Macroeconomic Dynamics, **17**, 2013, 89–122. Printed in the United States of America.
doi:10.1017/S1365100511000010

INVENTORIES, FLUCTUATIONS, AND GOODS SECTOR CYCLES

Louis J. Maccini
Johns Hopkins University

Adrian Pagan
University of Sydney

The paper looks at the role of inventories in U.S. cycles and fluctuations. It concentrates on the goods-producing sector and constructs a model that features both input and output inventories. A range of shocks are present in the model, including sales, technology, and inventory cost shocks. It is found that the presence of inventories does not change the average goods sector–cycle characteristics in the United States very much. The model is also used to examine whether new techniques for inventory control might have been an important factor contributing to the decline in the volatility of U.S. GDP growth. It is found that these would have had little impact upon the level of volatility.

Keywords: Business Cycles, Inventory Dynamics, Great Moderation

1. INTRODUCTION

It is not uncommon for commentators on the prospects for an economy to draw attention to recent inventory movements. Thus, if there has been a run down in stocks below what is perceived to be normal levels, this is taken as a sign of favorable output prospects in future periods; the reasoning behind this conclusion being that output needs to be produced not only to meet sales, but also to replenish stocks. Early in the history of business cycle research the question arose of whether the presence of inventory holdings by firms was a contributor to the "up and down" movements seen in economies. The classic analyses of this question were by Metzler (1941, 1947), who concluded that " An economy in which businessmen attempt to recoup inventory losses will always undergo cyclical fluctuations. . . ." His model emphasized that a business would attempt to keep inventories as a proportion of expected sales and so would rebuild these if they declined below that target level. Given that sales had to be forecast from their past history, he showed that output would follow a second-order difference equation, which would have

We would like to thank a referee especially for helpful comments on the previous version. Helpful comments on prior versions of the paper were also received from Margaret McConnell, Mateo Iacoviello, Fabio Schiantarelli, and Scott Schuh. The research was supported by ARC grant DP0342949. Address correspondence to: Adrian Pagan, School of Economics, Faculty of Arts and Social Sciences, Merewether Building (H04), The University of Sydney, NSW 2006, Australia; e-mail: Adrian.Pagan@sydney.edu.au.

© 2012 Cambridge University Press 1365-1005/12

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

complex roots in many cases. Consequently his model produced *oscillations* in output and this constituted the basis of his conclusion.

Turning points (peaks and troughs) in output can occur even if there are no oscillations. Thus a peak in output will be attained if positive growth turns into negative for some period of time. The chance of this occurring depends on the magnitude of long-run growth, the extent to which current growth depends on past growth rates, and the size of the shocks that are impinging on the macroeconomic system—see Harding and Pagan (2002). In contrast, the nature and extent of oscillations depend only upon the roots of the difference equation describing output and so are independent of any shocks. Consequently, there can be quite major differences between the cycle characteristics established by studying turning points in a series and those that analyze oscillations in the same series.

The question that then arises is what impact the presence of inventories has upon the nature of business cycles. An answer to this question is of interest for a number of reasons. First, such an analysis would generalize what Metzler did when he focused upon whether oscillations emerged from simple models of aggregate output. Second, there have always been comments that a large fraction of the change in output was accounted for by inventory movements—see, for example, Blinder and Maccini (1991). This has often been interpreted in a causal way as suggesting that inventories are the *cause* of the cycle. So it is interesting to see if that is a correct interpretation. Because the evidence is generally expressed in terms of the extent to which the movement in inventory investment during recessions is close to the actual fall in output, a turning point perspective is needed, given that it is necessary to define periods of recession. Finally, in the decades since the early eighties, expansions seemed to have lengthened around the world. One would expect that this would happen if the volatility of *output growth* declined, as that lessens the chance of negative growth, and so it has been suggested that advances in inventory technology might have been a major contributor to the Great Moderation (GM) in volatility that was seen in many countries for a substantial period after the 1980s.

In this paper we address the question of the impact of inventories on business cycles in two ways. One is to establish some facts about the nature of business cycles and then ask how the cycles would change if inventories were not present in the system. A second involves undertaking an analysis of whether advances in inventory management techniques were responsible for the decline in the volatility of GDP growth during the period from the mideighties until the advent of the global financial crisis. Because both approaches involve questions that can only be answered with a model of output and inventory choices, it is necessary to construct one that is a reasonable characterization of the data and that has enough structure to answer the questions posed above.

The model chosen is an extension of that in Humphreys et al. (2001), which has the attractive feature of emphasizing both raw materials and finished goods inventories. However, as that model was designed for stationary data, many of our extensions aim to ensure that the redesigned model is compatible with the unit

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

root and cointegration properties of the data. In particular, it is expressed in terms of various "shares" and ratios. This enables us to avoid filtering operations such as Hodrick–Prescott (HP) that would be inconsistent with the nature of the data implied by the model. Once the structural model has been developed, it is estimated using data prior to the GM—a period in which it was reasonable to assume constant parameters. This is novel because most of the literature addressing inventory issues "calibrates" the parameters, as in Kahn et al. (2002), Khan and Thomas (2007a, 2007b), and Wen (2008). Other authors have also developed structural inventory models, e.g., Ramey and Vine (2006), Iacoviello, Schiantarelli and Schuh (2007), and Kahn (2008).[1] Apart from handling nonstationarity and performing estimation, our model is distinctive in focusing on a single sector (the goods sector) and having two types of inventories (finished goods and materials), stocks that are held to reduce the cost to firms of lost sales and production disruptions, a production function that depends on the usage of materials rather than the stock of inventories, and a proper modeling of value added (corresponding to GDP) as well as gross output.

It is worth commenting on our choice to focus on the goods sector and two types of inventories, as this is unique in the literature.[2] In relation to the first, because inventories are mainly used in the production of goods, it seems natural that we focus upon cycles in this sector rather than in aggregate GDP, as service sector cycles are likely to be driven by separate factors and not much affected by inventories. The goods sector cycle is much shorter than that in aggregate GDP and many of the features cited for the overall economy are quite different for it. Thus Morley and Singh (2009) point out that, although the volatility of aggregate output growth exceeded that of "sales" in the period before the mid-1980s, it was not true in the post-1980s period. However, this inequality does not hold when one looks at the goods sector. There the volatility of output growth relative to sales growth has actually increased over the two periods. Another conceptual advantage of working with the goods sector is that most models used in the literature embody a goods production perspective, and so the data we use are much closer to what the model variables represent. Turning to the second choice, we present evidence that the factors influencing final goods inventories have not changed since the 1980s, but those for materials inventories have, as seen from a substantial decline in the ratio of materials inventories to output. Consequently, one needs a model that describes raw materials inventories, and not just those of final goods. However, in the literature the predominant approach has been to model the latter.

Two issues arise from our use of a single sector. First, because we are effectively working with an *incomplete* model of the macroeconomy, it is necessary to develop estimation methods that allow for this. Second, we need to consider whether such a model can answer the questions of interest. In our case, the selected model involves treating a number of variables as exogenous to the goods sector, most notably sales. This is in line with the traditional inventory literature. In a more general model, sales will be endogenous and will depend on a more primitive set of shocks than is present in our one-sector model. Thus we cannot precisely

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

describe how cycles in the goods sector vary with primitive shocks or address the broad question of whether the GM was due to better inventory management techniques, better monetary policy, or "good luck." But, for the questions we concentrate on—whether inventories are important in cycles and whether they had a role in the GM—it is not necessary to have more than a single-sector. We see our single-sector approach as advantageous in that it avoids the need to estimate relations for the household sector and the services sector, and monetary policy rules. Any failure to correctly specify all those relations would produce biased estimates of the parameters of the model, including those that represent inventory management techniques. Once the extent of the role that inventories play in cycles in the goods sector and the GM is determined, one can turn to constructing a general equilibrium model that addresses the relative contributions of inventory management, better monetary policy, and good luck to these phenomena.

Section 2 first reports some basic facts about the nature of the business cycle and the behavior of inventories over it. Section 3 then describes the model of the representative firm used to address the issues of the paper. After quantifying the parameters in Section 4, we use the model in Section 5 to simulate data, thereby studying the role of inventories in the business cycle and whether they were responsible for any changes in it. A final section concludes the paper.

The paper yields two main conclusions: First, the empirical work suggests that, on average, there is little contribution of inventories to the length and amplitude of the goods sector cycle. This is so despite the fact that the model produces predictions that are compatible with many of the features seen in the data relating to the behavior of inventories over the business cycle. Of course, inventory movements may be important in *particular* cycles. Second, advances in inventory management techniques seem to have contributed very little to the decline in the volatility of GDP growth that was apparent for some twenty-five years after the early eighties, and hence appear not to have been the source of the GM.

2. SOME CYCLE CHARACTERISTICS

2.1. Sectorial Cycles over Two Time Periods

It seems useful to reexamine the relation of inventories and the business cycle by utilizing the approach and techniques of the view of cycles described earlier, i.e., as reflecting turning points. We begin by thinking of aggregate economic activity as being usefully summarized by GDP—see Burns and Mitchell (1946, p. 72) for an early statement and NBER (2003) for a recent one. Because inventories are principally used in the production of goods, determining their role in the business cycle would seem to begin by splitting GDP into its goods and nongoods (services and structures) components, and then looking at the cycle in the goods component. Designating Q^g and $Q^s = Q - Q^g$ as the goods and nongoods components of GDP, respectively, a log linearization produces the approximation

$$\Delta q_t = \omega^g \Delta q_t^g + (1 - \omega^g)\Delta q_t^s,$$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

TABLE 1. U.S. business cycle characteristics (goods, services, and aggregate GDP: 1947/1–2005/4)

	Goods	Nongoods[a]	Aggregate
	Mean duration (quarters)		
Contractions	3.3	3.2	2.8
Expansions	12.3	36.5	20.7
	Mean amplitude (percentages[b])		
Contractions	−4.2	−1.2	−2.0
Expansions	18.0	32.9	21.7

[a] The nongoods sector includes services and structures.
[b] The amplitude in a contraction is the percentage decline in GDP from peak to trough and the amplitude in an expansion is the percentage increase in GDP from trough to peak.

where ω^g is the average of the ratio Q_t^g/Q_t and lower case letters indicate the log of a variable; i.e., $q_t = \ln Q_t$. Over 1947/1–2005/4 this average was 0.31 with a standard deviation of 0.02. The first and last observations on it were 0.32 and 0.36, respectively. So it has been a fairly stable ratio. The approximate value of q_t follows the actual data quite closely, so that one might build up the cycle in q_t by looking at those in the subaggregates q_t^g and q_t^s.

Table 1 sets out cycle characteristics using the modified BBQ algorithm set out in Harding and Pagan (2002) to find turning points in the series.[3] Because the most attention has been paid to cycles in the level of economic activity, as measured by variables such as GDP, it is interesting to examine the cycle in the subset of GDP that relates to goods. It is evident that the cycles in the subsectors are quite different, with the cycle in goods GDP being much shorter than that in the nongoods sector. Indeed, recessions in the goods sector have always been stronger and longer than those in aggregate GDP. It might be thought that this comes from a declining contribution of goods to aggregate GDP, but, as the ratios Q_t^g/Q_t presented earlier show, the opposite has happened in the chain-weighted data. Of course, in nominal terms, the ratio may well have declined, because the price of goods relative to nongoods has almost certainly declined. In any case, because we will be modeling q_t^g, the issue of how one assesses the importance of the goods sector does not arise.

Table 2 shows AR(2) processes of the form

$$\Delta q_t = \mu/(1-\rho_1-\rho_2) + \rho_1 \Delta q_{t-1} + \rho_2 \Delta q_{t-2} + \sigma \varepsilon_t$$

fitted to the growth rates in the three series over two periods—(I) 1947/1–1983/4 and (II) 1984/1–2005/4. An AR(2) was chosen for two reasons. First, oscillations can be checked by examining the roots of the polynomial $(1-\rho_1 L-\rho_2 L^2)=0$. Second, testing showed that only a second-order process was needed to fit the data. It is clear that the long-run growth in all quantities (μ) is much the same over the first period, but there is some difference in the second period, and it

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

TABLE 2. AR(2) processes fitted to the series over two sample periods

	μ	ρ_1	ρ_2	σ
GDP-I[a]	0.0088	0.314	0.050	0.011
GDP-II[b]	0.0079	0.183	0.358	0.005
Goods-I[a]	0.0086	0.102	0.074	0.022
Goods-II[b]	0.0096	0.106	0.260	0.011
Services-I[a]	0.0088	0.40	0.05	0.009
Services-II[b]	0.0071	0.16	0.34	0.003

[a] Sample period I is 1947-1 to 1983-4.
[b] Sample period II is 1984-1 to 2005-4.

seems as if both the volatility of the growth rate in GDP and its pattern of serial correlation changed as well. What is also interesting is that the serial correlation seen in aggregate GDP comes from the "services" component—the t-ratios for α_j for Δq_t^g are less than 1.61, so that there is virtually no serial correlation in it in either period.[4] For neither goods nor services is there any evidence of oscillations.

The different characteristics noted in Table 2 work in different directions when it comes to determining the impact upon cycles.[5] The much higher volatility in goods GDP growth will mean a shorter cycle in it than for services. Offsetting this, however, is the lower positive serial correlation in goods output, as simulations in Harding and Pagan (2002) point to this producing a longer cycle. Consequently, with the two factors operating in opposite directions, the relative length of cycles is indeterminate, although the very large differences in volatility, and smaller differences in serial correlation, suggest a much shorter cycle in goods GDP. The reduction in the volatility for goods-based GDP means that the cycle will have lengthened. Indeed, in the first period the cycle was 14.6 quarters long (expansions of 11.2 quarters) whereas in the second it was 29.5 quarters (expansions of 25 quarters).

2.2. Inventories and the Goods GDP Cycle: A Preliminary Analysis

To explore the impact of inventories upon the cycle, we focus on Δq_t^g in this paper, because it is the behavior of this series that will be affected by the presence of inventories. On a broad level it is worth exploring the question of how inventories might affect the goods cycle by examining how the data-generating process (DGP) of Δq_t^g is built up, i.e., what determines the parameters and the shocks in the AR(2) process. It would not be expected that the presence of inventories would affect long-run growth, μ, but we might expect that there could be some impact upon the dynamic response of q_t^g to shocks. Because ε_t will be built up from all the shocks of the macroeconomy, and one of these could be inventory cost shocks, this is another way in which inventories could affect the cycle.

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

TABLE 3. AR(2) fitted to Δx_t, Δy_t, Δq_t^g (1959/3–2005/4)

	Δx_t	Δy_t	Δq_t^g
μ	0.0082	0.0081	0.0093
ρ_1	0.384	0.379	0.015
ρ_2	−0.003	0.002	0.143
σ	0.013	0.013	0.017

To begin the investigation we start with the identity

$$X_t = Y_t - \Delta N_t, \tag{1}$$

where X_t is the level of gross sales in the goods sector, Y_t is the level of gross output in the goods sector, and N_t is the stock of finished goods inventories. If the holding of finished goods inventories were important to the cycle, we would expect that the cycle in $x_t = \ln X_t$ would be different from that in $y_t = \ln Y_t$. We constructed series on X_t and N_t and then found Y_t from the identity (1)—see Appendix A. It is worth mentioning that the X_t we construct here is not that referred to as "final sales" in the NIPA. The equivalent of the latter for the goods sector would be

$$FS_t = Q_t - \Delta N_t - \Delta M_t,$$

where M_t is the level of raw material inventories. Many investigations of inventories use FS_t to represent X_t, e.g., Wen (2005) and Morley and Singh (2009), but generally we cannot use the former as a proxy for the latter when attempting to quantify a model. It would seem that the series we use for X_t is only available for the goods sector of GDP, at least on a quarterly basis.

Value-added, Q_t^g, is the goods contribution to GDP, and this relates to Y_t and M_t through the identities

$$Q_t^g = Y_t - V_t U_t,$$
$$\Delta M_t = D_t - U_t,$$

where U_t is usage of raw materials, D_t is deliveries of raw materials, V_t is the price of raw materials relative to the price of output, and M_t is the stock of raw materials.[6] Thus inventories of raw materials may make the cycle in q_t^g different from that in y_t.[7]

From this discussion, it is clear that if inventories are to affect the cycle in Q_t^g (q_t^g), they must induce a change in the DGP of Δq_t^g from that of Δx_t.[8] Table 3 looks at the characteristics of the DGP for each of the series Δq_t^g, Δx_t, and Δy_t over the period 1959/3–2005/4. It suggests that finished goods inventories have little role in cycles, because the DGP of the growth rate of gross sales, Δx_t, is little different from that of the growth rate of gross output, Δy_t. In contrast, in the move from gross output to value added (GDP for the goods sector)

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

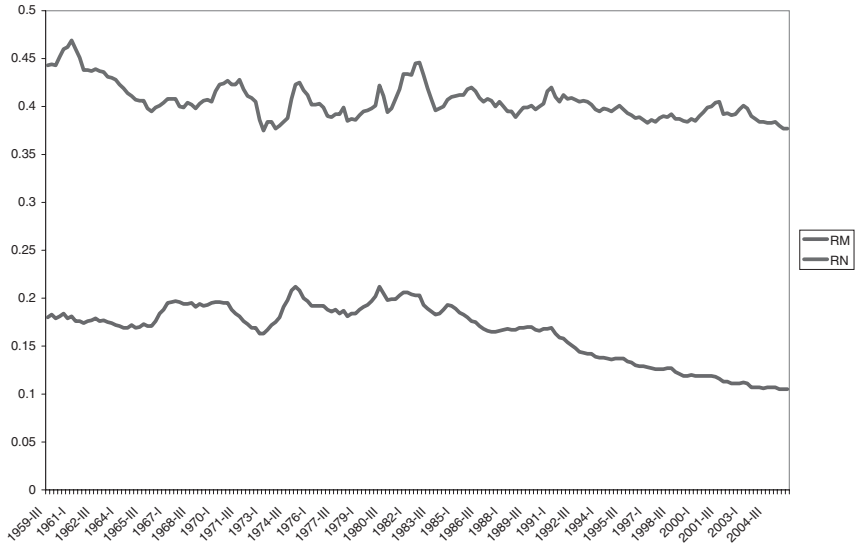


FIGURE 1. Ratios of N to X (RN) and V^*M to Y (RM).

there are significant differences, in that the DGP of goods sector GDP growth, Δq_t^g, is substantially different from that of the growth rate of gross output, Δy_t.[9] This points to a potential role for materials inventories in influencing the business cycle. However, whether the differences in the DGP's of Δq_t^g and Δy_t can be explained by movements in materials inventories, materials prices, or other forces needs to be determined. To do so, one needs to construct a model that can account for the separate effects.

Turning to the decline in the volatility of Δq_t^g, one factor proposed to account for this is the ability to economize on inventories with new technology. A quick look at the plausibility of this is available by looking at the ratios N_t/X_t and $V_t M_t/Y_t$ over time.[10] Figure 1 gives a plot of these. It is clear that there has been little change in the first ratio, but the second declined by about 50% after 1984, which is a substantial decline.[11] These stylized facts again point to the potential importance of materials inventories for looking at changes in cycles and, at least for the United States, these seem to have been more significant than finished goods inventories. Whether changes in the levels of inventories that are held can in fact explain changes in the cycle is a different matter, and once again points to the need to develop a model that explains Δq_t^g and that formally incorporates at least materials inventories.

Finally, it is worth dwelling upon inventory investment movements during business cycles. In particular, the view, expressed for example in Blinder and Maccini (1991), that a very large part of the reduction in GDP during recessions is associated with reductions in inventory investment. To measure this, it is useful to look at the ratio $\Sigma_{t=1}^{T}(1 - S_{gt})(\Delta^2 N_t + \Delta^2 M_t)/\Sigma_{t=1}^{T}(1 - S_{gt})\Delta Q_t^q$, where $S_{gt} = 1$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

if q_t^g is in an expansion phase and $S_{gt} = 0$ if it is in a contraction phase. Hence this statistic records the average fraction of GDP declines across a recession, which are associated with declines in inventory investment. It equals 0.77 for the 1959–2005 period (0.78 over 1959–1983). Applied to just materials inventory investment, the statistics would be 0.27 and 0.31. Hence the effect is strongest for finished goods inventory investment. This statistic is often used to support the view that inventory investment movements seem to "account" for a large part of output decline. *Prima facie* it seems to conflict with our tentative conclusion expressed above that finished goods inventories play little role in business cycles, but, in fact, there may be no inconsistency between the two perspectives. Because M, N are endogenous variables, it is possible that the absence of inventories might not change the cycle in Q_t^g much, but some common factor among the M, N series makes them cohere with Q_t^g. Again, a model is needed that enables one to see if it is possible to reconcile these different "stylized facts."

3. THE MODEL AND ITS EULER EQUATIONS

The model of the representative firm that we use is an extension of the one developed by Humphreys et al. (2001).[12] The model in Humphreys et al. has the advantage that it is a model of inventories broken down by stage of fabrication and thus distinguishes between finished goods—"output"—inventories and materials and supplies—"input"—inventories. The latter includes work in progress inventories as well; hereafter, we use the term "materials inventories" to refer to the sum of materials and supplies and work in progress inventories. The model thus permits an analysis of the role that each type of inventory stock plays in the production and sales process. This is an important advantage of the model, as finished goods and materials inventories may have played very different roles in the reduction of the volatility of GDP growth. Figure 1 reported that the materials–output ratio had declined about 50% since the mideighties, but the finished goods–sales ratio had remained about constant. This suggests that, to the extent that improved inventory management techniques have had a role to play in reducing the volatility of GDP growth, materials inventories may have been more important than finished goods inventories. Further, "just-in-time" techniques, which have become more widely used in recent years, are more applicable to materials inventory management than to that of finished goods.

3.1. The Production Function

We begin with a specification of the short-run production function, which is assumed to be Cobb–Douglas,

$$\begin{aligned} Y_t &= F(L_t, U_t, \epsilon_{yt}) \\ &= L_t^{\gamma_1} U_t^{\gamma_2} e^{\epsilon_{yt}}, \end{aligned} \tag{2}$$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

where $\gamma_1 + \gamma_2 < 1$, which implies strict concavity of the production function in materials usage and labor. Here, Y_t is output, L_t is labor input, U_t is materials usage, and ϵ_{yt} is a technology shock. Note that U_t is the *flow* of materials used in the production process. When production and inventory decisions are made, the capital stock is assumed to be taken as given by the firm and to be growing at a constant rate, which will be captured by a deterministic trend in the empirical work. Finally, the firm is assumed to purchase intermediate goods (work in process) from outside suppliers rather than producing them internally.[13] Thus, intermediate goods are analogous to materials and supplies, so work-in-process inventories can be lumped together with materials and supplies inventories. Because Y_t is gross output, we refer to equation (2) as the gross production function.

3.2. The Cost Structure

The firm's total cost structure consists of three major components: labor costs, inventory holding costs, and materials costs. This section describes each component.

Labor costs. Labor costs are

$$\begin{aligned} LC_t &= W_t L_t + W_t \widetilde{A}(L_t, L_{t-1}) \\ &= W_t L_t + W_t A(\Delta l_t - \Delta \bar{l}) L_{t-1}, \end{aligned} \tag{3}$$

where $\Delta l_t = \Delta \log L_t \approx \Delta L_t / L_{t-1}$ is the growth rate of labor and $\Delta \bar{l}$ is the steady state growth rate of labor. The first component, $W_t L_t$, is the standard wage bill where W_t is the real wage rate. The second component, $\widetilde{A}(L_t, L_{t-1})$, is an adjustment cost function intended to capture the hiring and firing costs associated with changes in labor inputs. The adjustment cost function has the usual properties: Specifically,

$$\begin{aligned} A' &\gtreqless 0, \qquad \text{as} \quad \Delta l_t \gtreqless \Delta \bar{l}, \\ A(0) = A'(0) &= 0, \qquad A'' > 0. \end{aligned}$$

Adjustment costs for labor accrue whenever the growth rate of the firm's labor force is different from the steady state growth rate. Further, adjustment costs exhibit rising marginal costs.

Inventory holding costs. Inventory holding costs for finished goods inventories are

$$HC_t^N = \Phi^N (N_{t-1}, X_t, \epsilon_{nt-1}) = \delta_1 \left(\frac{N_{t-1}}{X_t} \right)^{\delta_2} X_t + \delta_3 N_{t-1} + \epsilon_{nt-1} N_{t-1}, \tag{4}$$

$$\delta_1 > 0, \quad \delta_2 < 0, \quad \delta_3 > 0,$$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

where ϵ_{nt} is the shock to finished goods inventory holding costs. Inventory holding costs consist of two basic components. One, $\delta_1(N_{t-1}/X_t)^{\delta_2}X_t$, captures the idea that, given sales, higher inventories reduce costs in the form of lost sales because they reduce stockouts. The other, $\delta_3 N_{t-1}$, captures the idea that higher inventories raise costs because they raise holding costs in the form of storage costs, insurance costs, etc.[14] The effects of technological advances that improve inventory management methods can be captured, for example, through a change in δ_2 and changes in the volatility of the shock to finished goods inventory holding costs.[15]

Inventory holding costs for materials inventories are

$$\begin{aligned} HC_t^M &= \Phi^M\left(V_{t-1}M_{t-1}, Y_t, \epsilon_{mt-1}\right) \\ &= \tau_1\left(\frac{V_{t-1}M_{t-1}}{Y_t}\right)^{\tau_2} Y_t + \tau_3 V_{t-1}M_{t-1} + \epsilon_{mt-1}V_{t-1}M_{t-1}, \\ &\qquad \tau_1 > 0, \quad \tau_2 < 0, \quad \tau_3 > 0, \end{aligned} \tag{5}$$

where ϵ_{mt} is the shock to materials inventory holding costs. As above, there are two basic components: One, $\tau_1(V_{t-1}M_{t-1}/Y_t)^{\tau_2}Y_t$, captures the idea that, given output, higher inventories reduce costs in the form of lost output because they reduce stockouts and disruptions to the production process. The other, $\tau_3 V_{t-1}M_{t-1}$, captures the idea that higher inventories raises costs because they raise holding costs in the form of storage costs, insurance costs, etc. Note that materials inventory holding costs depend on production, rather than sales. This is because stocking out of materials inventories entails costs associated with production disruptions—lost production, so to speak—that are distinct from the costs associated with lost sales. Lost production may be manifested by reduced productivity or a failure to realize production plans. Again, the effects of technological advances that improve inventory management methods can be captured, for example, through a change in τ_2 and changes in the volatility of the shock to materials inventory holding costs.[16]

The finished goods and material inventory holding costs differ because the firm holds the two inventory stocks for different reasons. The firm stocks finished goods inventories to guard against random demand fluctuations. Finished goods inventories thus facilitate sales. On the other hand, the firm stocks materials inventories to guard against random fluctuations in productivity, materials prices and deliveries, and other aspects of production. Materials stocks thus facilitate production. Although sales and production are highly positively correlated, they perform different roles in the firm and differ enough at high frequencies to justify different specifications.

Materials costs. Finally, we turn to the cost of purchasing materials. We assume that the real cost of purchasing materials is given by

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

$$MC_t = V_t D_t + V_t \widetilde{\Gamma}(V_t D_t, Y_t) = V_t D_t + V_t \Gamma\left(\frac{V_t D_t}{Y_t}\right) D_t \tag{6}$$

$$= V_t D_t \left[1 + \Gamma\left(\frac{V_t D_t}{Y_t}\right)\right],$$

$$\Gamma' \geqq 0, \qquad \Gamma'' > 0$$

$$\Gamma(0) = 0,\ \Gamma\left(\frac{VD}{Y}\right) = 0,\ \Gamma'\left(\frac{VD}{Y}\right) = 0,$$

where V_t is a real "base price" for materials. The term, $V_t D_t$, is the value of purchases and deliveries valued at the base price. The term, $V_t \widetilde{\Gamma}(V_t D_t, Y_t)$, represents a premium that may need to be paid over and above the base price to undertake the level of purchases and deliveries, D_t. It is assumed to rise at an increasing rate with the amount purchased and delivered.

Two cases may be distinguished:

(1) Increasing marginal cost: $\Gamma' > 0$. In this case, the firm faces a rising supply price for materials purchases. When purchases are high relative to current stocks, the firm experiences increasing marginal costs due to higher premia that must be paid to acquire materials more quickly. A rationale for such a rising supply price is that the firm is a monopsonist in the market for materials. This is most likely to occur when materials are highly firm- or industry-specific and the firm or industry is a relatively large fraction of market demand.[17] The rising marginal cost of course gives rise to the "smoothing" of purchases.

(2) Constant marginal cost: $\Gamma' = 0$. In this case, the firm is in effect a price-taker in competitive input markets and is able to purchase all the materials it needs at the prevailing market price.

3.3. Cost Minimization

Assume that the representative firm takes sales (X_t) and factor prices (V_t and W_t) as exogenous. The firm's optimization problem is to minimize the discounted present value of total costs (TC),

$$E_0 \sum_{t=0}^{\infty} \beta^t TC_t = E_0 \sum_{t=0}^{\infty} \beta^t (LC_t + HC_t + MC_t), \tag{7}$$

where $\beta = (1 + r)^{-1}$ is the discount factor. The constraints are the production function, (2), and the two laws of motion governing inventory stocks. The identity for finished goods inventories is

$$\Delta N_t = Y_t - X_t, \tag{8}$$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

and the identity for materials inventories is

$$\Delta M_t = D_t - U_t. \tag{9}$$

The firm chooses $\{L_t, U_t, Y_t, M_t, N_t, D_t\}_{t=0}^{\infty}$ to minimize equation (7) subject to the constraints (2), (8), and (9).

3.4. Optimality Conditions

Assume that, when the representative firm makes decisions, current values of exogenous variables are in its information set. Then, the optimality conditions are

$$W_t + \Lambda_{1t} F_L(L_t, U_t, \epsilon_{yt}) + W_t \widetilde{A}_1(L_t, L_{t-1}) + E_t \beta W_{t+1} \widetilde{A}_2(L_{t+1}, L_t) = 0, \tag{10}$$

$$\Lambda_{1t} F_U(L_t, U_t, \epsilon_{yt}) + \Lambda_{3t} = 0, \tag{11}$$

$$V_t + V_t \widetilde{\Gamma}_D(V_t D_t, Y_t) - \Lambda_{3t} = 0, \tag{12}$$

$$\Phi_Y^M(V_{t-1} M_{t-1}, Y_t, \epsilon_{mt-1}) + \widetilde{\Gamma}_Y(V_t D_t, Y_t) - \Lambda_{1t} - \Lambda_{2t} = 0, \tag{13}$$

$$\Lambda_{2t} - \beta E_t \Lambda_{2t+1} + \beta E_t \Phi_N^N(N_t, X_{t+1}, \epsilon_{nt}) = 0, \tag{14}$$

$$\Lambda_{3t} - \beta E_t \Lambda_{3t+1} + \beta E_t \Phi_M^M(V_t M_t, Y_{t+1}, \epsilon_{mt}) = 0 \tag{15}$$

together with the production function, (2), and the accumulation equations, (8) and (9). Note that Λ_{1t}, Λ_{2t}, and Λ_{3t} are the Lagrangian multipliers associated with (2), (8), and (9), respectively.

To interpret the optimality conditions, observe that (10)–(12) imply that

$$\frac{F_L(L_t, U_t, \epsilon_{yt})}{F_U(L_t, U_t, \epsilon_{yt})} = \frac{W_t[1 + \widetilde{A}_1(L_t, L_{t-1}) + \beta E_t \frac{W_{t+1}}{W_t} \widetilde{A}_2(L_{t+1}, L_t)]}{V_t[1 + \widetilde{\Gamma}_D(V_t D_t, Y_t)]},$$

which states that the relative marginal products of labor and materials utilization must equal their relative marginal costs. In the case of labor, the latter is the current wage rate together with the marginal adjustment costs of hiring and firing labor. In the case of materials utilization, the marginal cost is the purchase price of an additional unit of materials together with any marginal premia due to rising supply prices that must be paid to acquire materials.

The optimality condition, (14), can be summed forward to get

$$\Lambda_{2t} = -E_t \sum_{i=0}^{\infty} \beta^{i+1} \Phi_N^N(N_{t+i}, X_{t+1+i}, \epsilon_{nt+i}),$$

which states that the imputed value to the firm of holding an additional unit of finished goods inventories must equal (minus) the expected discounted value of current and future marginal finished goods inventory holding costs.[18] The minus sign indicates that at the optimum the firm must operate on the downward-sloping component of the inventory-holding cost function for finished goods inventories.

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

This is the segment where the marginal value of holding finished goods inventories to avoid lost sales exceeds the marginal storage costs. Similarly, the optimality condition, (15), can be summed forward to get

$$\Lambda_{3t} = -E_t \sum_{i=0}^{\infty} \beta^{i+1} \Phi_M^M(V_{t+i} M_{t+i}, Y_{t+1+i}, \epsilon_{mt+i}),$$

which states that the imputed value to the firm of holding an additional unit of materials inventories must equal (minus) the expected discounted value of current and future marginal material inventory holding costs.[19] Again, the minus sign indicates that at the optimum the firm must operate on the downward-sloping component of the inventory-holding cost function for materials inventories. This is the segment where the marginal value of holding materials inventories to avoid lost output due to stockouts and disruptions of the production process exceeds the marginal storage costs.

Now, define the following shares and ratios:

$$\begin{aligned} S_{L,t} &= \frac{W_t L_t}{Y_t}, \quad S_{U,t} = \frac{V_t U_t}{Y_t}, \quad R_{N,t} = \frac{N_t}{X_t}, \quad R_{M,t} = \frac{V_t M_t}{Y_t}, \\ R_{D,t} &= \frac{V_t D_t}{Y_t}, \quad R_{Y,t} = \frac{Y_t}{X_t}, \quad R_{\Lambda_3,t} = \frac{\Lambda_{3t}}{V_t}. \end{aligned} \tag{16}$$

It is assumed that the variables in (16) are stationary with finite expected values, even if the variables they are constructed from have unit roots. It will also be assumed that V_t is a stationary random variable and this will make Λ_{3t}, Λ_{1t}, and Λ_{2t} stationary. Then, applying the functional form assumptions in (2), (4), (5), and (6), the resulting optimality conditions can be log-linearized around constant expected steady state values—Appendix C provides the details. In the notation, lower case letters are logarithms of upper case letters, so, for example, $l_t = \log L_t$, and thus the growth rate of a variable is $\Delta l_t = \Delta \log L_t \approx \Delta L_t / L_{t-1}$. A - above a variable denotes a constant expected steady state value. A $\wedge$ above an uppercase letter denotes a log deviation, whereas above a lowercase letter it denotes a simple (i.e., nonlogarithmic) deviation. So, for example, the log deviation of $R_{D,t}$ from its expected value is $\hat{R}_{D,t} = \log R_{D,t} - \log \overline{R}_D$, where $\overline{R}_D = E(R_{D,t})$, whereas the simple deviation of the growth rate of sales is $\Delta \widehat{x}_t = \Delta x_t - \Delta \overline{x}$. Similar notation applies to other variables. The log-linearized optimality conditions are then

$$\widehat{S}_{Lt} - \widehat{\Lambda}_{1t} + \varphi \Delta \widehat{l}_t - \beta \varphi E_t \Delta \widehat{l}_{t+1} = 0, \tag{17}$$

$$\widehat{S}_{Ut} - \widehat{\Lambda}_{1t} + \widehat{R}_{\Lambda_3,t} = 0, \tag{18}$$

$$\eta \widehat{R}_{D,t} - \widehat{R}_{\Lambda_3,t} = 0, \tag{19}$$

$$\mu_2 \overline{R}_M (1 - \Delta \overline{y}) [\widehat{R}_{M,t-1} - \Delta \widehat{y}_t] + \eta \overline{R}_D \widehat{R}_{D,t} + \overline{\Lambda}_1 \widehat{\Lambda}_{1t} + \overline{\Lambda}_2 \widehat{\Lambda}_{2t} = 0, \tag{20}$$

$$\overline{\Lambda}_2 \widehat{\Lambda}_{2t} - \beta \overline{\Lambda}_2 E_t \widehat{\Lambda}_{2t+1} + \beta \mu_1 E_t (\widehat{R}_{N,t} - \Delta \widehat{x}_{t+1}) + \beta \epsilon_{nt} = 0, \tag{21}$$

$$\widehat{R}_{\Lambda_3,t} - \beta E_t [\Delta \widehat{v}_{t+1} + \widehat{R}_{\Lambda_3,t+1}] + \beta \mu_2 E_t [\widehat{R}_{M,t} - \Delta \widehat{y}_{t+1}] + \beta \epsilon_{mt} = 0, \tag{22}$$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

$$\Delta\widehat{y}_t = \gamma_1 \Delta\widehat{l}_t + \gamma_2 \Delta\widehat{u}_t + \epsilon_{yt} - \epsilon_{yt-1}, \tag{23}$$

$$\widehat{R}_{N,t} - (1 - \Delta\overline{x})\,\widehat{R}_{N,t-1} + (1 - \Delta\overline{x})\,\Delta\widehat{x}_t - \frac{\overline{R}_Y}{\overline{R}_N}\widehat{R}_{Y,t} = 0, \tag{24}$$

$$\widehat{R}_{M,t} - (1 + \Delta\overline{v} - \Delta\overline{y})\,[\widehat{R}_{M,t-1} + \Delta\widehat{v}_t - \Delta\widehat{y}_t] - \frac{\overline{R}_D}{\overline{R}_M}\widehat{R}_{D,t} + \frac{\overline{S}_U}{\overline{R}_M}\widehat{S}_{U,t} = 0, \tag{25}$$

where $\varphi = A''(0)$, $\eta = \Gamma''(\overline{R}_D)\overline{R}_D^2$, $\mu_1 = (\delta_2 - 1)\delta_2\delta_1[\overline{R}_N(1 - \Delta\overline{x})]^{\delta_2 - 1}$, and $\mu_2 = (\tau_2 - 1)\tau_2\tau_1[\overline{R}_M(1 - \Delta\overline{y})]^{\tau_2 - 1}$.

Finally, other quantities can be derived in a similar way. Because we will ultimately want to reconstruct Q_t^g it is useful to find an expression for the log deviation of the ratio of GDP to gross output for the goods sector. Specifically, define $R_{Q^g,t} = Q_t^g / Y_t$. Then, to obtain $\widehat{R}_{Q^g,t}$, recall that the definition of GDP is

$$Q_t^g = Y_t - V_t U_t,$$

so that

$$\frac{Q_t^g}{Y_t} = 1 - \frac{V_t U_t}{Y_t}.$$

Then, using the definition of $R_{Q^g,t}$ and the utilization share, $S_{U,t} = V_t U_t / Y_t$, we have that

$$R_{Q^g,t} = 1 - S_{U,t}.$$

A log-linear approximation of this then yields

$$\overline{R}_{Q^g}\widehat{R}_{Q^g,t} = -\overline{S}_U \widehat{S}_{U,t}. \tag{26}$$

4. QUANTIFYING MODEL PARAMETERS

4.1. Matching Data and Model Variables

The model above is that of a representative firm. To apply the model to the goods sector as a whole, we assume that the representative firm behaves as if it is vertically integrated, so that it is representative of the whole goods sector of the economy. The representative firm holds materials inventory stocks that it uses in conjunction with labor (and capital) to produce output of finished goods, which it adds to finished goods inventories. The finished goods inventories may be held by manufacturers, wholesalers, or retailers. In effect, we treat the representative firm as managing the inventory stocks of finished goods whether they are held on the shelves of the manufacturer, the wholesaler, or the retailer. Appendix B deals with the exact measurement of the variables in the data.

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

4.2. Choosing a Sample Period for Estimation

If the GM were to be regarded as a consequence of changes in inventory technology it would be necessary to allow for changes in the model's parameters after it had occurred. Given that the ratios $R_{M,t}$ and $R_{N,t}$ were reasonably stable prior to the GM, it makes sense to estimate the model over the period 1959/1–1983/4. Even though the sample of data is relatively small, quite precise estimates of the parameters are obtained with it. Now one might also consider estimating the model over 1984/1–2005/4, but to do that it is necessary to decide how the parameters would have changed in this second period. Change they must have, given the behavior of $R_{M,t}$ and $R_{N,t}$. But how is less obvious, because there is a good deal of controversy over whether the GM occurred abruptly or gradually.[20] If we were to allow for parameter change then two models would be needed: one of output and inventories and one of how the parameters changed. At least in studying the GM it seems preferable to determine model parameter values that are compatible with data prior to the GM, and then to ask whether we can set new values that would replicate the GM. This is the strategy followed in the next section, and is the standard approach used in the literature: a benchmark set of parameter values is changed and the response of output volatility is noted. The only difference in what we do here is that the benchmark values are estimated rather than calibrated. Thus, in the remainder of this section we describe the estimation of the model parameters using data prior to the GM. We do, however, conclude the section with some commentary on what happens when the model is applied to 1984–2005 data.

4.3. Complete and Incomplete Model Estimation Strategies

As pointed out above, we quantify the model parameters using data over the period 1959/1–1983/4. Ideally one would like to have begun with 1947/1, but quarterly data on M_t and N_t were not available over that earlier period. Once estimated, the model can then be used to explore some of the questions raised in the Introduction.

The nine equations (17)–(25) describe the optimality conditions. These can be reduced to seven using (18) and (19) to eliminate $\hat{R}_{\Lambda_{3,t}}$ and $\widehat{\Lambda}_{1t}$. The seven remaining equations contain ten endogenous variables,

$$z_t' = [\,\widehat{S}_{L,t}\ \widehat{S}_{U,t}\ \Delta\hat{y}_t\ \widehat{R}_{D,t}\ \widehat{\Lambda}_{2t}\ \widehat{R}_{N,t}\ \widehat{R}_{M,t}\ \Delta\hat{u}_t\ \Delta\hat{l}_t\ \hat{R}_{Y,t}\,],$$

so that three extra equations need to be found. These come from identities describing $S_{L,t}$, $S_{U,t}$, and $R_{Y,t}$. They involve both endogenous and exogenous variables. As an example, we have $\Delta\hat{S}_{L,t} = \Delta\hat{w}_t + \Delta\hat{l}_t - \Delta\hat{y}_t$. Thus, it is necessary to specify the variables exogenous to the system, and there are six of these—v_t, Δw_t, Δx_t, ε_{yt}, ε_{nt}, ε_{mt}.

Later, in contrasting the estimation of complete and incomplete systems, it is useful to separate the exogenous variables into two groups—$\zeta_t' = [v_t\ \Delta w_t\ \Delta x_t]$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

and $\eta_t' = [\Delta\varepsilon_{yt}\ \varepsilon_{nt}\ \varepsilon_{mt}]$. The latter will be exogenous in a complete system as well as an incomplete one. But the former might become endogenous in a complete system. For stochastic assumptions about the exogenous variables, we will take v_t to be stationary, but, based on unit root tests, w_t and x_t are taken to be $I(1)$ (ADF tests of -2.2 and -0.72).[21] Hence they enter as changes in the definition of ζ_t. Therefore the goods model system of ten equations can be written in the form

$$B_{0z}z_t + B_{0\zeta}\zeta_t = Az^*_{t-1} + BE^+_t(z^*_{t+1}) + G\eta_t, \tag{27}$$

where $\bar{z}_t' = [z_t'\ \zeta_t']$ and E^+_t denotes the information used by the agents in forming expectations.

We now turn to the estimation of the parameters of (27) when it is an incomplete system, i.e., no *structural* equations are provided for ζ_t.[22] If there are no future expectations, this is traditionally done by "completing" the system in (27) with some *auxiliary* equations describing ζ_t. The combination of the equations in (27) and the auxiliary equations for ζ_t will be termed the *pseudo-system.* Now the limited information maximum likelihood (LIML) estimator chooses as the auxiliary equations the reduced-form equations for ζ_t that would be consistent with a complete system, and then applies MLE to the resulting pseudo-system. If all that is desired is a consistent rather than efficient estimator, it is not necessary to use all the variables (and lags) in the complete model reduced form to set up the auxiliary equations for ζ_t. Any subset will do. Based on this reasoning, one strategy is to make the auxiliary equations for the ζ_t depend only upon lags of ζ_t. For example, ζ_t could be described as a VAR and used as the auxiliary equations.

Now when future expectations are present the situation becomes more complicated. If one could estimate these then they could be treated as other endogenous variables. Suppose that we had a complete system and z^+_t were all the variables in it. Then the solution to the complete system would be a VAR in z^+_t. Consequently, $E^+_t(\bar{z}_{t+1})$ would be a linear function of $\bar{z}_t$ and its lags, e.g., in the case of a VAR(2), $E^+_t(\bar{z}_{t+1}) = A_1\bar{z}^+_t + A_2\bar{z}^+_{t-1}$. The weights A_j could be estimated by fitting a VAR(2) to the data on $\bar{z}^+_{t+1}$. Then we could treat the calculated expectation as an "observed" endogenous variable that was constructed through an identity. So expectations per se do not create any major estimation difficulties.

When we are estimating only a single sector, however, the expectation that can be formed in the way just described will involve a limited set of information $E_t(\bar{z}_{t+1})$, rather than what was potentially used by the agents, $E^+_t(\bar{z}_{t+1})$. Then (27) can be written as

$$B_{0z}z_t + B_{0\zeta}\zeta_t = A\bar{z}_{t-1} + BE_t(\bar{z}_{t+1}) + G\eta_t + B\xi_{t+1}, \tag{28}$$

where $\xi_{t+1} = E^+_t(\bar{z}_{t+1}) - E_t(\bar{z}_{t+1})$. Under a normality assumption for the shocks, the density function for $\bar{z}_t$ conditional on the limited information will be normal, with conditional mean $E_{t-1}(\bar{z}_t)$. But because $E_t(\xi_{t+1}) = 0$, $E_{t-1}(\bar{z}_t)$ will

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

be that from the incomplete model, ignoring $B\xi_{t+1}$. Thus the LIML estimator of the parameters of the incomplete model will be consistent. Efficiency is lost, though. In summary, partial equilibrium models such as our goods sector model can be validly estimated without specifying a complete general equilibrium model using other sectors.

In the description above of how expectations are handled, the complete model was only used to suggest what variables appeared in the VAR and what order it was. A subset of these variables will correspond to the information available from the goods sector alone. If the expected values of $\bar{z}_t$ are formed by estimating a VAR for $\bar{z}_t$ as described above, then the restrictions coming from the goods sector model have not been specifically incorporated into their construction unless the parameters of the VAR have been estimated to be consistent with them. To incorporate such restrictions, one finds the expectations implied by the pseudo-system. This is done by solving for z_t from the pseudo-system in the standard way. The program we use for estimation, DYNARE, computes expectations that are consistent with whatever model and information are specified. In our case the model is the pseudo-system involving the combination of the goods sector structural equations and some auxiliary equations describing ζ_t, whereas the information in the latter is limited to that from the goods sector.

4.4. Some Properties of the Data

In our model derivations, we have implicitly assumed that there are two cointegrating relations among any $I(1)$ variables. These are given by the finished goods inventory–sales ratio, $R_{N,t} = N_t/X_t$, and the materials inventory/output ratio, $R_{M,t} = V_t M_t/Y_t$. ADF tests on $\ln R_{M,t}$ and $\ln R_{N,t}$ were -3.09 and -3.17, easily rejecting the null hypothesis of a unit root at the 5% level of significance.[23] The labor share will be treated as unobserved in the empirical work and was assumed to be $I(0)$ in the model derivations. Taking these three ratios to be $I(0)$, we show in Appendix C that $S_{U,t}$, $R_{D,t}$, $R_{Y,t}$, $R_{\Lambda_{3t}}$, Λ_{1t}, and Λ_{2t} will then be $I(0)$ as well. Not unexpectedly, unit root tests applied to n_t, m_t, y_t, x_t, and w_t reveal these series to have unit roots. Two of these variables, x_t and w_t, are exogenous, and so can be taken as the permanent components underlying the $I(1)$ series. But, because there are two cointegrating vectors among the five $I(1)$ variables, there must be another common permanent component, and we identify this as the technology shock; i.e., ε_{yt} has a unit root.

It is necessary to describe the auxiliary equations for ζ_t and specify how the shocks η_t evolve. Because Δx_t and Δw_t are observable, we can fit processes to these series. An AR(1) was fitted to each, producing AR coefficients of 0.33 and 0.02, respectively.[24] Fitting an AR(2) to v_t gave

$$v_t = 1.21v_{t-1} - 0.29v_{t-2} + 0.0053\varepsilon_t^v.$$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

The shocks $\Delta\varepsilon_{yt}, \varepsilon_{nt}$, and ε_{mt} were all assumed to be AR(1) processes with coefficients ρ_y, ρ_n, and ρ_m, respectively, and these are to be estimated by MLE.

4.5. Estimation Results

The parameters in the Euler equations can be divided into three groups,

$$(I)\ \theta_1 = [\beta, \gamma_1, \gamma_2, \overline{R}_M, \overline{R}_N, \overline{R}_Y, \overline{R}_D], \tag{29}$$

$$(II)\ \theta_2 = [\tau_3, \delta_3, \overline{\Lambda}_1, \overline{\Lambda}_2], \tag{30}$$

$$(III)\ \theta_3 = [\tau_1, \tau_2, \delta_1, \delta_2, \varphi, \eta, \sigma_y, \sigma_n, \sigma_m, \rho_y, \rho_n, \rho_m], \tag{31}$$

where σ_y, σ_n, and σ_m are the standard deviations of the unobservable shocks $\varepsilon_y, \varepsilon_n$, and ε_m, respectively. The elements of θ_1 either are preset in the case of $\beta = 0.99$, $\gamma_1 = 0.22$, and $\gamma_2 = 0.66$, or are estimated using sample means for the ratios and steady state conditions, and are $\overline{R}_M = 0.187$, $\overline{R}_N = 0.413$, $\overline{R}_Y = 1.003$, and $\overline{R}_D = 0.718$. The γ_j parameters were set because the absence of capital in the model makes it difficult to estimate the parameters of a production process.[25]

Estimates of the four parameters in θ_2 are found from appropriate steady state conditions—see (C.36)–(C.44) in Appendix C. To obtain values for θ_2 from those steady state equations requires values for θ_3. Once a set of values for θ_3 is established, θ_1 and θ_2 are then fixed, so that it is only necessary to estimate θ_3 from the data. Finally, from the optimality conditions in (17) and (18), it is evident that τ_1, τ_2, δ_1, and δ_2 enter only through μ_1 and μ_2, so only two of these parameters can be identified. Consequently we set $\delta_1 = \tau_1 = 1$, which are common normalizations in the empirical literature on inventories. Doing so leaves the final set of parameters to be estimated as $\psi = [\tau_2, \delta_2, \varphi, \eta, \sigma_y, \sigma_n, \sigma_m, \rho_y, \rho_n, \rho_m]$.

Maximum likelihood estimation is then done to get estimates of ψ.[26] A complication arises from the fact that we only have data on six variables—$z_t^* = [S_{U,t}, R_{N,t}, R_{M,t}, \Delta x_t, v_t, \Delta w_t]$. If z_t had been observed then the likelihood would be just that of the VAR in z_t. When not all variables in z_t are observed, it is necessary to add an observation equation connecting the observed and unobserved variables, i.e., $z_t^* = Hz_t$, to the state dynamics equations, as summarized by the VAR in z_t. The likelihood of the resulting state space form formed from this equation and (28) is then that for the observed z_t^*, and it is found recursively by an application of the Kalman predictor. This is a standard way of performing MLE on DSGE models [see Ireland (2004)] and is adopted in the DYNARE program that we utilize for estimation.[27]

Maximum likelihood estimates of the parameters are available in Table 4. All estimates of the parameters have the predicted theoretical signs and are statistically significant. Observe that $\delta_2 = -0.026$ and $\tau_2 = -0.015$, which indicates that the (absolute value of the) elasticity of the benefits to the firm of holding finished

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

TABLE 4. Model parameter estimates

	τ_2	δ_2	φ	η	σ_y
Estimate	−0.015	−0.026	1.16	5.7	0.008
t-statistic	2.8	2.9	4.3	4.9	10.1
	σ_n	σ_m	ρ_n	ρ_m	ρ_y
Estimate	0.002	0.005	0.96	0.88	0.11
t-statistic	3.5	3.6	26.5	10.6	5.0

goods inventories exceeds that of materials inventories. The implied estimates of τ_3 and δ_3 from the steady state equations are 0.07 and 0.04, respectively.[28]

4.6. Matching the Model and the Data

At this point, it is useful to look at the adequacy of the model in reproducing some of the more interesting characteristics of the data. Our objective was to produce a set of reasonable estimates for the model parameters rather than to rely on some calibration strategy. Accordingly, we provide an examination of the match with a set of characteristics that are most relevant to the questions we are asking. As mentioned earlier, because the goods sector cycle involves the first two moments of Δq_t^g, these are given most attention.

First, the model implies that the standard deviation for Δq_t^g, $\sigma_{\Delta q^g}$, would be 0.0243, versus the value of 0.021 in the data.[29] The test statistic that these are different is 2.4, so that the model seems to produce a reasonable fit to the variance of Δq_t^g, although a little too high. The parameter estimates imply a first-order serial correlation coefficient in Δq_t^g of 0.15, whereas the data say there is virtually no serial correlation. The standard error on this estimate is, however, 0.10 and so there is no significant difference between them. In terms of cycle outcomes, the durations of expansions and contractions for Q^g are 9.45/3.43(data) and 9.28/4.23(model).[30]

Other features of interest relate to the first-order serial correlation coefficients in Δm_t and Δn_t. There is quite high serial correlation in these, in contrast to the situation for Δq_t^g. Specifically, they are 0.625 and 0.318, respectively. The model predicts them to be 0.610 and 0.344, respectively, so it captures the fact that the serial correlation patterns in inventories and goods-sector GDP are quite different.

Finally, as we observed earlier, one of the reasonably constant features of the data is the association between the decline in inventory investment and the fraction of the decline in GDP over recessions. For 1959–1983 this was 0.78 for total inventory investment and 0.31 for materials inventory investment (again for the goods sector). The model predicts these to be 0.88 and 0.39, respectively (found by averaging over 500 sets of simulated data with 98 observations in each set). The range of variation that comes out of the simulations is quite large, meaning that the predictions are consistent with the data at an extremely low level of

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

significance. Consequently the model seems to capture many of the features of inventory movements and their relations to the goods cycle quite well.

4.7. The Model in the Post-1983 Period

We attempted to fit the model to the 1984–2005 period, assuming that there was a new set of parameters. However, the autoregressive parameters of both the raw material and final good inventory shocks tended to unity. Because these are constrained to be less than unity, a maximum to the likelihood was not found. Thus it is clear that a model with a single change in parameters cannot fit the post-1983 data. But this might have been expected because, with non–time varying parameters (and stationary shocks), ratios such as $R_{M,t}$ need to be stationary processes. Now it was mentioned earlier that this is true of the data over 1959–1984, but it is not true for the post-1984 period. Indeed, ADF tests for the logs of $R_{N,t}$ and $R_{M,t}$ found to be -3.17 and -3.09 in the first sample, become -1.95 and -0.37 in the second. Sometimes ADF tests can be misleading, so that computing the first-order AR coefficient of the series being tested is useful. For $R_{N,t}$ it is 0.93, and so we might think that it is highly persistent but still $I(0)$. However, the same coefficient is 0.998 for $\ln R_{M,t}$, and it would make no sense to treat it as $I(0)$. The nonstationarity of $R_{M,t}$ therefore shows up as a unit root in the raw material shocks. Of course, because the steady state value of $R_{M,t}$ depends directly on the model parameters, this just means they have changed. If they were to be allowed to change to track $R_{M,t}$, it would be necessary that they evolve stochastically. On a practical level, such evolution introduces significant stochastic nonlinearities, and such a model would be hard to solve under rational expectations. Moreover, the distinction between a shock and a parameter becomes blurred if both are unobserved and evolve stochastically. For our purposes, there seems to be only marginal value in trying to accommodate these changes, particularly when addressing questions relating to the GM. But even when assessing how important inventories are to the cycle, it seems more informative to simply look at how the cycle would change if the parameter values were chosen to replicate the 1984–2005 sample values of $R_{N,t}$ and $R_{M,t}$. In doing so, we find that it does not change our conclusions.

5. EXPERIMENTS WITH THE MODEL

5.1. Analysis of Fluctuations and the Great Moderation

The causes for the Great Moderation have been much debated in the literature. When this feature was observed, it was natural that one look at changes taking place in the economy that might lead to such an outcome. Because there had been great advances in inventory control methods, in particular the development of "just in time" philosophies relating to production, it seemed possible that this might be a source of the changes; e.g., see Kahn et al. (2002). To assess this possibility, we first adopt the above estimated model as the "base model" and then ask how $\sigma_{\Delta q^g}$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

varies with changes in selected parameters of the model. One set of parametric changes that we are interested in is what happens when δ_2 and τ_2 change. This implies that fewer finished goods inventories and materials inventories are held as a ratio to sales and output, respectively. Consider, for example, a decline in the absolute value of τ_2, that shifts the materials inventory holding cost function. Intuitively, such a decline captures the idea that computerization, just-in-time procedures, or other technological advances in inventory management techniques imply that the firm can experience the same level of lost production with a smaller level of materials inventories, given the level of output. Or, alternatively, a given materials inventory/output ratio will generate a smaller amount of lost production. A similar interpretation applies to a decline in the absolute value of δ_2. To compute values of Δq_t^g when a parameter changes, we reverse the estimation strategy, and now solve for ratios such as R_N and R_M as functions of the estimated model parameters. A second set of parametric changes of interest is the impact of changes in the volatility of the observed and unobserved shocks, including the shocks to inventory holding costs. Reductions in the volatility of the shocks to finished goods and materials inventory holding costs are other possible measures of advances in inventory management methods. We examine the effects of such measures as well.

$|\tau_2|$ was reduced by 20%. This produced a decline in the VM/Y ratio that roughly matches what was seen over the period 1984/1–2005/4. Because, as noted above, N_t/X_t changed only minimally after 1984 we make a smaller change in $|\delta_2|$. Similarly, the reduction in standard deviations of all shocks was set to 50%, as that was roughly what happened to the observable shocks over that period. So these experiments are about how we might have expected fluctuations to have changed over the second period given that the volatility reduction in observed shocks was matched by that in unobserved ones. The experiments involving reductions in the standard deviations of shocks also give some insight into what the main sources of fluctuations would be. Finally, we present an experiment in which δ_2 and τ_2 are just 1/100 of the values in Table 4. Such a reduction means that inventories have very low value to the firm, in the sense of reducing the costs of lost sales or lost output, and thus emulates a situation where inventories are not held and are thus not present in the system. Table 5 gives the results of these experiments.[31]

It is clear that, if the change in inventory technology can be thought of as involving a change in the magnitude of τ_2, so that smaller materials stocks relative to output are an optimal choice, then this produces only slightly smaller fluctuations in goods GDP. As Table 5 indicates, a 20% decline in τ_2 produces only a 1% reduction in $\sigma_{\Delta q^g}$. If a change in inventory technology is reflected in a reduction in the volatility of the shock to materials stocks, then it too has little impact. A 50% reduction in σ_m also produces only a 1% reduction in $\sigma_{\Delta q^g}$. Similarly, changes in δ_2 or σ_n also produce very small changes in $\sigma_{\Delta q^g}$. Hence changes in parameters that can be interpreted as capturing advances in inventory management techniques cannot be the source of the reduced volatility in Δq_t^g.

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

TABLE 5. Effects on $\sigma_{\Delta q^g}$ of parameter perturbations

	$\sigma^a_{\Delta q^g}$	$\rho^a_{\Delta q^g}$
Base	0.0243	0.158
$0.9\delta_2$	0.0242	0.156
$0.8\tau_2$	0.0241	0.156
$0.5\sigma_x$	0.0204	0.014
$0.5\sigma_w$	0.0243	0.158
$0.5\sigma_v$	0.0233	0.16
$0.5\sigma_n$	0.0242	0.157
$0.5\sigma_m$	0.0241	0.16
$0.5\sigma_y$	0.0194	0.315
$0.01(\delta_2, \tau_2)$	0.0208	0.156

[a] Effect of cuts in parameters.

Major declines in $\sigma_{\Delta q^g}$ are produced only by substantial declines in the volatilities of sales, the technology shock, and, to a lesser degree, materials prices. To quantify this, decomposing the variance of Δq^g_t into the various shocks produces the result that 48% is due to the technology shock, 39% is due to the sales shock, 10% is due to the materials price shock, and 1% is due to the materials inventory shock. Neither wages nor the finished goods inventory shocks are important. If the volatility of all shocks was reduced by 50%, $\sigma_{\Delta q^g}$ would become 0.012, which is very close to the actual reduction over the 1984/1–2005/4 period—see Table 5. If only the observed ones were reduced, $\sigma_{\Delta q^g}$ would only have dropped to 0.019, so that the unobservable shocks are critical to explain this phenomenon. Specifically, a large decline in the volatility of technology shocks will be needed.

An alternative viewpoint, expressed in Kahn et al. (2002), is that the information set of firms may have changed as a consequence of computerization. In particular, sales may now be known with greater accuracy. To assess this, we considered an experiment in which it was assumed that only past values of sales were known to the agents, as contrasted with the experiments above, where current information is available.[32] To conduct this experiment, the model was reestimated using the new information set and the implied $\sigma_{\Delta q^g}$ was still found to be 0.0243. Hence the reduction in the volatility of Δq^g_t as a result of improved information about sales is negligible, because the base case in Table 5 represents what it would be with the expanded information.

In light of these results, it is useful to consider briefly the debate over whether monetary policy had an impact on $\sigma_{\Delta q^g}$. One might expect that this effect would work through sales and, although the decline in the volatility of the latter has made a contribution, it would not have led to the observed decline in volatility if technology shocks had not changed as well. Thus it is hard to see monetary policy as being the major driving force in the reduction in the volatility in the goods sector.

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

TABLE 6. Effects on cycles of parameter perturbations

	Durations (quarters)[a]		Amplitudes (percentages)[a]	
	Contractions	Expansions	Contractions	Expansions
Base	4.23	9.28	−5.84	16.02
$0.9\delta_2$	4.24	9.32	−5.81	16.03
$0.8\tau_2$	4.23	9.34	−5.77	16.02
$0.5\sigma_x$	3.80	11.19	−4.14	15.6
$0.5\sigma_w$	4.23	9.34	−5.84	16.09
$0.5\sigma_v$	4.22	9.57	−5.58	16.02
$0.5\sigma_n$	4.24	9.27	−5.81	15.99
$0.5\sigma_m$	4.23	9.31	−5.79	16.02
$0.5\sigma_y$	4.11	9.86	−4.73	15.29
$0.01(\delta_2, \tau_2)$	4.31	10.57	−4.89	15.60

[a] Effect of cuts in parameters.

5.2. Analysis of Cycles

Although the nature of the business cycle depends upon the volatility of Δq_t^g, it also depends upon the mean of this process and the nature of any serial correlation in it. Consequently, the experiments above were repeated to determine their effects upon the cycle in q_t^g. Table 6 shows how the durations and amplitudes of expansions and contractions in q_t^g would change. It should be noted that over the period of estimation, 1959/1–1983/4, the durations of contractions and expansions were 3.43 and 9.45 quarters, and so the length of expansions using the estimated model parameters (the "base" simulation) is quite close to that actually observed.

Given that we know that cycle length depends upon the volatility and serial correlation properties of Δq_t^g—Harding and Pagan (2002)—the results in Table 6 are largely consistent with, and predictable by, the outcomes in Table 5. As in the experiments above, changes in parameters that govern inventory holding behavior, that is, changes in τ_2, δ_2, σ_m, or σ_n, have very small effects on the length or amplitude of the cycle. An exception occurs for the relative effects of the experiments involving a reduction in sales and technology shock volatilities. Here the cycle becomes longer with the first experiment, even though the volatility decrease is slightly less than in the second experiment. This shows that the degree of serial correlation in Δq_t^g is also important for cycle outcomes.

The final experiment, where $|\delta_2|$ and $|\tau_2|$ were reduced to 1% of their value, creates an environment where the firm has essentially no incentive to hold inventories. In effect, this emulates an environment where inventories are not present in the system. This experiment shows that the presence of inventories in the system results in more and shorter cycles, although on average they are shorter by only a little more than one quarter. Hence, overall, the importance of inventories to the average cycle is limited, even though it may be that for particular cycles their

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

presence has a greater effect. It should be noted that in no case are there complex roots in the Δq_t^g process and so there are no oscillations.

6. CONCLUSION

In this paper, we have developed a model of the optimal holding of finished goods and materials inventories by a goods-producing firm and have used it to analyze a number of questions that have come up about the role of inventories. The paper yields two main conclusions. First, we showed that inventories have had only a small effect upon the average duration of expansions and contractions of the business cycle in the U.S. economy. To show the latter, we looked at business cycles in terms of the turning points in the level of goods GDP, which is a very different perspective from the traditional work on inventory cycles, which has looked at oscillations in aggregate activity. This conclusion emerges despite the fact that the model produces predictions that are consistent with many of the features observed in the data regarding the behavior of inventories over the business cycle. Second, we showed that changes in inventory management technology have had little effect upon the volatility of GDP growth in the goods sector of the U.S. economy. The model we developed allows a role for materials prices in producing cycles and we found that the latter did have some impact, but the main drivers of changes in the business cycle were technology and sales variations. In particular, the decline in the volatility of GDP growth in the goods sector during the GM cannot be explained without substantial declines in the volatility of both sales and technology.

We note that we have treated both sales and technology shocks as exogenous. Extensions of the model that relax these assumptions may allow more of a role for advances in inventory technology. For example, in line with Basu, Fernald and Kimball (2006), if advances in inventory management affected total factor productivity, then there might be an indirect effect of inventory management on technology and thus GDP growth. For another, we allow the benefits to holding finished goods inventories to enter the firm's decision process through inventory holding costs. If, instead, sales are made endogenous, and finished goods inventories affect sales directly, as in Bils and Kahn (2000), this may alter the parameters that govern the benefits of holding finished goods inventories, and thus influence GDP growth. The latter seems unlikely, given that the finished goods inventory/sales ratio seems not to have declined during the GM.

The approach we take in this paper is to develop a partial equilibrium model of the goods sector of the economy. This enables us to focus on the basic question of whether advances in inventory technology were responsible for the decline in the volatility of GDP growth in the goods sector with a minimal number of assumptions about the rest of the economy. Our approach is quite different from the VAR approaches and the more recent general equilibrium models of the economy as a whole. Nonetheless, the different approaches that have been used to address the basic question appear to be coming to similar conclusions, namely, that advances in inventory management techniques played a relatively small role

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

in the Great Moderation and that the presence of inventories in the system neither is a major source of shocks nor substantially modifies the propagation mechanism. This is reassuring in that knowledge is advanced when very different approaches arrive at essentially similar conclusions regarding important questions.

NOTES

1. VAR models have also been used to look at questions arising from inventories, e.g., Blanchard and Simon (2001), Stock and Watson (2002, 2003), Ahmed et al. (2004), Herrera and Pesavento (2005, 2009), Irvine and Schuh (2005), and McCarthy and Zakrajsek (2007).

2. Iacoviello et al. (2007) also build and estimate a model with two types of inventories. But they work with a model where finished goods inventories are held by households for utility services, the stock of input inventories enters directly into the production function, and it is estimated with Bayesian methods. In contrast, we focus on the goods sector alone, treat the reasons for holding input and output inventories as methods of reducing the costs of production disruptions and lost sales, respectively, assume that production depends on the usage of material inputs, and estimate the model with maximum likelihood procedures.

3. The modification involves a more efficient algorithm developed by James Engel for locating turning points. GAUSS and MATLAB programs for it are available at http://www.ncer.edu.au/data/.

4. However, there is quite a high standard deviation for $\hat{\rho}_j$ for the smaller sample, and so the point estimates are not significantly different from one another.

5. The volatility of services-based GDP is so low over the second period that no recession was evident in that sector during the 1990s and 2000s. Recessions in these periods were essentially a phenomenon in the goods sector.

6. There are missing elements in this definition, such as energy usage and imports. Also, we have assumed that the price of materials used is the same as that of the stock of raw materials.

7. We can measure the quantities in the identities above in the following way. First, V_t is taken to be the implicit price deflator for raw materials used in the private business sector divided by the implicit price deflator for goods GDP. Second, U_t may then be recovered from $(Y_t - Q_t^g)/V_t$.

8. Cycles in Q_t^g and q_t^g are identical, as the log transformation is monotonic, and so turning points in each series will be at the same points in time.

9. The data on x_t are not available before 1959/1.

10. We look at $V_t M_t/Y_t$, because any change in V_t would be expected to offset M_t/Y_t so that movements in the latter may simply reflect a response to relative price changes rather than a technological change. Of course, even this ratio may not fully control for such a price effect. Note that essentially the same pattern occurs for M_t/Y_t. It too declines by about 45% after 1984, but slightly less precipitously.

11. The ratios presented in Figure 1 are constructed with chain-weighted data for the relevant variables. An alternative is to use ratios constructed with nominal data. We re-computed the ratios with the latter and found that they give essentially the same patterns as those shown in Figure 1. Ramey and Vine (2006) provide further discussion of ratios based on chain-weighted versus nominal data.

12. The model presented here extends the Humphreys et al. (2001) model by defining the two types of inventory holding costs so that they are in "ratio" form. This implies that the steady state values are ratios and can be taken to be constant in the long run. It should be noted that the formulation presented here does not assume particular target stocks. Rather, these are implied by the optimality conditions.

13. To allow for production of intermediate goods within the firm requires extending the production function to incorporate joint production of final and intermediate goods. This extension is a substantial modification of the standard production process that we leave for future work.

14. These two components underlie the rationale for the quadratic inventory holding costs in the standard linear–quadratic model. The formulation above separates the components and assumes

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

constant-elasticity functional forms, which facilitates log-linearization around constant steady states. We assume that the firm minimizes discounted expected costs and thereby abstract from market structure issues. The latter are dealt with by Bils and Kahn (2000).

15. Observe that (3.3) implies a "target stock" of finished goods inventories that minimizes finished goods inventory holding costs. Assuming that $\epsilon_{nt-1} = 0$, the target stock, N_t^*, is

$$N_t^* = -\left(\frac{\delta_3}{\delta_1 \delta_2}\right)^{\frac{1}{\delta_2 - 1}} X_t,$$

making the implied target stock proportional to sales. This is analogous to the target stock assumed in the standard linear–quadratic model. Note that the target stock is not the steady state-stock of finished goods inventories. The steady state stock minimizes total costs in steady state, whereas the target stock merely minimizes inventory holding costs. The steady state stock will be derived below.

16. Similarly, observe that (3.4) implies a "target stock" of materials inventories that minimizes materials inventory holding costs. Assuming $\epsilon_{mt-1} = 0$, the target stock, $V_t M_t^*$, is

$$V_t M_t^* = -\left(\frac{\tau_3}{\tau_1 \tau_2}\right)^{\frac{1}{\tau_2 - 1}} Y_t,$$

so that the implied target stock is proportional to output. Note here as well that the target stock of materials inventories is not the steady state stock of materials inventories. The latter will be derived below.

17. This is analogous to the literature on adjustment cost models for investment in plant and equipment where external adjustment costs are imposed in the form of a rising supply price for capital goods. See, e.g., Abel (1979).

18. This assumes that the transversality condition, $\lim_{T \to \infty} \beta^T E_t \Lambda_{2t+T} = 0$, is satisfied.

19. This assumes that the transversality condition, $\lim_{T \to \infty} \beta^T E_t \Lambda_{3t+T} = 0$, is satisfied.

20. See McConnell and Perez-Quiros (2000) for an example where the GM is taken to occur as an abrupt shift. Blanchard and Simon (2001) and, more recently, Davis and Kahn (2008) are examples where it occurs as a slow adjustment. It should be said that the continuous decline in $R_{M,t}$ does suggest that any change in parameters was not abrupt, although the responses might have taken time to work out.

21. The v_t process is a very persistent one, but the sharp rise in oil prices (and associated raw material prices) in 1974 had a major effect upon the unit root tests. Putting in a dummy variable for that event suggested that v_t was stationary, and so we decided to treat it as $I(0)$. As a robustness check, we also redid the estimation when v_t was assumed to have a unit root. The likelihood that was attained was slightly lower, and the parameter estimates and the results of the experiments were essentially the same as those reported in a later section.

22. The following discussion follows that of Fukač and Pagan (2010).

23. These tests are based on using the SBC criterion to choose the best lag length for the ADF test. The result is lags of 2 and 3, respectively.

24. We did run regressions of Δx_t against lags of variables such as $S_{u,t-1}$, $R_{N,t-1}$, and y_{t-1}, but these provided little explanatory power over lagged values of Δx_t, so we chose the simpler specification.

25. If we set $\gamma_2 = 0.71$ to reflect the share of raw materials as measured in our data, then it seems virtually impossible to allow for a role for capital, as a realistic share for labor would mean that the sum of the raw material and labor shares would exceed unity. In addition, it was found that setting γ_j below the preset values above resulted in a substantial decline in the likelihood, so these values seemed a reasonable compromise.

26. Because $S_{U,t}$ and $R_{M,t}$ are $I(1)$ over the post-1983 sample, it is not possible to estimate the model over the period 1984/1–2005/1, because we have used that assumption many times in the derivations. If one attempts to do so, all the unobservable shocks have unit roots and the estimation basically breaks down. Essentially, we would need to allow the parameters to vary in order to fit the data after 1983/4, and, as we said earlier, this would require some assumptions about how they vary.

27. We use version 3.065 of DYNARE, written by S. Adjemian, M. Juillard, and O. Kamenik.

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

28. Recall that in notes 11 and 12 we stated the formulae for calculating the "target stock" for both finished goods inventories and materials and supplies inventories. These minimize the *holding costs* for the respective types of inventory. The parameter estimates would then imply that the "target stock" for finished goods inventories is $N_t^*/X_t = 0.65$ whereas the "target stock" for materials and supplies inventories is $V_t M_t^*/Y_t = 0.21$. These might be compared with the steady state finished goods inventory/sales ratio, which is $\overline{R}_N = 0.413$, and the steady state materials inventory/output ratio, which is $\overline{R}_M = 0.187$. However, it should be borne in mind that the the latter reflect what the ratios would be that minimize *total costs* in steady state and so will imply levels of inventories (relative to sales or output) different from those coming from minimizing the holding cost functions alone.

29. The characteristics of Δq_t^g reported in Table 2 for the sample period 1947–1983 are virtually the same as those from 1959–1983.

30. Note that this is for the subperiod 1959/3–1983/4, and it is quite a short period to measure cycle characteristics.

31. If one wishes to find the impact of combinations of parameter changes rather than a single one, the approach used in assessing the sensitivity of computable general equilibrium model solutions to parameters can be followed. Given that the model solution is a function of the parameters, $y_t = g(\theta)$, where θ are the parameters, linearization around some base set of parameters produces $y_t - y^* = \frac{\partial g}{\partial \theta}(\theta^*)(\theta - \theta^*)$. Of course this is only a local approximation, but experiments show that it is quite accurate for the likely range of variation in the coefficients. $\frac{\partial g}{\partial \theta}(\theta^*)$ can be measured from Table 5; e.g., a 50% reduction in σ_x leads to a 16% reduction in $\sigma_{\Delta q^g}$, so that a 1% change in σ_x would lead to a 0.32% change in $\sigma_{\Delta q^g}$.

32. This means that $E_t(\Delta \hat{x}_{t+1})$ in the Euler equations is replaced by $\rho^2 \Delta \hat{x}_{t-1}$ rather than $\rho \Delta \hat{x}_t$.

33. The Bureau of Economic Analysis series on manufacturing and trade inventories experienced discontinuities in the late 1990s due to the conversion from SIC to NAICS classification systems. We dealt with this problem by constructing inventory stock series by extending backward each series from current stocks using growth rates.

34. Note that there are no materials and supplies and work in progress inventories in wholesale trade or retail trade.

REFERENCES

Abel, A. (1979) *Investment and the Value of Capital*, New York: Garland.

Ahmed, S., A. Levin, and B. A. Wilson (2004) Recent U.S. macroeconomic stability: Good luck, good policies or good practices? *Review of Economics and Statistics* 86, 824–832.

Basu, S., J. G. Fernald, and M.S. Kimball (2006) Are technology improvements contractionary. *American Economic Review* 96, 1418–1448.

Bils, M. and J. A. Kahn (2000), What inventory behavior tells us about business cycles. *American Economic Review*, 90, 458–480.

Blanchard, O. and J. Simon (2001) The long and large decline in U.S. output volatility. *Brookings Papers on Economic Activity* 1, 135–174.

Blinder, A. and L. Maccini (1991) Taking stock: A critical assessment of recent research on inventories. *Journal of Economic Perspectives* 5, 73–96.

Burns, A. F., and W. C. Mitchell (1946) *Measuring Business Cycles*. New York: NBER.

Davis, S. and J. Kahn (2008) Interpreting the Great Moderation: Changes in the volatility of economic activity as the macro and micro levels. *Journal of Economic Perspectives* 22(4), 155–180.

Fukač, M. and A. R. Pagan (2010) Limited information estimation and evaluation of DSGE models. *Journal of Applied Econometrics* 25, 55–70.

Harding, D. and A. R. Pagan (2002) Dissecting the cycle: A methodological investigation. *Journal of Monetary Economics* 49, 365–381.

Herrera, A. and E. Pesavento (2005) The decline in U.S. output volatility: Structural changes and inventory investment. *Journal of Business and Economic Statistics* 23, 462–472.

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

Herrera, A. and E. Pesavento (2009) Oil price shocks, systematic monetary policy, and the Great Moderation. *Macroeconomic Dynamics* 13, 107–137.
Humphreys, B. R., L. J. Maccini, and S. Schuh (2001) Input and output inventories. *Journal of Monetary Economics* 47, 347–375.
Iacoviello, M., F. Schiantarelli, and S. Schuh (2007) Input and Output Inventories in General Equilibrium. Working paper in economics 658, Boston College.
Ireland, P. (2004) A method for taking models to the data. *Journal of Economic Dynamics and Control* 28, 1205–1226.
Irvine, O. and S. Schuh (2005) The Role of Co-movement and Inventory Investment in the Reduction of Output Volatility. Federal Reserve Bank of Boston, Working Paper No. 05-09.
Kahn, J. (2008) Durable Goods Inventories and the Great Moderation. Federal Reserve Bank of New York, Staff Reports No. 325.
Kahn, J., M. McConnell, and G. Perez-Quiros (2002) On the causes of the increased stability of the U.S. economy. *Federal Reserve Bank of New York Economic Policy Review* 8(1), 183–206.
Khan, A. and J. Thomas (2007a) Explaining inventories: A business cycle assessment of the stockout avoidance and (*S*,*s*) motives. *Macroeconomic Dynamics* 11, 638–664.
Khan, A. and J. Thomas (2007b) Inventories and the business cycle: An equilibrium analysis of (*S*,*s*) policies. *American Economic Review* 97, 1165–1188.
McCarthy, J. and E. Zakrajsek (2007) Inventory dynamics and business cycles: What has changed? *Journal of Money, Credit and Banking* 39, 591–613.
McConnell, M. and G. Perez-Quiros (2000) Output fluctuations in the United States: What has changed since the early 1980s. *American Economic Review* 90, 1464–1476.
Metzler, L. (1941) The nature and stability of inventory cycles. *Review of Economics and Statistics* 23, 113–129.
Metzler, L. (1947) Factors governing the length of the inventory cycle. *Review of Economics and Statistics* 29, 1–15.
Morley, J. and A. Singh (2009) Inventory Mistakes and the Great Moderation. Mimeo, Washington University.
NBER (2003) *The NBER's Recession Dating Procedure* available at http://www.nber.org/cycles/recessions.html).
Ramey, V. and D. Vine (2006) Tracking the source of the decline in GDP volatility: An analysis of the automobile industry. *American Economic Review* 96, 1876–1889.
Stock, J. and M. Watson (2002) Has the business cycle changed and why? In M. Gertler and K. Rogoff (eds.), *NBER Macroeconomics Annual 2002*. Cambridge, MA, MIT Press.
Stock, J. and M. Watson (2003) Has the business cycle changed? Evidence and expanations? In *Monetary Policy and Uncertainty: Adapting to a Changing Economy*, Federal Bank Of Kansas City Symposium, 9–56.
Wen, Y. (2005) Understanding the inventory cycle. *Journal of Monetary Economics* 52, 1533–1555.
Wen, Y. (2008) Input and Output Inventory Dynamics. *Working paper* 2008-008, Federal Reserve Bank of St Louis.
Whelan, K. (2002) A guide to U.S. chain aggregated NIPA data. *Review of Income and Wealth* 48, 217–233.

APPENDIX A: VARIABLE DEFINITIONS

A summary of definitions of variables is as follows:

Q_t = Real GDP
Q_t^g = Real GDP for the goods sector
Q_t^s = Real GDP for the services sector
X_t = Gross sales

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

Y_t = Gross output
N_t = Finished goods inventories
M_t = Materials inventories = materials and supplies inventories + work-in-progress inventories
U_t = Utilization of materials in production
D_t = Deliveries of materials
V_t = Materials prices
L_t = Labor services
W_t = Wage rate
$R_{Nt} = N_t/X_t$ = Finished goods inventory/sales ratio
$R_{Mt} = V_t M_t/Y_t$ = Materials inventory/output ratio
$R_{Yt} = Y_t/X_t$ = Output/sales ratio
$R_{D,t} = V_t D_t/Y_t$ = Deliveries/output ratio
$S_{L,t} = W_t L_t/Y_t$ = Labor share of output
$S_{U,t} = V_t U_t/Y_t$ = Materials utilization share of output
Λ_{1t}, Λ_{2t}, and Λ_{3t} are Lagrangian multipliers
ϵ_{yt} = Technology shock
ϵ_{nt} = Shock to finished goods inventory holding costs
ϵ_{mt} = Shock to materials inventory holding costs

APPENDIX B: DATA DESCRIPTION

In the model we assume that the representative firm behaves as if it is vertically integrated, so that it is representative of the goods sector of the whole economy. We assume that the representative firm holds materials inventory stocks, which it uses in conjunction with labor (and capital) to produce output of finished goods, which it adds to finished goods inventories. The finished goods inventories may then be held by manufacturers, wholesalers, or retailers. In effect, we treat the representative firm as managing the inventory stocks of finished goods whether they are held on the shelves of the manufacturer, the wholesaler, or the retailer.

The data on inventories, sales, and GDP are quarterly, seasonally adjusted, (2000) chain-weighted series in billions of dollars, and cover the period 1959/1 through 2005/4. Value added or GDP is the real value of GDP for the goods sector of the economy; it was obtained directly as an aggregate from the Bureau of Economic Analysis. To construct aggregates of chain-weighted data, we used the appropriate Tornqvist approximation for chain-weighted data as suggested by Whelan (2002). The Tornqvist approximation weights growth rates of components of an aggregate by the average nominal share in the two periods for which the growth rate is calculated. Accordingly, we constructed an aggregate stock of finished goods inventories by aggregating the real value of finished goods inventories in manufacturing, wholesale trade, and retail trade using the Tornqvist approximation.[33] The aggregate stock of materials inventories is constructed by aggregating the materials and supplies and work in progress inventories held by manufacturers again using the Tornqvist approximation.[34] A data series for gross output of the goods sector is not available from government statistical agencies. We therefore constructed an approximate measure of gross output for the goods sector by aggregating gross sales or shipments for the manufacturing and trade sectors of the economy and the changes in finished goods inventory investment for those sectors

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

again using the Tornqvist approximation. GDP and gross output are of course measured as quarterly flows, and inventories are measured as end-of-quarter stocks.

The series on W is the ratio of the average hourly earnings for goods producing industries divided by the implicit price deflator for sales of the business sector. V is found by dividing the implicit price deflator for input inventories by the implicit price deflator for the sales of the business sector.

APPENDIX C: DERIVATION OF OPTIMALITY CONDITIONS

Recall again that lowercase letters are the logarithms of uppercase letters; so, for example, $l_t = \log L_t$, and thus the growth rate of a variable is $\Delta l_t = \Delta \log L_t \approx \Delta L_t / L_{t-1}$. Then, using the functional form assumptions, the optimality conditions, (10)–(15), together with the production function, (2), and the accumulation equations, (8) and (9) become

$$W_t + \Lambda_{1t}\gamma_1 \frac{Y_t}{L_t} + W_t A'(\Delta l_t - \Delta\bar{l}) + \beta E_t W_{t+1} A(\Delta l_{t+1} - \Delta\bar{l}) \qquad \textbf{(C.1)}$$

$$-\beta E_t W_{t+1} \frac{L_{t+1}}{L_t} A'\left(\Delta l_{t+1} - \Delta\bar{l}\right) = 0,$$

$$\Lambda_{1t}\gamma_2 \frac{Y_t}{U_t} + \Lambda_{3t} = 0, \qquad \textbf{(C.2)}$$

$$V_t\left[1 + \Gamma\left(\frac{V_t D_t}{Y_t}\right) + \frac{V_t D_t}{Y_t}\Gamma'\left(\frac{V_t D_t}{Y_t}\right)\right] - \Lambda_{3t} = 0, \qquad \textbf{(C.3)}$$

$$(1-\tau_2)\,\tau_1 \left(\frac{V_{t-1}M_{t-1}}{Y_t}\right)^{\tau_2} - \left(\frac{V_t D_t}{Y_t}\right)^2 \Gamma'\left(\frac{V_t D_t}{Y_t}\right) - \Lambda_{1t} - \Lambda_{2t} = 0, \qquad \textbf{(C.4)}$$

$$\Lambda_{2t} - \beta E_t \Lambda_{2t+1} + \beta E_t \delta_2 \delta_1 \left(\frac{N_t}{X_{t+1}}\right)^{\delta_2 - 1} + \beta\delta_3 + \beta\epsilon_{nt} = 0, \qquad \textbf{(C.5)}$$

$$\Lambda_{3t} - \beta E_t \Lambda_{3t+1} + \beta E_t \tau_2 \tau_1 V_t \left(\frac{V_t M_t}{Y_{t+1}}\right)^{\tau_2 - 1} + \beta V_t \tau_3 + \beta V_t \epsilon_{mt} = 0,$$

$$Y_t = (L_t)^{\gamma_1}(U_t)^{\gamma_2} e^{\epsilon_{yt}}, \qquad \textbf{(C.6)}$$

$$N_t - N_{t-1} - Y_t + X_t = 0, \qquad \textbf{(C.7)}$$

$$M_t - M_{t-1} - D_t + U_t = 0. \qquad \textbf{(C.8)}$$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

Rearranging terms, we have that

$$1+\Lambda_{1t}\gamma_1\frac{Y_t}{W_tL_t}+A'\left(\Delta l_t-\Delta\bar{l}\right)+\beta E_t\frac{W_{t+1}}{W_t}\left[A\left(\Delta l_{t+1}-\Delta\bar{l}\right)\right] \tag{C.9}$$
$$-\beta E_t\frac{W_{t+1}}{W_t}\frac{L_{t+1}}{L_t}A'\left(\Delta l_{t+1}-\Delta\bar{l}\right)=0,$$

$$\Lambda_{1t}\gamma_2\frac{Y_t}{V_tU_t}+\frac{\Lambda_{3t}}{V_t}=0, \tag{C.10}$$

$$1+\Gamma(\frac{V_tD_t}{Y_t})+\frac{V_tD_t}{Y_t}\Gamma'(\frac{V_tD_t}{Y_t})-\frac{\Lambda_{3t}}{V_t}=0, \tag{C.11}$$

$$(1-\tau_2)\,\tau_1\left(\frac{V_{t-1}M_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t}\right)^{\tau_2}-\left(\frac{V_tD_t}{Y_t}\right)^2\Gamma'\left(\frac{V_tD_t}{Y_t}\right)-\Lambda_{1t}-\Lambda_{2t}=0, \tag{C.12}$$

$$\Lambda_{2t}-\beta E_t\Lambda_{2t+1}+\beta E_t\delta_2\delta_1\left(\frac{N_t}{X_t}\frac{X_t}{X_{t+1}}\right)^{\delta_2-1}+\beta\delta_3+\beta\epsilon_{nt}=0, \tag{C.13}$$

$$\frac{\Lambda_{3t}}{V_t}-\beta E_t\frac{V_{t+1}}{V_t}\frac{\Lambda_{3t+1}}{V_{t+1}}+\beta E_t\tau_2\tau_1\left(\frac{V_tM_t}{Y_t}\frac{Y_t}{Y_{t+1}}\right)^{\tau_2-1}+\beta\tau_3+\beta\epsilon_{mt}=0, \tag{C.14}$$

$$\left(\frac{Y_t}{Y_{t-1}}\right)=\left(\frac{L_t}{L_{t-1}}\right)^{\gamma_1}\left(\frac{U_t}{U_{t-1}}\right)^{\gamma_2}e^{\epsilon_{yt}-\epsilon_{yt-1}}, \tag{C.15}$$

$$\frac{N_t}{X_t}-\frac{N_{t-1}}{X_{t-1}}\frac{X_{t-1}}{X_t}-\frac{Y_t}{X_t}+1=0, \tag{C.16}$$

$$\frac{V_tM_t}{Y_t}-\frac{V_t}{V_{t-1}}\frac{V_{t-1}M_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t}-\frac{V_tD_t}{Y_t}+\frac{V_tU_t}{Y_t}=0. \tag{C.17}$$

Now, define the growth rate of, for example, X_t , by $X_t=(1+\Delta x_t)X_{t-1}$, and similarly for other variables. Further, use the approximation $1/(1+\Delta x_t)\approx 1-\Delta x_t$ to rewrite the optimality conditions as

$$1+\Lambda_{1t}\gamma_1\frac{Y_t}{W_tL_t}+A'(\Delta l_t-\Delta\bar{l})+\beta E_t\,(1+w_{t+1})\,A(\Delta l_{t+1}-\Delta\bar{l}) \tag{C.18}$$
$$-\beta E_t\,(1+w_{t+1})\,(1+l_{t+1})\,A'\left(\Delta l_{t+1}-\Delta\bar{l}\right)=0,$$

$$\Lambda_{1t}\gamma_2\frac{Y_t}{V_tU_t}+\frac{\Lambda_{3t}}{V_t}=0, \tag{C.19}$$

$$1+\Gamma\left(\frac{V_tD_t}{Y_t}\right)+\frac{V_tD_t}{Y_t}\Gamma'\left(\frac{V_tD_t}{Y_t}\right)-\frac{\Lambda_{3t}}{V_t}=0, \tag{C.20}$$

$$(1-\tau_2)\,\tau_1\left[\frac{V_{t-1}M_{t-1}}{Y_{t-1}}\,(1-\Delta y_t)\right]^{\tau_2}-\left(\frac{V_tD_t}{Y_t}\right)^2\Gamma'\left(\frac{V_tD_t}{Y_t}\right)-\Lambda_{1t}-\Lambda_{2t}=0, \tag{C.21}$$

$$\Lambda_{2t}-\beta E_t\Lambda_{2t+1}+\beta E_t\delta_2\delta_1\left[\frac{N_t}{X_t}\,(1-\Delta x_{t+1})\right]^{\delta_2-1}+\beta\delta_3+\beta\epsilon_{nt}=0, \tag{C.22}$$

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

$$\frac{\Lambda_{3t}}{V_t} - \beta E_t\,(1+\Delta v_{t+1})\,\frac{\Lambda_{3t+1}}{V_{t+1}} + \beta E_t \tau_2 \tau_1 \left[\frac{V_t M_t}{Y_t}\,(1-\Delta y_{t+1})\right]^{\tau_2 - 1} + \beta\tau_3 + \beta\epsilon_{mt} = 0, \tag{C.23}$$

$$\Delta y_t = \gamma_1 \Delta l_t + \gamma_2 \Delta u_t + \epsilon_{yt} - \epsilon_{yt-1}, \tag{C.24}$$

$$\frac{N_t}{X_t} - \frac{N_{t-1}}{X_{t-1}}\,(1-\Delta x_t) - \frac{Y_t}{X_t} + 1 = 0, \tag{C.25}$$

$$\frac{V_t M_t}{Y_t} - (1+\Delta v_t)\,(1-\Delta y_t)\,\frac{V_{t-1} M_{t-1}}{Y_{t-1}} - \frac{V_t D_t}{Y_t} + \frac{V_t U_t}{Y_t} = 0. \tag{C.26}$$

Then, using the definitions of the shares and ratios stated in (16), the optimality conditions may be written as

$$1 + \Lambda_{1t}\gamma_1 S_{L,t}^{-1} + A'\left(\Delta l_t - \Delta\bar{l}\right) + \beta E_t\,(1+\Delta w_{t+1})\,A\left(\Delta l_{t+1} - \Delta\bar{l}\right) - \beta E_t\,(1+\Delta w_{t+1})\,(1+\Delta l_{t+1})\,A'\left(\Delta l_{t+1} - \Delta\bar{l}\right) = 0, \tag{C.27}$$

$$\Lambda_{1t}\gamma_2 S_{U,t}^{-1} + R_{\Lambda_3,t} = 0, \tag{C.28}$$

$$1 + \Gamma(R_{D,t}) + R_{D,t}\Gamma'(R_{D,t}) - R_{\Lambda_3,t} = 0, \tag{C.29}$$

$$(1-\tau_2)\,\tau_1 [R_{M,t-1}\,(1-\Delta y_t)]^{\tau_2} - R_{D,t}^2 \Gamma'(R_{D,t}) - \Lambda_{1t} - \Lambda_{2t} = 0, \tag{C.30}$$

$$\Lambda_{2t} - \beta E_t \Lambda_{2t+1} + \beta E_t \delta_2 \delta_1 \left[R_{N,t}\,(1-\Delta x_{t+1})\right]^{\delta_2 - 1} + \beta\delta_3 + \beta\epsilon_{nt} = 0, \tag{C.31}$$

$$R_{\Lambda_3,t} - \beta E_t\,(1+\Delta v_{t+1})\,R_{\Lambda_3,t+1} + \beta E_t \tau_2 \tau_1 \left[R_{M,t}\,(1-\Delta y_{t+1})\right]^{\tau_2 - 1} + \beta\tau_3 + \beta\epsilon_{mt} = 0, \tag{C.32}$$

$$\Delta y_t = \gamma_1 \Delta l_t + \gamma_2 \Delta u_t + \epsilon_{yt} - \epsilon_{yt-1}, \tag{C.33}$$

$$R_{N,t} - R_{N,t-1}\,(1-\Delta x_t) - R_{Y,t} + 1 = 0, \tag{C.34}$$

$$R_{M,t} - (1+\Delta v_t - \Delta y_t)\,R_{M,t-1} - R_{D,t} + S_{U,t} = 0, \tag{C.35}$$

where again Λ_{1t}, Λ_{2t}, and Λ_{3t} are Lagrangian multipliers associated with (2), (8), and (9), respectively.

In Section 4.2 it was assumed that $R_{N,t}$, $R_{M,t}$, and $S_{L,t}$ are all $I(0)$. If $R_{N,t}$ and $R_{M,t}$ are $I(0)$, S_{Ut}, $R_{D,t}$, $R_{Y,t}$, $R_{\Lambda_{3t}}$, Λ_{1t}, and Λ_{2t} are $I(0)$ as well. To see this, observe that if $R_{N,t}$ and $R_{M,t}$ are $I(0)$, then (C.34), (C.31), and (C.32) imply that $R_{N,t}$, $\Lambda_{2,t}$ and $R_{\Lambda_{3,t}}$ are $I(0)$. Using (C.28) and (C.29) give $S_{U,t}$ and $R_{D,t}$ as $I(0)$. Using (C.30) we have $\Lambda_{1,t}$ as $I(0)$ and so from (C.28) $S_{U,t}$ is $I(0)$. Note that (C.34) makes $R_{Y,t}$ an $I(0)$ variable.

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press

We assume that in steady state the expected values of the factor input shares, $S_{U,t}$ and $S_{L,t}$, the ratios $R_{N,t}$, $R_{M,t}$, $R_{D,t}$, $R_{Y,t}$, and $R_{\Lambda_3,t}$, and the growth rates of variables are constants. The nonstochastic steady state conditions are then

$$1 + \overline{\Lambda}_1 \gamma_1 \overline{S}_L^{-1} = 0, \tag{C.36}$$

$$1 + \overline{\Lambda}_1 \gamma_2 \overline{S}_U^{-1} = 0, \tag{C.37}$$

$$\overline{R}_{\Lambda_3} = 1, \tag{C.38}$$

$$(1 - \tau_2)\, \tau_1 \left[\overline{R}_M\, (1 - \Delta\overline{y})\right]^{\tau_2} - \overline{\Lambda}_1 - \overline{\Lambda}_2 = 0, \tag{C.39}$$

$$(1 - \beta)\, \overline{\Lambda}_2 + \beta\delta_2\delta_1 \left[\overline{R}_N\, (1 - \Delta\overline{x})\right]^{\delta_2 - 1} + \beta\delta_3 = 0, \tag{C.40}$$

$$[1 - \beta\, (1 + \Delta\overline{v})] + \beta\tau_2\tau_1 \left[\overline{R}_M\, (1 - \Delta\overline{y})\right]^{\tau_2 - 1} + \beta\tau_3 = 0, \tag{C.41}$$

$$\gamma_1 \Delta\overline{l} + \gamma_2 \Delta\overline{u} = \Delta\overline{y}, \tag{C.42}$$

$$1 + \Delta\overline{x}\overline{R}_N = \overline{R}_Y, \tag{C.43}$$

$$\overline{R}_D + (\Delta\overline{v} - \Delta\overline{y})\, \overline{R}_M = \overline{S}_U. \tag{C.44}$$

Again, in Section 4.2 above on estimation strategy, we stated that estimates of the four parameters in θ_2 are found from appropriate steady state conditions. Specifically, they are found from the four equations corresponding to the steady state conditions (C.37) and (C.38)–(C.41).

Now, log-linearizing the optimality conditions, (C.27)–(C.35), around the constant steady state values, (C.36)–(C.44), yields the log-linearized optimality conditions stated above in (17)–(25).

https://doi.org/10.1017/S1365100511000010 Published online by Cambridge University Press