates and the interest rates are modulated by the Markov chain, and the Markov switching diffusion is assumed to be independent of the underlying Brownian motion. The correlation between the risky asset and the liability is considered. The objective is to minimize the risk (measured by variance) of the terminal wealth subject to a given expected terminal wealth level. Using the Lagrange multiplier technique and the linear-quadratic control technique, we get the expressions of the optimal portfolio and the mean–variance efficient frontier in closed forms. Further, the results of our special case without liability is consistent with those results of Zhou and Yin [Zhou, X. Y., Yin, G., 2003. Markowitz's mean–variance portfolio selection with regime switching: A continuous-time model, *SIAM J. Control Optim.* **42** (4), 1466-1482].

Xin, Z. & Siu, T. K. *Optimal investment and reinsurance of an insurer with model uncertainty.* 81-88. We introduce a novel approach to optimal investment–reinsurance problems of an insurance company facing model uncertainty via a game theoretic approach. The insurance company invests in a capital market index whose dynamics follow a geometric Brownian motion. The risk process of the company is governed by either a compound Poisson process or its diffusion approximation. The company can also transfer a certain proportion of the insurance risk to a reinsurance company by purchasing reinsurance. The optimal investment–reinsurance problems with model uncertainty are formulated as two-player, zero-sum, stochastic differential games between the insurance company and the market. We provide verification theorems for the Hamilton–Jacobi–Bellman–Isaacs (HJBI) solutions to the optimal investment–reinsurance problems and derive closed-form solutions to the problems.

Yang, J., Qi, Y. & Wang, R. *A class of multivariate copulas with bivariate Fréchet marginal copulas.* 139-147. In this paper, we present a class of multivariate copulas whose two-dimensional marginals belong to the family of bivariate Fréchet copulas. The coordinates of a random vector distributed as one of these copulas are conditionally independent. We prove that these multivariate copulas are uniquely determined by their two-dimensional marginal copulas. Some other properties for these multivariate copulas are discussed as well. Two applications of these copulas in actuarial science are given.

Zhao, X. B., Zhou, X. & Wang, J. L. *Semiparametric model for prediction of individual claim loss reserving.* 1-8. The estimation of loss reserves for incurred but not reported (IBNR) claims presents an important task for insurance companies to predict their liabilities. Conventional methods, such as ladder or separation methods based on aggregated or grouped claims of the so-called "run-off triangle", have been illustrated to have some drawbacks. Recently, individual claim loss models have attracted a great deal of interest in actuarial literature, which can overcome the shortcomings of aggregated claim loss models. In this paper, we propose an alternative individual claim loss model, which has a semiparametric structure and can be used to fit flexibly the claim loss reserving. Local likelihood is employed to estimate the parametric and nonparametric components of the model, and their asymptotic properties are discussed. Then the prediction of the IBNR claim loss reserving is investigated. A simulation study is carried out to evaluate the performance of the proposed methods.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

INSURANCE: MATHEMATICS & ECONOMICS

Volume **45** (2), 2009

CAO, Y. & WAN, N. *Optimal proportional reinsurance and investment based on Hamilton–Jacobi–Bellman equation.* 157-162. In the whole paper, the claim process is assumed to follow a Brownian motion with drift and the insurer is allowed to invest in a risk-free asset and a risky asset. In addition, the insurer can purchase the proportional reinsurance to reduce the risk. The paper concerns the optimal problem of maximizing the utility of terminal wealth. By solving the corresponding Hamilton–Jacobi–Bellman equations, the optimal strategies about how to purchase the proportional reinsurance and how to invest in the risk-free asset and risky asset are derived respectively.

CHOO, W. & DE JONG, P. *Loss reserving using loss aversion functions.* 271-277. This article discusses the determination of risk capital based on "aversion" functions. Aversion functions weigh different outcomes according to perceived severity. Many practical and popular risk measures are usefully viewed in terms of aversion functions including those arising from distortion operators and risk margin loadings. The approach of this paper builds on, unifies, and extends existing disparate approaches discussed in the literature. Analytical and computer generated illustrations are given as well as suggestions for the practical determination of aversion functions.

CORRADINI, M. & GHENO, A. *Incomplete financial markets and contingent claim pricing in a dual expected utility theory framework.* 180-187. This paper investigates the price for contingent claims in a dual expected utility theory framework, the dual price, considering arbitrage-free financial markets. A pricing formula is obtained for contingent claims written on n underlying assets following a general diffusion process. The formula holds in both complete and incomplete markets as well as in constrained markets. An application is also considered assuming a geometric Brownian motion for the underlying assets and the Wang transform as the distortion function.

COSTABILE, M., GAUDENZI, M., MASSABÒ, I. & ZANETTE, A. *Evaluating fair premiums of equity-linked policies with surrender option in a bivariate model.* 286-295. We tackle the problem of computing fair periodical premiums of an equity-linked policy with a maturity guarantee and an embedded surrender option. We consider the policy as a Bermudan-style contingent claim that can be exercised at the premium payment dates. The evaluation framework is based on a discretization of a bivariate model that considers the joint evolution of the equity value with stochastic interest rates. To deeply reduce the computational complexity of the pricing problem we use the singular points framework that allows us to compute accurate upper and lower estimates of the policy premiums.

D'AMICO, G., GUILLEN, M. & MANCA, R. *Full backward non-homogeneous semi-Markov processes for disability insurance models: A Catalunya real data application.* 173-179. In this paper a stochastic model for disability insurance contracts is presented. The model is based on a discrete time non-homogeneous semi-Markov process to which the backward recurrence time process is joined. This permits us to study in a more complete way the disability evolution and to face the duration problem in a more effective way. The model is applied to a sample of contracts drawn at random from a mutual insurance company.

DALKILIC, T. E., TANK, F. & KULA, K. S. *Neural networks approach for determining total claim amounts in insurance.* 236-241. In this study, we present an approach based on neural networks, as an alternative to the ordinary least squares method, to describe the relation between the dependent and independent variables. It has been suggested to construct a model to describe the relation between dependent and independent variables as an alternative to the

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

ordinary least squares method. A new model, which contains the month and number of payments, is proposed based on real data to determine total claim amounts in insurance as an alternative to the model suggested by Rousseeuw *et al.* (1984) [Rousseeuw, P., Daniels, B., Leroy, A., 1984. Applying robust regression to insurance. *Insurance: Math. Econom.* **3**, 67-72] in view of an insurer.

Dassios, A. & Wu, S. *On barrier strategy dividends with Parisian implementation delay for classical surplus processes.* 195-202. In this paper, we apply a single barrier strategy to optimise dividend payments in the situation where there is a time lag $d < 0$ between decision and implementation. Using a classical surplus process with exponentially distributed jumps, we obtain the optimal barrier b* which maximises the expected present value of dividends.

Elliott, R. J., Chen, Z. & Duan, Q. *Insurance claims modulated by a hidden Brownian marked point process.* 163-172. Aimed at better modeling insurance claims in an economic environment driven by business cycles, a new Markov-modulated Poisson process model is proposed, and an algorithm is derived to estimate the hidden Markov process by using the observed information. Our method differs from existing ones in the following ways: the new hidden process can model more efficiently the cyclic state of the economic environment; our theory is based on a variation of the law of large numbers and is easy to understand; the Fourier expansion-based parameter estimation algorithm is flexible and can be more easily implemented than other algorithms. Simulation results not only demonstrate the practicality of our model and algorithm, but also show the efficiency and robustness of the estimation algorithm.

Feng, R. *On the total operating costs up to default in a renewal risk model.* 305-314. The paper proposes a new approach to study a general class of ruin-related quantities in the context of a renewal risk model. While the classical approaches in Sparre Andersen models have their own merits, the approach presented in this paper has its advantages from the following perspectives. (1) The underlying surplus process has the flexibility to reflect a broad range of scenarios for surplus growth including dividend policies and interest returns. (2) The solution method provides a general framework to unify a great variety of existing ruin-related quantities such as Gerber–Shiu functions and the expected present value of dividends paid up to ruin, and facilitates derivations of new ruin-related quantities such as the expected present value of total claim costs up to ruin, etc. In the end, many specific examples are explored to demonstrate its application in renewal risk models.

Haberman, S. & Renshaw, A. *On age-period-cohort parametric mortality rate projections.* 255-270. An enhanced version of the Lee–Carter modelling approach to mortality forecasting, which has been extended to include an age modulated cohort index in addition to the standard age modulated period index, is described and tested for prediction robustness. Life expectancy and annuity value predictions, at pensioner ages and for various periods are compared, both with and without the age modulated cohort index, for the England & Wales male mortality experience. The simulation of prediction intervals for these indices of interest is discussed in detail.

Hernández-Bastida, A., Fernández-Sánchez, M. P. & Gómez-Déniz, E. *The net Bayes premium with dependence between the risk profiles.* 247-254. In Bayesian analysis it is usual to assume that the risk profiles T1 and T2 associated with the random variables "number of claims" and "amount of a single claim", respectively, are independent. A few studies have addressed a model of this nature assuming some degree of dependence between the two random variables (and most of these studies include copulas). In this paper, we focus on the collective and Bayes net premiums for the aggregate amount of claims under a compound model assuming some degree of dependence between the random variables T1 and T2. The degree of dependence is modelled using the Sarmanov–Lee family of distributions [Sarmanov,

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

O. V., 1966. Generalized normal correlation and two-dimensional Frechet classes. *Doklady (Soviet Mathematics)* **168**, 596-599 and Ting-Lee, M. L., 1996. Properties and applications of the Sarmanov family of bivariate distributions. *Communications Statistics: Theory and Methods* **25** (6), 1207-1222], which allows us to study the impact of this assumption on the collective and Bayes net premiums. The results obtained show that a low degree of correlation produces Bayes premiums that are highly sensitive.

KALLENBERG, W. C. M. *Estimating copula densities, using model selection techniques.* 209-233. Recently a new way of modeling dependence has been introduced considering a sequence of parametric copula models, covering more and more dependency aspects and thus giving a closer approximation to the true copula density. The method uses contamination families based on Legendre polynomials. It has been shown that in general after a few steps accurate approximations are obtained. In this paper selection of the adequate number of steps is considered, and estimation of the unknown parameters within the chosen contamination family is established, thus obtaining an estimator of the unknown copula density. There should be a balance between the complexity of the model and the number of parameters to be estimated. High complexity gives a low model error, but a large stochastic or estimation error, while a very simple model gives a small stochastic error, but a large model error. Techniques from model selection are applied, thus letting the data tell us which aspects are important enough to capture into the model. Natural and simple estimators of the involved Fourier coefficients complete the procedure. Theoretical results show that the expected quadratic error is reduced by the selection rule to the same order of magnitude as in a classical parametric problem. The method is applied on a real data set, illustrating that the new method describes the data set very well: the error involved in the classical Gaussian copula density is reduced with no fewer than 50%.

KUDRYAVTSEV, A. A. *Using quantile regression for rate-making.* 296-304. Regression models are popular tools for rate-making in the framework of heterogeneous insurance portfolios; however, the traditional regression methods have some disadvantages particularly their sensitivity to the assumptions which significantly restrict the area of their applications. This paper is devoted to an alternative approach–quantile regression. It is free of some disadvantages of the traditional models. The quality of estimators for the approach described is approximately the same as or sometimes better than that for the traditional regression methods. Moreover, the quantile regression is consistent with the idea of using the distribution quantile for rate-making. This paper provides detailed comparisons between the approaches and it gives the practical example of using the new methodology.

MALEVERGNE, Y. & REY, B. *On cross-risk vulnerability.* 224-229. We introduce the notion of cross-risk vulnerability to generalize the concept of risk vulnerability introduced by Gollier and Pratt [Gollier, C., Pratt, J. W., 1996. Risk vulnerability and the tempering effect of background risk. *Econometrica* **64**, 1109-1124]. While risk vulnerability captures the idea that the presence of an unfair financial background risk should make risk-averse individuals behave in a more risk-averse way with respect to an independent financial risk, cross-risk vulnerability extends this idea to the impact of a non-financial background risk on the financial risk. It provides an answer to the question of the impact of a background risk on the optimal coinsurance rate and on the optimal deductible level. We derive necessary and sufficient conditions for a bivariate utility function to exhibit cross-risk vulnerability both toward an actuarially neutral background risk and toward an unfair background risk. We also analyze the question of the sub-additivity of risk premia and show to what extent cross-risk vulnerability provides an answer.

OHLSSON, E. & LAUZENINGKS, J. *The one-year non-life insurance risk.* 203-208. A major part of the literature on non-life insurance reserve risk has been devoted to the ultimo risk, the risk in

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

the full run-off of the liabilities. This is in contrast to the short time horizon in internal risk models at insurance companies, and the one-year risk perspective taken in the Solvency II project of the European Community. This paper aims at clarifying the one-year risk concept and describing simulation approaches, in particular for the one-year reserve risk. We also discuss the one-year premium risk and its relation to the premium reserve. Finally, we initiate a discussion on the role of risk margins and discounting for the reserve and premium risk, with focus on the Cost-of-Capital method. We show that risk margins do not affect the reserve risk and show how reserve duration can be used for easy calculation of risk margins.

Renaud, J.-F. *The distribution of tax payments in a Lévy insurance risk model with a surplus-dependent taxation structure.* 242-246. We study the distribution of tax payments in the model of Kyprianou and Zhou [Kyprianou, A. E., Zhou, X., 2009. General tax structures and the Lévy insurance risk model. *J. Appl. Probab.* (in press)], that is a Lévy insurance risk model with a surplus-dependent tax rate. More precisely, after a short discussion on the so-called tax identity, we derive a recursive formula for arbitrary moments of the discounted tax payments until ruin and we identify the distribution of the tax payments when there is no force of interest.

Segovia-Gonzalez, M. M., Guerrero, F. M. & Herranz, P. *Explaining functional principal component analysis to actuarial science with an example on vehicle insurance.* 278-285. Given the high competitiveness in the vehicle insurance market, the need arises for an adequate pricing policy. To this end, insurance companies must select risks in a way that allows the expected claims ratio to come as close as possible to the real claims ratio. The use of new analytical tools which provide more information is of great interest. In this paper it is shown how functional principal component analysis can be useful in actuarial science. An empirical study is carried out with data from a Spanish insurance company to estimate the risk of occurrence of a claim in terms of the driver's age, whilst taking into account other relevant variables.

Xu, G. & Zheng, H. *Approximate basket options valuation for a jump-diffusion model.* 188-194. In this paper we discuss the approximate basket options valuation for a jump-diffusion model. The underlying asset prices follow some correlated diffusion processes with idiosyncratic and systematic jumps. We suggest a new approximate pricing formula which is the weighted sum of Roger and Shi's lower bound and the conditional second moment adjustments. We show that the approximate value is always within the lower and upper bounds of the option and is very sharp in our numerical tests.

Yang, Z. *Urban public pension, replacement rates and population growth rate in China.* 230-235. This paper uses an overlapping generations model to investigate the urban public pension in China. It examines the effects of the replacement rates and population growth rate on the capital–labor ratio, pension benefits, consumption and utility, and finds the optimal replacement rate. It is shown that raising the individual account benefit replacement rate only induces the increase in the individual account benefits. Raising the social pool benefit replacement rate induces the increase in the social pool benefits and retirement-period consumption, while the decrease in the capital–labor ratio, individual account benefits, working-period consumption and utility. The fall in the population growth rate leads to the increase in the capital–labor ratio, social pool benefits, individual account benefits, working-period consumption and utility, and leads to a decrease in the retirement-period consumption. The optimal social pool benefit replacement rate depends on the individual discount factor, social discount factor, capital share of income and population growth rate, and it decreases in the case of falling population growth rates. It will do more good than harm to raise the individual account benefit replacement rate, reduce the social pool benefit replacement rate and strictly implement China's population policy.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

INSURANCE: MATHEMATICS & ECONOMICS

Volume **45** (3), 2009

ALBRECHER, H. & KORTSCHAK, D. *On ruin probability and aggregate claim representations for Pareto claim size distributions.* 362-373. We generalize an integral representation for the ruin probability in a Crámer–Lundberg risk model with shifted (or also called US-)Pareto claim sizes, obtained by Ramsay (2003), to classical Pareto(a) claim size distributions with arbitrary real values $a > 1$ and derive its asymptotic expansion. Furthermore an integral representation for the tail of compound sums of Pareto-distributed claims is obtained and numerical illustrations of its performance in comparison to other aggregate claim approximations are provided.

BADESCU, A., ELLIOTT, R. J. & SIU, T. K. *Esscher transforms and consumption-based models.* 337-347. The Esscher transform is an important tool in actuarial science. Since the pioneering work of Gerber and Shiu (1994), the use of the Esscher transform for option valuation has also been investigated extensively. However, the relationships between the asset pricing model based on the Esscher transform and some fundamental equilibrium-based asset pricing models, such as consumption-based models, have so far not been well-explored. In this paper, we attempt to bridge the gap between consumption-based models and asset pricing models based on Esscher-type transformations in a discrete-time setting. Based on certain assumptions for the distributions of asset returns, changes in aggregate consumptions and returns on the market portfolio, we construct pricing measures that are consistent with those arising from Esscher-type transformations. Explicit relationships between the market price of risk, and the risk preference parameters are derived for some particular cases.

BARGÈS, M., COSSETTE, H. & MARCEAU, É. *TVaR-based capital allocation with copulas.* 348-361. Because of regulation projects from control organisations such as the European solvency II reform and recent economic events, insurance companies need to consolidate their capital reserve with coherent amounts allocated to the whole company and to each line of business. The present study considers an insurance portfolio consisting of several lines of risk which are linked by a copula and aims to evaluate not only the capital allocation for the overall portfolio but also the contribution of each risk over their aggregation. We use the tail value at risk (TVaR) as risk measure. The handy form of the FGM copula permits an exact expression for the TVaR of the sum of the risks and for the TVaR-based allocations when claim amounts are exponentially distributed and distributed as a mixture of exponentials. We first examine the bivariate model and then the multivariate case. We also show how to approximate the TVaR of the aggregate risk and the contribution of each risk when using any copula.

BRAZAUSKAS, V. & KLEEFELD, A. *Robust and efficient fitting of the generalized Pareto distribution with actuarial applications in view.* 424-435. Due to advances in extreme value theory, the generalized Pareto distribution (GPD) emerged as a natural family for modeling exceedances over a high threshold. Its importance in applications (e.g., insurance, finance, economics, engineering and numerous other fields) can hardly be overstated and is widely documented. However, despite the sound theoretical basis and wide applicability, fitting of this distribution in practice is not a trivial exercise. Traditional methods such as maximum likelihood and method-of-moments are undefined in some regions of the parameter space. Alternative approaches exist but they lack either robustness (e.g., probability-weighted moments) or efficiency (e.g., method-of-medians), or present significant numerical problems (e.g., minimum-divergence procedures). In this article, we propose a computationally tractable method for fitting the GPD, which is applicable for all parameter values and offers competitive trade-offs between robustness and efficiency. The method is based on 'trimmed moments'. Large-sample properties of the new estimators are provided, and their small-sample

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

behavior under several scenarios of data contamination is investigated through simulations. We also study the effect of our methodology on actuarial applications. In particular, using the new approach, we fit the GPD to the Danish insurance data and apply the fitted model to a few risk measurement and ratemaking exercises.

Chadjiconstantinidis, S. & Papaioannou, A. D. *Analysis of the Gerber–Shiu function and dividend barrier problems for a risk process with two classes of claims.* 470-484. In this paper we consider a risk model with two independent classes of insurance risks. We assume that the two independent claim counting processes are, respectively, the Poisson and the generalized Erlang(2) process. We prove that the Gerber–Shiu function satisfies some defective renewal equations. Exact representations for the solutions of these equations are derived through an associated compound geometric distribution and an analytic expression for this quantity is given when the claim severities have rationally distributed Laplace transforms. Further, the same risk model is considered in the presence of a constant dividend barrier. A system of integro-differential equations with certain boundary conditions for the Gerber–Shiu function is derived and solved. Using systems of integro-differential equations for the moment-generating function as well as for the arbitrary moments of the discounted sum of the dividend payments until ruin, a matrix version of the dividends-penalty is derived. An extension to a risk model when the two independent claim counting processes are Poisson and generalized Erlang( ), respectively, is considered, generalizing the aforementioned results.

Dhaene, J., Denuit, M. & Vanduffel, S. *Correlation order, merging and diversification.* 325-332. We investigate the influence of the dependence between random losses on the shortfall and on the diversification benefit that arises from merging these losses.

We prove that increasing the dependence between losses, expressed in terms of correlation order, has an increasing effect on the shortfall, expressed in terms of an appropriate integral stochastic order. Furthermore, increasing the dependence between losses decreases the diversification benefit.

We also consider merging comonotonic losses and show that even in this extreme case a strictly positive diversification benefit will often arise.

Gao, Q. & Hu, C. *Dynamic mortality factor model with conditional heteroskedasticity.* 410-423. In most methods for modeling mortality rates, the idiosyncratic shocks are assumed to be homoskedastic. This study investigates the conditional heteroskedasticity of mortality in terms of statistical time series. We start from testing the conditional heteroskedasticity of the period effect in the naïve Lee–Carter model for some mortality data. Then we introduce the Generalized Dynamic Factor method and the multivariate BEKK GARCH model to describe mortality dynamics and the conditional heteroskedasticity of mortality. After specifying the number of static factors and dynamic factors by several variants of information criterion, we compare our model with other two models, namely, the Lee–Carter model and the state space model. Based on several error-based measures of performance, our results indicate that if the number of static factors and dynamic factors is properly determined, the method proposed dominates other methods. Finally, we use our method combined with Kalman filter to forecast the mortality rates of Iceland and period life expectancies of Denmark, Finland, Italy and Netherlands.

Huang, J.-T., Lee, K.-J., Liang, H. & Lin, W.-F. *Estimating value at risk of portfolio by conditional copula-GARCH method.* 315-324. Copula functions represent a methodology that describes the dependence structure of a multi-dimension random variable and has become one of the most significant new tools to handle risk factors in finance, such as Value-at Risk (VaR), which is probably the most widely used risk measure in financial institutions. Combining copula and the forecast function of the GARCH model, this paper proposes a new

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

method, called conditional copula-GARCH, to compute the VaR of portfolios. This work presents an application of the copula-GARCH model in the estimation of a portfolio's VaR, composed of NASDAQ and TAIEX. The empirical results show that, compared with traditional methods, the copula model captures the VaR more successfully. In addition, the Student-t copula describes the dependence structure of the portfolio return series quite well.

LI, J. *Comparative higher-degree Ross risk aversion.* 333-336. We provide generalized comparative global conditions for higher-degree Ross risk aversion, which are similar to those studied by Ross for risk aversion. This generalization corresponds to the special cases of comparative risk aversion as developed by Ross (1981) and of comparative downside risk aversion as developed by Modica and Scarsini (2005).

LOISEL, S., MAZZA, C. & RULLIÈRE, D. *Convergence and asymptotic variance of bootstrapped finite-time ruin probabilities with partly shifted risk processes.* 374-381. In the classical risk model, we prove the weak convergence of a sequence of empirical finite-time ruin probabilities. In an earlier paper (see Loisel *et al.* (2008)), we proved an equivalent result in the special case where the initial reserve is zero, and checked that numerically the general case seems to be true. In this paper, we prove the general case (with a nonnegative initial reserve), which is important for applications to estimation risk. So-called partly shifted risk processes are introduced, and used to derive an explicit expression of the asymptotic variance of the considered estimator. This provides a clear representation of the influence function associated with finite time ruin probabilities and gives a useful tool to quantify estimation risk according to new regulations.

PLAT, R. *On stochastic mortality modeling.* 393-404. In the last decennium a vast literature on stochastic mortality models has been developed. All well-known models have nice features but also disadvantages. In this paper a stochastic mortality model is proposed that aims at combining the nice features from the existing models, while eliminating the disadvantages. More specifically, the model fits historical data very well, is applicable to a full age range, captures the cohort effect, has a non-trivial (but not too complex) correlation structure and has no robustness problems, while the structure of the model remains relatively simple. Also, the paper describes how to incorporate parameter uncertainty in the model. Furthermore, a risk neutral version of the model is given, that can be used for pricing.

SONG, Y. & YAN, J.-A. *Risk measures with comonotonic subadditivity or convexity and respecting stochastic orders.* 459-465. This paper proposes some new classes of risk measures, which are not only comonotonic subadditive or convex, but also respect the (first) stochastic dominance or stop-loss order. We give their representations in terms of Choquet integrals w.r.t. distorted probabilities, and show that if the physical probability is atomless then a comonotonic subadditive (resp. convex) risk measure respecting stop-loss order is in fact a law-invariant coherent (resp. convex) risk measure.

SORDO, M. A. *Comparing tail variabilities of risks by means of the excess wealth order.* 466-469. There is a growing interest in the actuarial community in employing certain tail conditional characteristics as measures of risk, which are informative about the variability of the losses beyond the value-at-risk (one example is the tail conditional variance, introduced by Furman and Landsman (2006a, 2006b)). However, comparisons of tail risks based on different measures may not always be consistent. In addition, conclusions based on these conditional characteristics depend on the choice of the tail probability p, so different p's also may produce contradictory conclusions. In this note, we suggest comparing tail variabilities of risks by means of the excess wealth order, which makes judgments only if large classes of tail conditional characteristics imply the same conclusion, independently of the choice of p.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

Van Haastrecht, A., Lord, R., Pelsser, A. & Schrager, D. *Pricing long-dated insurance contracts with stochastic interest rates and stochastic volatility.* 436-448. We consider the pricing of long-dated insurance contracts under stochastic interest rates and stochastic volatility. In particular, we focus on the valuation of insurance options with long-term equity or foreign exchange exposures. Our modeling framework extends the stochastic volatility model of Schöbel and Zhu (1999) by including stochastic interest rates. Moreover, we allow all driving model factors to be instantaneously correlated with each other, i.e. we allow for a general correlation structure between the instantaneous interest rates, the volatilities and the underlying stock returns. As insurance products often incorporate long-term exposures, they are typically more sensitive to changes in the interest rates, volatility and currencies. Therefore, having the flexibility to correlate the underlying asset price with both the stochastic volatility and the stochastic interest rates, yields a realistic model which is of practical importance for the pricing and hedging of such long-term contracts. We show that European options, typically used for the calibration of the model to market prices, and forward starting options can be priced efficiently and in closed-form by means of Fourier inversion techniques. We extensively discuss the numerical implementation of these pricing formulas, allowing for a fast and accurate valuation of European and forward starting options. The model will be especially useful for the pricing and risk management of insurance contracts and other exotic derivatives involving long-term maturities.

Wang, Y. *Quantile hedging for guaranteed minimum death benefits.* 449-458. Quantile hedging for contingent claims is an active topic of research in mathematical finance. It plays a role in incomplete markets when perfect hedging is not possible. Guaranteed minimum death benefits (GMDBs) are present in many variable annuity contracts, and act as a form of portfolio insurance. They cannot be perfectly hedged due to the mortality component, except in the limit as the number of contracts becomes infinitely large. In this article, we apply ideas from finance to derive quantile hedges for these products under various assumptions.

Yuan, H. & Hu, Y. *Optimal consumption and portfolio policies with the consumption habit constraints and the terminal wealth downside constraints.* 405-409. In this paper, we consider the optimal consumption and portfolio policies with the consumption habit constraints and the terminal wealth downside constraints, that is, here the consumption rate is greater than or equal to some nonnegative process, and the terminal wealth is no less than some positive constant. Using the martingale approach, we get the optimal consumption and portfolio policies.

Zhou, M. & Cai, J. *A perturbed risk model with dependence between premium rates and claim sizes.* 382-392. This paper considers a dependent risk model with diffusion for the surplus of an insurer, in which a current premium rate will be adjusted after a claim occurs and the adjusted rate is determined by the amount of the claim. At the same time, the diffusion is changed correspondingly. Using Rouché's theorem, we first derive the closed-form solution for the Laplace transform of the survival probability in the dependent risk model. Then, using the Laplace transform, we derive a defective renewal equation satisfied by the survival probability. For the exponential claim sizes, we present the explicit recursion expression for the survival probability, by which we can exactly solve the survival probability step-by-step. We also illustrate the influence of the model parameters in the dependent risk model on the survival probability by numerical examples.

Insurance: Mathematics & Economics

Volume **46** (1), 2010

Albrecherl, H., Constantinescu, C., Pirsic, G., Regensburger, G. & Rosenkranz, M. *An algebraic operator approach to the analysis of Gerber–Shiu functions.* 42-51. We introduce

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

an algebraic operator framework to study discounted penalty functions in renewal risk models. For inter-arrival and claim size distributions with rational Laplace transform, the usual integral equation is transformed into a boundary value problem, which is solved by symbolic techniques. The factorization of the differential operator can be lifted to the level of boundary value problems, amounting to iteratively solving first-order problems. This leads to an explicit expression for the Gerber–Shiu function in terms of the penalty function.

Bauer, D., Börger, M. & Ruß, J. *On the pricing of longevity-linked securities.* 139-149. For annuity providers, longevity risk, i.e. the risk that future mortality trends differ from those anticipated, constitutes an important risk factor. In order to manage this risk, new financial products, so-called longevity derivatives, may be needed, even though a first attempt to issue a longevity bond in 2004 was not successful.
While different methods of how to price such securities have been proposed in recent literature, no consensus has been reached. This paper reviews, compares and comments on these different approaches. In particular, we use data from the United Kingdom to derive prices for the proposed first longevity bond and an alternative security design based on the different methods.

Biffis, E. & Kyprianou, A. E. *A note on scale functions and the time value of ruin for Lévy insurance risk processes.* 85-91. We examine discounted penalties at ruin for surplus dynamics driven by a general spectrally negative Lévy process; the natural class of stochastic processes which contains many examples of risk processes which have already been considered in the existing literature. Following from the important contributions of [Zhou, X., 2005. On a classical risk model with a constant dividend barrier. *North Am. Act. J.*, 95-108] we provide an explicit characterization of a generalized version of the Gerber–Shiu function in terms of scale functions, streamlining and extending results available in the literature.

Biffis, E. & Morales, M. *On a generalization of the Gerber–Shiu function to path-dependent penalties.* 92-97. The Expected Discounted Penalty Function (EDPF) was introduced in a series of now classical papers ([9], [11] and [12]). Motivated by applications in option pricing and risk management, and inspired by recent developments in fluctuation theory for Lévy processes, we study an extended definition of the expected discounted penalty function that takes into account a new ruin-related random variable. In addition to the surplus before ruin and deficit at ruin, we extend the EDPF to include the surplus at the last minimum before ruin. We provide an expression for the generalized EDPF in terms of convolutions in a setting involving a subordinator and a spectrally negative Lévy process. Some expressions for the classical EDPF are recovered as special cases of the generalized EDPF.

Biffis, E. & Blake, D. *Securitizing and tranching longevity exposures.* 186-197. We consider the problem of optimally designing longevity risk transfers under asymmetric information. We focus on holders of longevity exposures that have superior knowledge of the underlying demographic risks, but are willing to take them off their balance sheets because of capital requirements. In equilibrium, they transfer longevity risk to uninformed agents at a cost, where the cost is represented by retention of part of the exposure and/or by a risk premium. We use a signalling model to quantify the effects of asymmetric information and emphasize how they compound with parameter uncertainty. We show how the cost of private information can be minimized by suitably tranching securitized cashflows, or, equivalently, by securitizing the exposure in exchange for an option on mortality rates. We also investigate the benefits of pooling several longevity exposures and the impact on tranching levels.

Blake, D., De Waegenaere, A., MacMinn, R. & Nijman, T. *Longevity risk and capital markets: The 2008-2009 update.* 135-138. This issue of Insurance: Mathematics and Economics contains 11 contributions to the academic literature all dealing with longevity risk and

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

capital markets. Draft versions of the papers were presented at Longevity Four: The Fourth International Longevity Risk and Capital Markets Solutions Conference that was held in Amsterdam on 25-26 September 2008. It was hosted by Netspar and the Pensions Institute (at Cass Business School), and organized by PensionSummit.

Longevity risk and related capital market solutions have grown increasingly important in recent years, both in academic research and in the real world life markets. Mortality improvements around the world are putting more and more pressure on governments, pension funds, life insurance companies as well as individuals, to deal with the longevity risk they face. At the same time, capital markets can, in principle, provide vehicles to hedge longevity risk effectively. Many new investment products have been created both by the insurance/reinsurance industry and by the capital markets. Mortality catastrophe bonds are an example of a successful insurance-linked security. Some new innovative capital market solutions for transferring longevity risk include longevity (or survivor) bonds, longevity (or survivor) swaps and mortality (or q-) forward contracts. The aim of the International Longevity Risk and Capital Markets Solutions Conferences is to bring together academics and practitioners from all over the world to discuss and analyze these exciting new developments.

Chen, H. & Cummins, J. D. *Longevity bond premiums: The extreme value approach and risk cubic pricing.* 150-161. The purpose of this study is to analyze the securitization of longevity risk with an emphasis on longevity risk modeling and longevity bond premium pricing. Various longevity derivatives have been proposed, and the capital market has experienced one unsuccessful attempt by the European Investment Bank (EIB) in 2004. After carefully analyzing the pros and cons of previous securitizations, we present our proposed longevity bonds, whose payoffs are structured as a series of put option spreads. We utilize a random walk model with drift to fit small variations of mortality improvements and employ extreme value theory to model rare longevity events. Our method is a new approach in longevity risk securitization, which has the advantage of both capturing mortality improvements within sample and extrapolating rare, out-of- sample longevity events. We demonstrate that the risk cubic model developed for pricing catastrophe bonds can be applied to mortality and longevity bond pricing and use the model to calculate risk premiums for longevity bonds.

Cheung, E. C. K. & Landriault, D. *A generalized penalty function with the maximum surplus prior to ruin in a MAP risk model.* 127-134. In this paper, a risk model where claims arrive according to a Markovian arrival process (MAP) is considered. A generalization of the well-known Gerber–Shiu function is proposed by incorporating the maximum surplus level before ruin into the penalty function. For this wider class of penalty functions, we show that the generalized Gerber–Shiu function can be expressed in terms of the original Gerber–Shiu function (see e.g. [Gerber, Hans U., Shiu, Elias, S. W., 1998. On the time value of ruin. *North American Actuarial Journal* **2** (1), 48-72]) and the Laplace transform of a first passage time which are both readily available. The generalized Gerber–Shiu function is also shown to be closely related to the original Gerber–Shiu function in the same MAP risk model subject to a dividend barrier strategy. The simplest case of a MAP risk model, namely the classical compound Poisson risk model, will be studied in more detail. In particular, the discounted joint density of the surplus prior to ruin, the deficit at ruin and the maximum surplus before ruin is obtained through analytic Laplace transform inversion of a specific generalized Gerber–Shiu function. Numerical illustrations are then examined.

Cheung, E. C. K., Landriault, D., Willmot, G. E. & Woo, J.-K. *Structural properties of Gerber–Shiu functions in dependent Sparre Andersen models.* 117-126. The structure of various Gerber–Shiu functions in Sparre Andersen models allowing for possible dependence between claim sizes and interclaim times is examined. The penalty function is assumed to depend on some or all of the surplus immediately prior to ruin, the deficit at ruin, the minimum surplus before ruin, and the surplus immediately after the second last claim before ruin. Defective

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

joint and marginal distributions involving these quantities are derived. Many of the properties in the Sparre Andersen model without dependence are seen to hold in the present model as well. A discussion of Lundberg's fundamental equation and the generalized adjustment coefficient is given, and the connection to a defective renewal equation is considered. The usual Sparre Andersen model without dependence is also discussed, and in particular the case with exponential claim sizes is considered.

Chi, Y., Jaimungal, S. & Lin, X. S. *An insurance risk model with stochastic volatility*. 52-66. In this paper, we extend the Cramér–Lundberg insurance risk model perturbed by diffusion to incorporate stochastic volatility and study the resulting Gerber–Shiu expected discounted penalty (EDP) function. Under the assumption that volatility is driven by an underlying Ornstein–Uhlenbeck (OU) process, we derive the integro-differential equation which the EDP function satisfies. Not surprisingly, no closed-form solution exists; however, assuming the driving OU process is fast mean-reverting, we apply the singular perturbation theory to obtain an asymptotic expansion of the solution. Two integro-differential equations for the first two terms in this expansion are obtained and explicitly solved. When the claim size distribution is of phase-type, the asymptotic results simplify even further and we succeed in estimating the error of the approximation. Hyper-exponential and mixed-Erlang distributed claims are considered in some detail.

Cox, S. H., Lin, Y. & Pedersen, H. *Mortality risk modeling: Applications to insurance securitization*. 242-253. This paper proposes a stochastic mortality model featuring both permanent longevity jump and temporary mortality jump processes. A trend reduction component describes unexpected mortality improvement over an extended period of time. The model also captures the uneven effect of mortality events on different ages and the correlations among them. The model will be useful in analyzing future mortality dependent cash flows of life insurance portfolios, annuity portfolios, and portfolios of mortality derivatives. We show how to apply the model to analyze and price a longevity security.

Dickson, D. C. M. & Li, S. *Finite time ruin problems for the Erlang(2) risk model*. 12-18. We consider the Erlang(2) risk model and derive expressions for the density of the time to ruin and the joint density of the time to ruin and the deficit at ruin when the individual claim amount distribution is (i) an exponential distribution and (ii) an Erlang(2) distribution. We also consider the special case when the initial surplus is zero.

Gerber, H. U., Shiu, E. S. W. & Yang, H. *An elementary approach to discrete models of dividend strategies*. 109-116. The paper studies a discrete counterpart of Gerber *et al.* (2006). The surplus of an insurance company (before dividends) is modeled as a time-homogeneous Markov chain with possible changes of size $+1, 0, -1, -2, -3, \ldots$. If a barrier strategy is applied for paying dividends, it is shown that the dividends-penalty identity holds. The identity expresses the expected present value of a penalty at ruin in terms of the expected discounted dividends until ruin and the expected present value of the penalty at ruin if no dividends are paid. For the problem of maximizing the difference between the expected discounted dividends until ruin and the expected present value of the penalty at ruin, barrier strategies play a prominent role. In some cases an optimal dividend barrier exists. The paper discusses in detail the special case where the distribution of the change in surplus does not depend on the current surplus (so that in the absence of dividends the surplus process has independent increments). A closed-form result for zero initial surplus is given, and it is shown how the relevant quantities can be calculated recursively. Finally, it is shown how optimal dividend strategies can be determined; typically, they are band strategies.

Gong, G. & Webb, A. *Evaluating the Advanced Life Deferred Annuity — An annuity people might actually buy*. 210-221. Although annuities provide longevity insurance that should be

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

attractive to households facing an uncertain lifespan, rates of voluntary annuitization remain extremely low. We evaluate the Advanced Life Deferred Annuity, an annuity purchased at retirement, providing an income commencing in advanced old age. Using numerical optimization, we show that it would provide a substantial proportion of the longevity insurance provided by an immediate annuity, at much lower cost. At plausible levels of actuarial unfairness, households should prefer it to both immediate and postponed annuitization and an optimal decumulation of unannuitized wealth. Few households would suffer significant losses were it used as a 401(k) plan default.

KOGURE, A. & KURACHI, Y. *A Bayesian approach to pricing longevity risk based on risk-neutral predictive distributions.* 162-172. We present a Bayesian approach to pricing longevity risk under the framework of the Lee–Carter methodology. Specifically, we propose a Bayesian method for pricing the survivor bond and the related survivor swap designed by Denuit *et al.* (2007). Our method is based on the risk neutralization of the predictive distribution of future survival rates using the entropy maximization principle discussed by Stutzer (1996). The method is illustrated by applying it to Japanese mortality rates.

LOEFFEN, R. L. & RENAUD, J.-F. *De Finetti's optimal dividends problem with an affine penalty function at ruin.* 98-108. In a Lévy insurance risk model, under the assumption that the tail of the Lévy measure is log-convex, we show that either a horizontal barrier strategy or the take-the-money-and-run strategy maximizes, among all admissible strategies, the dividend payments subject to an affine penalty function at ruin. As a key step for the proof, we prove that, under the aforementioned condition on the jump measure, the scale function of the spectrally negative Lévy process has a log-convex derivative.

MING, R.-X., WANG, W.-Y. & XIAO, L.-Q. *On the time value of absolute ruin with tax.* 67-84. Consider a compound Poisson surplus process of an insurer with debit interest and tax payments. When the portfolio is in a profitable situation, the insurer may pay a certain proportion of the premium income as tax payments. When the portfolio is below zero, the insurer could borrow money at a debit interest rate to continue his/her business. Meanwhile, the insurer will repay the debts from his/her premium income. The negative surplus may return to a positive level except that the surplus is below a certain critical level. In the latter case, we say that absolute ruin occurs. In this paper, we discuss absolute ruin quantities by defining an expected discounted penalty function at absolute ruin. First, a system of integro-differential equations satisfied by the expected discounted penalty function is derived. Second, closed-form expressions for the expected discounted total sum of tax payments until absolute ruin and the Laplace–Stieltjes transform (LST) of the total duration of negative surplus are obtained. Third, for exponential individual claims, closed-form expressions for the absolute ruin probability, the LST of the time to absolute ruin, the distribution function of the deficit at absolute ruin and the expected accumulated discounted tax are given. Fourth, for general individual claim distributions, when the initial surplus goes to infinity, we show that the ratio of the absolute ruin probability with tax to that without tax goes to a positive constant which is greater than one. Finally, we investigate the asymptotic behavior of the absolute ruin probability of a modified risk model where the interest rate on a positive surplus is involved.

PANG, G. & WARSHAWSKY, M. *Optimizing the equity-bond-annuity portfolio in retirement: The impact of uncertain health expenses.* 198-209. This paper derives optimal equity-bond-annuity portfolios for retired households who face stochastic capital market returns, differential exposures to mortality risk and uncertain uninsured health expenses, and differential Social Security and defined benefit pension coverage. The results show that the health spending risk drives household portfolios to shift from risky equities to safer assets and enhances the demand for annuities due to their increasing-with-age superiority over bonds in hedging against life-contingent health spending and longevity risks. Households with higher income

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

have a greater incremental demand for life annuities. The annuities in turn provide greater leverage for equity investment in the remaining asset portfolios.

SCHMIDLI, H. *On the Gerber–Shiu function and change of measure.* 3-11. We consider several models for the surplus of an insurance company mainly under some light-tail assumptions. We are interested in the expected discounted penalty at ruin. By a change of measure we remove the discounting, which simplifies the expression. This leads to (defective) renewal equations as they had been found by different methods in the literature. If we use the change of measure such that ruin becomes certain, the renewal equations simplify to ordinary renewal equations. This helps to discuss the asymptotics as the initial capital goes to infinity. For phase-type claim sizes, explicit formulae can be derived.

STEVENS, R., DE WAEGENAERE, A. & MELENBERG, B. *Longevity risk in pension annuities with exchange options: The effect of product design.* 222-234. We consider defined benefit pension plans that, at retirement age, allow the participant to choose between a single life annuity and a joint and survivor annuity. We compare two plans that differ in terms of how pension rights are accrued. In one plan, the participant accrues the right to receive a single life annuity, and can exchange that annuity for an actuarially equivalent joint and survivor annuity at retirement date. The opposite holds in the other plan. We show that both plans are affected by longevity risk in two ways. First, the participants' choices at retirement age affect the ratio of survivor benefits over single life benefits, and, therefore, affect the natural hedge potential that arises from combining single life and survivor annuities. Second, uncertainty in the rate at which the participant will be allowed to exchange one type of annuity for the other at retirement date induces uncertainty in the level of the nominal rights for single life and survivor annuities, respectively. We compare the two plans, and show that longevity risk is substantially lower in case rights are accrued in the form of a joint and survivor annuity.

TANG, Q. & LI, W. *Asymptotic aspects of the Gerber–Shiu function in the renewal risk model using Wiener–Hopf factorization and convolution equivalence.* 19-31. We study the asymptotic behavior of the Gerber–Shiu expected discounted penalty function in the renewal risk model. Under the assumption that the claim-size distribution has a convolution-equivalent density function, which allows both heavy-tailed and light-tailed cases, we establish some asymptotic formulas for the Gerber–Shiu function with a fairly general penalty function. These formulas become completely transparent in the compound Poisson risk model or for certain choices of the penalty function in the renewal risk model. A by-product of this work is an extension of the Wiener–Hopf factorization to include the times of ascending and descending ladders in the continuous-time renewal risk model.

TSAI, J. T., WANG, J. L. & TZENG, L. Y. *On the optimal product mix in life insurance companies using conditional value at risk.* 235-241. This paper proposes a Conditional Value-at-Risk Minimization (CVaRM) approach to optimize an insurer's product mix. By incorporating the natural hedging strategy of Cox and Lin (2007) and the two-factor stochastic mortality model of Cairns *et al.* (2006b), we calculate an optimize product mix for insurance companies to hedge against the systematic mortality risk under parameter uncertainty. To reflect the importance of required profit, we further integrate the premium loading of systematic risk. We compare the hedging results to those using the duration match method of Wang *et al.* (forthcoming), and show that the proposed CVaRM approach has a narrower quantile of loss distribution after hedging — thereby effectively reducing systematic mortality risk for life insurance companies.

WILLMOT, G. E. & WOO, J.-K. *Surplus analysis for a class of Coxian interclaim time distributions with applications to mixed Erlang claim amounts.* 32-41. Gerber–Shiu analysis with the generalized penalty function proposed by Cheung *et al.* (in press-a) is considered in the

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

Sparre Andersen risk model with a Kn family distribution for the interclaim time. A defective renewal equation and its solution for the present Gerber–Shiu function are derived, and their forms are natural for analysis which jointly involves the time of ruin and the surplus immediately prior to ruin. The results are then used to find explicit expressions for various defective joint and marginal densities, including those involving the claim causing ruin and the last interclaim time before ruin. The case with mixed Erlang claim amounts is considered in some detail.

Wills, S. & Sherris, M. *Securitization, structuring and pricing of longevity risk.* 173-185. Pricing and risk management for longevity risk have increasingly become major challenges for life insurers and pension funds around the world. Risk transfer to financial markets, with their major capacity for efficient risk pooling, is an area of significant development for a successful longevity product market. The structuring and pricing of longevity risk using modern securitization methods, common in financial markets, have yet to be successfully implemented for longevity risk management. There are many issues that remain unresolved for ensuring the successful development of a longevity risk market. This paper considers the securitization of longevity risk focusing on the structuring and pricing of a longevity bond using techniques developed for the financial markets, particularly for mortgages and credit risk. A model based on Australian mortality data and calibrated to insurance risk linked market data is used to assess the structure and market consistent pricing of a longevity bond. Age dependence in the securitized risks is shown to be a critical factor in structuring and pricing longevity linked securitizations.

Yang, S. S., Yue, J. C. & Huang, H.-C. *Modeling longevity risks using a principal component approach: A comparison with existing stochastic mortality models.* 254-270. This research proposes a mortality model with an age shift to project future mortality using principal component analysis (PCA). Comparisons of the proposed PCA model with the well-known models — the Lee–Carter model, the age–period–cohort model (Renshaw and Haberman, 2006), and the Cairns, Blake, and Dowd model — employ empirical studies of mortality data from six countries, two each from Asia, Europe, and North America. The mortality data come from the human mortality database and span the period 1970-2005. The proposed PCA model produces smaller prediction errors for almost all illustrated countries in its mean absolute percentage error. To demonstrate longevity risk in annuity pricing, we use the proposed PCA model to project future mortality rates and analyze the underestimated ratio of annuity price for whole life annuity and deferred whole life annuity product respectively. The effect of model risk on annuity pricing is also investigated by comparing the results from the proposed PCA model with those from the LC model. The findings can benefit actuaries in their efforts to deal with longevity risk in pricing and valuation.

Insurance: Mathematics & Economics

Volume **46** (2), 2010

Asimit, A. V., Furman, E. & Vernic, R. *On a multivariate Pareto distribution.* 308-316. A multivariate distribution possessing arbitrarily parameterized Pareto margins is formulated and studied. The distribution is believed to allow for an adequate modeling of dependent heavy tailed risks with a non-zero probability of simultaneous loss. Numerous links to certain existing probabilistic models, as well as seemingly useful characteristic results are proved. Expressions for, e.g., decumulative distribution functions, densities (joint) moments and regressions are developed. An application to the classical pricing problem is considered, and some formulas are derived using the recently introduced economic weighted premium calculation principles.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

CHEN, H., COX, S. H. & WANG, S. S. *Is the Home Equity Conversion Mortgage in the United States sustainable?: Evidence from pricing mortgage insurance premiums and non-recourse provisions using the conditional Esscher transform.* 371-384. The purpose of this paper is to build a modeling and pricing framework to investigate the sustainability of the Home Equity Conversion Mortgage (HECM) program in the United States under realistic economic scenarios, i.e., whether the premium payments cover the fair premiums for the inherent risks in the HECM program. We note that earlier HECM models use static mortality tables, neglecting the dynamics of mortality rates and extreme mortality jumps. The earlier models also assume housing prices follow a geometric Brownian motion, which contradicts the fact that housing prices exhibit strong autocorrelation and varying volatility over time. To solve these problems, we propose a generalized Lee–Carter model with asymmetric jump effects to fit the mortality data, and model the house price index via an ARIMA–GARCH process. We then employ the conditional Esscher transform to price the non-recourse provision of reverse mortgages and compare it with the calculated mortgage insurance premiums. The HECM program turns out to be sustainable based on our model setup and parameter settings.

CHI, Y. *Analysis of the expected discounted penalty function for a general jump–diffusion risk model and applications in finance.* 385-396. In this paper, we extend the Cramér–Lundberg risk model perturbed by diffusion to incorporate the jumps of surplus investment return. Under the assumption that the jump of surplus investment return follows a compound Poisson process with Laplace distributed jump sizes, we obtain the explicit closed-form expression of the resulting Gerber–Shiu expected discounted penalty (EDP) function through the Wiener–Hopf factorization technique instead of the integro-differential equation approach. Especially, when the claim distribution is of Phase-type, the expression of the EDP function is simplified even further as a compact matrix-type form. Finally, the financial applications include pricing barrier option and perpetual American put option and determining the optimal capital structure of a firm with endogenous default.

FURMAN, E. & LANDSMAN, Z. *Multivariate Tweedie distributions and some related capital-at-risk analyses.* 351-361. We study a multivariate extension of the univariate exponential dispersion Tweedie family of distributions. The class, referred to as the multivariate Tweedie family (MTwF), on the one hand includes multivariate Poisson, gamma, inverse Gaussian, stable and compound Poisson distributions and on the other hand introduces a high variety of new dependent probabilistic models unstudied so far. We investigate various properties of MTwF and discuss its possible applications to financial risk management.

HUANG, H.-C. & LEE, Y.-T. *Optimal asset allocation for a general portfolio of life insurance policies.* 271-280. Asset liability matching remains an important topic in life insurance research. The objective of this paper is to find an optimal asset allocation for a general portfolio of life insurance policies. Using a multi-asset model to investigate the optimal asset allocation of life insurance reserves, this study obtains formulae for the first two moments of the accumulated asset value. These formulae enable the analysis of portfolio problems and a first approximation of optimal investment strategies. This research provides a new perspective for solving both single-period and multiperiod asset allocation problems in application to life insurance policies. The authors obtain an efficient frontier in the case of single-period method; for the multiperiod method, the optimal asset allocation strategies can differ considerably for different portfolio structures.

LAURINI, M. P. & MOURA, M. *Constrained smoothing B-splines for the term structure of interest rates.* 339-350. The constrained smoothing B-splines (COBS) is proposed as a nonparametric approach to estimate the term structure of interest rate. Compared to the existing methods in the literature, COBS' main innovation lies in its incorporation of important constraints imposed by no-arbitrage, such as monotonically decreasing and boundary conditions for the

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

discount function, positive forward and spot rates. In addition, by estimating the conditional median function, COBS is less sensible to outliers in reduced samples than other common methods in the literature. Estimation for high and low liquidity markets together with simulation exercises puts COBS in an intermediate position between usual parametric and nonparametric methods in the literature. It has more flexibility than parametric methods and, compared to other nonparametric methods, satisfies no-arbitrage constraints and generates parsimonious shapes of the term structure of interest rates.

Lim, A. E. B. & Wong, B. *A benchmarking approach to optimal asset allocation for insurers and pension funds.* 317-327. We solve the optimal asset allocation problem for an insurer or pension fund by using a benchmarking approach. Under this approach the objective is an increasing function of the relative performance of the asset portfolio compared to a benchmark. The benchmark can be, for example, a function of an insurer's liability payments, or the (either contractual or target) payments of a pension fund. The benchmarking approach tolerates but progressively penalizes shortfalls, while at the same time progressively rewards outperformance. Working in a general, possibly non-Markovian setting, a solution to the optimization problem is presented, providing insights into the impact of benchmarking on the resulting optimal portfolio. We further illustrate the results with a detailed example involving an option based benchmark of particular interest to insurers and pension funds, and present closed form solutions.

Lin, T. & Tzeng, L. Y. *An additive stochastic model of mortality rates: An application to longevity risk in reserve evaluation.* 423-435. The paper proposes an additive continuous-time stochastic mortality model which revises that (B&H model) of Ballotta and Haberman (2006). The structure of the B&H model implies that the future hazard rate is proportional to the stochastic component, thus inducing two questionable features. First, in the B&H model, the uncertainty of the future hazard rate will be enlarged as the base hazard rate increases. However, an increase in the base hazard rate may not cause a dramatic increase suggested by the exponential component of B&H (2006). Second, in the B&H model, the uncertainty of the future hazard rate will be larger in the group which is older and will be greatly augmented by the interaction of age and time. But the uncertainty of the future hazard rate may not increase with an increase in age. The problems can be resolved by our additive structure which is the sum of a deterministic estimator and a stochastic component. Since using the additive structure will contribute to the fact that the stochastic component is independent of age and the base hazard rate, in our model the uncertainty of the future hazard rate will not be affected by an increase in age or in the base hazard rate. We further demonstrate an application of our model by calculating reserves of longevity risks for pure endowments and various common annuity products in the UK. We also compare our results with those of the B&H model.

Mulero, J., Pellerey, F. & Rodríguez-Griñolo, R. *Stochastic comparisons for time transformed exponential models.* 328-333. Different sufficient conditions for stochastic comparisons between random vectors have been described in the literature. In particular, conditions for the comparison of random vectors having the same copula, i.e., the same dependence structure, may be found in Müller and Scarsini (2001). Here we provide conditions for the comparison, in the usual stochastic order sense and in other weaker stochastic orders, of two time transformed exponential bivariate lifetimes having different copulas. Some examples of applications are provided too.

Najafabadi, A. T. P. *A new approach to the credibility formula.* 334-338. The usual credibility formula holds whenever, (i) claim size distribution is a member of the exponential family of distributions, (ii) prior distribution conjugates with claim size distribution, and (iii) square error loss has been considered. As long as, one of these conditions is violent, the usual credibility formula no longer holds. This article, using the mean square error minimization

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

technique, develops a simple and practical approach to the credibility theory. Namely, we approximate the Bayes estimator with respect to a general loss function and general prior distribution by a convex combination of the observation mean and mean of prior, say, approximate credibility formula. Adjustment of the approximate credibility for several situations and its form for several important losses are given.

QU, X., ZHOU, J. & SHEN, X. *Archimedean copula estimation and model selection via l1-norm symmetric distribution.* 406-414. Based on the relationship between Archimedean copulas and l1-norm symmetric distributions, we propose a method to not only estimate the copula parameter but also select the copula model through the observation data in this paper. The strong consistency of the estimator is proved, and a Radial Information Criteria (RIC) is provided to select the appropriate Archimedean copula model fitting the data best. It can be extended to the multivariate cases conveniently because the selection is achieved by using the one-dimensional radial distribution to capture the dependence structure for multivariate data. The Monte Carlo simulation experiments illustrate that the proposed approach works well in parameter estimation and model selection for both bivariate and multivariate cases. An application in modelling the dependence structure of real stock indices is carried out with good performance as well.

SCHMIDLI, H. *Conditional law of risk processes given that ruin occurs.* 281-289. A risk process that can be Markovised is conditioned on ruin. We prove that the process remains a Markov process. If the risk process is a PDMP, it is shown that the conditioned process remains a PDMP. For many examples the asymptotics of the parameters in both the light-tailed case and the heavy-tailed case are discussed.

TANG, Q., WANG, G. & YUEN, K. C. *Uniform tail asymptotics for the stochastic present value of aggregate claims in the renewal risk model.* 362-370. Consider an insurer who is allowed to make risk-free and risky investments. The price process of the investment portfolio is described as a geometric Lévy process. We study the tail probability of the stochastic present value of future aggregate claims. When the claim-size distribution is of Pareto type, we obtain a simple asymptotic formula which holds uniformly for all time horizons. The same asymptotic formula holds for the finite-time and infinite-time ruin probabilities. Restricting our attention to the so-called constant investment strategy, we show how the insurer adjusts his investment portfolio to maximize the expected terminal wealth subject to a constraint on the ruin probability.

XIE, J.-H. & ZOU, W. *Expected present value of total dividends in a delayed claims risk model under stochastic interest rates.* 415-422. In this paper, a compound binomial risk model with a constant dividend barrier under stochastic interest rates is considered. Two types of individual claims, main claims and by-claims, are defined, where every by-claim is induced by the main claim and may be delayed for one time period with a certain probability. In the evaluation of the expected present value of dividends, the interest rates are assumed to follow a Markov chain with finite state space. A system of difference equations with certain boundary conditions for the expected present value of total dividend payments prior to ruin is derived and solved. Explicit results are obtained when the claim sizes are Kn distributed or the claim size distributions have finite support. Numerical results are also provided to illustrate the impact of the delay of by-claims on the expected present value of dividends.

ZENG, X. *Optimal reinsurance with a rescuing procedure.* 397-405. We consider a large insurance company whose reserve is modeled by a diffusion process. The management of the insurance company makes a decision on reinsurance in order to reduce the insurance risk. An optimal decision is the one which minimizes the expected time to reach a goal before the reserve reaches a ruin level. We introduce a rescuing procedure to deal with the case that the company

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

is "too big to fail". We disclose that the optimal decision of the management heavily depends on how much time the company needs to wait for rescuing when it gets in trouble.

ZHAO, X. B. & ZHOU, X. *Applying copula models to individual claim loss reserving methods.* 290-299. The estimation of loss reserves for incurred but not reported (IBNR) claims presents an important task for insurance companies to predict their liabilities. Recently, individual claim loss models have attracted a great deal of interest in the actuarial literature, which overcome some shortcomings of aggregated claim loss models. The dependence of the event times with the delays is a crucial issue for estimating the claim loss reserving. In this article, we propose to use semi-competing risks copula and semi-survival copula models to fit the dependence structure of the event times with delays in the individual claim loss model. A nonstandard two-step procedure is applied to our setting in which the associate parameter and one margin are estimated based on an *ad hoc* estimator of the other margin. The asymptotic properties of the estimators are established as well. A simulation study is carried out to evaluate the performance of the proposed methods.

ZHOU, C., WU, W. & WU, C. *Optimal insurance in the presence of insurer's loss limit.* 300-307. In this paper we consider the optimal insurance problem when the insurer has a loss limit constraint. Under the assumptions that the insurance price depends only on the policy's actuarial value, and the insured seeks to maximize the expected utility of his terminal wealth, we show that coverage above a deductible up to a cap is the optimal contract, and the relaxation of insurer's loss limit will increase the insured's expected utility.
When the insurance price is given by the expected value principle, we show that a positive loading factor is a sufficient and necessary condition for the deductible to be positive. Moreover, with the expected value principle, we show that the optimal deductible derived in our model is not greater (lower) than that derived in Arrow's model if the insured's preference displays increasing (decreasing) absolute risk aversion. Therefore, when the insured has an IARA (DARA) utility function, compared to Arrow model, the insurance policy derived in our model provides more (less) coverage for small losses, and less coverage for large losses.
Furthermore, we prove that the optimal insurance derived in our model is an inferior (normal) good for the insured with a DARA (IARA) utility function, consistent with the finding in the previous literature. Being inferior, the insurance can also be a Giffen good. Under the assumption that the insured's initial wealth is greater than a certain level, we show that the insurance is not a Giffen good if the coefficient of the insured's relative risk aversion is lower than 1.

Reproduced with the permission of Elsevier Science.
Subscription details available from: Elsevier Science, PO Box 311, 1000 AE Amsterdam, The Netherlands. E-mail: nlinfo-f@elsevier.nl

JOURNAL OF RISK AND INSURANCE

Volume **76** (4), 2009

BARTH, M. M. & ECKLES, D. L. *An Empirical Investigation of the Effect of Growth on Short-Term Changes in Loss Ratios.* 867-885. Given the use of premium growth as a risk measure in regulatory and private risk assessment models, the impact of growth on underwriting profitability is an important question. Our results show a negative relationship between premium growth and changes in loss ratios, suggesting that premium growth alone does not necessarily result in higher underwriting risk. Further, there is a positive relationship between claim count growth and changes in loss ratios, suggesting that claim count growth may be a preferred measure of underwriting risk.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

Boucher, J.-P., Denuit, M. & Guillen, M. *Number of Accidents or Number of Claims? An Approach with Zero-Inflated Poisson Models for Panel Data.* 821-846. The hunger for bonus is a well-known phenomenon in insurance, meaning that the insured does not report all of his accidents to save bonus on his next year's premium. In this article, we assume that the number of accidents is based on a Poisson distribution but that the number of claims is generated by censorship of this Poisson distribution. Then, we present new models for panel count data based on the zero-inflated Poisson distribution. From the claims distributions, we propose an approximation of the accident distribution, which can provide insight into the behavior of insureds. A numerical illustration based on the reported claims of a Spanish insurance company is included to support this discussion.

Colquitt, L. L., Sommer, D. W. & Ferguson, W. L. *A Citation Analysis of Risk, Insurance, and Actuarial Research: 2001 Through 2005.* 933-953. The bibliographies of 17 risk journals were evaluated to determine the relative influence of these risk journals on risk, insurance, and actuarial research published during the years 2001 through 2005. Tables are provided that show the frequency with which each of these journals cites itself and the other sample journals. The journals are ranked, within two groups (risk and insurance group and actuarial group), based on their total influence (total citations including and excluding self-citations) and their per article influence (per article citations including and excluding self-citations). Finally, the most frequently cited articles from each risk journal are reported.

Darkiewicz, G., Deelstra, G., Dhaene, J., Hoek, J. Van der & Vanmaele, M. *Bounds for Right Tails of Deterministic and Stochastic Sums of Random Variables.* 847-866. We investigate lower and upper bounds for right tails (stop-loss premiums) of deterministic and stochastic sums of nonindependent random variables. The bounds are derived using the concepts of comonotonicity, convex order, and conditioning. The performance of the presented approximations is investigated numerically for individual life annuity contracts as well as for life annuity portfolios, where mortality is modeled by Makeham's law, whereas investment returns are modeled by a Brownian motion process.

Gatzert, N., Hoermann, G. & Schmeiser, H. *The impact of the secondary market of life insurers' surrender profits.* 887-908. Life insurers often claim that the life settlement industry reduces their surrender profits and leads to an adverse shift in their portfolio of insured risks; that is, high risks remain in the portfolio instead of surrendering. In this article, we aim to quantify the effect of altered surrender behavior — subject to the health status of an insured — in a portfolio of life insurance contracts on the surrender profits of primary insurers. Our model includes mortality heterogeneity by applying a stochastic frailty factor to a mortality table. We additionally analyze the impact of the premium payment method by comparing results for annual and single premium payments.

Harrington, S. E. *The financial crisis, systemic risk, and the future of insurance regulation.* 785-819. This article considers the role of American International Group (AIG) and the insurance sector in the 2007-2009 financial crisis and the implications for insurance regulation. Following an overview of the causes of the crisis, I explore the events and policies that contributed to federal government intervention to prevent bankruptcy of AIG and the scope of federal assistance to AIG. I discuss the extent to which insurance in general poses systemic risk and whether a systemic risk regulator is desirable for insurers or other nonbank financial institutions. The last two sections of the article address the financial crisis's implications for proposed optional and/or mandatory federal chartering and regulation of insurers and for insurance regulation in general.

Tsai, C., Kuo, W. & Chiang, D. M.-H. *The Distributions of Policy Reserves Considering the Policy-Year Structures of Surrender Rates and Expense Ratios.* 909-931. In this article, we

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

examine how the policy-year structures of expense ratios and surrender rates affect the distributions of policy reserves. Our results show that a convex expense ratio curve, though reduces the mean and the uncertainty of reserves, could make the beneficial impact of surrenders on insurers become detrimental. Our results also show that the convexity of the surrender rate curve is favorable to insurers while the volatilities of surrender rates are unfavorable. We further find that neglecting the policy-year structures of surrender rates and expense ratios may result in overestimation of the mean and the uncertainty of reserves.

Reproduced with the permission of the American Risk and Insurance Association.
Subscription details available from: the American Institute for CPCU, 720 Providence Road, Malvern, PA 19355, USA. E-mail: aria@cpcuiia.org

North American Actuarial Journal

Volume **13** (4), 2009

Ai, J., Brockett, P. L. & Golden, L. L. *Assessing consumer fraud risk in insurance claims: An unsupervised learning technique using discrete and continuous predictor variables.* 438-458. We present an unsupervised learning method for classifying consumer insurance claims according to their suspiciousness of fraud vs nonfraud. The predictor variables contained within a claim file that are used in this analysis can be binary, ordinal categorical, or continuous variates. They are constructed such that the ordinal position of the response to the predictor variable bears a monotonic relationship with the fraud suspicion of the claim. Thus, although no individual variable is of itself assumed to be determinative of fraud, each of the individual variables gives a 'hint' or indication as to the suspiciousness of fraud for the overall claim file. The presented method statistically concatenates the totality of these 'hints' to make an overall assessment of the ranking of fraud risk for the claim files without using any a priori fraud-classified or -labelled subset of data. We first present a scoring method for the predictor variables that puts all the variables (whether binary 'red flag indicators', ordinal categorical variables with different categories of possible response values, or continuous variables) onto a common −1 to 1 scale for comparison and further use. This allows us to aggregate variables with disparate numbers of potential values. We next show how to concatenate the individual variables and obtain a measure of variable worth for fraud detection, and then how to obtain an overall holistic claim file suspicion value capable of being used to rank the claim files for determining which claims to pay and the order in which investigate claims further for fraud. The proposed method provides three useful outputs not usually available with other unsupervised methods: (1) an ordinal measure of overall claim file fraud suspicion level, (2) a measure of the importance of each individual predictor variable in determining the overall suspicion levels of claims, and (3) a classification function capable of being applied to existing claims as well as new incoming claims. The overall claim file score is also available to be correlated with exogenous variables such as claimant demographics or high-volume physician or lawyer involvement. We illustrate that the incorporation of continuous variables in their continuous form helps classification and that the method has internal and external validity via empirical analysis of real data sets. A detailed application to automobile bodily injury fraud detection is presented.

Chen, K. & Hardy, M. R. *The DB underpin hybrid pension plan: Fair valuation and funding.* 407-424. In this paper the defined benefit underpin guarantee is valued as a financial option, within the traditional funding paradigms of actuarial science. Assuming fixed interest rates, and assuming that salaries can be treated as a tradable asset, we value the guarantee using fair value principles. Contribution rates are developed for the Entry Age Normal, Projected Unit Credit, and Traditional Unit Credit funding methods. In addition, for the accruals methods,

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

we demonstrate the implied hedging strategy. The traditional unit credit offers the best method of these three, as it is consistent with the principles of financial economics, and the resulting contributions more naturally follow the cost of the emerging benefit, without creating expensive barriers to new hires. The method generates significant contribution volatility, and we demonstrate how this can be reduced with suitable benefit design and ongoing risk management.

CHEUNG, E. C. K. & LANDRIAULT, D. *Analysis of a generalized penalty function in a semi-Markovian risk model.* 497-513. In this paper an extension of the semi-Markovian risk model studied by Albrecher and Boxma (2005) is considered by allowing for general interclaim times. In such a model, we follow the ideas of Cheung *et al.* (2010b) and consider a generalization of the Gerber–Shiu function by incorporating two more random variables in the traditional penalty function, namely, the minimum surplus level before ruin and the surplus level immediately after the second last claim prior to ruin. It is shown that the generalized Gerber–Shiu function satisfies a matrix defective renewal equation. Detailed examples are also considered when either the interclaim times or the claim sizes are exponentially distributed. Finally, we also consider the case where the claim arrival process follows a Markovian arrival process. Probabilistic arguments are used to derive the discounted joint distribution of four random variables of interest in this risk model by capitalizing on an existing connection with a particular fluid flow process.

FURMAN, E. & ZITIKIS, R. *Weighted pricing functionals with applications to insurance: An overview.* 483-496. We explore the role of weighted distributions in pricing insurance risks. In particular, we relate the distributions to actuarial and economic premium calculation principles and in this way provide a unifying methodology for constructing new principles and analyzing known ones.

KELLY, M., ISOTUPA, S. & KLEFFNER, A. *The impact of adjuster moral hazard on driving records.* 425-437. In a first-party recovery scheme for automobile property damage, the first-party insurer compensates not-at-fault vehicular damage. In this scheme, adjusters may not have the incentive to assign liability when the driver is, in fact, at fault for the accident. This is due to adjusters not having to coordinate with a third-party adjuster, and, for insureds that carry collision coverage, the assignment of fault does not appreciably affect the compensation paid out. This in turn reduces the effectiveness of the experience-rating component of the insurance premium. Empirical evidence that supports the presence of incorrect fault assignment is provided. A stochastic model of experience rating analyzing the impact of incorrect fault assignment on driving record classes confirms that low-risk insureds pay more for insurance than if fault was correctly assigned.

TAN, K. S., WENG, C. & ZHANG, Y. *VaR and CTE criteria for optimal quota-share and stop-loss reinsurance.* 459-482. It is well known that reinsurance can be an effective risk management tool for an insurer to minimize its exposure to risk. In this paper we provide further analysis on two optimal reinsurance models recently proposed by Cai and Tan. These models have several appealing features including (1) practicality in that models could be of interest to insurers and reinsurers, (2) simplicity in that optimal solutions can be derived in many cases, and (3) integration between banks and insurance companies in that the models exploit explicitly some of the popular risk measures such as value-at-risk and conditional tail expectation. The objective of the paper is to study and analyze the optimal reinsurance designs associated with two of the most common reinsurance contracts: the quota share and the stop loss. Furthermore, as many as 17 reinsurance premium principles are investigated. This paper also highlights the critical role of the reinsurance premium principles in the sense that, depending on the chosen principles, optimal quota-share and stop-loss reinsurance may or may not exist. For some cases we formally establish the sufficient and necessary (or just

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

sufficient) conditions for the existence of the nontrivial optimal reinsurance. Numerical examples are presented to illustrate our results.

Available for downloading from: http://www.soa.org/news-and-publications/publications/journals/naaj/naaj-detail.aspx

Reproduced with the permission of the Society of Actuaries.
Subscription details available from: Society of Actuaries, 475 N. Martingale Road, Schaumburg, ILL 60173 USA, www.soa.org

SCANDINAVIAN ACTUARIAL JOURNAL

Volume **4,** 2009

BJORKWALL, S., HOSSJER, O. & OHLSSON, E. *Non-parametric and parametric bootstrap techniques for age-to-age development factor methods in stochastic claims reserving.* 306-331. In the literature, one of the main objects of stochastic claims reserving is to find models underlying the chain-ladder method in order to analyze the variability of the outstanding claims, either analytically or by bootstrapping. In bootstrapping these models are used to find a full predictive distribution of the claims reserve, even though there is a long tradition of actuaries calculating the reserve estimate according to more complex algorithms than the chain-ladder, without explicit reference to an underlying model. In this paper we investigate existing bootstrap techniques and suggest two alternative bootstrap procedures, one non-parametric and one parametric, by which the predictive distribution of the claims reserve can be found for other age-to-age development factor methods than the chain-ladder, using some rather mild model assumptions. For illustration, the procedures are applied to three different development triangles. Keywords: Bootstrap; Chain-ladder; Development factor method; Development triangle; Stochastic claims reserving.

BOYLE, P. & TIAN, W. *Optimal design of equity-linked products with a probabilistic constraint.* 253-280. There is a rich variety of tailored investment products available to the retail investor. These products combine upside participation in bull markets with downside protection in bear markets. Examples include the equity-linked products sold by insurance companies and the structured products marketed by banks. This paper examines a particular contract design for products of this nature. The paper finds the optimal design from the investor's viewpoint. It is assumed that the investor wishes to maximize expected utility of the terminal wealth subject to certain constraints. These constraints include a guaranteed rate of return as well as the opportunity to outperform a benchmark portfolio with a given probability. We derive the explicit form of the optimal design assuming both constraints apply and we illustrate the nature of the solution using some specific examples. Keywords: Equity-Indexed Annuities; Structured products; Optimal design; Optimal portfolio selection.

FROSTIG, E. & DENUIT, M. *Ruin probabilities and optimal capital allocation for heterogeneous life annuity portfolios.* 295-305. A large part of the actuarial literature is devoted to the derivation of ruin probabilities in various non-life insurance risk models. On the contrary, very few papers deal with ruin probabilities for life insurance portfolios. The difficulties arise from the dependence and non-stationarity of the annual payments made by the insurance company. This paper shows that the ruin probability in case of life annuity portfolios can be computed from algorithms derived by De Pril (1989) and Dhaene & Vandebroek (1995). Approximations for ruin probabilities are discussed. The present article complements the works of Frostig *et al.* (2003) who considered whole life, endowment, and temporary assurances, and of Denuit & Frostig (2008) who considered homogeneous life annuities

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

portfolios. Here, heterogeneous portfolios (with respect to age and/or face amounts) are studied. Particular attention is paid to the capital allocation problem. The total amount of reserve is shared among the risk classes in order to minimize the ruin probability. It is then fair to charge a higher margin to the risk classes requiring more capital. Keywords: Life insurance; Aggregate claim; Recursive formulas.

Wu, X. & Li, S. *On the discounted penalty function in a discrete time renewal risk model with general interclaim times.* 281-294. In this paper a discrete time renewal risk model with arbitrary interclaim times is discussed. We show that the expected discounted penalty function satisfies a recursive formula. In particular, the probability generating function of the time of ruin, as a function of the initial surplus, has a compound geometric tail. When the claim amounts follow a geometric distribution, explicit expression for the Gerber–Shiu function can be obtained for the specially chosen penalty function. The constant claim amounts and mixed geometric claim amounts are also examined. Keywords: Sparre Andersen risk process; Gerber–Shiu function; Recursive formula; Generating function; Mixture of geometric distributions.

Scandinavian Actuarial Journal

Volume **1,** 2010

Adamic, P. F. *Modeling multiple risks in the presence of double censoring.* 68-81. Self-consistent (SC) iterative algorithms will be proposed to non-parametrically estimate the cause-specific cumulative incidence functions in a multiple decrement, doubly censored context. Double censoring is defined to include both left and right censored observations, in addition to exact observations. The algorithms are a generalisation of the classical univariate algorithms of Efron and Turnbull. Unlike any previous competing risk models proposed in the literature to date, the proposed algorithms will be fully non-parametric while also explicitly allowing for the possibility of masked models of failure, whereby failure is known only to occur due to a subset from the set of all possible causes. In short, the method is useful in any actuarial application that encounters censored and/or masked risks. The paper concludes by showing how the method can be applied to employee benefits modeling.

Bai, L. & Guo, J. *Optimal dividend payments in the classical risk model when payments are subject to both transaction costs and taxes.* 36-55. In this paper, we study optimal dividend problem in the classical risk model. Transaction costs and taxes are required when dividends occur. The problem is formulated as a stochastic impulse control problem. By solving the corresponding quasi-variational inequality, we obtain the analytical solutions of the optimal return function and the optimal dividend strategy when claims are exponentially distributed. We also find a formula for the expected time between dividends. The results show that, as the dividend tax rate decreases, it is optimal for the shareholders to receive smaller but more frequent dividend payments.

Giovanni, D. de *Lapse rate modeling: a rational expectation approach.* 56-67. The surrender option embedded in many life insurance products is a clause that allows policyholders to terminate the contract early. Pricing techniques based on the American Contingent Claim (ACC) theory are often used, though the actual policyholders' behaviour is far from optimal. Inspired by many prepayment models for mortgage backed securities, this paper builds a Rational Expectation (RE) model describing the policyholders' behaviour in lapsing the contract. A market model with stochastic interest rates is considered, and the pricing is carried out through numerical approximation of the corresponding two-space-dimansional parabolic partial differential equation. Extensive numerical experiments show the differences in terms of

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

pricing and interest rate elasticity between the ACC and RE approaches as well as the sensitivity of the contract price with respect to changes in the policyholders' behaviour.

MacDonald, A. & McIvor, K. R. *Pensions and genetics: can longevity genes be reliable risk factors for annuity pricing?* 1-14. We consider a number of gene variants that have been found to affect longevity. Their effects have been modelled using Cox or logistic regressions, whose fitted parameters have simple asymptotic sampling distributions. The expected present value of a life annuity allowing for these genetic risk estimates inherits a sampling distribution, which can be found by simulation. We find that possibly significant uncertainty about annuity premiums may be overlooked if the standard errors of parameters estimated in medical studies are ignored by medical underwriters. Such considerations may play an important part when the acceptability of using a risk factor in underwriting is conditional on proof of its relevance and reliability. This is the current position in respect of genetic information in many countries, most prominently in the UK.

Preisel, M., Jarner, S. F. & Eliasen, R. *Investing for retirement through a with-profits pension scheme: a client's perspective.* 15-35. Saving for retirement by with-profits pension contracts is markedly different from traditional savings vehicles in at least two aspects: premia committed to the company are managed according to the preferences of the company — which may not coincide with the long-term interests of the client — and the return on investments is not directly transferred to client's savings but awaits a decision by the company to spend it as bonus. We show that a management's general aversion to (short term) insolvency risk results in investment strategies dynamically scaling investment risk up and down with the current funding status of the company. The resultant dynamics hugely impacts the long-term funding status of the pension company and thereby the investment outcome of with-profits contracts. We show that for a one-period optimizing company there exists a stationary regime only for moderately aggressive investment strategies, and we derive an analytical approximation to the stationary funding ratio distribution when it exists. in contrast to the one-period case, we show that the highest expected bonus level in stationarity is not achieved for the most aggressive investment strategy available. The reason being that if investments become too aggressive the company will spend a lot of time at low funding ratios where bonus cannot be attributed impairing the average bonus.

Reproduced with the permission of Taylor & Francis, www.tandf.no/saj

South African Actuarial Journal

Volume **9,** 2009

Lowther, M. W., McMillan, W. J. & Venter, F. *Education for actuarial quality must develop more than technical competence.* This paper is located in the field of actuarial professional education. It draws on current literature and empirical evidence to argue the need for developing generic 'normative' capabilities and values in the actuarial profession. The paper examines three themes of normative education for actuaries — the intended purpose of an education programme, the range of capabilities and values to be developed and educational 'best practice'. A literature review of normative education in various professions provided a theoretical framework for data analysis, and for making recommendations.

Data were collected from three sources: interviews with stakeholders; documentary evidence regarding current education of actuaries and accountants; and advertisements for employment of nearly or newly qualified actuaries. The paper draws on the literature and the empirical evidence to argue that the purpose of an actuarial education programme is to ensure the delivery of a service of quality. For this delivery to be achieved, attention must be given to

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press

technical, normative and organisational learning. The paper then reviews normative capabilities and values for actuaries. Thereafter, educational considerations for the development of normative capabilities and values are discussed. The paper concludes by drawing on the study to outline principles to guide curriculum planning for the normative component of the new South African actuarial qualification.

Available for downloading from: http://ajol.info/index.php/saaj/issue/archive

TERBLANCHE, W. *Demand for actuarial resources in South Africa.* A key thrust of the Actuarial Society of South Africa's strategic plan is to promote diversity in actuarial demography. The aim is that the membership profile of the actuarial profession increasingly reflects the demographic profile of the country. More broadly, it is useful for a profession to consider how it has evolved, and how changes in the environment in which it operates may affect demand for its services in the future. In the last few years, as a result of economic development and growth in the services sector, as well as changes in the financial and insurance environment, including regulatory and technological developments as well as social and demographic changes, the demand for actuaries in South Africa has grown significantly. South Africa faces a number of challenges, such as a poorly educated workforce with high levels of unemployment and skills shortages. Poor socio-economic conditions for a large majority of the population result in high crime levels and emigration of skilled workers — a negative spiral. These are all interrelated and all affect economic growth and the demand for actuaries. While deterioration in these conditions negatively affects demand, global developments, such as international accounting standards and Solvency II, are creating demand. At the same time, emigration of actuaries and actuarial students creates shortages and a much tighter actuarial resource market. This paper covers the current demographics of the membership of the Actuarial Society of South Africa, factors affecting demand, the results of research performed among South African employers of actuarial resources and models developed for projecting future demand. The models indicate moderate growth in demand in the short term.

Available for downloading from: http://ajol.info/index.php/saaj/issue/archive

Reproduced with the permission of the Actuarial Society of South Africa.

https://doi.org/10.1017/S1748499500000750 Published online by Cambridge University Press