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LOGICAL CONTEXTUALITY IN FREGE

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Abstract. Logical universalism, a label that has been pinned on to Frege, involves the conflation of two features commonly ascribed to logic: universality and radicality. Logical universality consists in logic being about absolutely everything. Logical radicality, on the other hand, corresponds to there being the one and the same logic that any reasoning must comply with. The first part of this paper quickly remarks that Frege's conception of logic makes logical universality prevail and does not preclude the admission of different contexts of discourse. The paper then aims to make it clear how Frege's universalism can make sense of contextuality. Drawing on a suggestion made by Frege in his discussion of Hilbert, it shows that a properly Fregean notion of model can be devised. Taking up a suggestion from Wilfrid Hodges and William Demopoulos that the non-logical constants of a formal language can be compared to indexicals, this paper shows, *pace* Hodges and Demopoulos, that such an understanding of non-logical constants is not beyond Frege's horizon. A formal framework, based on the modern tool of fibrations, is set out to explain and justify this point. This framework allows one to compare Frege and Tarski, by formalizing Frege's suggestion and by presenting Tarski's semantics in a generalized setting.

The basic working hypothesis guiding this paper is that some insights and devices of Frege's logic are still to be reactivated. In the first part, I will compare the respective ways in which Kant and Frege combine two different features commonly ascribed to logic, and will then criticize the unqualified "logical universalism" that has been ascribed to Frege by authors such as van Heijenoort and Hintikka. Many Frege scholars now have argued that Frege's universalism should not be conflated with a stiff absolutism. My specific claim is that the defense of the universality of logic is even fully compatible with the admission of variable contexts of discourse.

The last two parts of the paper are the positive counterpart of the first one. Drawing on a suggestion made by Frege in his "Foundations of Geometry," it aims to show how precisely Frege's perspective can countenance particular contexts of discourse and sets out a formal framework to illustrate that point. This framework will make it possible to bring out a specifically Fregean notion of model and to compare it with the Tarskian one. I will then conclude, *pace* Wilfrid Hodges and William Demopoulos, that Frege can, as well as Tarski, make sense of non-logical constants.

§1. Frege's "logical universalism".

1.1. Kant and Frege. Logic is traditionally conceived of as a theory that precedes any other theory. Such a precedence consists in the absolutely general bearing of logic on thought in general. The juridiction of logic knows no bound. But this traditional

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characterization of logic points to two distinct features, namely the *universality* of logic, on the one hand, and the *radicality* of logic, on the other.

The universality of logic consists in its being about absolutely everything, i.e., in the existence of a single unrestricted range of values for entity variables. The radicality of logic, on the other hand, corresponds to there being the one and the same logic that any reasoning must comply with, if it is to be accepted as a reasoning at all. It is connected, in Frege, to the acknowledgement of the principles of logic as being laws. Logical laws are such that one cannot but presuppose them, even to challenge them, so that their rejection is ultimately self-refuting.

Thus, the bearing of logic has an objectual aspect, corresponding to universality, and a discursive aspect, corresponding to radicality. Otherwise put, the universality of logic consists in that nothing can escape it; The radicality of logic consists in that nobody can escape it. The universality and the radicality of logic have been associated since at least Kant's *Logic*:

> If [. . .] we put aside all cognition that we have to borrow from *objects* and merely reflect on the use just of the understanding, we discover those of its rules which are necessary without qualification, for every purpose and without regard to any particular objects of thought, because without them we would not think at all. (Kant (1992), "The Jäsche logic," p. 528; I, Ak. IX, 12)

Kant indiscriminately describes (formal) logic as the "science of necessary laws of thought, without which no use of the understanding or of reason takes place at all"[1] (radicality) and as "a science a priori of the necessary laws of thought, not in regard to particular objects, however, but to all objects in general"[2] (universality).

Even though Kant and Frege obviously do not conceive of the generality of logic in the same way, they concur, according to John MacFarlane,[3] in characterizing logic through its normative generality. Logic provides one with the norms with which any thought whatsoever must comply as such, independently of its object and of the particular laws governing that object. However, MacFarlane's analysis somewhat conceals the asymmetry of Kant's and Frege's respective agendas.

To Kant, radicality is a way of accounting for formal universality from within a nonformal conception of logic. Formal logic is in a sense more general than transcendantal logic, because it does not presuppose the possibility of experience, yet the domain ruled by formal logic cannot be seen as extending the domain ruled by trancendental logic, since no domain properly speaking can enlarge the totality of possible experience. In that setting, the generality of formal logic has to be reckoned through something else than its universality, namely: Formal logic states conditions of possibility of thought as such.

Frege's move goes the other way round: The universality of logic allows Frege to express radicality from within an anti-psychologistic conception of logic. The universality of logic provides a way of reckoning the existence of laws of thought while dwelling neither on the possibly psychological connotation of the notion of "thought," nor on the slippery notion of "law." Absolutely unrestricted universality is Frege's way out to render the overhanging

[1] Kant (1992), Ak. IX, 13, p. 529.
[2] Kant (1992), Ak. IX, 16, p. 531.
[3] MacFarlane (2002, pp. 35–38).

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status of logic without committing himself to some kind of necessity that could not be captured formally.

To sum up, both Kant and Frege, despite their substantial disagreement in logic, bestow on logic two distinctive features: universality and radicality. And they both agree to consider those features as inseparable. But, while Kant puts the stress on radicality to account for the universality of logic, Frege puts the stress on universality to account for its radicality. In Frege, logical universality clearly prevails over logical radicality.

1.2. The invention of logical universalism. Logical universalism is not the association of logical universality and logical radicality. It is more than that, the sheer conflation of both, as well as the further claim that "one cannot consider one's language from the outside." Although a label that has been pinned on to Frege, logical universalism is *not* epitomized by Frege himself. Logical universalism is more of a reconstruction by authors who *fought*, under this term, a family of philosophical stances expressed by Frege, Russell, Wittgenstein and, to some extent, Carnap or even the young Tarski.

Indeed, a tradition initiated by Jean van Heijenoort[4] has tended to put in the same "universalist" bag four independent claims, namely: (i) Logic resorts to absolutely unrestricted variables; (ii) Logic refers to one single context of discourse; (iii) The language of logic is unescapable; (iv) Metatheoretical studies, and semantics to begin with, are impracticable and, for that reason, banned. Jaakko Hintikka is one of the commentators who, most stalwartly, identified three tenets that he claims to recognize in all the universalists (from Frege to Quine), namely: the universality of logical variables, the radicality of logical truths, and the "inescapability" of logical language—the package of these three tenets resulting in the "ineffability of semantics."[5]

Hintikka conflates two theses, which he imputes both to "the universalist:" the universal scope of logical variables and the universality of the language of logic as a "universal medium" in which one can say everything and in which one must say anything. This obviously does not square with the distinction that Frege emphasizes in the *Grundgesetze* between the "expository language" (*Darlegungssprache*) and the "auxiliary language" (*Hilfessprache*) of his logical system. The latter is certainly not universal in the sense of the ordinary language, since it is rudimentary to the extreme: The point of logicism is precisely that it is still sufficient to develop the whole of arithmetic.[6] As to the expository language, it eludes in some respects the formal rules set out in the system, as the problematic logical status of such an expression as "the function $\Phi(\xi)$" shows—which proves that, in a way, one can (and even must) escape the regimentation of logic.[7] Ironically, the confusion of the universality of logic with the radicality of the language of logic comes rather from Tarski's 1935 essay on truth.[8]

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[4] van Heijenoort (1967), particularly pp. 326–327.

[5] See Hintikka (1997, pp. 216–219).

[6] Thanks to Marco Panza for pointing this out to me.

[7] On this, see footnote to §4 of Frege (1893). Considering, on the contrary, the expository language, not as a mere way to give hints, but as a genuine theoretic language, i.e., as falling under the rules of the *Begriffschrift*, actually leads to some version or another of the concept 'horse' paradox, as discussed by Frege in his "On concept and object."

[8] One well-known result of the essay is that an adequate definition of the truth predicate for some language can only be given in a metalanguage whose order is strictly greater than the order of the language under consideration. This excludes the colloquium language, because of its "universality"—in Tarski's sense of a language in which whatever can be expressed in whatever language, can be expressed (see Tarski, 1983, pp. 164 and 248). Indeed, the order of a language

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Actually, the universality of logic does not preclude one from considering local contexts of discourse, in particular for metatheoretical purposes. As early as in the *Principles* (see §430), Russell was well aware that an axiomatic system lends itself to various different interpretations: For instance, Russell knew very well that any progression is a possible interpretation of Dedekind's arithmetic. Besides,[9] Russell's theory of types in *Principia Mathematica* can be seen as a common platform for various epistemic realizations—which ties in with Russell's perspectivist reconstruction of reality in *Our Knowledge of the External World*. Quite generally, Rouilhan[10] has formally established that Frege and Russell did not, but at least *could have* introduced models of a theory within a universal logical system (a system based on the use of universal variables) and thus *could have* built up, within such a system, the equivalent of Tarskian model theory.

Logical universality ("absolute generality") is certainly incompatible with ZFC-based model theory, whereas logical radicality understood à la Hintikka is certainly incompatible with any semantic metatheory. But the defense of logical universality does *not* imply logical universalism and, since Frege's conception of logic makes logical universality prevail, it is maybe *de facto* incompatible with Tarskian model theory as such, but should not be deemed *de jure* incompatible with any semantic metatheory.

Frege himself provides an illustration of this point at the end of his 1903 "On the Foundations of Geometry:"[11]

> [. . .] Euclidean geometry presents itself as a special case of a more inclusive system which allows for innumerable other special cases—innumerable geometries, if that word is still admissible. [. . .] If one wanted to use the word "point" in each of these geometries, it would become equivocal. To avoid this, we should have to add the name of the geometry, e.g., "point of the *A*-geometry," "point of the *B*-geometry," etc. Something similar will hold for the words "straight line" and "plane." (Frege, 1971, p. 37)

Obviously, all special geometries are not subdomains of a single geometric domain, but different specifications of the meaning of fundamental geometric concepts, and thus correspond to different theoretical contexts. Actually, Frege was not insensitive to the idea of context variation, despite his "logical universalism." In the rest of this paper, I would like to show how Frege's upholding of the universality of logic is perfectly compatible with making sense of contextuality. The main evidence from Frege's work is, despite its

being defined as the maximal order of all its semantical categories, the colloquium language is of infinite order. However, as Philippe de Rouilhan pointed out (*in* de Rouilhan, 1998, pp. 85–102), Tarski replaced, in the post-scriptum, the original notion of order with a new one, drawn from the cumulative hierarchy of ZF set theory. The order of a variable (and, consequently, the order of a language) is not defined as a syntactic type any more, but by the set-theoretical rank of its possible values. Thus, the order of a language becomes the least upper bound of the ranks of all the sets whose existence follows from the axioms (see Tarski, 1983, p. 271, note 1). This change precipitated a set-theoretical conception of the hierarchy of languages: The classes to which the expressions of the object-language refer must all belong to classes to which expressions of the metalanguage refer. So the domain of the metalanguage must be strictly wider than the domain of the object-language. As a consequence, a universal domain cannot lend itself to any metatheory. This result, however, follows only from a set-theoretic artefact.

9 See Halimi (2011).

10 de Rouilhan (2012, pp. 573–574).

11 Frege (1903).

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being deeply critical of Hilbert's independence results, the 1906 "On the Foundations of Geometry" (especially the final section), Frege's second essay of that title.[12]

§2. Logical contextuality.

2.1. The 1906 "On the Foundations of Geometry". The richness of Frege's 1906 essay is hardly a scoop. Many Frege scholars[13] showed, against the van Heijenoort-Dreben-Hintikka-Ricketts interpretation of Fregean logicism,[14] that there was no objection of principle in Frege against metatheoretic considerations, and most of them refer to the 1906 "On the Foundations of Geometry" to that end. In their view, Frege's universalist conception of logic clearly does not bar any metatheoretic perspective, simply because Frege's logical works themselves actually witness to it: Genuinely semantical reflections can be found in Frege, and even though they require distinguishing between logic as the most fundamental science and its particular implementation in the form of a given symbolic calculus,[15] they certainly cannot be brushed aside as mere didactic elucidations.

However, the focus of the reconsideration of Frege against the "no-metaperspective" interpretation has remained mostly *truth*. For instance, Peter Sullivan describes the Fregean conception of logic, that he calls "internalism," as the "claim that the notion of truth belongs *within* the language, and is not to be ceded to any external vantage point on it."[16] Sullivan's paper focuses primarily on the issue of truth, as do Stanley's and Tappenden's papers: Frege's metatheoretic samples pertain to semantics understood as the theory of reference and truth.[17] The now very substantial material adduced by the no-"no-metaperspective" interpretation calls for a stronger claim: Not only Frege's conception of logic countenances metatheoretic considerations, but Frege's suggestions in his 1906 essay hint at a genuine model-theoretic framework, although of course not of the Tarskian sort. No commentator upholding the first point is willing to consider the second and envisage a Fregean model theory.[18] In contrast, this paper aims to prove that Frege's conception of logic does not preclude some kind of model theory, by actually building up a notion of model extending Frege's suggestions. This will show how Frege's conception of logic harbors an original notion of context of discourse for logical reasoning, or "logical context," for short.

There are good reasons to turn towards Frege's 1906 "Foundations of Geometry," which precisely illustrates that a universal domain of quantification does not prevent one from introducing various "systems of thoughts" that can be conceived of as reinterpretations of the language of mathematics.[19] Frege's paper is devoted to the discussion of the independence proofs given in Hilbert's *Foundations of Geometry*. Against Hilbert's axiomatic

12 Frege (1906).

13 *Cf.* Stanley (2005), Landini (1998), Antonelli & May (2005), Tappenden (2000), Sullivan (2005), Tappenden (2005), Heck (2007), Blanchette (2014), Korhonen (2012).

14 Besides Hintikka's writings already quoted, see Dreben & van Heijenoort (1986, p. 44) and Ricketts (2005, pp. 137–141).

15 Korhonen (2012, p. 602).

16 Sullivan (2005, p. 90).

17 See Sullivan (2005, pp. 87–93 and 98–101), Stanley (2005) (passim) and Tappenden (2005, pp. 200–201 and 217–219). An exception is Korhonen (2012, pp. 605–607) but remains to be developed.

18 See, in particular, Antonelli & May (2005, p. 169) and Blanchette (2005, pp. 41–42).

19 "What Frege objected to was Hilbert's claim that content can be bestowed upon a sign *simply* by indicating a range of alternative interpretations. In some sense, it seems to me, Frege thought that the concept of an interpreted language was more basic than that of an uninterpreted one—and it

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method, Frege insists upon the important distinction to be made between real propositions (that can be said to be true or false) and pseudo-propositions (merely formal sequences of signs).[20] It is noteworthy, however, that Frege is nonetheless willing in the end to formalize dependence and independence through the use of variable reinterpretations, which Frege introduces as *internal translations* and calls "vocabularies:"

> Imagine a vocabulary: not, however, one in which words of one language are opposed to corresponding ones of another, but where on both sides there stand words of the same language but having different senses. [. . .] We may say in general that words with the same grammatical function are to stand opposite one another. Each word occurring on the left has its determinate sense—at least we assume this—and likewise for each one occurring on the right. Now by means of this opposition the senses of the words on the left are also correlated with the senses of the words on the right. Let this correlation be one-to-one, so that on neither the left nor the right is the same thing expressed twice. We can now translate; not, however, from one language to another, whereby the same sense is retained; but into the very same language, whereby the sense is changed. (Frege, 1971, pp. 107–108)

Frege eventually defines dependence as follows:

> Let us now consider whether a thought G is dependent upon a group of thoughts Ω. We can give a negative answer to this question if, according to our vocabulary, to the thoughts of group Ω there corresponds a group of true thoughts Ω', while to the thought G there corresponds a false thought G'. For if G were dependent upon Ω, then, since the thoughts of Ω' are true, G' would also have to be dependent upon Ω' and consequently G' would be true. (Frege, 1971, p. 110)

Frege's definition of logical dependence among actual (contentful) thoughts clearly suggests Tarski's definition of logical consequence. But it does not prefigure it, for one important reason.

> Crucially, for Frege, thoughts are evaluated as having the truth-values they actually have. No true thought is treated as false or projected into counterfactual circumstances in which it is false. (Axioms will be paired with other thoughts some of which may be actually false.) (Tappenden, 2000, p. 274)

In other words, the vocabularies $e \mapsto e'$ mentioned by Frege relate expressions e and e' belonging to the *same* logically regimented *interpreted* language L. They are not arbitrary maps from one domain (or L-structure) to another, contrary to Tarski's logical semantics, based on the consideration of all possible interpretations of some formal language.

Frege's discussion of Hilbert's *Foundations of Geometry* has been recently reconsidered, and partly vindicated against the view that Frege had it wrong because he simply missed Hilbert's achievements.[21] Still, Frege's positive proposal, based on vocabularies,

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is hard not to be sympathetic. But it simply does not follow that one cannot intelligibly consider other interpretations of the dis-interpreted symbols of a given language." (Heck, 2007, pp. 43–44)

[20] See Frege (1971, p. 102).

[21] See Tappenden (2000), in particular.

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to account for independence, has never been taken seriously, let alone envisaged as an alternative to Tarski's approach, and to that extent Frege's papers on geometry have never been vindicated against the view that Frege had it wrong in comparison with Tarski.[22]

2.2. Two problems with the 1906 "Foundations". The peculiar kind of model theory sketched by Frege in his "Foundations of Geometry" shows that Frege's perspective is not incompatible in principle with the admission of various contexts of discourse. Admittedly, such admission, in Frege's article, pertains to geometry, rather than to arithmetic and logic, and geometry pertains only to a restricted domain: It is not properly universal as arithmetic is.[23] Still, nothing precludes the extension of Frege's suggestion to a general framework, beyond the strict discussion of geometry.[24] The language of geometry is part of the language, and the particular case of geometry precisely shows that, more generally, the language lends itself, as a whole, to partial reinterpretations. This has nothing to do with the fact that geometry is not as general in scope as arithmetic and logic. Frege rejects only vocabularies which would change the sense of a word standing for a logical constant (such as "not," or "any").[25] Logical terms should be left invariant by any vocabulary. And that is precisely what happens in modern semantics as well: The signature of a formal language only consists of its non-logical constants, whereas the constants of the underlying logic are not open to various interpretations.

Still, the delineation of what is logical appears to Frege as a problem. Actually, Frege mentions two obstacles in following the strategy based on permutations of non-logical vocabulary, and these problems have to be tackled before anything else:

> We will find that this final basic law [the formality of logical laws according to which each object or first-level concept can be replaced, as far as logic is concerned, with any other] which I have attempted to elucidate by means of the above-mentioned vocabulary still needs more precise formulation, and that to give this will not be easy. Furthermore, it will have to be determined what counts as a logical inference and what is proper to logic. (Frege, 1906, p. 110)

The first obstacle, the lack of "precise formulation," can be understood as the claim that no syntactic result is able to establish the kind of independence that Frege has in mind:

> Because each thought is generally expressible by a number of different sentences, there is much more to the relation of *provability* than is evidenced by the relation of *derivability*. Where p is a thought and s a sentence expressing it, Π a set of thoughts and Σ a set of sentences expressing Π: the derivability of s from Σ guarantees that p is a consequence of Π, but the fact that s is not derivable from Σ is no guarantee that p is not a consequence of Π. (Blanchette, 2005, pp. 35–36)[26]

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22 See Resnik (1974, pp. 398–400) and Demopoulos (1994).

23 On this question, see Tappenden (1995, pp. 333–336).

24 Frege's notion of a vocabulary is actually echoed by the §26 of *Foundations of Arithmetic*, where Frege refers to the duality principle in projective geometry to illustrate his claim that the objective content conveyed by mathematics is beyond intuition. This claim is general and shows that Frege is not considering geometry, in that paragraph, as a mere special case.

25 Frege (1971, p. 110).

26 See also Ricketts (2005, p. 150) and Blanchette (2014) (passim).

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Thus, each syntactic representation of logical relations between thoughts by a system of derivations between sentences corresponds to a certain decomposition of logically complex notions, linked to a particular choice of primitives. Different formal systems, corresponding to different decompositions based on different bases of primitives, bring out different logical relations, and in particular different logical dependencies. So there is no way to exclude, about a thought purporting to be independent of others, that a certain syntactic representation of the former could reveal its logical dependence upon the latter. This is the weakness common to both Hilbert's and 1906 Frege's methods to establish the independence of an axiom. In both cases, the various interpretations or reinterpretations of the language put a certain syntactic form to the fore, but logical structure is something deeper.[27] However, being able to distinguish all logical connections between the relevant terms, and thus to unfold the full logical complexity thereof, would amount to possessing, as alrealy completed, the logical analysis that formal systems are precisely geared to guide and to convey as work-in-progress.

So, enlightening as Blanchette's explanation of the first obstacle may be, it precisely shows how this first obstacle is not an obstacle that it would make sense to try to overcome. Indeed, the resort to syntactic representations constitutes the very core of logic, and Frege knows very well about it. The ultimate logical analysis of a mathematical concept is certainly out of reach, and logical analysis cannot be carried out if not by the mediation of a formal system. Certain syntactic representations are more faithful than others, and the Begriffsschrift itself is the paramount example of the kind of faithfulness that Frege is seeking. But if relations of derivability are mere indications (or approximations) of logical connections, logical connections can be accessed only through the process of bringing out relations of derivability within a formal calculus. To posit logical connections independently of any particular syntactic framework would be, instead of upholding logical universalism, lapsing into logical "absolutism," namely the claim that a certain formal theory fully captures logic as a science and fully expresses all absolute logical connections between concepts of a certain class—but this is not Frege's conception. Admittedly, no independence proof can be more than relative to some syntactic representation. However, no absolute independence proof can be envisaged anyway, so this cannot be used against the framework of the 1906 "Foundations" per se. Establishing facts about derivability w.r.t. a certain syntactic representation of logic is not a second-best option, even in Frege's view, and this approach is precisely what the framework suggested by Frege aims to support.[28]

Frege's second worry has to do with the delineation of logical terms to be held fixed in any vocabulary. As outlined by Blanchette, this worry leads one back to the previous one:

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[27] On this point, see the conclusion of Blanchette (2014): "From Frege's point of view, in order to systematically and effectively apply the 1906 test, then, one would need a way to distinguish terms whose contents bear logical connections to one another from those that don't. One will need, that is, to be able to distinguish the 'logical' from the 'non-logical' in the broad sense of that term. And here it's clear that Frege does not think there is a systematic or straightforward way to do this."

[28] See Landini (1998), p. 35, in particular: "Independence is a syntactic matter concerning derivability, and derivability within a calculus for logic must not be confused with 'implication.' It is important that the former track the latter. After all, one can adopt rules for a formal calculus that do not track logic at all. Nonetheless, results concerning derivability can be useful and Frege and Russell were simply over-zealous in their concern that changing the meaning of logical particles of a purported formal calculus for logic severs the connection between the calculus and logic itself." Landini's remark can even be strengthened: Studying derivability in a formal calculus that purports to (partially) capture logic is not only useful, but the only thing possible to do.

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All the genuinely logically primitive terms should be held fixed, but those include all terms whose content is germane to logical connections of content in general—which amounts to saying that many terms that the model-theoretic conception reckons as non-logical turn out to be logical in Frege's view.[29] According to both Blanchette and Ricketts, this is the main problem that Frege's strategy has to face.[30] The situation is not as intractable as it may seem, however. There are indeed at least two options. Both options proceed impredicatively by characterizing logical notions as the largest class of terms satisfying a certain overall condition. The first option, drawing on Tarski's method to characterize logical notions by means of permutations, has been set out by A. Antonelli and R. May.[31] The second option consists in drawing on Carnap's syntactic criterion to distinguish between logical and descriptive expressions of a language, as set out in the *Logical Syntax of Language*,[32] and in transposing it to the context of an interpreted language, as follows.

DEFINITION 2.1. *Given two terms a, b with the same syntactic type,* v_a^b *is the vocabulary* (*i.e., the reinterpretation of the language*) *induced by the replacement of 'a' with 'b'. For any sentence p of the language,* $v_a^b(p)$ *is the* (*possibly*) *new sentence generated from p by this vocabulary.*

A thought expressed by a sentence p is valid (*resp.* contravalid) *iff there exists some constituent u of p such that, for any term x of the same syntactic type as u, the thought expressed by* $v_u^x(p)$ *is true* (*resp. false*).

A thought is (logically) determinate *if it is either valid or contravalid.*

DEFINITION 2.2. *Let* $\mathfrak{R}_1$ *be the product of all classes* $\mathfrak{R}_i$ *of expressions of the language which fulfill the following four conditions.* 1. *If* $\mathfrak{U}_1$ *belongs to* $\mathfrak{R}_i$*, then* $\mathfrak{U}_1$ *is not empty and there exists a sentence which can be subdivided into partial expressions in such a way that all belong to* $\mathfrak{R}_i$ *and one of them is* $\mathfrak{U}_1$. 2. *Every sentence which can be thus subdivided into expressions of* $\mathfrak{R}_i$ *is determinate.* 3. *The expressions of* $\mathfrak{R}_i$ *are as small as possible, that is to say, no expression belongs to* $\mathfrak{R}_i$ *that can be subdivided into several expressions of* $\mathfrak{R}_i$. 4. $\mathfrak{R}_i$ *is as comprehensive as possible, that is to say, it is not a proper sub-class of a class which fulfills both* (1) *and* (2).

A term is called logical *if it is capable of being subdivided into expressions of* $\mathfrak{R}_1$*; Otherwise it is called* non-logical.

The intuitive rationale of this definition is the clause 2: Logical terms are those, the composition of which can elicit but sentences whose truth value is logically determined. The class of logical terms thus delineated is not obvious, but clearly contains all paradigmatically logical constants such as the usual connectives ("not," "and," …) and fits Frege's description of logical truths as fully general truths. The actual specification of this class is still lacking, but suffice it to note that Frege's suggestion, at least, is not doomed to failure: Both obstacles that stand in the way of its development can be overcome.

2.3. Non-logical constants as indexicals. Let us take stock. Frege's universalism lends itself to being implemented in a much more supple way than his foes make it out to do.

[29] Blanchette (2005, pp. 41–42).

[30] Ricketts (2005, p. 151) and Blanchette (2014, pp. 115–116).

[31] Antonelli & May (2005, pp. 177–181).

[32] Carnap (2002, §50, pp. 177–178). See Bonnay (2009) as a defense of Carnap's syntactic option, to the effect that the technical difficulties faced by Carnap's characterization are not insuperable. This reassessment of Carnap's criterion applies *mutatis mutandis* to its transposition to the present case.

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To get back to Hintikka's diagnosis, the universality of the language does not preclude one at all from speaking "of how its semantical relations to the world could be changed:" Only, instead of changing the references of the words, one interchanges the meanings of some words. The universality of the language of logic does not block at all "a systematic variation of the interpretation of a language:" Only, propositions are always interpreted ones and reinterpreting the language is not understood according to a referentialist conception any more, but as an internal operation. The universality of the language of logic does not make at all "model theory impossible:" Only, a different kind of model theory is proposed, that has been upstaged by Tarskian model theory even before it could be recognized and developed. Actually, Frege's suggestion can be compared to a sketchy variant, reminiscent of Bolzano's conception, of the "interpretational semantics" that John Etchemendy ascribes to Tarski.[33]

Extracting an alternative model theory from Frege's suggestions given in his 1906 "Foundations" would deepen the way in which many commentators reconsidered Frege's conception of logic as countenancing a metatheoretical perspective.[34] In what follows, I precisely would like to explore how much of Frege's proposal can be formally extended as an original approach towards "models" (in a non-Tarskian sense of the word, obviously), and show that Frege's conception of logic harbors a genuine notion of *context of discourse* which challenges the notion of "universe of discourse" stemming from the Skolem-Tarski tradition.

Any account of contextuality cannot be better tested than through the account of non-logical constants that it carries: Non-logical constants are the terms that are not held fixed under context shifting. A model theory of some sort, an implementation of contextuality within logic and an account of non-logical constants go together, and I claim that such a triptych can actually be extracted from Frege's 1906 essay. This last point runs afoul of the claim, put forward by Wilfrid Hodges and William Demopoulos, that a Fregean language is unable to make room for contextuality and, as a consequence, to account for non-logical constants. Let us see first what Hodges and Demopoulos say about Frege.

Frege and Peano conceived of a rigourous language as a logically regimented fragment of natural language. As a consequence, a sentence of a Frege-Peano language is interpreted from the outset and thus is in itself true or false. On the contrary, modern formal first-order languages have to be interpreted. The interpretation of the logical part of a formal language is built into the semantical rules themselves. But the workings of the non-logical part are different. Hodges[35] suggests that non-logical constants of a formal language can be seen as indexicals within the space of structures for the language. For example, the symbol '$\cdot$' of the group operation, in the language L of group theory, is interpreted, in the *context* of each L-structure, by a specific operation in that structure: In any group G, "the symbol '$\cdot$' automatically refers to the operation labelled '$\cdot$' in the group G." So non-logical constants can be given a meaning by being applied to a particular structure, exactly as the word

[33] Etchemendy (1990), Chapters 3 and 4. Among the obstacles that Frege sees as standing in the way, the "persistence" requirement that the class of logically true sentences should persist through expansions of the language is not mentioned. This is certainly due to the fact that Frege's suggestion originally applies to contexts, such as projective geometry, whose expressive resources are not the point.

[34] In the absence of any genuine alternative Fregean semantical metatheory, Frege is doomed to appear to be the poor relation of metatheory as developed by Tarski: See Tappenden (2005, p. 191).

[35] See Hodges (1986, pp. 148–150).

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"yesterday" can be given a meaning by being used in a particular temporal context. The conceptual payoff of this suggestion is that "there is nothing *sui generis* about truth in a structure." Truth in a structure is just truth, being granted the semantical resources of contexts for the interpretation of non-logical constants as indexicals.

According to Hodges, a Frege-Peano language has logical constants, variables, and a set of natural language expressions endowed with their intended meanings. Non-logical constants are not recognized as such and their status oscillates between constants and variables. Demopoulos' diagnosis is that Frege's failure to account for the non-logical constants of a syntactically rigorous language can be traced back to his failure to account for indexicals in natural language:

> One of the features of natural language that simply has no analogue in a Frege-Peano language is the presence of indexicals. Non-logical constants, being like indexical expressions in the way they function, would for this reason alone make it difficult to express Huntington's axiomatization within a Frege-Peano language, and Hodges's account therefore rather naturally suggests itself: Frege rejects the notion of a non-logical constant because his basic tool for the analysis of concepts—his 'concept-script'—ignores the category of expression to which they are most similar. Rejecting non-logical constants, Frege *must* reject Hilbert-style axiomatizations [. . .]. (Demopoulos, 1994, p. 216)

Furthermore, Frege's blindness to indexicals would itself stem from his blindness to contextuality:

> The notion of sense appropriate for the expressions of a Frege-Peano language is one that determines the reference of expressions independently of a structured context, whether this consists of locations in space-time, or of structures for a formalized fragment of the language. [. . .] for Frege the sense of a designating expression determines its reference 'absolutely', i.e., independently of such contextual considerations as are relevant in the case of non-logical constants. (Demopoulos, 1994, p. 218)

I now would like to challenge such assessment, and to show, by going in formal detail, that the Fregean notion of vocabulary is rich enough to substantiate a genuine semantics of contextuality, and thereby a genuine account of non-logical constants.

§3. Frege's model theory.

3.1. Analysis of Frege's suggestion. Each vocabulary, in Frege's sense, performs a context change, the replacement of certain semantic values with others: The meaning of a word is assigned as the new meaning of some other word. This understanding of contextuality presupposes the framework of an interpreted language.

In that framework, each context variation induces a systematic reinterpretation of language. Indeed, an interpreted language is not a mere collection of words, but a system of meanings and references, so that any change about one of them is always a simultaneous change of several interconnected meanings and references. This is what complex names embody, were they object-names or function-names. Names in general and referentiality are an essential issue throughout Frege's *Grundgesetze*, of which a constant task is to formulate new names in order to name new contents. The introduction of complex names is ruled by the conditions stated at §29–31, which ensure the referentiality of each newly

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constructed name. To give but an example, in the definition of the application operator given at §34,[36] the definiens represents a combination of signs that can be considered as an autonomous syntactic unit, which thus can be substituted as a whole to any object name. The structure of the correct formation of a complex name is what the definition shows, and the introduction of a new simple sign by the definition does not nullify but on the contrary presupposes it. Each context change has to reflect these dependencies that follow from the way complex names are formed.

So any technical apparatus adequate to represent contexts and context change so construed has to possess the following features. Firstly, each context should appear as one item within some space of all possible (partial) reinterpretations of the language, among which the identity (which does not reinterpret anything). Secondly, to each context should correspond a certain structured collection of semantic values: a certain system of interconnected references of different kinds.[37] Finally, each context change should be represented as a uniform variation respecting the connections involved in the context on which the context change operates. The relationships between any two contexts should be mirrored by the relationships existing between the respective references of complex names as they are interpreted in both contexts.

A general technical framework naturally suggests itself: that of "fibrations." A *fibration* over a category B can be described as the assignment, to each object b of B, of a category E_b, in such a way that any morphism $u : b \to b'$ in B gives rise to a functor $u^* : E_{b'} \to E_b$, called a "reindexing functor."[38] The category B is called the "base category" and, for each object b of B, the category $E_b := p^{-1}(b)$ is called the "fiber" above b, which generalizes the set-theoretic notion of pre-image. The fundamental extra-ingredient carried by a fibration, in comparison with pre-images, is the fact that the base category B is endowed with some structure (as represented by the morphisms of B), and that the existence of a fibration, owing to the correspondence $u \mapsto u^*$, requires a systematic connection between the relations between any two objects in the base, and the relations between the corresponding fibers.

The picture extracted from Frege's 1906 suggestion naturally fits the definition of a fibration. Each Fregean vocabulary is defined as a *variant* of the regular use of (certain) names, and this essential variational feature should be made explicit through the introduction of some base variation space: This is the horizontal dimension of the picture. But, on top of each vocabulary, stands the complete referential system induced by the reinterpretation of certain terms: This is the vertical dimension of the picture, and it is correlated to the horizontal one, since each horizontal variation is matched by a correlative variation of what stands above each vocabulary. Hence a fundamentally two-dimensional picture, which fibrations precisely help to articulate. Indeed, each Fregean vocabulary is a partial translation shifting the current system of all semantic values: This can be rephrased by taking Fregean contexts to be objects of some base category; the referential system based on a context to be the corresponding fiber above that context; and context change to be a functorial transformation between such fibers. In that view, Fregean vocabularies are structured transformations (reindexing functors) between indexed structured systems (fibers). This remains to be formalized.

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[36] Frege (2013, p. 53).

[37] About the different kinds of arguments and of functions, see Frege (2013), §23.

[38] This definition is actually the (sketchy) definition of what is called an "indexed category." For a detailed definition of a fibration (or "fibered category"), and of the relationship between fibrations and indexed categories, see Jacobs (1999, pp. 19–27 and 50–51).

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3.2. Formalization. Let L be a fixed language, a sublanguage of what is taken to be *the* language L_0 (for instance, L is the Frege-Peano version of the language of group theory) and σ be the signature of L (in the case of group theory, $\sigma = \{e, \cdot, (-)^{-1}\}$).

A possible objection here is that the very modern idea of signature of a formal language is foreign to Frege, because, as emphasized by Blanchette (see above), the syntax of a formal language can never reveal by itself which is the *real* signature of a mathematical theory. For instance, the real signature of arithmetic is actually empty, in Frege's view. So one is brought back to Frege's first objection, to the effect that absolute logical status and logical connections cannot be read off from any syntactic representation in a formal system. This objection, already addressed, simply urges one to distinguish between what Frege would have been willing to do, and what one could do in order to pursue Frege's suggestion. What follows does not contend to reflect the former, but only the latter. Once logical constants have been marked out, thanks precisely to the tool of vocabularies (along the lines of Definitions 2.1 and 2.2 above), nothing bars one from considering a specific nonempty signature and from applying to it a Fregean semantic apparatus.

A *Fregean σ-structure* S consists of a set Ω of expressions such that $\sigma \subseteq \Omega$, together with a vocabulary $f : \Omega \to \Omega_0$, where Ω_0 is the set of all expressions of the language L_0. Each such σ-structure $S = \langle \Omega, f \rangle$ can be understood as a context (in the broad sense) for the use of all the elements of σ. The map f maps any expression e in Ω to the expression $f(e)$ that e "translates" in Ω. In other words, e means w.r.t. S (i.e., as reinterpreted according to f) what $f(e)$ means *normally* (i.e., in L_0). In case $f(c) = c$, S is said to adopt the intended interpretation of 'c'. So Fregean structures are not interpretations of a formal language, but internal deformations of "the" language along the part of the language indicated by the signature at stake.

For instance, let us suppose that one shifts from Euclidean plane geometry to projective plane geometry. The signature σ of the language then contains in particular the words 'plane', 'line', and 'incident'. The vocabulary f_p that is part of the σ-structure $S_p = \langle \Omega_p, f_p \rangle$ representing projective geometry is specified by: f_p('plane') = 'surface of a sphere', f_p('line') = 'great circle' and f_p('incident') = 'incident'. When one uses the word 'plane' in the context of S_p, one actually means the surface of a sphere. On the other hand, the intended (Euclidean) interpretation of geometrical terms corresponds to the trivial σ-structure $S_0 = \langle \Omega_0, f_0 \rangle$ given by $f_0 = \mathrm{id}$. Then the thought expressed in S_0 by 'Any two lines in the plane are incident' is false, whereas the same sentence, as reinterpreted in S_p, expresses a thought that comes out true, namely the thought *Any two great circles on the sphere are incident*.

A *morphism* $g : S = \langle \Omega, f : \Omega \to \Omega_0 \rangle \to S' = \langle \Omega', f' : \Omega' \to \Omega_0 \rangle$ *of Fregean σ-structures* is simply a vocabulary $g : \Omega' \to \Omega$ such that $f \circ g = f'$. Let $\mathcal{F}$ be the category of Fregean σ-structures thus defined. A σ-structure S is *σ-congruent with* a σ-structure S', written $S \equiv_\sigma S'$, iff there is a morphism $S \to S'$ of σ-structures such that $\sigma \subseteq \mathrm{Fix}(g)$—which means that S and S' reinterpret all the expressions of σ in the same way. The relation of σ-congruence (or, at least, its symmetrical closure) is an equivalence relation. A *context* (in the narrow sense) is a σ-congruence class of σ-structures, for some implicit signature σ: It is a complete class of σ-structures sharing the same reinterpretation of all the members of σ. All σ-structures of the same context as S can be regarded as alternative possible worlds in that context.

It is natural to consider, above each σ-structure S, the system R_S mentioning the reference of all simple and complex proper names, as well as the value-ranges of all function-names (of all levels), as these names are reinterpreted by S, together with all

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the dependencies and connections between them. Note that R_S thus defined encodes the value-range of the (equivalent of) Tarskian satisfaction relation. This system R_S can easily be given the structure of a category: Its objects are all referential pairs $\langle n, r\rangle$, where either 'n' is a proper name and r its reference, or 'n' is a function-name and r its value-range. The morphisms of R_S are determined by the following rule: There is a morphism $\langle n, r\rangle \to \langle n', r'\rangle$ if 'n' forms part of 'n'' or coincides with it.[39]

Any morphism $g : S = \langle \Omega, f : \Omega \to \Omega_0\rangle \to S' = \langle \Omega', f' : \Omega' \to \Omega_0\rangle$ in $\mathcal{F}$ induces a natural mapping $g^* : R_{S'} \to R_S$: Given a complex name $n(x_i)$ whose constituents are the x_i's,

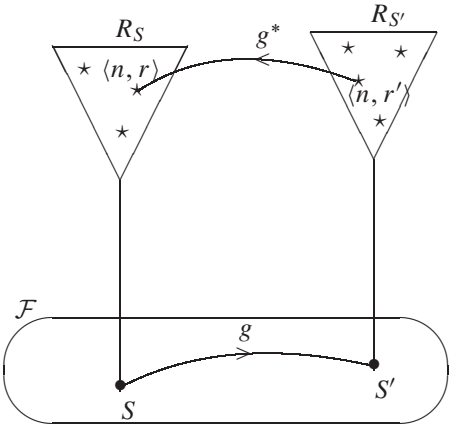
$$g^*(\langle n(x_i), r'\rangle) = \langle n(x_i), r\rangle,$$

wherein r' is the reference of n w.r.t. S', namely the normal reference of $n(f'(x_i)) = n((fg)(x_i))$, and r is the reference of n w.r.t. S, namely the normal reference of $n(f(x_i))$. For instance, $f_p = \mathrm{id}_{\Omega_0} \circ f_p$, so f_p is a morphism from S_0 to S_p and

$$(f_p)^*(\langle \text{'plane'}, \text{sphere}\rangle) = \langle \text{'plane'}, \text{plane}\rangle.$$

In case the two structures S and S' are σ-congruent, each non-logical constant c in the signature σ is preserved by g^*.

Since each morphism g between σ-structures induces a functor g^* between the hierarchies based on those σ-structures, the correspondence $S \mapsto R_S$ is functorial. And since any functor from C to the category of (small) categories defines a fibration over C,[40] one gets a fibration over $\mathcal{F}$—let us call it *Frege's fibration*. Vocabulary shift (in Frege's sense) is thus interpreted as a reindexing functor g^* in Frege's fibration:



Frege's fibration

3.3. Comparison with Tarski. By way of comparison, let us see how things work in Tarski's semantics. The natural category attached to the language L (of signature σ) is the category $\mathcal{L}$ whose objects are all L-structures, and whose morphisms $h : M \to M'$ are all

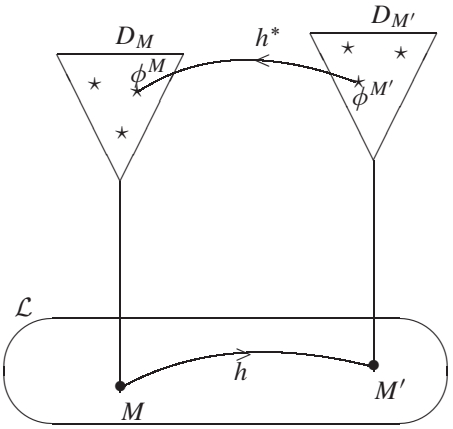
[39] See Frege (2013, §26, pp. 43–44).

[40] Such a functor actually constitutes a "presheaf of categories" over C, which is a special (and simpler) type of fibration.

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L-elementary embeddings $M \prec M'$, in other words all maps $h : |M| \to |M'|$ such that $M \vDash \phi[\vec{a}]$ if $M' \vDash \phi[h(\vec{a})]$ for all L-formulae ϕ and all tuples $\vec{a}$ of M.

It is quite natural to consider, above each L-structure M, the hierarchy D_M of all definable subsets of (a Cartesian product of) M, i.e., all the subsets of $|M|^n$ (for some $n \geq 1$) equal to $\phi^M = \{(a_1, \ldots, a_n) : M \vDash \phi[a_1, \ldots a_n]\}$ for some L-formula ϕ with n free variables. Each individual constant c is represented in M by the definable subset $(x = c)^M = \{c^M\}$. This hierarchy can naturally be turned into a category:[41] The objects of D_M are all definable subsets of $|M|$ and its morphisms are all definable maps between them (i.e., all maps whose graph is a definable subset of $|M|$). Any elementary embedding $h : M \to M'$ induces a mapping $h^* : \phi^{M'} \mapsto h^{-1}(\phi^{M'})$ from $D_{M'}$ to D_M. So again, as in Frege's case, the correspondence $M \mapsto D_M$ allows one to define a fibration over $\mathcal{L}$, *Tarski's fibration*:



Tarski's fibration

An important point is that, in Tarski's semantics, *each* L-structure is a context of its own: Each L-structure provides one with an appropriate fixed interpretation of all the non-logical constants of L. So, in fact, each structure M is the unique possible world that can be considered in its own context. As soon as another L-structure M' is considered, the reference of each non-logical constant changes, so that M' cannot be said to share the *same* context as M. Otherwise put, in two-dimensional parlance, only "diagonal" or "primary intentions" can be represented. Actually, from Tarski's perspective, it does not make much sense anyway to consider different L-structures in the same context. This is not what Tarskian L-structures are meant for. The whole point, in Tarski's semantics, is to consider all L-structures whatsoever (or all models of some L-theory), independently of any contextual specification, so as to delineate the logically valid sentences (or the logical consequences of a theory).

Tarski's semantics should not be confused with Tarski's fibration. Tarski's semantics corresponds to the limit case of Tarski's fibration where no constraint rules the change of the current structure. In Tarski's semantics, indeed, when going across all the possible interpretations of L, one is entirely free to shift from an L-structure M to another

[41] See MacLane & Moerdijk (1992, pp. 542–543).

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L-structure M', regardless of the connections between M and M'. Any L-structure is accessible from any other L-structure, whereas in Tarski's fibration, specific accessibility paths (namely, elementary chains) are prescribed in the base category and, accordingly, specific connections between fibers are distinguished. For each formal language L, contexts correspond to bundles of elementary chains and express the natural way of structuring the universe of all L-structures. This is exactly what Frege's fibration puts to the fore as well: Each selection of a particular set of primitives (each particular signature σ) induces a collection of contexts, each of which corresponds to an admissible path across structures.

This leads one back to the issue of non-logical constants: The apparatus of fibrations comes in useful to account for them. A *section* of Frege's fibration is the choice, above *each* σ-structure S of the base category $\mathcal{F}$, of a certain item $s(S)$ in the corresponding fiber R_S, in such a way that, for any two σ-structures S and S', $s(S) = g^*(s(S'))$ for some morphism $g : S \to S'$ of Fregean σ-structures. So the choice made in each fiber is constrained insofar as all the choices thus made should comply with the conditions indicated by the morphisms of the base. The definition of a section of Tarski's fibration is exactly analogous, with the constraints indicated by elementary embeddings instead. So a section of Frege's fibration consists of a collection of items (referential pairs), one for each σ-structure, so that any two σ-congruent σ-structures share the same item as soon as this item interprets an expression of σ. Similarly, a section of Tarski's fibration consists of a collection of items (definable subsets), one for each L-structure, so that any two L-structures with a morphism between them have matching items as soon as these items interpret the same expression of σ. But this is exactly Hodges' description of a non-logical constant. Thus, in Frege's fibration as well as in Tarski's, a non-logical constant becomes nothing but a *section* of the fibration—or rather, a family of sections (a section per maximal chain of morphisms in the base category).

Of course, this view of non-logical constants as sections is almost tautological, since the preservation of σ is precisely built into the morphisms of Frege's fibration and into the morphisms of Tarskis's fibration as well.[42] Still, it allows to picture and highlight non-logical constants as being indexicals. Indeed, the *role* or "character" of a demonstrative, as defined by Kaplan, says John Perry, is "a rule taking us from an occasion of utterance to a certain object."[43] In the case of a non-logical constant, the rule at stake corresponds to the *constraints* set upon the available or admissible choices to build up a particular section of the fibration. This stands in sharp contrast with the assignments of values to a variable in Tarski's semantics: The choices of the respective values assigned to the same variable in two different structures are completely independent. In the same way, the respective interpretations c^M and $c^{M'}$ of the same constant, or the respective interpretations φ^M and $\varphi^{M'}$ of the same formula in different structures can be considered independently of each other: Nothing in Tarski's semantics prescribes special connections for their consideration. On the contrary, a non-logical constant, in both Frege's and Tarski's *fibrations*, consists of successive choices (along the morphisms of the base category) that have to make up a section over some distinguished path.

Two conclusions can be drawn from the formal reconstruction of Frege's suggestion and from its comparison with Tarski's semantics. Firstly, understanding non-logical constants as indexicals *is not* beyond Frege's logical horizon. Secondly, however, a non-logical

[42] In the case of Tarski's fibration, replacing L-elementary embeddings with L-homomorphisms would be sufficient to ensure the preservation of the non-logical constants of L.

[43] Perry (1977, p. 479).

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constant, understood as a section of a certain fibration, does not work in the same way at all in Frege and in Tarski. A change of context, in Frege, is not a change of place, but *an internal reorganization of the language*. From the viewpoint of axiomatic method, axioms of geometry do not express propositions (they can be true or false only in particular structures) and thus cannot be evaluated individually (but only as parts of axiomatic systems). Remarkably, Frege's rejection of the first point is based on his full embrace of the second.

By comparison, each non-logical constant of a formal language, in Tarskian semantics (as opposed to Tarski's fibration), can be reinterpreted independently of the rest of the language (as long as the language only has to be interpreted, and no axiomatic system is to be satisfied). Accordingly, Tarski's semantics conflates contexts with domains and merges both notions into the mixed notion of "universe of discourse." This conflation is exactly the target of Frege's criticism of the invocation of "circumstances" of truth:

> A proposition that holds only under certain circumstances is not a real proposition. However, we can express the circumstances under which it holds in antecedent propositions and add them as such to the proposition. So supplemented, the proposition will no longer hold only under certain circumstances but will hold quite generally. [. . .] If we suppose that a proposition can hold under certain circumstances but not under others, then we allow ourselves to be led by the nose by self-induced inexactitudes of expression. (Frege, 1971, p. 98)

To Frege, the confusion of pseudo-propositions (formal sentences) with real (interpreted) propositions, and the confusion of contexts with "circumstances,"[44] are two facets of the same fundamental confusion. On the contrary, Frege's vocabularies make it clear that a change of context is irreducible to a change of domain and to a change of circumstances as well. To put it another way: In the context of consistency and independence proofs, Hilbert put forward the general concept of *reinterpretation* of a formal theory. Tarski's semantics actually constitutes an implementation of Hilbert's concept of reinterpretation, but only a particular one:[45] Nothing precludes the possibility of another one. And the Fregean road opened by the notion of vocabulary is precisely another one, that the framework of fibrations is geared to bringing out and to developing.

§4. Conclusion. In his first *Logical Investigation*, Frege seems to be bogged down by the contextuality elicited by demonstratives such as 'I'. Demopoulos' diagnosis, following Hodges', is that contextuality is, as such, downright foreign to Frege's conception and practice of logic. I will not delve here into the detailed discussion of the First *Logical Investigation* that Dummett, Evans and Perry, among others, have pursued. The emphasis laid on demonstratives and the sensitivity to contextuality evinced by Frege's first *Logical Investigation* are, I think, to be traced back to Frege's reaction to Hilbert's axiomatic method in mathematics. In Frege's text, the problem raised by indexicals is essentially the problem raised by Hilbert's variable interpretation of non-logical constants. On that score, it is true, as suggested by Demopoulos, that the main issue of Frege's discussion of demonstratives pertains to Frege's understanding of non-logical constants.

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[44] On this, see (Ricketts, 2005, p. 145).

[45] This is underlined in particular by Antonelli & May (2005, p. 169): "On our view, the necessary step for obtaining metatheoretical results—soundness, completeness, independence, and so on—is externalism. The open question is whether we also need to make the narrower assumption embedded in a Tarskian metatheory. [. . .] Frege's answer is that it is not."

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However, Frege's logical universalism has prompted many commentators to decide beforehand that Frege could not make sense of contextuality. This is exactly the hasty conclusion that I have opposed in this paper: Frege's logical universalism has to be reconsidered, to begin with. Indeed, logical universality is perfectly compatible with contextuality, not only in principle, but also explicitly in Frege's work itself. Believing that logical universality excludes contextuality amounts to confusing a context variation with a domain variation. But a context variation is not a domain variation, unless the principle of Tarskian semantics is presupposed. On that score, it is not true that, as Demopoulos and Hodges contended, Frege was doomed to reject non-logical constants. And it is not true that, contrary to Tarski, Frege could not make any sense of the idea of a context. It is the other way around: Frege's idea, at the end of his 1906 "Foundations of Geometry," makes it possible to analyze contextuality in a more satisfying way than Tarski's semantics does.

In the last part of this paper, using the modern tool of fibrations, I have shown how variable contexts can be built up from within a Frege-Peano interpreted language. The framework of fibrations is a very natural setting to present both Frege's vocabularies and Tarski's models in a unified way and leads one to understand non-logical constants as sections of a fibration. This understanding of non-logical constants allows one to implement Frege's suggestion, to separate Tarski's semantics from Tarski's fibration and to compare Fregean sections with Tarskian ones, so as to get a better grasp of the difference between both perspectives.

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