Why Is There Universal Macrobehavior? Renormalization Group Explanation as Noncausal Explanation

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Renormalization group (RG) methods are an established strategy to explain how it is possible that microscopically different systems exhibit virtually the same macrobehavior when undergoing phase transitions. I argue—in agreement with Robert Batterman—that RG explanations are noncausal explanations. However, Batterman misidentifies the reason that RG explanations are noncausal: it is not the case that an explanation is noncausal if it ignores causal details. I propose an alternative argument, according to which RG explanations are noncausal explanations because their explanatory power is due to the application of mathematical operations, which do not serve the purpose of representing causal relations.

1. Introduction. It is a puzzling and highly nontrivial fact that the macrobehavior of systems with many microcomponents is largely independent of the microbehavior in the sense that the macrobehavior is universal. Macrobehavior is universal if the same macrobehavior can be realized by microscopically different systems. "Universality" is a technical term used in the physics literature—the more familiar philosophical term for the phenomenon is, at least for the purpose of this discussion, "multiple realizability." One prominent example in the recent literature consists in microscopically different physical systems (such as various liquids, gases, magnets) that display the same

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macrobehavior when undergoing phase transitions (Batterman 2000, 2002). Scientists and philosophers alike believe that the universality of macrobehavior cries out for an explanation: how can we explain the remarkable fact that there is universal macrobehavior? Robert Batterman (2000, 11) provides a recent and influential attempt to answer this question. Batterman motivates and illustrates the question by referring to Jerry Fodor's catchy way of articulating the request for an explanation: "Damn near everything we know about the world suggests that unimaginably complicated to-ings and fro-ings of bits and pieces at the extreme micro-level manage somehow to converge on stable macro-level properties. . . . The 'somehow', really is entirely mysterious . . . why there should be (how there could be) macro level regularities at all in a world where, by common consent, macro level stabilities have to supervene on a buzzing, blooming confusion of micro level interactions" (Fodor 1997, 161). Fodor demands an explanation for how it is possible that universal (or multiply realized) macroregularities obtain given the "confusion of micro level interactions."

Batterman addresses Fodor's challenge by appealing to a method that physicists use in order to explain universality. Batterman's paradigm example of such a method is the renormalization group (RG) method, which enables us to explain why it is the case that microscopically different systems display the same macrobehavior when undergoing phase transitions (e.g., gases, fluids, and magnets). The guiding idea of RG is to ignore various microscopic details and interactions that are irrelevant for the macrobehavior in question. RG is a general explanatory strategy to distinguish relevant and irrelevant microdetails. In short, Batterman's response to Fodor's challenge is that microscopically diverse materials can realize the same macrobehavior, because many differences in the microdetails simply do not matter for the macrobehavior (Morrison 2012).

The primary focus of this paper is to inquire what kind of an explanation the RG-based explanation of universal macrobehavior is and, in particular, whether such an explanation is a causal explanation. The goal of this paper is to defend a noncausal interpretation of RG explanations by arguing that RG explanation is a specific kind of mathematical explanation. The rough idea of a mathematical explanation is that the explanation works in virtue of mathematical facts and mathematical operations, which do not represent causal relations in the world. Which relations are counted as causal is judged by using the so-called folk notion and the associated folk features of causation (such as being asymmetric and time-asymmetric relations, or being relations holding between tokens or types of events).[1]

1. See Norton (2007, 36–38) and Ladyman and Ross (2007, 268). As these philosophers point out, causation is often characterized by these features not only in ordinary discourse but also in special science discourse.

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More precisely, I proceed as follows: In section 2 I outline three essential elements of an RG explanation. I agree with Batterman that the RG method does not provide a causal explanation. However, I argue that his argument for this claim is ill founded (sec. 3). The goal of the paper is to explore whether there is an alternative argument to support a noncausal interpretation of RG explanations. I propose an alternative argument for the claim that the RG explanation is noncausal (sec. 4). The alternative argument builds on and significantly extends Marc Lange's (2013) concept of a mathematical explanation in order to argue that RG explanations are noncausal in virtue of being mathematical explanations.

2. RG Explanations of Universality. In this section I outline how an RG explanation works. The discussion here will be largely nontechnical as the paper is concerned with a nontechnical question (I mostly follow the exposition in Batterman 2000). Batterman's prime examples of universal behavior are phase transitions in fluids, gases, and magnets (prominently discussed in Batterman 2000, 2002, 2010). His main focus is on explaining the surprising fact that materially different systems (various gases, fluids, magnets) display the same macrobehavior when undergoing phase transitions (e.g., transitions from a liquid to a vaporous phase, or transitions from a ferromagnetic to a paramagnetic phase near the critical temperature). If microscopically different systems (e.g., fluids and magnets) display the same macrobehavior when undergoing phase transitions, then "sameness" is characterized by the same critical exponent (a dimensionless number; see Batterman 2000, 125–26). The explanandum of interest is this sameness in character of macrobehavior.[2]

It is useful to understand the workings of RG explanations of universality in terms of three explanatory steps: firstly, system-specific laws governing the interactions among the microcomponents of a physical system (Hamiltonians); secondly, RG transformations; and thirdly, the flow of Hamiltonians. Let me briefly present these steps in slightly more detail (as indicated above, I am not able to do justice to the elegant technical details of RG in this short paper; for a detailed survey see Fisher 1982, chap. 5; Batterman 2000, 137–44).

2.1. First Step: Hamiltonians. RG explains the universal macrobehavior of gases and fluids by representing the physical system in question using a Hamiltonian—a function characterizing, among other things, the

2. Owing to space constraints, I do not discuss whether there is a general model of explanation unifying causal and noncausal explanation. See Woodward (2003, 220–21) and Strevens (2008, 179–80) for inspiring but not elaborated suggestions regarding the unifying notion of counterfactual dependence and difference making.

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interactions between the components (or degrees of freedom[3]) of the system. One specific epistemic problem with the Hamiltonian of a 'real' physical system undergoing phase transition (say, a heating pot of water) is that each component of such a system interacts not only with its nearby neighbors but also with distant components. Hence, keeping track of the interaction between all the components of, say, a liquid undergoing phase transitions is—given the large number of components—epistemically intractable.

2.2. Second Step: Transformations. The second element of the RG explanation deals with this epistemic intractability: a particular transformation on the Hamiltonian (the "RG transformation"). Batterman describes the purpose of this kind of transformation as "chang[ing] an initial physical Hamiltonian describing a real system into another Hamiltonian in the space [of possible Hamiltonians]. The transformation preserves, to a certain extent, the form of the original Hamiltonian so that when the thermodynamic parameters are properly adjusted (renormalized) the new renormalized Hamiltonian describes a system exhibiting similar behavior" (Batterman 2000, 126–27). Operations such as spatial contraction and the renormalization of parameters that are involved in RG transformations allow us to represent one and the same fluid F in a different way: the number of interacting components of F (or degrees of freedom) is effectively reduced. That is, the transformed Hamiltonian of F describes the interaction of fewer components (or fewer degrees of freedom). Repeatedly applying RG transformations amounts to a description of the system, say, fluid F, on larger and larger length scales; the RG transformation is a coarse-graining procedure.[4] Carrying out the transformation repeatedly comes with an epistemic benefit: "the transformation effects a reduction in the number of coupled components or degrees of freedom within the correlation length. Thus, the new renormalized Hamiltonian describes a system that presents a more tractable problem and is easier to deal with. By repeated application of this renormalization group transformation the problem becomes more and more tractable" (Batterman 2000, 126–27). In other words, the RG transformation solves the epistemic problem of intractability (see above). Essentially, RG transformations eliminate microdetails irrelevant for the explanation of phase transitions.

2.3. Third Step: Flow of Hamiltonians. Using Batterman's terminology, suppose we start with the "initial physical manifold" or, equivalently, the "real physical" Hamiltonian H of a fluid F (undergoing a phase transition near the critical temperature). Then one repeatedly applies the RG transformation and obtains other Hamiltonians describing the same system F with fewer

3. Wilson (2010) provides a detailed discussion of the concept of degrees of freedom.

4. Shimony (1993, 208) discussed RG transformations and their applicability conditions.

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component interactions than H. Interestingly, these different Hamiltonians "flow" into a fixed point (in the space of possible Hamiltonians), which describes a specific behavior characterized by a critical exponent (Batterman 2000, 143). Now suppose there is another fluid F^* and its behavior (during phase transition) is described by the initial Hamiltonian H^*. Repeatedly applying the RG transformation to H^* generates other, equivalent Hamiltonians (with fewer component interactions than H^*). If the Hamiltonians representing fluid F^* and fluid F turn out to "flow" to the same fixed point, then their behavior, when undergoing phase transition, is characterized by the same critical exponent (Fisher 1982, 85; Batterman 2000, 143).

Hence, we have arrived at the explanandum of an RG explanation: the three elements of an RG explanation provide a method to determine under which conditions two microscopically different systems (i.e., systems with different initial "real physical" Hamiltonians) belong to the same "universality class," that is, are characterized by the same critical exponent (Fisher 1982, 87). Two systems belong to the same universality class if reiterating RG transformations reveals that both systems "flow" to the same fixed point.

Batterman certainly has a point when he claims that the RG method explains by showing how various details about component interactions are irrelevant for the macrobehavior of systems (Batterman 2000, 127). RG explanations not only reveal what is irrelevant but also provide information about what is relevant for a specific macrobehavior. As Batterman states, "For instance it turns out that the critical exponent can be shown to depend on the spatial dimension of the system and on the symmetry properties of the order parameter. So, for example, systems with one spatial dimension or quasi-one dimensional systems such as polymers, exhibit different exponents than (quasi-) two dimensional systems like films" (Batterman 2000, 127).

This is, roughly, how Batterman thinks that RG explanations of universal behavior work. I have no ambition to challenge his exposition. The main question of this article is whether any of the three elements of an RG explanation warrants a noncausal interpretation of such an explanation.[5]

5. I will not address two separable issues regarding RG explanations: (1) whether RG explanations are reductive explanations, and (2) which role limit theorems play in RG explanations. Some philosophers have recently argued—contra Batterman (2002) and Morrison (2012)—that RG explanations are Nagelian reductions (Butterfield 2011; Norton 2012) and reductive microexplanations (Hüttemann, Kühn, and Terzidis, forthcoming; Reutlinger, forthcoming). Even if these views are correct, it is possible to distinguish causal elements (Hamiltonians) and potentially noncausal elements (RG transformations and the flow of Hamiltonians) of an RG explanation. Moreover, Norton (2012) holds that (1) limit theorems involve strong idealizations and require careful interpretation and (2) the idealizations in question are, ultimately and contrary to Batterman and Morrison, dispensable for RG explanations of universality. All I wish to stress is that the issue of

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3. Against Batterman's Anticausal Argument. Batterman presents a principled objection—his anticausal argument—to subsuming RG explanations under the causal model of scientific explanation. Batterman's main argument runs as follows: according to the causal model, explanations do not and cannot ignore causal "micro details" because an explanation provides "detailed causal-mechanical accounts of the workings of the mechanisms leading to the occurrence of the explanandum phenomenon" (Batterman 2000, 28). A causal explanation "tells us all of the gory details" (28) about why a particular effect occurs (Batterman 2010, 2 and 21). However, ignoring certain details about the interactions of components of a physical system seems to be essential for the second step of the RG explanation (i.e., the RG transformations). Batterman concludes that the causal model of scientific explanation cannot accommodate RG explanations.

To be fair, suppose that causal relations are characterized by the so-called folk role (Norton 2007; Price and Corry 2007). The folk role characterizes causal relations as holding between tokens of events (in the case of actual causation) or between types of events (in the case of type-level causation). Moreover, the folk notion assigns specific features to causal relations: relations of this kind are, among other things, asymmetric and time-asymmetric (dependence) relations. Batterman seems to be right that if explanation is just providing information about causes and if causation is a (counterfactual, probabilistic, nomic, etc.) dependence relation between (tokens or types of) events, then RG explanations do not easily fit into a causal model of scientific explanation. However, there are at least two objections to Batterman's anticausal argument that the causal model does not fit RG explanations because the former cannot account for abstracting away from details.

First, as many philosophers working on causation in the special sciences have pointed out, many perfectly legitimate causal explanations in these sciences do in fact frequently ignore lower-level details (Cartwright 1989; Woodward 2003; Strevens 2008). For instance, a microeconomic explanation of an increase of the price of a commodity cites causes but ignores many psychological, biological, and physical details about the agents that interact on a market. Similarly, even a neuroscientific micromechanistic explanation of long-term potentiation refers to the activities of neurons but abstracts from various physical details without losing the causal character of the explanation (Craver 2007, 65–72). These examples support the claim that it is compatible to explain by citing causes and mechanisms and, at the same time, to ignore details. Therefore, these examples undermine Batterman's anticausal argument.

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whether RG explanations involve idealizations is also independent of the matter whether RG explanations are causal or not.

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Second, Batterman's characterization of the causal model of scientific explanation—that is, not being able to ignore details—is inaccurate as a general characterization. This characterization applies only to a specific version of the causal account (such as Railton's account, to which Batterman alludes; Batterman 2002, 28). In contrast to Batterman's view, a number of recent influential causal models of explanation are explicitly designed to account for the fact that many excellent scientific explanations "ignore details." For instance, Strevens (2008) and Franklin-Hall (2014) explicitly deny that explanation merely consists in citing causes; they add an "optimizing procedure" (Strevens 2008) and a "biggest bang-for-your-buck" principle (Franklin-Hall 2014), which are procedures to omit irrelevant causal information. Woodward's (2010) notion of causal specificity plays a similar role in his theory of causal explanation; it determines the accurate explanatory level of abstraction from (causal) details. Hence, either Batterman's key claim that the causal model of explanation cannot account for explanations that involve abstractions from details is controversial (to say the least), or Batterman's anticausal argument induces a merely verbal dispute regarding whether the cases—which Strevens, Franklin-Hall, and Woodward discuss—deserve to be called causal explanations.

If these objections are striking and Batterman's anticausal argument fails, where does this leave us? At least two reactions are possible. One reaction might be to defend the view that one of the above-mentioned causal models of explanation applies, contra Batterman, to RG explanations. I will not pursue this strategy. My main reason not to pursue this strategy is that causal explanation can, at best, explain why a particular behavior of a system, say, a fluid *F*, occurs by citing a cause or an underlying causal micromechanism. This seems to hold independently of whether a causal explanation abstracts from details or not. As a matter of fact, Strevens, Franklin-Hall, and Woodward are primarily and explicitly concerned with questions of how one can simplify a system-specific causal model, why the behavior of one kind of a system depends on few macrovariables, and exactly how one can reasonably abstract from the microdetails of one kind of system. For instance, Strevens (2003, 2008) cares about methodological principles enabling ecologists to ignore certain causal microdetails of the interactions between a specific population of foxes and a specific population of rabbits and, ultimately, to determine a small number of macrovariables, which can be used to explain and predict the growth of these specific populations of foxes and rabbits. Call questions of this kind 'system-specific questions'. Let us grant that these philosophers provide satisfactory answers to these system-specific questions. However, it is at least not obvious that causal models of explanation are also suited to successfully address the question of why two different systems with different underlying causal mechanisms, say, two fluids *F* and *F**, display the same macro-

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behavior (similarly, Batterman 2002, 23–24). The point I wish to stress is that it is, at least, a challenge even for those causal models of explanation with a built-in abstraction principle to accommodate RG explanations, because the latter address the question of why microcausally different systems behave similarly on the macro level. Call this challenge the 'inter-systems challenge'. I take this challenge to be good enough to motivate a different reaction to the failure of Batterman's anticausal argument.

I explore a strategy according to which the conclusion of Batterman's anticausal argument is true (i.e., RG explanations are noncausal explanations) and this conclusion is supported by an alternative argument. The next section elaborates the alternative argument for the claim that RG explanation is not causal (in addition to the inter-systems objection presented in the previous paragraph) and provides a positive characterization of noncausal explanations.

4. RG Explanation as Mathematical Explanation. In order to decide whether RG explanations are noncausal, one needs a criterion to distinguish causal and (at least some sorts of) noncausal explanations. Marc Lange (2013) provides a useful candidate for such a criterion. Lange argues that one ought to distinguish between (1) explanations that explain in virtue of describing cause-effect relations in the world and (2) explanations whose explanatory powers stem from "distinctively mathematical" facts. In order to avoid misunderstandings, let me clarify that I will not adopt Lange's approach straightforwardly. Instead, I propose an amended and enriched account of mathematical explanation, which preserves only some of Lange's core ideas and includes one of Batterman's (2010) core insights regarding mathematical explanations. It is argued that this account of mathematical explanation provides good reasons to believe that RG explanations are noncausal explanations because of being mathematical explanations.

Lange illustrates how a mathematical explanation works by using the following toy example: the empirical fact that Marc has three children and 23 strawberries, as well as the distinctively mathematical fact that 23 cannot be divided evenly by 3, explains why Marc failed when he tried a moment ago to distribute his strawberries evenly among his children without cutting any.[6] Lange points out that a distinctively mathematical explanation may include some causal information. For instance, the explanation may include information about which beliefs and desires regarding his three children caused Marc to distribute the strawberries, information about the proper functioning of the physiological mechanism of his body and the bodies of his children during the time that the distribution of strawberries takes place, and so on. For the purposes of this paper, it is instructive that Lange thinks

6. Lange (2013, 18–19) also discusses a scientific example in detail: a distinctively mathematical explanation of why a simple double pendulum has four equilibrium configurations.

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of causal information—in the context of mathematical explanations—as a presupposition of the explanation-seeking "why" question (Lange 2013, 13). For instance, in the case of the strawberry example the "why" question is as follows: presupposing that Marc's beliefs and desires caused him to distribute the strawberries and presupposing the proper functioning of the physiological mechanism of his body and the bodies of his children during the time of the distribution, and so on, why did Marc fail to distribute the strawberries evenly? The explanation of why he failed to distribute the strawberries evenly among his three children is, however, noncausal, as the explanatory force is exclusively derived from a distinctively mathematical fact (i.e., the fact that 23 cannot be divided evenly by 3). In other words, if an explanation derives its explanatory power from distinctively mathematical facts (and not from a description of causes of the explanandum), then the explanation is noncausal.

I anticipate a worry at this point. One might be concerned that 'appealing to mathematics' is not sufficient for rendering an explanation noncausal. This would be a devastating result for an account of mathematical explanation. However, it is, of course, not the case—and Lange explicitly and repeatedly stresses this point—that if an explanation is formulated in terms of mathematics, then it is noncausal. One can appeal to the folk features of causation in order to argue that some explanatory mathematical statements are correctly interpreted as causal statements. For instance, in the recent causal modeling literature, causal explanations refer to causal generalizations or so-called structural equations (Woodward 2003). These generalizations are mathematical statements about functional dependencies between variables. Explanations that refer to these functional dependencies are causal explanations because these dependencies hold between types of events (expressed as the values of random variables) and the dependencies are typically taken to have the features associated with the folk notion of causation (such as being asymmetric and time-asymmetric dependence relations). Hence, explanations can perfectly well refer to causal generalizations, which are couched in mathematical language, without losing their causal character. Similarly, the fact that billiard ball A has a certain velocity might be causally explained by A's collision with another billiard ball B. Assume that the behavior of these billiard balls is governed by Newtonian laws of motion. If it is the case that the nomic relations in this scenario hold between the types or tokens of events, and if the nomic relations instantiate sufficiently many other folk features of causation, then these mathematically formulated laws causally explain why the billiard ball A has a particular velocity.

Taking this worry into account, I adopt the following preliminary definition of a noncausal explanation, to be refined later on: an explanation is noncausal if and only if the explanans contains at least one noncausal element *e*, and *e* ensures the success of the explanation. In the case of a math-

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ematical explanation, the noncausal component *e* is a claim stating a mathematical fact (e.g., that 23 cannot be divided by 3). As I discuss in more detail below, my disagreement with Lange concerns the question whether Lange's 'distinctively' mathematical facts exhaust the class of explanatory noncausal mathematical facts. Following Batterman (2010), I argue that the class of explanatory facts also includes (contingent) facts about the applicability of mathematical operations.

Let us see whether this criterion helps us decide whether RG explanations are noncausal in virtue of being distinctively mathematical. I will go through the three steps of an RG explanation. The first element of an RG explanation is the Hamiltonian of a system. Since the Hamiltonian describes the interaction between the components of a system (a gas, a fluid, a magnet), this component contains causal information.[7] Hence, this step of an RG explanation is at least unlikely to warrant a noncausal interpretation of RG explanations.

Do the remaining two steps of an RG explanation warrant a noncausal qua mathematical interpretation? Recall that the fact to be explained is why it is the case that two systems S and S^*—which are microscopically different and, therefore, characterized by different Hamiltonians H and H^*—exhibit the same macrobehavior when undergoing phase transitions. To see that the demand for an explanation is to an extent independent of the Hamiltonians, one is able to phrase the explanation-seeking "why" question in the following way: the Hamiltonians H and H^* play the role of, to adopt Lange's terminology, presuppositions of the explanation-seeking "why" question. That is, presupposing that S has the initial Hamiltonian H and S^* has the initial Hamiltonian H^*, why is it the case that S and S^* display universal macrobehavior (characterized by the same critical exponent)? As in the strawberry case, the explanation-seeking "why" question presupposes that a particular causal structure is in place. In the strawberry case, the presupposition concerns beliefs and desires that are effective, the working of physiological mechanisms, and so on; in the RG case, it is presupposed that Hamiltonians describe the causal interactions among the components of a system. I argue that the RG answer to this "why" question is mathematical. The mathematical explanatory power is derived from the remaining two elements of an RG explanation: the transformations and flow of Hamiltonians. If this is correct, then RG explanations are noncausal explanations.

Which facts count as mathematical facts? Applying Lange's concept of a distinctively mathematical explanation to RG explanations reveals a disanalogy between the strawberry scenario and the RG explanation. Lange appeals to mathematical facts in a strict or "distinctive" sense: for instance, a number (such as 23) and its features (such as not being evenly divisible by 3).

7. For brevity's sake and for the sake of the argument, I ignore Russellian arguments here, according to which fundamental physical laws are not causal laws (Price and Corry 2007).

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However, the mathematical elements figuring in RG explanations are unlike mathematical facts in the strict sense: the former are mathematical operations rather than mathematical facts. I take this as a forceful suggestion to extend Lange's original account by enriching the class of mathematically explanatory facts. Here I follow Batterman (2010, 7–8), who introduces a helpful distinction of kinds of explanatory mathematical facts: (metaphysically necessary) facts about (abstract) mathematical entities such as numbers, sets, and graphs (and their properties); and facts about mathematical operations (taking the thermodynamic limit is his primary example of a mathematical operation with explanatory import).[8] Batterman (2010) does not exploit this distinction in order to strengthen his claim that RG explanations are noncausal; his goal is to undermine the indispensability argument for platonism and the "mapping account" of explanatory mathematical models. I use Batterman's distinction between entities and operations for deciding whether RG explanations are mathematical in character and, hence, noncausal.

I anticipate another worry at this point. One might object that extending Lange's account by allowing mathematical facts and operations to be explanatory misses a crucial feature of Lange's notion of distinctively mathematical explanations. Namely, it leaves out the feature that distinctively mathematical facts possess a kind of modality that is stronger than physical modality. For example, Lange appeals to an explanatory mathematical fact, according to which it is metaphysically necessary (and not merely physically necessary) that 23 cannot be evenly divided by 3.[9] By contrast, a mathematical operation (such as an RG transformation) does not apply to a physical system with metaphysical necessity; such an operation applies given physically contingent conditions (Shimony 1993, 208). In response, I do not deny that—given the strawberry case and the RG case—there is such a difference in the modal character of the explanatory facts, but I do not follow Lange in taking metaphysical necessity as the distinctive feature of (noncausal) explanatory mathematics. Instead, it is my claim, for which I argue in the remainder of this section, that one need not appeal to metaphysical necessity in order to claim that mathematical facts explain in a

8. Following Bueno and French's (2011) criticism, I do not agree with Batterman's claim that there is a sharp distinction between facts and operations. However, I find it useful to draw attention to (1) a kind of explanatory mathematics (such as in the second and third steps of RG explanations) that is often ignored and (2) different modal strengths of "distinctively mathematical" facts and facts about applying operations.

9. However, when discussing the distinctively mathematical explanation of the behavior of a simple double pendulum, Lange weakens this condition and acknowledges instances of distinctively mathematical explanations, according to which the relevant explanatory mathematics is "not a mathematical, conceptual, metaphysical, or logical truth" (Lange 2013, 20).

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noncausal way. All one needs to establish is that the mathematics does not explain by referring to causal facts (i.e., facts about relations that count as causal in the light of the folk notion of causation). Philosophers who are skeptical of metaphysical necessity (and other kinds of necessity that are stronger than physical necessity) might even regard this strategy as having an advantage over Lange's approach.

Which steps render the RG explanation a mathematical explanation? As presented in section 2, the RG method explains the universality of critical behavior by invoking two elements besides the Hamiltonian of a physical system: (1) RG transformations on the Hamiltonians in question that enable physicists to ignore aspects of the interactions between microcomponents of, say, a fluid, and (2) a "flow" or mapping of transformed Hamiltonians (of fluids F and F*) to the same fixed point that allows one to calculate the critical exponent. Operations such as spatial contraction and the renormalization of parameters, of which the RG transformation consists, reduce the number of interacting components (or degrees of freedom). This coarse-graining procedure helps to solve the problem of epistemic intractability (see sec. 2). The "flow" or mapping of the Hamiltonians, which are generated by iterated RG transformations, is an operation that ultimately determines the critical exponents of microscopically different systems and, based on the sameness of critical exponents, groups physical systems into universality classes. Both the transformations and the 'flow' are mathematical operations, which, ultimately, serve the purpose to reveal something that two fluids have in common despite the fact that their "real physical" Hamiltonians (or "initial physical manifolds") are strikingly different. Recall that I use the folk notion of causation in order to decide whether an explanatory fact is causal or not. Let us apply this test to the case at hand. Neither the RG transformations nor the flow of Hamiltonians relate tokens or types of events. Rather, both operations relate entire Hamiltonians in the space of possible Hamiltonians. Hence, both operations relate the wrong kinds of entities for being causal relations. Moreover, neither RG transformations nor the flow of Hamiltonians relate entities in the way required for causal relations: both operations obviously fail to relate entities in a time-asymmetric and asymmetric way. To see why, suppose once more that the Hamiltonians H and H* represent the interactions of the microcomponents of some fluid F, and H* is the product of an RG transformation on the "real physical" H. If this is the situation, then it is not the case that Hamiltonian H occurs before (or after) Hamiltonian H*, because Hamiltonians simply do not stand in temporal relations. It also seems to be inaccurate to say that H and H* stand in any significantly asymmetric dependence relation because H and H* are equivalent representations of the same fluid F (when undergoing phase transition). Therefore, neither of the two mathematical operations involved in RG explanations is best under-

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stood as directly revealing information about cause-effect relations. If this observation is correct, then, according to RG, the key to understanding why two microscopically different fluids display the same macrobehavior consists in the application of two noncausal mathematical operations. Hence, RG explanations are noncausal explanations in virtue of being mathematical explanations.

5. Conclusion. I started out with presenting the scientific demand for an explanation of how it is possible that microscopically different systems exhibit virtually the same macrobehavior when undergoing phase transitions. RG methods provide a well-established explanation of this phenomenon in physics. The main goal of this paper was to argue—in agreement with Batterman—that RG explanations are noncausal explanations. It was argued that Batterman misidentifies the reason that RG explanations are noncausal: he is wrong to claim that if an explanation ignores causal (micro) details, then it is not a causal explanation. I proposed an alternative argument for the claim that RG explanations are noncausal explanations. RG explanations are noncausal explanations because their explanatory power is due to the application of mathematical operations, which do not serve the purpose of representing causal relations.

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