13

Neutrinos

Neutrinos are elusive particles: for many years their very existence was only inferred from the part that they play in β-decay. However, we have seen in Chapter 10 and Chapter 11 that they are of great importance in astrophysics, and in the forging of the nuclei of the heavy elements in supernovae. Apart from β-decay, other experimental results on neutrinos are accumulating. In this chapter we describe some of these results, and their possible interpretation.

13.1 Neutrino cross-sections

To design neutrino detectors, for example to measure the flux of neutrinos striking the Earth, it is important to know their interaction cross-sections with atomic nuclei and electrons. Unless the neutrino energy is so high that its de Broglie wavelength $\lambda = (2\pi\hbar/p)$ is comparable with or less than the nuclear radius, the nuclear cross-sections for processes which convert a neutrino to its charged lepton partner will involve matrix elements of the same form as those which appear in the theory of β-decay. For example the total cross-section for the reaction

$$\bar{\nu}_e + p \rightarrow n + e^+,$$

with unpolarised protons, at a neutrino energy above the threshold energy for the reaction, is (Problem 12.4)

$$\sigma = \frac{G_w^2}{\pi(\hbar c)^4}[1 + 3g_A^2](cp_e)E_e. \tag{13.1}$$

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In this expression p_e and E_e are the electron momentum and electron energy in the centre-of-mass system. (Note that the energy of the electron is determined by the energy of the neutrino.)

More generally, the total cross-section for the reaction

$$\nu_e + A \rightarrow B + e^-,$$

when the nuclear matrix elements correspond to an allowed transition, and the neutrino energy is above threshold, is given by

$$\sigma = \frac{G_w^2}{\pi(\hbar c)^4}\left[|M_F|^2 + |M_{GT}^x|^2 + |M_{GT}^y|^2 + |M_{GT}^z|^2\right](cp_e)E_e F(Z, E_e), \tag{13.2}$$

where Z is the atomic number of the final nuclear state, and $F(Z, E)$ is the Coulomb correction factor introduced in equation (12.6).

The same formula holds for the anti-neutrino reaction,

$$\bar{\nu}_e + B \rightarrow A + e^+.$$

The reactions described above involve the exchange of a virtual $W^{\pm}$ boson. By the exchange of a virtual neutral Z boson, a neutrino can scatter elastically from a nucleus. However for neutrino detection the important elastic scattering is that from electrons (see §10.5). The processes involved in electron neutrino scattering from electrons are represented diagrammatically in Fig. 13.1. At low energies the W boson contribution to the scattering comes from matrix elements of the form

$$H_{f0} = G_F \int \phi_{ef}^*(\mathbf{r})\phi_{eo}(\mathbf{r})\phi_{\nu f}^*(\mathbf{r})\phi_{\nu o}(\mathbf{r})d^3\mathbf{r}. \tag{13.3}$$

Reference to spin has been suppressed. The form echoes the Fermi interaction in β-decay, but since only leptons are involved it is the Fermi constant G_F which appears. The contribution from the interaction with the Z boson involves a new parameter of the Standard Model: the Weinberg angle θ_W. For neutrino energies $E_\nu \gg m_e c^2$ (but less than M_W) in the centre-of-mass system, the total cross-section σ_ν for scattering from unpolarised electrons is calculated to be

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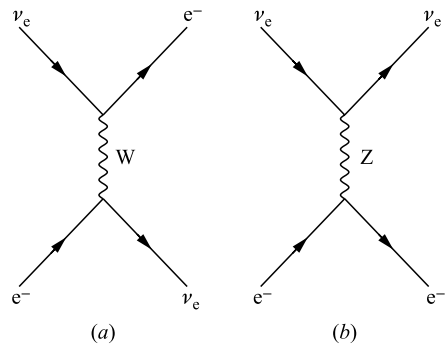


Fig. 13.1 Processes contributing to the elastic scattering of electron neutrinos by electrons: (a) exchange of a virtual charged W boson; (b) exchange of a neutral Z boson.

$$\sigma_\nu = \frac{G_F^2}{3\pi(\hbar c)^4}[3 + 12\sin^2\theta_W + 16\sin^4\theta_W](cp_e)E_e. \tag{13.4a}$$

The corresponding cross-section for anti-neutrinos is

$$\sigma_{\bar{\nu}} = \frac{G_F^2}{3\pi(\hbar c)^4}[1 + 4\sin^2\theta_W + 16\sin^4\theta_W](cp_e)E_e. \tag{13.4b}$$

In these expressions p_e and E_e are the electron momentum and energy in the centre-of-mass system. The accepted value of $\sin^2\theta_W$ from experiment is

$$\sin^2\theta_W = 0.231\,24(24).$$

13.2 The mass of the electron neutrino

In standard unified theories the neutrino masses are assumed to be zero. It is clearly important to test this assumption experimentally. A finite neutrino mass of even a few eV/c^2 would have significant consequences in, for example, cosmology. The signature of a finite neutrino mass would in β-decay appear in the shape of the electron energy spectrum near maximum energy. From expression (12.5), this shape depends sensitively on whether $m_\nu = 0$ or $m_\nu \neq 0$. The difference is clearer in a *Kurie plot* of

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$$\left[\frac{\mathrm{d}R/\mathrm{d}E_\mathrm{e}}{F(Z_\mathrm{d}, E_\mathrm{e})E_\mathrm{e}(E_\mathrm{e}^2 - m_\mathrm{e}^2c^4)^{\frac{1}{2}}}\right]^{\frac{1}{2}}$$

against electron energy since from equation (12.5)

$$\left[\frac{\mathrm{d}R/\mathrm{d}E_\mathrm{e}}{F(Z_\mathrm{d}, E_\mathrm{e})E_\mathrm{e}(E_\mathrm{e}^2 - m_\mathrm{e}^2c^4)^{\frac{1}{2}}}\right]^{\frac{1}{2}}$$
$$= (\text{constant})(E_0 - E_\mathrm{e})^{\frac{1}{2}}[(E_0 - E_\mathrm{e})^2 - m_\nu^2c^4]^{\frac{1}{4}}.$$

If $m_\nu = 0$, this plot gives a straight line $(E_0 - E_\mathrm{e})$ passing through E_0; if $m_\nu \neq 0$ the line is curved and the tangent at maximum energy is vertical.

A much-studied decay in this context is that of tritium

$$^3_1\mathrm{H} \rightarrow {}^3_2\mathrm{He} + \mathrm{e}^- + \bar{\nu} + 18.6\ \mathrm{keV}.$$

The low electron kinetic energies in this decay are experimentally advantageous. Figure 13.2 shows experimental data and there is remarkable overall agreement between the data and the fitted theoretical spectrum. A Kurie plot of data near $E_\mathrm{e} = E_0$ is also shown. The difficulty of the experiment is evident: the conclusion is that $m_\nu < 60\ \mathrm{eV}/c^2$. More recent tritium experiments (Belesev, A. I. *et al.* (1995), *Phys. Lett. B* **350**, 263) suggest that

$$m_\nu < 4.35\ \mathrm{eV}$$

with high probability.

We conclude there is not yet direct evidence that the electron neutrino has mass, and there is no direct evidence of mass for the muon neutrino or the tau neutrino. However, there is a growing body of experimental results which suggest that neutrinos do have mass, but on a much smaller scale than that probed by direct experiments.

In §2.4 and §3.6 we introduced all of the elementary fermions: the leptons and the quarks. The three charged leptons and the six charged quarks all have mass. Are the neutrinos the exception?

13.3 Neutrino mixing and neutrino oscillations

Let us assume there are three basic neutrino types, which we denote by $|\nu_1\rangle$, $|\nu_2\rangle$, $|\nu_3\rangle$, having definite masses m_1, m_2, m_3 respectively. These states

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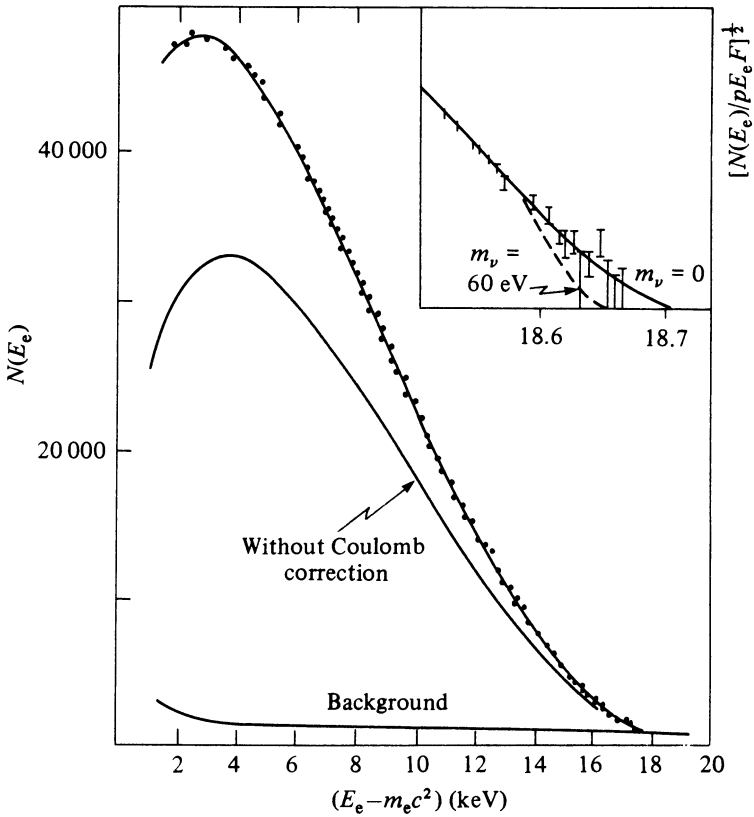


Fig. 13.2 The electron energy spectrum from the decay of tritium. The experimental points give the number of electrons $N(E_e)$ observed in small energy 'bins' from a very large number of decays. (Taken from Lewis, V. E. (1970), *Nucl. Phys.* **A151**, 120.) The spectrum is well fitted using equations (12.5) and (12.6). Also shown for comparison is the curve without the Coulomb correction. The inset shows a Kurie plot of the spectrum near the electron end point. (For this data see Bergkuist, K. E. (1972), *Nucl. Phys.* **B39**, 317.) The theoretical curves in the inset include the effect of the finite size of the energy 'bins'.

are eigenstates of the mass operator, and may be taken to be orthogonal and normalised. The mass operator acts on internal degrees of freedom of the neutrino, and the mass eigenstates may be represented by 3×1 column matrices. The formalism is somewhat similar to that of intrinsic electron spin, in which 'spin-up' and 'spin-down' states are eigenstates of the operator s_z.

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Mixing means that the electron neutrino state $|\nu_e\rangle$ produced in a β-decay is not a basic neutrino, but a linear combination of the three mass eigenstates. Similarly the muon neutrino and the tau neutrino are linear combinations of the mass eigenstates, orthogonal to each other and to the electron neutrino.

For mathematical simplicity we shall take a two-component model and suppose that $|\nu_e\rangle$ is well represented by

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle. \tag{13.5}$$

Orthogonal to $|\nu_e\rangle$ is the state

$$|\nu_x\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle, \tag{13.6}$$

as the reader may readily verify. $|\nu_x\rangle$ might be a muon neutrino or a tau neutrino. θ is known as the *mixing angle*.

If the electron neutrino was created at $t = 0$ with momentum p in the plane wave state $e^{ipz/\hbar}$, then at time t the state will have evolved to

$$|\nu_e\rangle_t = e^{-iE_1 t/\hbar}\cos\theta|\nu_1\rangle + e^{-iE_2 t/\hbar}\sin\theta|\nu_2\rangle, \tag{13.7}$$

where $E_1 = \sqrt{p^2c^2 + m_1^2c^4}$, $E_2 = \sqrt{p^2c^2 + m_2^2c^4}$.

We can solve equations (13.5) and (13.6) for $|\nu_1\rangle$ and $|\nu_2\rangle$ in terms of $|\nu_e\rangle$ and $|\nu_x\rangle$:

$$|\nu_1\rangle = \cos\theta|\nu_e\rangle - \sin\theta|\nu_x\rangle,$$
$$|\nu_2\rangle = \sin\theta|\nu_e\rangle + \cos\theta|\nu_x\rangle,$$

and then substitute in (13.7) to obtain

$$|\nu_e\rangle_t = e^{-iE_1 t}[(\cos^2\theta + e^{i(E_1-E_2)t/\hbar}\sin^2\theta)|\nu_e\rangle - \sin\theta\cos\theta(1 - e^{i(E_1-E_2)t})|\nu_x\rangle].$$

If after time t the neutrino is detected, the probability of it being an electron neutrino is, by the usual rules of quantum mechanics,

$$\begin{aligned} P_e(t) &= |(\cos^2\theta + e^{i(E_1-E_2)t/\hbar}\sin^2\theta)|^2 \\ &= 1 - \sin^2 2\theta \sin^2\{(E_2 - E_1)t/2\hbar\}, \end{aligned} \tag{13.8}$$

and the probability of it being an x neutrino is

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$$\begin{aligned} P_x(t) &= \sin^2\theta\cos^2\theta|(1-\mathrm{e}^{\mathrm{i}(E_1-E_2)t/\hbar})|^2 \\ &= \sin^2 2\theta\sin^2\{(E_2-E_1)t/2\hbar\}. \end{aligned} \tag{13.9}$$

Unless $m_1 = m_2$, so that $E_1 = E_2$, it is evident that $P_\mathrm{e} < 1$ if there is mixing.

A neutrino may be detected and identified by its conversion into its charged leptonic partner (§10.5). In the case of neutrinos produced from nuclear β-decay, or from thermonuclear reactions in the Sun, the neutrino energies are too small to produce a muon or tau lepton. Thus the x neutrino will not be detected. If there is neutrino mixing, it will be seen as a reduction by a factor P_e in the anticipated flux of electron neutrinos. Note that electron lepton number is no longer conserved (see §2.5): we are straying outside the Standard Model of particle physics.

To analyse P_e further, we assume that the neutrinos are highly relativistic. We can then take the first term in the Taylor expansion and write

$$E_2 - E_1 = \sqrt{p^2c^2 + m_2^2c^4} - \sqrt{p^2c^2 + m_1^2c^4} \approx \frac{(\Delta m^2)c^4}{2pc}, \tag{13.10}$$

where $\Delta m^2 = m_2^2 - m_1^2$.

A relativistic neutrino wave packet will travel with velocity $\approx c$ so that, at a distance z from the source,

$$P_\mathrm{e}(z) = P_\mathrm{e}(t = z/c) = 1 - \sin^2 2\theta\sin^2\{(E_2-E_1)z/2\hbar c\}. \tag{13.11}$$

Using (13.10) this becomes

$$P_\mathrm{e}(z) = 1 - \sin^2 2\theta\sin^2(\pi z/L), \tag{13.12}$$

where the *oscillation length* L is given by

$$L = \frac{4\pi(pc)(\hbar c)}{(\Delta m^2)c^4}, \tag{13.13}$$

which can be written as

$$L = 2.48\left(\frac{pc}{1\ \mathrm{MeV}}\right)\left(\frac{1\ \mathrm{eV}^2}{(\Delta m^2)c^4}\right)\mathrm{m}. \tag{13.14}$$

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Table 13.1. *The ratio of measured neutrino flux to neutrino flux predicted by the standard solar model, in five independent experiments*

Experiment	Type of detector	Result/Theory
Homestake (USA)	^{37}Cl	0.33 ± 0.029
Kamiokande (Japan)	H_2O	0.54 ± 0.07
GALLEX (Italy)	^{71}Ga	0.60 ± 0.06
SAGE (Russia)	^{71}Ga	0.52 ± 0.06
SuperKamiokande (Japan)	H_2O	0.474 ± 0.020

(See Bahcall, J. N. *et al.* (1998), *Phys, Rev. D* **58**, 096016.)

It is the sinusoidal nature of P_e as a function of z which gives rise to the name *neutrino oscillations* for this phenomenon.

13.4 Solar neutrinos

Table 13.1 shows the results of five independent measurements of the ratio of the solar neutrino detection rate, to the expected rate calculated from the standard solar model and the Standard Model of particle physics. Within the context of neutrino oscillations, this can be taken as a measure of P_e. P_e is consistently less than one. Also, the measurements are sensitive to different neutrino energy bands (§10.5), and P_e appears to be energy dependent.

Can these results be explained by neutrino oscillations? Within our two-neutrino model, if $(\Delta m^2)c^4 \sim 1\ \text{eV}^2$ we see from equation (13.14) that the oscillation length L is then very much smaller than the size of the Sun's thermonuclear core, where solar neutrinos are created. (For solar neutrinos, $pc \leq 10$ Mev.) Averaging over the core,

$$P_e = 1 - \tfrac{1}{2}\sin^2 2\theta.$$

This is independent of neutrino energy, and the maximum possible suppression is $P_e = \frac{1}{2}$. Thus $(\Delta m^2)c^4 \sim 1\ \text{eV}^2$ would appear to be inconsistent with the experimental data.

An alternative relevant length scale is the mean Earth–Sun distance of $\approx 1.496 \times 10^{11}\text{m} = 1$ astronomical unit. We can rewrite (13.12) as

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$$P_e(z) = 1 - \sin^2 2\theta \sin^2\left[1.9\left(\frac{z}{1\text{ AU}}\right)\left(\frac{1\text{ MeV}}{pc}\right)\left(\frac{(\Delta m^2)c^4}{10^{-11}\text{ eV}^2}\right)\right] \tag{13.15}$$

and $0 < pc \leq 10$ MeV, $z = 1$ AU (ignoring the diurnal and annual variations of z). Unless $(\Delta m^2)c^4 \sim 10^{-11}$ eV^2, the range of neutrino momentum values will lead to an averaging of the sine-squared term, and again we shall have a factor of $\frac{1}{2}$. However, a detailed analysis suggests that the experimental data can be fitted by this model if $(\Delta m^2)c^4 \sim 6.5 \times 10^{-11}$ eV^2, and $\sin^2 2\theta \sim 0.75$.

Another interesting possible mechanism for explaining the solar neutrino deficit arises from the interaction of neutrinos with matter. As neutrinos pass through matter they acquire an effective mass. An electron neutrino can interact with electrons in matter through the exchange of a virtual W boson, as in Fig. 13.1(*a*). At low energies this gives the Fermi interaction matrix elements (13.3). These can be interpreted as arising from a neutrino effective mass $G_F n_e/c^2$, where n_e is the number density operator for electrons. Taking spin into account gives a factor $\sqrt{2}$. The effective mass from the Fermi interaction is thus given by

$$m_{\nu e}c^2 = \sqrt{2} n_e G_F = 1.27(1\text{ Å}^3 n_e) \times 10^{-13}\text{ eV}. \tag{13.16}$$

The Gamow–Teller interaction contributes only in ferromagnetic materials. This effective mass term is unique to the electron neutrino, because matter has zero muon density and zero tau density. The neutrino interaction with electrons through the neutral Z boson (Fig. 13.1(*b*)) contributes to the effective mass for all neutrino types, but does not induce mass differences. It is therefore of little interest in the context of neutrino oscillations.

These effective masses are calculated in the Standard Model of particle physics. They are small, both in terrestrial materials and in the Sun, as equation (13.16) indicates. However, if we postulate the existence of intrinsic neutrino masses and neutrino mixing, new resonance phenomena appear. In passing through matter of varying electron number density, as happens to a neutrino created in the core of the Sun (Fig. 10.1), the matter modification to the mass of the electron neutrino can cause a large neutrino oscillation, even though the vacuum mixing angle is very small. As the neutrino leaves the Sun, the oscillation is then effectively frozen, since further oscillations have a small amplitude. It is found that,

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if these effects are to account for the solar neutrino deficit, $(\Delta m^2)c^4 \sim 10^{-5}$ eV^2, $\sin^2 2\theta \sim 10^{-2}$.

13.5 Atmospheric neutrinos

A quite different scale on which to search for neutrino oscillations is given by *atmospheric neutrinos*. The Earth is continually bombarded by cosmic rays, which consist for the most part of high-energy protons and electrons. The protons, in their collision with nuclei in the upper atmosphere, produce π-mesons. The π-mesons while still in the upper atmosphere decay by the chains

$$\begin{aligned} \pi^+ &\to \mu^+ + \nu_\mu \\ &\quad\ \big\lfloor\!\to \mathrm{e}^+ + \nu_\mathrm{e} + \bar{\nu}_\mu \end{aligned}$$

$$\begin{aligned} \pi^- &\to \mu^- + \bar{\nu}_\mu \\ &\quad\ \big\lfloor\!\to \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu \end{aligned}$$

The neutrinos and antineutrinos are produced at a mean height $H \sim 20$ km, with energies extending to the multi-GeV region.

From the expressions in §13.1 the cross-sections for neutrino and anti-neutrino scattering increase with energy, so that the detection of these uncharged leptons becomes easier at higher energy. In water detectors such as SuperKamiokande charged leptons are produced through reactions essentially of the form

$$\begin{aligned} &\nu_e + \mathrm{n} \to \mathrm{e}^- + \mathrm{p}, \qquad \bar{\nu}_\mathrm{e} + \mathrm{p} \to \mathrm{e}^+ + \mathrm{n}; \\ &\nu_\mu + \mathrm{n} \to \mu^- + \mathrm{p}, \quad \bar{\nu}_\mu + \mathrm{p} \to \mu^+ + \mathrm{n} \end{aligned}$$

which take place within 22.5 kilotonnes of water.

The charged leptons give Cerenkov radiation which provides information on the energy, direction, and identity of the incident uncharged lepton. At high energy, the direction of the charged lepton is closely correlated with that of the incident neutrino or anti-neutrino. Electrons and muons may be distinguished by characteristics of their Cerenkov signals. (Electrons are scattered more than muons in their passage through the water. See §14.2.)

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Since neutrinos traverse the Earth almost unimpeded, the angle θ_z of a neutrino's direction with the local vertical at the detector (the zenith angle) determines its distance z from its point of production in the upper atmosphere. To a good approximation z is given (using elementary geometry) by

$$z = \left\{R_\oplus^2 \cos^2\theta_z + 2R_\oplus H + H^2\right\}^{1/2} - R_\oplus \cos\theta_z,$$

where $R_\oplus = 6380$ km is the Earth's mean radius, and $H \sim 20$ km; z varies between $z = H$ when $\theta_z = 0$, and $z = 2R_\oplus + H$ when $\theta_z = \pi$.

In Fig. 13.3, the ratio of observed to expected events is plotted as a function of $\cos\theta_z$ for electron-like events and for muon-like events. Data from SuperKamiokande and Kamiokande has been combined.

In the electron data there is no sign of an oscillation. This is consistent with both of the scenarios we have sketched for solar neutrinos. If either $(\Delta m^2)c^4 \sim 10^{-11}$ eV^2 or $(\Delta m^2)c^4 \sim 10^{-5}$ eV^2, no oscillation would be expected over a distance of $\sim R_\oplus$.

The muon data is quite different. There is a clear suppression for $\cos\theta_z < -0.2$, which suggests an oscillation length for the muon neutrino comparable to $R_\oplus$. In a two-neutrino model we can write

$$P_\mu = 1 - \sin^2 2\theta_\mu \sin^2\left[8.1\left(\frac{z}{R_\oplus}\right)\left(\frac{1\text{ GeV}}{pc}\right)\left(\frac{(\Delta m_\mu^2)c^4}{10^{-3}\text{ eV}^2}\right)\right].$$

Δm_μ^2 might be the difference in the squared masses of the tau neutrino and the muon neutrino. There is no sign in the data of the muon neutrino oscillating into an electron neutrino. Detailed fits to the data suggest $(\Delta m_\mu^2)c^4 \sim 10^{-3}$ eV^2, and a large value for $\sin^2 2\theta_\mu$.

In neutrino physics, we see that nuclear physics, astrophysics, particle physics, and indeed cosmology, come together, and present challenging experimental and theoretical problems.

Problems

13.1 Consider allowed β-decays which have a large energy release E_0 (e.g. the decay of ^{8}B, §10.4). In such decays, the effects of Coulomb corrections and finite lepton masses are small. Show that, neglecting these effects,

(*a*) the mean life depends on E_0 as E_0^{-5},

(*b*) the mean electron energy is $E_0/2$.

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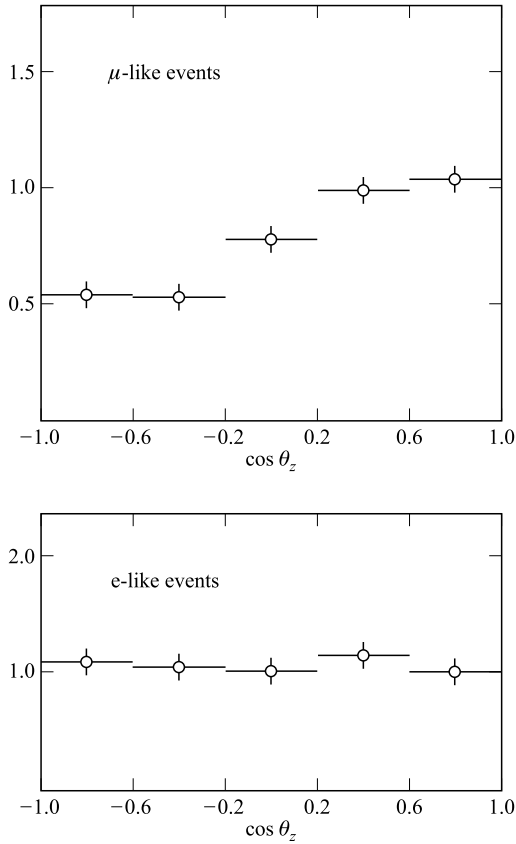


Fig. 13.3 The ratio of observed to expected events plotted as a function of $\cos\theta_z$, for muon-like and electron-like events. (See Harrison, P. F. *et al.* (1999), *Phys. Lett. B* **458**, 79.)

To examine the effect of a finite neutrino mass on the energy spectrum, only decays with energy in a small range $\Delta E_e \sim m_\nu c^2$ at the end-point $E_e \approx E_0$ are significant. Show that the proportion of such decays is very small, of order $10(\Delta E_e/E_0)^3$.

13.2 In the K-capture

$$^{7}_{4}\mathrm{Be(atom)} \rightarrow {}^{7}_{3}\mathrm{Li(atom)} + \nu,$$

with the beryllium source at rest, the recoil energy of the lithium atoms (mass 6536 MeV/c^2) was measured to be (55.9 ± 1.0) eV (Davis, R. (1952), *Phys. Rev.* **86**, 976). The mass difference between the two

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atoms is 0.862 MeV/c^2. Show that this experiment implied the neutrino mass to be less than 160 keV/c^2.

13.3 The cross-sections for neutrino interactions are typically of order

$$\sigma \sim \frac{G_F^2}{(\hbar c)^4} E_\nu^2 \sim \left(\frac{E_\nu}{1\ \text{MeV}}\right)^2 10^{-17}\ \text{fm}^2,$$

where E_ν is the neutrino energy in the centre-of-mass frame.

(*a*) In the core of a supernova, a neutrino will scatter mostly from neutrons. Show that, in a core of radius 10 km and nucleon number 2.4×10^{57}, the mean free path of a neutrino is

$$l \sim \left(\frac{1\ \text{MeV}}{E_\nu}\right)^2 175\ \text{m}.$$

(*b*) Consider scatterings in which a neutrino stays a neutrino (as will usually be the case for muon and tau neutrinos). Show that the time taken for a neutrino to diffuse from the centre to the surface of this core is of order

$$\left(\frac{E_\nu}{1\ \text{MeV}}\right)^2 2 \times 10^{-3}\ \text{s}.$$

(Assume the neutrino path is a random walk, so that $(R/l)^2$ steps are needed to diffuse a distance R.)

13.4 For neutrinos in thermal equilibrium, the power emitted per unit area at a surface is given approximately by a formula similar to the Stefan–Boltzmann law for photons:

$$\text{power per unit area} = a_\nu T^4,$$

where $a_\nu = 3 \times (7/8) \times a_{\text{photon}}$ and $a_{\text{photon}} = \dfrac{\pi^2}{60} \dfrac{k_B^4}{c^2 \hbar^3}$.
(The factor 3 comes from the three neutrino types, and the factor (7/8) from the Fermi–Dirac statistics.) Show that the core of a star with radius 10 km and surface temperature T given by

$$T^4 = T_0^4 e^{-t/\tau},$$

where $k_B T_0 = 4.2$ MeV, $\tau = 4.6$ s, will radiate a total of 3×10^{58} MeV by neutrino emission.

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