PARTICLES OF HALF-ODD INTEGRAL SPIN

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Introduction

In three papers (1, 2, 3) Bhabha has set up a new theory for relativistic particles of any spin, and has suggested that a particular wave equation of half-odd spin, classified as $(\frac{3}{2}, \frac{1}{2})$, may be applicable to the proton. It is therefore of interest to study this equation and to calculate results which could perhaps be compared with experiment; since in the non-relativistic limit the particle obeys Dirac's equation, it is sufficient to consider the behaviour at energies large compared to the rest mass. Here the important problem is the calculation of scattering cross-sections for cosmic-ray processes.

As a first step it is necessary to quantize Bhabha's theory for the case of half-odd spin and, since in the non-relativistic limit the particle conforms to the exclusion principle, the quantization is in terms of 'plus' brackets. We meet at once an apparent inconsistency, for (eqn. (30)) it is necessary to give negative values to some of the plus brackets. This is just the situation which arises when we try to quantize theories of spin 0 or 1 in accordance with the exclusion principle, and is the basis of the statement (4) that particles of integral spin cannot have Fermi-Dirac statistics. In Bhabha's theory the negative signs occur because the eigenvalues of the matrix $D\alpha^0$, which defines the charge density, are of either sign. However, as pointed out by Bhabha at the 1946 Cambridge conference, it is possible to introduce an indefinite metric in the Hilbert space of the quantized system, and the formalism of Pauli (5) is available. Although the formalism has not the detailed physical interpretation possible with a positive-definite metric, the method of procedure for the solution of particular problems is clear enough. Nevertheless, there remains a blemish on an otherwise elegant theory.

A recent paper by Madhavarao and others (6) materially simplifies the algebra of the α matrices, and the calculation of the scattering of mesons by Bhabha's protons becomes feasible. This problem is discussed in § 6 and the treatment applies equally to protons described by Dirac's equation, which corresponds to the one-rowed representation of the operators ξ, introduced in (6).

The necessary formalism is set up in §§ 1–5, where we discuss the general case of half-odd integral spin. In § 2 we give the generalization of the result that in Dirac's theory the eigenvalues of the velocity of an electron are $\pm c$.

1. The free-particle eigenstates

Bhabha's equation is
$$(\alpha^\mu p_\mu - \chi)\,\psi(x) = 0, \tag{1}$$
and the conjugate complex is
$$\psi^\dagger D(\alpha^\mu p_\mu - \chi) = 0. \tag{2}$$

Here $p_\mu = i(\partial/\partial x^\mu)$ and the suffix μ runs from 0 to 3, i from 1 to 3. We take
$$g^{00} = -g^{11} = -g^{22} = -g^{33} = 1, \quad \hbar = c = 1, \quad x = (x^0 \dots x^3).$$

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The operators α^μ obey the commutation rules

$$(\alpha^\mu, \alpha^\nu) = I^{\mu\nu}, \quad (\alpha^\mu, I^{\nu\sigma}) = g^{\mu\nu}\alpha^\sigma - g^{\mu\sigma}\alpha^\nu, \tag{3}$$

from which it follows (1) that ψ satisfies the equation

$$\left\{\Box + \left(\frac{\chi}{n}\right)^2\right\}\left\{\Box + \left(\frac{\chi}{n-1}\right)^2\right\} \dots \left\{\Box + \left(\frac{\chi}{\frac{1}{2}}\right)^2\right\}\psi = 0, \tag{4}$$

so that the momentum eigenstates of equation (1) are of type

$$\psi(x) = \psi(r, j, \mathbf{k})\, e^{-ik_\mu x^\mu}, \tag{5}$$

with

$$k_\mu k^\mu = (k^0)^2 - \mathbf{k}^2 = \left(\frac{\chi}{j}\right)^2 \quad (\mathbf{k} = k^1, k^2, k^3), \tag{6}$$

and j has one of the values

$$\pm n, \quad \pm(n-1), \dots, \quad \pm\tfrac{1}{2}. \tag{7}$$

n is half-odd integral and defines the maximum spin of the particle.

The ambiguity of sign in equation (6) may be resolved by giving j the sign of k^0; then, for every state equation (5), j is a well-defined Lorentz invariant and the mass of the particle is χ/j.

$\psi(r, j, \mathbf{k})$ is independent of x and satisfies the equation

$$(\alpha^\mu k_\mu - \chi)\,\psi(r, j, \mathbf{k}) = 0. \tag{8}$$

Likewise

$$\psi^\dagger(r, j, \mathbf{k})\, D(\alpha^\mu k_\mu - \chi) = 0. \tag{9}$$

Hence

$$\psi^\dagger(r, j, \mathbf{k})\, D\alpha^0(k_0 - k_0')\,\psi(r', j', \mathbf{k}) = 0,$$

so that

$$\psi^\dagger(r, j, \mathbf{k})\, D\alpha_0(r', j', \mathbf{k}) = 0 \quad \text{if} \quad j \neq j'. \tag{10}$$

Now pass by a succession of infinitesimal Lorentz transformations to the frame in which the particle is at rest.

$$k_\mu \to \left(\frac{\chi}{j}, 0, 0, 0\right), \quad \mathbf{k} \to \mathbf{0}, \quad \psi(r, j, \mathbf{k}) \to \psi(r, j, \mathbf{0}),$$

and (8) becomes

$$(\alpha^0 \chi/j - \chi)\,\psi(r, j, \mathbf{0}) = 0.$$

The $\psi(r, j, \mathbf{0})$ form a complete set of orthogonal eigenvectors belonging to the eigenvalue j of α^0; since D is a function of α^0 they must satisfy

$$\psi^\dagger(r, j, \mathbf{0})\, D\alpha^0 \psi(r', j, \mathbf{0}) = 0 \quad \text{unless} \quad r = r'. \tag{11}$$

r is an eigenvalue which specifies the spin state; for example, $\psi(r, j, \mathbf{0})$ may be an eigenstate of I_{12} and of $I_{12}^2 + I_{23}^2 + I_{31}^2$. If we are working with a representation of order z of the matrices α^μ, then just z combinations of the parameters r, j are required. Now $\psi^\dagger(r, j, \mathbf{0})\, D\alpha^i \psi(r', j, \mathbf{0})$ vanishes since α^i has only matrix elements connecting values of j differing by ± 1, so that, consistently with the assumption that $\psi^\dagger$ is the conjugate complex of ψ, we normalize the ψ to

$$\psi^\dagger(r, j, \mathbf{0})\, D\alpha^\mu \psi(r', j, \mathbf{0}) = D(j)\,\delta_{rr'}(\chi/j, 0, 0, 0),$$

where $D(j)$ is the eigenvalue of D corresponding to the eigenvalue j of α^0. Because the left-hand side is a 4-vector, this implies that

$$\psi^\dagger(r, j, \mathbf{k})\, D\alpha^\mu \psi(r', j, \mathbf{k}) = D(j)\,\delta_{rr'} k^\mu,$$

so that, using also (10), we have

$$(k^0)^{-1}\,\psi^\dagger(r, j, \mathbf{k})\, D\alpha^0 \psi(r', j', \mathbf{k}) = D(j)\,\delta_{rr'}^{jj'} \tag{12}$$

which is the only possible Lorentz invariant normalization of the eigenvectors.

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Equation (12) implies that for a given $\mathbf{k}$ the matrix composed of the z vectors $\psi(r, j, \mathbf{k})$ is of rank z, and an arbitrary vector ψ_ρ of z components ($\rho = 1, \ldots, z$) can therefore be expanded in terms of these basic vectors†. It follows that there exists an inverse relationship

$$\sum^{r,j} \psi_\rho(r, j, \mathbf{k})\, D(j)\, (k^0)^{-1}\, (\psi^\dagger(r, j, \mathbf{k})\, D\alpha_0)_\sigma = \delta_{\rho\sigma}, \tag{13}$$

and also that

$$\sum^{r,j} D(j)\, (k^0)^{-1}\, \psi^\dagger(r, j, \mathbf{k})\, D\alpha^0 \,.\, O\psi(r, j, \mathbf{k}) = \text{Spur } O, \tag{14}$$

where O is any $z \times z$ matrix.

2. The 'velocity'

Dirac (*Quantum Mechanics*, 2nd ed. p. 260) has given a physical argument to show that the eigenvalues of the velocity of a relativistic particle must be $\pm c$; in the case of spin $\frac{1}{2}$ this is easily verified by consideration of the commutation rules for the α^s. In the general case we have to show that the eigenvalues of $(\alpha_0)^{-1}\alpha^i$ are ± 1.

If $k = |\mathbf{k}|$, $\mathbf{i} = k^{-1}\mathbf{k}$, (8) gives

$$\left\{\frac{j'}{|j'|}(k^2 + \chi^2/j'^2)^{\frac{1}{2}} + (\alpha^0)^{-1}(\boldsymbol{\alpha} \,.\, \mathbf{i}k - \chi)\right\} \psi(r', j', \mathbf{k}) = 0.$$

Hence

$$\Pi_j \left\{\frac{j}{|j|}(k^2 + \chi^2/j^2)^{\frac{1}{2}} + (\alpha^0)^{-1}(\boldsymbol{\alpha} \,.\, \mathbf{i}k - \chi)\right\} \psi(r', j', \mathbf{k}) = 0, \tag{15}$$

which follows from the previous result, since all the factors of the product commute. Equation (15) is true for all r', j', and, since the $\psi(r', j', \mathbf{k})$ form a complete set, we have, combining terms belonging to the same value of $|j|$,

$$\Pi_{|j|}\{-(k^2 + \chi^2/j^2) + [(\alpha^0)^{-1}(\boldsymbol{\alpha} \,.\, \mathbf{i}k - \chi)]^2\} = 0.$$

This is true for all k, so that, equating to zero the highest power of k, we have

$$\Pi_{|j|}\{[(\alpha_0)^{-1}\boldsymbol{\alpha} \,.\, \mathbf{i}]^2 - 1\} = 0 \quad \text{or, by (7),} \quad \{[(\alpha_0)^{-1}\boldsymbol{\alpha} \,.\, \mathbf{i}]^2 - 1\}^{n+\frac{1}{2}} = 0, \tag{16}$$

which shows that $(\alpha^0)^{-1}\boldsymbol{\alpha} \,.\, \mathbf{i}$ has eigenvalues ± 1. Note that the various factors of equation (16) do not vanish separately; nevertheless, (16) is of great value for the manipulation of the α^s.

3. The momentum representation

The general solution of (1) may be expanded as

$$\Psi(x) = (2\pi)^{-\frac{3}{2}} \sum^{r,j} \int \frac{d^3k}{k^0}\, U(r, j, \mathbf{k})\, \psi(r, j, \mathbf{k})\, e^{-ik_\mu x^\mu}, \tag{17}$$

and the general solution of (2) as

$$\Psi^*(x) = (2\pi)^{-\frac{3}{2}} \sum^{r,j} \int \frac{d^3k}{k^0}\, U^*(r, j, \mathbf{k})\, \psi^\dagger(r, j, \mathbf{k})\, e^{ik_\mu x^\mu}. \tag{18}$$

The $U(r, j, \mathbf{k})$ are Lorentz invariant, that is, if $j\mathbf{k} \to j\mathbf{k}'$ then

$$U'(r, j, \mathbf{k}') = U(r, j, \mathbf{k}).$$

† This is not quite trivial, since, according to equation (8), the $\psi(r, j, \mathbf{k})$ are eigenvectors of $(\alpha^0)^{-1}(\alpha^i k_i - \chi)$, which is non-Hermitian.

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Bhabha derives the charge current vector

$$s^\mu = e\Psi^* D\alpha^\mu \Psi,$$

and the energy momentum tensor

$$t^0_\mu = \Psi^* D\alpha^0 i \frac{\partial}{\partial x^\mu} \Psi.$$

In the momentum representation the total charge, energy and momentum are given by

$$\begin{aligned} Q &= \frac{e}{(2\pi)^3} \sum^{r,j} \sum^{r',j'} \iint \frac{d^3k}{k^0} \frac{d^3k'}{k^{0'}} U^*(r', j', \mathbf{k}') \\ &\qquad \times \psi^\dagger(r', j', \mathbf{k}') D\alpha^0 \psi(r, j, \mathbf{k})\, U(r, j, \mathbf{k}) \exp\left[-i(k_\mu - k'_\mu)\, x^\mu\right] \\ &= e \sum^{r,j} \int \frac{d^3k}{k^0} D(j)\, U^*(r, j, \mathbf{k})\, U(r, j, \mathbf{k}) \end{aligned} \tag{19}$$

by (12), and similarly

$$(E, \mathbf{G}) = T^{0\mu} = \sum^{r,j} \int \frac{d^3k}{k^0} D(j)\, U^*(r, j, \mathbf{k})\, U(r, j, \mathbf{k})\, k^\mu. \tag{20}$$

4. Commutation rules

We quantize the theory by regarding $\Psi_\rho(x)$, $\Psi^*_\rho(x)$, not as conjugate-complex variables, but as adjoint operators; their behaviour under Lorentz transformations is unchanged. Take as Hamiltonian for the total system

$$H = -\int \Psi^*(x)\, D(\alpha^i p_i - \chi)\, \Psi(x)\, d^3x. \tag{21}$$

The rule $i(\partial\xi/\partial x^0) = (\xi, H)$ leads to the correct equations of motion for Ψ and Ψ^* if we adopt the following commutation rules, valid for $x^0 = x^{0'}$, and chosen so as to lead to Fermi statistics:

$$\left.\begin{aligned} &[\Psi_\rho(x), (\Psi^*(x')\, D\alpha^0)_\sigma] = \delta_{\rho\sigma}\delta(\mathbf{x}-\mathbf{x}'), \\ &[\Psi_\rho(x), (\Psi_\sigma(x')] = [\Psi^*_\rho(x), \Psi^*_\sigma(x')] = 0. \end{aligned}\right\} \tag{22}$$

Here we use the notation

$$\left.\begin{aligned} &[P, Q] = PQ + QP \\ &(P, Q) = PQ - QP. \end{aligned}\right\} \tag{23}$$

while

In the U space the corresponding commutation rules are

$$[U(r, j, \mathbf{k}), U^*(r', j', \mathbf{k}')] = k_0 D(j)\, \delta^{jj'}_{rr'}\, \delta(\mathbf{k}-\mathbf{k}'), \tag{24}$$

for these imply

$$\begin{aligned} (2\pi)^{-3} \Big[&\sum_{r,j} \int \frac{d^3k}{k_0} U(r, j, \mathbf{k})\, \psi_\rho(r, j, \mathbf{k})\, e^{-ik_\mu x^\mu}, \sum_{r',j'} \int \frac{d^3k}{k'_0} \\ &\qquad \times U^*(r', j', \mathbf{k}')\, (\psi^\dagger(r, j', \mathbf{k}')\, D\alpha^0)_\sigma \exp[ik'_\mu x^{\mu'}] \Big] \\ &= (2\pi)^{-3} \sum_{r,j} \int \frac{d^3k}{k_0} D(j)\, \psi_\rho(r, j, \mathbf{k})\, (\psi^\dagger(r, j, \mathbf{k})\, D\alpha_0)_\sigma \exp\left[ik_\mu(x'-x)^\mu\right] \\ &= \delta_{\rho\sigma}\delta(\mathbf{x}-\mathbf{x}') \end{aligned}$$

by (13), and this is just equivalent to (22) above.

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The commutation rules are best discussed by approximating to the continuous range of $\mathbf{k}$ states by a discrete set. Divide the $\mathbf{k}$ space into small regions $\tau_a, \tau_b, \ldots, \tau_r, \ldots$ drawn about the points $\mathbf{k}_a, \mathbf{k}_b, \ldots, \mathbf{k}_r, \ldots$, and define

$$V(r, j, \mathbf{k}_a) = \sqrt{\frac{|k_a^0|}{\tau_a}} \int^{\tau_a} U(r, j, \mathbf{k}) \frac{d^3k}{k^0} \tag{25}$$

and V^* likewise.

The commutation rule (equation (24)) becomes

$$[V(r, j, \mathbf{k}_a), V^*(r', j', \mathbf{k}_b)] = D(j)\, \delta_{ab} \delta^{jj'}_{rr'} \frac{|k_a^0|}{k_a^0} = s(j)\, \delta_{ab} \delta^{jj'}_{rr'}, \tag{26}$$

where we define

$$s(j) = D(j) \frac{|k_a^0|}{k_a^0} = D(j)\{\text{sign of } j\}. \tag{27}$$

In this discrete representation we have also to replace (17) by

$$\Psi(x) = (2\pi)^{-\frac{3}{2}} \sum^{a,r,j} \sqrt{(\tau_a)}\, V(r, j, \mathbf{k}_a) \phi(r, j, \mathbf{k}_a) \exp[-ik_\mu^a x^\mu], \tag{28}$$

with

$$\phi(r, j, \mathbf{k}_a) = |k_a^0|^{-\frac{1}{2}} \psi(r, j, \mathbf{k}_a).$$

Then equations (12), (13) become

$$\left.\begin{aligned} \phi^\dagger(r, j, \mathbf{k}_a)\, D\alpha^0 \phi(r', j', \mathbf{k}_a) &= s(j)\, \delta^{jj'}_{rr'}, \\ \sum^{rj} \phi_\rho(r, j, \mathbf{k}_a)\, s(j)\, (\phi^\dagger(r, j, \mathbf{k}_a)\, D\alpha^0)_\sigma &= \delta_{\rho\sigma}; \end{aligned}\right\} \tag{29}$$

and equation (14) is similarly modified.

It will be convenient to use the symbol $'a$ or sometimes just a for the triad $(r', j', \mathbf{k}_a)$. Equation (26) may then be written as

$$[V_a, V_b^*] = s_a \delta_{ab}, \quad [V_a, V_b] = [V_a^*, V_b^*] = 0. \tag{30}$$

In Dirac's theory of the electron we have $D \equiv \alpha^0$, so that $S(j) \equiv 1$ and V and V^* are interpreted as Hermitian conjugate operators. In Bhabha's theory, for 'spins' n greater than one half, $s(j)$ takes the values ± 1, and this interpretation is inconsistent with (30). It is just this inconsistency which has been used by Pauli (4) to show that fields of integral spin cannot be quantized with plus brackets.

Equation (30) implies that

$$N_a = s_a V_a^* V_a \tag{31}$$

satisfies the characteristic equation

$$N_a^2 = N_a, \tag{32}$$

so that its eigenvalues are 0 and 1. We have also

$$(N_a, N_b) = 0. \tag{33}$$

We proceed to set up a Jordan-Wigner representation of the V_a, V_a^*.

Introduce auxiliary operators v_a, v_a^* which satisfy the commutation rules

$$[v_a, v_a^*] = s_a, \quad v_a v_a = v_a^* v_a^* = 0. \tag{34}$$

But v_a, v_a^* commute with v_b, v_b^* for $a \neq b$. Then

$$n_a = s_a v_a^* v_a \tag{35}$$

satisfies the equations

$$n_a^2 = n_a \quad (1 - 2n_a)^2 = 1, \tag{36}$$

$$(n_a, n_b) = 0, \tag{37}$$

so that its eigenvalue n_a' takes the values 0 and 1 only.

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We adopt a definite but arbitrary order for the $a, b, \ldots, r, \ldots$ and take as basic states the simultaneous eigenstates $\langle n'_a n'_b \ldots n'_r \ldots |$ of the operators n_a, n_b, ... Then the eigenvalue equation $\langle n'_a \ldots n'_r \ldots | n'_r | \rangle = n'_r \langle n'_a \ldots n'_r \ldots | \rangle$, and the commutation rules (34) are satisfied by

$$\left.\begin{aligned} \langle n'_a \ldots n'_r \ldots | v_r | \rangle &= \begin{cases} s_r \langle n'_a \ldots (n'_r+1) \ldots | \rangle & \text{if} \quad n'_r = 0, \\ 0 & \text{if} \quad n'_r = 1, \end{cases} \\ \langle n'_a \ldots n'_r \ldots | v_r^* | \rangle &= \begin{cases} 0 & \text{if} \quad n'_r = 0, \\ \langle n'_a \ldots (n'_r-1) \ldots | \rangle & \text{if} \quad n'_r = 1. \end{cases} \end{aligned}\right\} \tag{38}$$

As usual we write, for any $|\rangle$ vector, $\langle | n'_a \ldots n'_r \ldots \rangle = \overline{\langle n'_a \ldots n'_r \ldots | \rangle}$, and we assume also that

$$\left.\begin{aligned} \langle | v_r | n'_a \ldots n'_r \ldots \rangle &= \begin{cases} 0 & \text{if} \quad n'_r = 0, \\ \langle | n'_a \ldots (n'_r-1) \ldots \rangle & \text{if} \quad n'_r = 1, \end{cases} \\ \langle | U_r^* | n'_a \ldots n'_r \ldots \rangle &= \begin{cases} \langle | n'_a \ldots (n'_r+1) \ldots \rangle s_r & \text{if} \quad n'_r = 0, \\ 0 & \text{if} \quad n'_r = 1. \end{cases} \end{aligned}\right\} \tag{39}$$

With these definitions $\langle | v_r$ and $v_r^* | \rangle$ are conjugate-complex vectors. Similarly for $\langle | v_r^*$ and $v_r | \rangle$.

We now define the scalar product of two states $\langle p |$ and $| q \rangle$ as

$$\langle p | q \rangle = \sum_{n'_a \ldots n'_r \ldots} \langle p | n'_a \ldots n'_r \ldots \rangle \eta_a(n'_a) \ldots \eta_r(n'_r) \ldots \langle n'_a \ldots n'_r \ldots | q \rangle, \tag{40}$$

and the $\eta_a(n'_a)$, defined for $n'_a = 0, 1$, only may be different functions of the n'_a for the different states a.

Consider

$$\langle p | (v_r | q \rangle) = \sum_{n'_a \ldots n'_r \ldots}^{n'_r = 0} \langle p | n'_a \ldots 0 \ldots \rangle \eta_a(n'_a) \ldots \eta_r(0) \ldots s_r \langle n'_a \ldots 1 \ldots | q \rangle$$

and

$$(\langle p | v_r) | q \rangle = \sum_{n'_a \ldots n'_r \ldots}^{n'_r = 1} \langle p | n'_a \ldots 0 \ldots \rangle \eta_a(n'_a) \ldots \eta_r(1) \ldots \langle n'_a \ldots 1 \ldots | q \rangle.$$

For consistency these two expressions must be equal, which requires that

$$\eta_r(0)\, s_r = \eta_r(1), \tag{41}$$

and the same condition ensures that

$$\langle p | (v_r^* | q \rangle) = (\langle p | v_r^*) | q \rangle.$$

s_r has been normalized to take the values ± 1 only, so that, consistently with (41), the same condition may be applied to the $\eta_r(n'_r)$.

It remains to express the V_r, V_r^*, satisfying (30), in terms of the v_r, v_r^*. Because $(1 - 2n_r)$ anticommutes with v_r, v_r^* and commutes with all other v_s, v_s^*, we take

$$\left.\begin{aligned} V_r &= (1-2n_a) \ldots (1-2n_{r-1})\, v_r, \\ V_r^* &= v_r^* (1-2n_{r-1}) \ldots (1-2n_a). \end{aligned}\right\} \tag{42}$$

Because of (36), $$n_r \equiv N_r,$$

so that the $\langle \ldots n'_r \ldots | \rangle$ and $\langle \ldots N'_r \ldots | \rangle$ representations are identical.

The charge, energy and momentum defined by equations (19) and (20) may now be expressed, with the help of (25) and (31), as

$$Q = e \sum_a^a N_a, \quad (E, \mathbf{G}) = \sum_a^a (k^0, \mathbf{k}_a) N_a, \tag{43}$$

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and the operators N_a have eigenvalues 0 and 1. These expressions are seen to be the same whether we consider the familiar case of spin $\frac{1}{2}$ or consider spins greater than $\frac{1}{2}$. But the normalization of the wave function by means of the equation

$$\langle p \,|\, p\rangle = \text{constant}$$

is essentially different in the two cases, since for spins greater than $\frac{1}{2}$ the bilinear form $\langle p \,|\, p\rangle$, defined by (40), is of indefinite sign. As emphasized by Pauli (5), this circumstance leads to difficulties with the physical interpretation of the formalism. Nevertheless, the theory is self-consistent, and the Lagrangian formulation ensures that, in interaction with other fields, the usual conservation laws are satisfied; it should therefore be capable of some physical interpretation.

We shall assume that the result of a measurement of an observable, defined by a self-adjoint linear operator O, is one of its eigenvalues; and that the expectation value of O for the state $|\, p\rangle$ is

$$\langle p \,|\, O \,|\, p\rangle / \langle p \,|\, p\rangle, \tag{44}$$

although, in general, for some states $|\, p\rangle$ this rule leads to negative probabilities of obtaining particular eigenvalues of O. The procedure is necessarily tentative; however, it is possible to make some progress with the calculation of transition probabilities and this is the most pressing problem.

5. Perturbation theory

We now suppose the particles defined by equation (21) to interact with some field defined by dynamical variables β_λ say, which in the absence of interaction are constants of the motion. The V_a, V_a^*, β_λ will now vary with time and must be specified as $V_a(x_0)$, etc. We also specialize the discrete representation by supposing all fields to be confined in a cube of volume Ω, so that we have in (28)

$$(2\pi)^{-\frac{3}{2}} \tau_a^{\frac{1}{2}} = \Omega^{-\frac{1}{2}}.$$

The interaction Hamiltonian is necessarily self-adjoint and, in the cases of interest, is of the form

$$H^{\text{int.}} = \sum^{\lambda} \int^{\Omega} d^3x \, \Psi^*(x) \, W^\lambda \Psi(x) \, \beta_\lambda \exp\left[-ik_\mu^\lambda x^\mu\right] \tag{45}$$

$$= \sum^{\lambda, l, m} V_l^* H_{lm}^\lambda V_m \, \beta_\lambda \exp\left[i(k_0^l - k_0^m - k_0^\lambda)\, x^0\right]. \tag{46}$$

W^λ is a Hermitian matrix of order z defined as a function of the α^μ and k_μ^λ, and does not involve x so that, by (28), we have

$$H_{lm}^\lambda = \phi_{(l)}^\dagger W^\lambda \phi(m), \tag{47}$$

with a condition

$$\mathbf{k}^l = \mathbf{k}^m + \mathbf{k}^\lambda. \tag{48}$$

The equations of motion for the V, V^* are

$$i \frac{\partial V(l)}{\partial x^0} = (V, H^{\text{int.}}) = \sum^{\lambda, m} s_l H_{lm}^\lambda V_m \beta_\lambda \exp\left[i(k_0^l - k_0^m - k_0^\lambda)\, x^0\right],$$

and since $H^{\text{int.}}$ is self-adjoint

$$-i \frac{\partial V^*(l)}{\partial x_0} = \sum^{\lambda, m} s_l \beta_\lambda^* V_m^* (H_{lm}^\lambda)^\dagger \exp\left[i(k_0^m - k_0^l + k_0^\lambda)\, x^0\right].$$

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The state of the complete system is represented by a fixed vector $|\,\rangle$ which we suppose to be an eigenstate of the $N_l(0)$ and of the variables $\beta^*_\lambda(0)\,\beta_\lambda(0)$. Let the corresponding eigenvalues be

$$N'_a(0) = 1, \quad N'_l(0) = 0 \quad \text{for} \quad a \neq l, \quad \text{and} \quad (\beta^*_\lambda(0)\,\beta_\lambda(0))'.$$

Then, with the usual approximations,

$$V_b(x_0)\,|\,\rangle = -\sum^{\lambda} s_b H^\lambda_{ba} \beta_\lambda(0) \frac{\exp[i(k^b_0 - k^a_0 - k^\lambda_0)\,x^0] - 1}{k^b_0 - k^a_0 - k^\lambda_0} V_a(0)\,|\,\rangle,$$

$$\langle|\, V^*_b(x_0) = \langle|\, \sum^{\lambda} s_b V^*_a(0)\,\beta^*_\lambda(0)(H^\lambda_{ba})^\dagger \frac{\exp[i(k^a_0 - k^b_0 + k^\lambda_0)\,x^0] - 1}{k^a_0 - k^b_0 + k^\lambda_0},$$

so that

$$\langle|\, N_b(x^0)\,|\rangle = 2\sum^{\lambda} s_a s_b (H^\lambda_{ba})^\dagger (H^\lambda_{ba}) \frac{1 - \cos(k^b_0 - k^a_0 - k^\lambda_0)\,x^0}{(k^b_0 - k^a_0 - k^\lambda_0)^2} \langle|\, \beta^*_\lambda(0)\,\beta_\lambda(0)\,N_a(0)\,|\rangle,$$

or

$$\frac{\langle|\, N_b(x^0)\,|\rangle}{\langle|\rangle} = 2\sum^{\lambda} s_a s_b (H^\lambda_{ba})^\dagger (H^\lambda_{ba})\,(\beta^*_\lambda(0)\,\beta_\lambda(0))' \frac{1 - \cos(k^b_0 - k^a_0 - k^\lambda_0)\,x^0}{(k^b_0 - k^a_0 - k^\lambda_0)^2}.$$

The right-hand side is large if $\quad k^b_0 - k^a_0 - k^\lambda_0 = 0. \qquad (49)$

If $\rho\,dk_0$ is the number of final states (of given particle spin and polarization of the β field) in the range k_0 to $k_0 + dk_0$ of $(k^b_0 - k^a_0 - k^\lambda_0)$, then the expectation value at time x^0 for the total number of particles in the neighbourhood of the final state is $\sum^{\lambda} P^\lambda_{ba}$, where

$$P^\lambda_{ba} = 2\pi\,|\,x^0\,|\,s_a s_b (H^\lambda_{ba})^\dagger H^\lambda_{ba} \rho(\beta^*_\lambda(0)\,\beta_\lambda(0))'. \qquad (50)$$

In the usual theory $s_a = s_b = 1$, so that P^λ_{ba} is positive-definite and may be defined as the probability of a transition from state a to state b with a corresponding change in the β field. In the present theory equation (50) is of indefinite sign, and such a detailed interpretation is not possible. However, we shall sum P^λ_{ba} over all the states $'b$ having a given value of $\mathbf{k}^b$ and of the sign of k^b_0 and average over all the states $'a$ having given values of $\mathbf{k}^a$ and of the sign of k^a_0 and, taking into account equation (44), we interpret this quantity Q^λ_{ba} as the probability of transition of a particle from an initial state a of definite momentum and charge (since the sign of the charge may be regarded as specified by that of k_0) to a similarly specified final state b.

We have considered a first-order process. If all H^λ_{ba} connecting states a and b vanish, the compound matrix elements are formed according to the rule

$$\sum^{c} \frac{H^\lambda_{bc}\,s_c\,H^\mu_{ca}}{k^a_0 + k^\mu_0 - k^c_0}. \qquad (51)$$

6. Scattering of mesons by protons

As an example, consider the interaction of charged vector mesons with a proton-neutron which obeys the equation proposed by Bhabha (2) (classified as $\frac{3}{2}, \frac{1}{2}$). Since in the non-relativistic limit the nucleon obeys Dirac's equation, we can assert that the only co-variants, bilinear in the proton-neutron wave functions, which can be com-

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bined with the vector and tensor of the meson field to form interaction terms non-vanishing in the non-relativistic limit, are

$$\left.\begin{aligned} &\text{Scalar:} && \Psi_N^* D \Psi_P && = \omega \text{ say},\\ &\text{Vector:} && \Psi_N^* D\alpha^\mu \Psi_P && = v^\mu \text{ say},\\ &\text{Skew tensor:} && \Psi_N^* DiI^{\mu\nu} \Psi_P && = U^{\mu\nu}. \end{aligned}\right\} \tag{52}$$

Note that D, $D\alpha^\mu$, $DiI^{\mu\nu}$ are Hermitian matrices.

In the notation of Kemmer (7) we may define an interaction Lagrangian

$$L^{\text{int.}} = -g(v^\mu \phi_\mu^* + v^{\mu *}\phi_\mu) - \tfrac{1}{2} f(U^{\mu\nu}\chi_{\mu\nu}^* + U^{\mu\nu *}\chi_{\mu\nu}). \tag{53}$$

This is not the most general interaction possible in Bhabha's theory, but, since the coupling constants have to be determined by a study of the nuclear binding forces, it seems reasonable to exclude interactions which would be appreciable only at high energies. The interesting problem is that of the scattering of mesons by nucleons at energies large compared with the rest energy of a nucleon. For $f \neq 0$ the problem is quite excessively complicated, so that we now specialize further and consider only the g interaction.

Carrying out the Fourier analysis and quantization of the meson field by the usual method, we obtain matrix elements of the type of equation (47),

$$\left.\begin{aligned} &P_a \to N_b + Y^+_{\text{long.}}, && \kappa^{-1}(2\Omega\epsilon)^{-\frac{1}{2}} \phi^\dagger_{(b)} Dg(-\epsilon\boldsymbol{\alpha}.\mathbf{i} + \alpha^0 p)\phi(a),\\ &P_a + Y^-_{\text{long.}} \to N_b, && \kappa^{-1}(2\Omega\epsilon)^{-\frac{1}{2}} \phi^\dagger_{(b)} Dg(\epsilon\boldsymbol{\alpha}.\mathbf{i} - \alpha^0 p)\phi(a),\\ &P_a \to N_b + Y^+_{\text{trans.}}, && \kappa^{-1}(2\Omega\epsilon)^{-\frac{1}{2}} \phi^\dagger_{(b)} Dg(-i\boldsymbol{\alpha}.\mathbf{j})\phi(a),\\ &P_a + Y^-_{\text{trans.}} \to N_b, && \kappa^{-1}(2\Omega\epsilon)^{-\frac{1}{2}} \phi^\dagger_{(b)} Dg(i\boldsymbol{\alpha}.\mathbf{j})\phi(a). \end{aligned}\right\} \tag{54}$$

Here $\mathbf{p}$, ϵ denote the momentum and energy of the meson Y. $\epsilon^2 = \mathbf{p}^2 + \kappa^2$ and κ is the rest mass of the meson.

$\mathbf{p}$ is $\pm$ the wave vector $\mathbf{k}$ for $\pm$ mesons,

$$p = |\mathbf{p}| \quad \text{and} \quad \mathbf{i} = \mathbf{p}/|\mathbf{p}|. \tag{55}$$

Also $\mathbf{j}$ is the unit polarization vector for transverse mesons. Then the selection rules (equations (48), (49)) express the conservation of momentum and energy.

Total charge $+2$. Consider the scattering of a positive meson by a proton:

$$P_a + Y_a^+ \to N_c + Y_b^+ + Y_a^+ \to P_b + Y_b^+.$$

Here $\mathbf{p}_a$ is the momentum of the initial meson, $\mathbf{K}_a = -\mathbf{p}_a$ that of the initial proton; $\mathbf{p}_b$ is the momentum of the final meson, $\mathbf{K}_b = -\mathbf{p}_b$ that of the final proton.

Momentum is conserved so that for the intermediate state

$$\mathbf{K}_c = -(\mathbf{p}_a + \mathbf{p}_b) = \mathbf{K}_a + \mathbf{K}_b. \tag{56}$$

Energy is conserved so, neglecting χ and κ in comparison with $|\mathbf{p}_a|$, $|\mathbf{p}_b|$, we have

$$|\mathbf{p}_a| = |\mathbf{p}_b| = p = \epsilon = |\mathbf{K}_a| = |\mathbf{K}_b|. \tag{57}$$

If k_0^a, k_0^b, K_0^a, K_0^b are the initial and final 'frequencies' of meson and proton, the compound matrix element for scattering from state a to state b is, according to (51),

$$H_{ba} = \sum^c \frac{H_{bc} s_c H_{ca}}{K_0^a - K_0^c - k_0^b}.$$

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But since we neglect χ, κ, we have $K_0^a - k_0^b = 0$, and also $K_0^c = \pm |\mathbf{K}^c|$. Hence

$$H_{ba} = -\sum^c \frac{H_{bc} s_c H_{ca}}{K_0^c} = -|\mathbf{p}_a + \mathbf{p}_b|^{-2} \sum^c H_{bc} s_c K_0^c H_{ca}.$$

We shall sum the transition probability over the possible final meson polarizations and average over the initial polarizations. In the extreme relativistic case we need therefore retain only the matrix elements H_{ba} corresponding to the interaction with longitudinal mesons. Then, by (54) and (57),

$$H_{ba} = -\frac{g^2 p}{2\kappa^2 \Omega} |\mathbf{p}_a + \mathbf{p}_b|^{-2} \sum \phi^\dagger_{(b)} D(-\boldsymbol{\alpha} . \mathbf{i}_a + \alpha^0) \phi(c) s_c K_0^c \phi^\dagger_{(c)} D(-\boldsymbol{\alpha} . \mathbf{i}_b + \alpha^0) \phi(a).$$

Now neglecting χ, we have, according to (8),

$$(K_0^c - (\alpha^0)^{-1} \boldsymbol{\alpha} . \mathbf{K}^c) \phi(c) = 0,$$

so by (56)

$$H_{ba} = +\frac{g^2 p}{2\kappa^2 \Omega} |\mathbf{p}_a + \mathbf{p}_b|^{-2} \sum^c \phi^\dagger_{(b)} D(-\boldsymbol{\alpha} . \mathbf{i}_a + \alpha^0) \times (\alpha^0)^{-1} \boldsymbol{\alpha} . (\mathbf{p}_a + \mathbf{p}_b) \phi(c) s_c \phi^\dagger_{(c)} D(-\boldsymbol{\alpha} . \mathbf{i}_b + \alpha^0) \phi(a).$$

Since c is a state of fixed $\mathbf{K}_c$, we may apply (29) and obtain

$$H_{ba} = +\frac{g^2 p}{2\kappa^2 \Omega} |\mathbf{p}_a + \mathbf{p}_b|^{-2} \phi^\dagger_{(b)} D(-\boldsymbol{\alpha} . \mathbf{i}_a + \alpha^0) (\alpha^0)^{-1} \boldsymbol{\alpha} . (\mathbf{p}_a + \mathbf{p}_b) (\alpha^0)^{-1} (-\boldsymbol{\alpha} . \mathbf{i}_b + \alpha^0) \phi(a),$$

and, since $K_0^a > 0$, by (8), (55) and (57), we have

$$(\alpha^0 + \boldsymbol{\alpha} . \mathbf{i}_a) \phi(a) = 0, \tag{58}$$

$$H_{ba} = +\frac{g^2 p^2}{2\kappa^2 \Omega} |\mathbf{p}_a + \mathbf{p}_b|^{-2} \phi^\dagger_{(b)} D\{\boldsymbol{\alpha} . (\mathbf{i}_a + \mathbf{i}_b) (\alpha^0)^{-1} \boldsymbol{\alpha} . (\mathbf{i}_a + \mathbf{i}_b) (\alpha^0)^{-1} \boldsymbol{\alpha} . (\mathbf{i}_b + \mathbf{i}_a)\} \phi(a).$$

Introduce the unit vector

$$\mathbf{i}_c = \frac{\mathbf{i}_a + \mathbf{i}_b}{|\mathbf{i}_a + \mathbf{i}_b|}. \tag{59}$$

With the notation $\alpha^c = \boldsymbol{\alpha} . \mathbf{i}_c$, etc., we have

$$H_{ba} = +\frac{g^2 |\mathbf{i}_a + \mathbf{i}_b|}{2\kappa^2 \Omega} \phi^\dagger_{(b)} D\alpha^0 \{(\alpha^0)^{-1} \alpha^c\}^3 \phi(a). \tag{60}$$

For the transition probability we need

$$Q_{ab} = \Sigma\Sigma s_b \phi^\dagger_{(b)} D\alpha^0 \{(\alpha^0)^{-1} \alpha^c\}^3 \phi(a)\, s_a \phi^\dagger_{(a)} D\alpha^0 \{(\alpha^0)^{-1} \alpha^c\}^3 \phi(b), \tag{61}$$

in which we sum over the spins $r_b, j_b > 0$ in the final state b and average over the spins $r_a, j_a > 0$ in the initial state a. These sums and averages are confined to positive values of $K_0^{(a)}$, $K_0^{(b)}$, but may be extended over both signs of j by the introduction of annihilation operators. Equation (58) applies to $K_0^{(a)}$ positive; in general we have

$$\frac{\alpha^0 - \boldsymbol{\alpha} . \mathbf{i}_a}{2\alpha^0} \phi(a) = \phi(a) \quad \text{if } a \text{ is a state of positive } K_0^a,$$
$$= 0 \quad \text{if } a \text{ is a state of negative } K_0^a.$$

Then

$$Q_{ab} = \frac{1}{2z} \Sigma\Sigma s_b \phi^\dagger_{(b)} D\alpha^0 \{(\alpha^0)^{-1} \alpha^c\}^3 (1 - (\alpha^0)^{-1} \alpha^a) \times \phi(a) s_a \phi^\dagger_a D\alpha^0 \{(\alpha^0)^{-1} \alpha^c\}^3 (1 - (\alpha^0)^{-1} \alpha^b) \phi(b),$$

in which a factor $2/z$ has been introduced since there are $z/2$ states a with a given value of $\mathbf{K}_a$ and positive K_0^a.

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By (14) and (29),

$$Q_{ab} = \frac{1}{2z}\mathrm{Spur}\{\{(\alpha^0)^{-1}\alpha^c\}^3(1-(\alpha^0)^{-1}\alpha^a)\{(\alpha^0)^{-1}\alpha^c\}^3(1-(\alpha^0)^{-1}\alpha^b)\}. \tag{62}$$

To evaluate equation (62) introduce a unit vector

$$\mathbf{i}_d = (\mathbf{i}_a - \mathbf{i}_b)/|\,\mathbf{i}_a + \mathbf{i}_b\,|,$$

so that
$$\mathbf{i}_a = \lambda\mathbf{i}_c + \mu\mathbf{i}_d \quad \text{and} \quad \mathbf{i}_b = \lambda\mathbf{i}_c - \mu\mathbf{i}_d, \tag{63}$$

with
$$\lambda = \tfrac{1}{2}|\,\mathbf{i}_a + \mathbf{i}_b\,| \quad \text{and} \quad \mu = \tfrac{1}{2}|\,\mathbf{i}_a - \mathbf{i}_b\,|. \tag{64}$$

Then
$$Q_{ab} = \frac{1}{2z}\mathrm{Spur}\{\{(\alpha^0)^{-1}\alpha^c\}^3(1-\lambda(\alpha^0)^{-1}\alpha^c - \mu(\alpha^0)^{-1}\alpha^d) \times\{(\alpha^0)^{-1}\alpha^c\}^3(1-\lambda(\alpha^0)^{-1}\alpha^c+\mu(\alpha^0)^{-1}\alpha^d)\}.$$

We now introduce the factorization of the α algebra due to Madhavarao and others (6), and write
$$\alpha^0 = \eta_0\xi_0, \quad \text{but} \quad \alpha^c = i\eta_c\xi_c \quad \text{and} \quad \alpha^d = i\eta_d\xi_d.$$

The ξ and η obey the commutation rules given by these authors. The factor i is introduced since the α^c, α^d are anti-Hermitian.

Then

$$Q_{ab} = \frac{1}{2z}\mathrm{Spur}\{-i\eta_0\eta_c(\xi_0^{-1}\xi_c)^3(1-i\lambda\eta_0\eta_c\xi_0^{-1}\xi_c - i\mu\eta_0\eta_d\xi_0^{-1}\xi_d)(-i\eta_0\eta_c)(\xi_0^{-1}\xi_c)^3 \times(1-i\lambda\eta_0\eta_c\xi_0^{-1}\xi_c+i\mu\eta_0\eta_d\xi_0^{-1}\xi_d)\}$$

$$= \frac{1}{2z}\mathrm{Spur}\{(\xi_0^{-1}\xi_c)^3(1-i\lambda\eta_0\eta_c\xi_0^{-1}\xi_c+i\mu\eta_0\eta_d\xi_0^{-1}\xi_d)(\xi_0^{-1}\xi_c)^3 \times(1-i\lambda\eta_0\eta_c\xi_0^{-1}\xi_c+i\mu\eta_0\eta_d\xi_0^{-1}\xi_d)\}.$$

Since $\mathrm{Spur}\,\eta_0\eta_c = 0$, but $\mathrm{Spur}\,(\eta_0\eta_c)^2 = -4$, etc.,

$$Q_{ab} = \frac{2}{z}\mathrm{Spur}\{(\xi_0^{-1}\xi_c)^6 + \lambda^2(\xi_0^{-1}\xi_c)^8 + \mu^2(\xi_0^{-1}\xi_c)^3(\xi_0^{-1}\xi_d)(\xi_0^{-1}\xi_c)^3(\xi_0^{-1}\xi_d)\}. \tag{65}$$

We simplify this result by using (16), which implies that

$$\{(\xi_0^{-1}\xi_c)^2 - 1\}^2 = 0, \tag{66}$$

which can also be proved directly from the commutation rules of the ξ_μ. It follows that the eigenvalues of $(\xi_0^{-1}\xi_c)^2$ are all $+1$, so that (65) becomes

$$Q_{ab} = \frac{2}{z}\{(1+\lambda^2)\,\mathrm{Spur}\,1 + \mu^2\,\mathrm{Spur}\,[(\xi_0^{-1}\xi_c)^3\xi_0^{-1}\xi_d(\xi_0^{-1}\xi_c)^3\xi_0^{-1}\xi_d]\}. \tag{67}$$

If we use an m-rowed representation of the ξ_μ, since $z = 4m$,

$$Q_{ab} = \frac{1}{2}\left\{(1+\lambda^2) + \frac{\mu^2}{m}\mathrm{Spur}\,[A]\right\}. \tag{68}$$

After considerable calculation we find

$$\mathrm{Spur}\,A = \tfrac{16}{9}\mathrm{Spur}\{9 + \tfrac{9}{16} + 12\xi_d - 36\xi_{cd} - 48\xi_{cd0}\},$$

and using the table of Spurs given in (6), we have

Representation	1 row	4 row	5 row	
$\frac{1}{m}\mathrm{Spur}\,A$	1	$6\frac{1}{3}$	$43\frac{2}{3}$	(69)

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Then, if we introduce θ, the angle between the initial and final momenta of the meson, according to equation (64),

$$\lambda^2 = \tfrac{1}{2}(1+\cos\theta) \quad \text{and} \quad \mu^2 = \tfrac{1}{2}(1-\cos\theta),$$

and so (68) becomes

$$\left.\begin{aligned} &m = 1 \quad \text{(Dirac's equation)} \quad Q_{ab} = 1, \\ &m = 4 \qquad\qquad\qquad\quad Q_{ab} = \tfrac{7}{3}(1-\tfrac{4}{7}\cos\theta), \\ &m = 5 \qquad\qquad\qquad\quad Q_{ab} = \tfrac{35}{3}(1-\tfrac{32}{35}\cos\theta). \end{aligned}\right\} \tag{70}$$

It is satisfactory that, in all cases, Q_{ab} is positive.

Introducing $\rho = (\Omega/8\pi^2)\,p^2 d(\cos\theta)$ we have, according to (50), (60) and (61), after averaging over the initial meson polarizations, a scattering cross-section

$$\begin{aligned} d\phi &= \frac{1}{3}\,2\pi\,\frac{g^4}{4\kappa^4}\,2(1+\cos\theta)\,\frac{p^2}{8\pi^2}\,d(\cos\theta)\,Q_{ab} \\ &= \frac{1}{24\pi}\,\frac{g^4}{\kappa^4}\,p^2(1+\cos\theta)\,Q_{ab}\,d(\cos\theta), \end{aligned} \tag{71}$$

which in the case $m = 1$ is a well-known result.

Equation (71) applies also to the scattering of negative mesons by neutrons and protons and of positive mesons by neutrons.

Equation (70) shows that the angular distribution of scattering is quite different for the various proton wave equations. For the total cross-section ϕ we have, by integration over θ,

$$m = 1\colon\ \phi = \frac{g^4p^2}{12\pi\kappa^4}, \quad m = 4\colon\ \phi = \frac{g^4p^2}{12\pi\kappa^4}\,\frac{17}{9}, \quad m = 5\colon\ \phi = \frac{g^4p^2}{12\pi\kappa^4}\,\frac{73}{9}. \tag{72}$$

DISCUSSION

Equation (72) shows that the scattering cross-section is largest for the particle described by the five-rowed representation of the ξ operators. The corresponding representation of the α matrices is of order 20, and in Bhabha's scheme the theory is classified as $(\frac{3}{2},\frac{3}{2})$. Since in the non-relativistic limit this particle has spin $\frac{3}{2}$, the equation cannot be used to describe a proton and we need not consider it any further.

The four-rowed representation of the ξ operators corresponds to the equation $(\frac{3}{2},\frac{1}{2})$ proposed by Bhabha for the proton; the one-rowed representation leads to Dirac's equation. In the non-relativistic limit these equations are identical and, since the coupling constant g is determined from the observed nucleon binding potentials by a non-relativistic calculation, g must have the same value in both theories. At low energies both theories therefore give the same scattering cross-sections. But at high energies Bhabha's equation leads to a value of the total cross-section for the charge scattering of mesons that is greater, by a factor 17/9, than the value given by Dirac's equation. In the present state of our knowledge of cosmic rays and mesons, a factor of the order 17/9 cannot significantly be compared with experiment. Indeed, the calculation of § 6 takes no account of the divergence difficulty, but it is reasonable to suppose that in a divergence free theory Bhabha's equation would still give the larger cross-section.

It would be of interest to evaluate the scattering cross-section including the dipole

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(f) interaction, but the algebraic expressions involved are so excessively complicated as to suggest that the theory cannot be a correct description of Nature.

The work of §§ 4 and 5 shows that the use of a negative metric involves no formal inconsistencies, though the formalism has not the detailed physical interpretation customary in quantum theory.

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