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ARTICLE

The dilemma between repugnancy and sadism extends beyond critical-level utilitarianism

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Abstract
We consider axioms under which population principles face an unavoidable choice between accepting the Repugnant Conclusion or the Very Sadistic Conclusion. Our analysis reveals that this dilemma persists even without imposing the axiom of Anonymity. This significantly broadens the ethical implications when considering populations that differ in temporal, spatial or probabilistic dimensions or when including non-human populations. Imposing Existence Independence, combined with an Archimedean axiom that we term the Strict Limit Property, is sufficient for the dilemma to arise. These findings substantially generalize previous impossibility results in population ethics and highlight the inescapable trade-offs faced by any coherent population principle.

Keywords: population ethics; Repugnant Conclusion; Very Sadistic Conclusion; utilitarianism

JEL classification: D31; D71

1. Introduction

Population ethics is the study of welfare evaluations that govern decisions affecting the size and composition of populations. It addresses questions such as whether it is better to create more happy people or to improve the well-being of a smaller number of individuals and how to compare different possible populations. Classical 'total' utilitarianism, which aims to maximize the total sum of individual utilities, leads to paradoxes like the Repugnant Conclusion, where a vast population with lives barely worth living is deemed better than a smaller, flourishing one. To address these issues, alternative theories, such as critical-level utilitarianism (CLU), have been developed (Blackorby and Donaldson 1984; Blackorby *et al.* 1995). This is a celebrated principle for evaluating choices between variable populations.

CLU modifies total utilitarianism by introducing a critical level of utility or well-being. This critical level represents a threshold such that adding a person with utility at precisely this level is considered morally indifferent. Consequently, a life only

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contributes positively to overall welfare if its utility exceeds this critical level. The distinguishing feature of CLU is that it employs a constant critical level that remains independent of both existing utility distributions and population sizes when evaluating the addition of a new life. CLU offers several appealing theoretical features. It provides a balanced approach between total and average utilitarianism and aligns well with many intuitive judgements about population well-being. Although it raises important questions about where the threshold should be set, CLU is regarded as a key approach in population ethics, especially in policy domains such as climate change and social welfare, where both quality and quantity of life are significant considerations.

However, CLU poses a fundamental dilemma of population ethics, regardless of which threshold is chosen:

- Setting the critical level at (or below) the individually neutral utility level implies the Repugnant Conclusion (Parfit 1984) where, for any population with excellent lives, there is a population with lives barely worth living that is better, provided that the latter includes sufficiently many people.
- Setting the critical level strictly above the individually neutral utility level implies the Very Sadistic Conclusion (Arrhenius 2000*a*,*b*, forthcoming), where, for any population with terrible lives not worth living, there is a population with good lives that is worse, provided that the latter includes sufficiently many people.

Arguably, a population principle that entails the Repugnant Conclusion or the Very Sadistic Conclusion is considered implausible (at least to some degree).

In this paper, we show how this dilemma extends far beyond the class of CLU population principles. In particular, equal treatment of people through the imposition of an axiom of Anonymity is not needed. This is important if (potential) people differ with respect to time, location or probability of existence or if animal populations are to be included. Moreover, Paretian principles, as well as continuity and an assumption of a critical level, are superfluous. Indeed, we establish that the dilemma arises as long as we keep the axiom of *Existence Independence* and add an Archimedean axiom which we call the *Strict Limit Property*.

The two axioms that we impose are considered basic in population ethics. Existence Independence is an axiom of the separability of persons. This axiom asserts that the relative goodness of two possible states of affairs should depend only on the individuals who are affected by the choice between the two states, meaning the addition of identical groups of people who are not affected to different populations should not change their relative ranking. The Archimedean axiom in population ethics states that any welfare difference, no matter how high or low, can be outweighed by a sufficiently large number of small welfare changes. This mathematical property ensures that no welfare gain for any finite population is infinitely valuable, allowing for consistent quantitative comparisons across different population sizes and welfare distributions. The Strict Limit Property captures this idea.

Our result has an interesting contrast with the mere addition paradox, which is suggested by Parfit (1984). This is formally shown by Ng (1989) under a framework

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with Anonymity. According to Ng's impossibility theorem, any population principle that satisfies the mere addition property and anti-non-egalitarianism entails the Repugnant Conclusion. To be precise, the mere addition property posits that adding a person with a positive utility level is not bad, while anti-non-egalitarianism states that, other things being equal, a perfectly equal distribution of utility among individuals is at least as good as a possibly unequal distribution with a lower total utility. Each of the two is considered plausible (at least to some degree) as an ideal population theory, and thus, this impossibility result shows the fundamental difficulty for population ethics. This result has been extended by Carlson (1998). A possible resolution has been proposed by Sider (1991), but it is 'equality-averse'; see also Pivato (2020).

The Very Sadistic Conclusion has been regarded as a variant of the Sadistic Conclusion, which states that there exist some populations A, A', B such that adding people A with positive utility levels to B is worse than adding people A' with negative utility levels. Note that the Sadistic Conclusion only requires the existence of populations. This essentially means that the Very Sadistic Conclusion is more problematic than the Sadistic Conclusion. Regarding the Sadistic Conclusion, Arrhenius (2000*a*,*b*, forthcoming) has offered a series of impossibility results which show the dilemma between avoidance of the Repugnant Conclusion and avoidance of the Sadistic Conclusion. This dilemma is strengthened by Cato and Harada (2023), who employ a non-anonymous framework of Blackorby *et al.* (1998). Essentially, it shows that the dilemma arises as long as the Weak Limit Property (defined in Section 5.5) and Existence Independence are imposed. However, it has been an open question whether and how the Very Sadistic Conclusion, a more devastating problem, can be avoided along with avoidance of the Repugnant Conclusion.

As alluded to above, one notable aspect of our framework and results is their non-anonymous nature. Indeed, many existing works rely on the axiom of Anonymity, which requires that the identities of 'people' do not matter. As is well known, concerns regarding people's identities are central to Parfit's population ethics. The world in which individuals 1 and 2 receive 20 and 10, respectively, is not necessarily equally as good as the world in which individuals 2 and 3 receive 20 and 10. This is because individual 2, who exists in both worlds, has a higher utility in the latter, and some may argue that this makes the latter morally better. By demonstrating that the fundamental dilemma remains even in a non-anonymous setting, we have provided, in a sense, a synthesis of arguments in population ethics since Parfit (1984).

Our framework is also compatible with the existence of different species. One of the earliest works on multi-species population ethics is done by Blackorby *et al.* (1997). A more formal analysis is offered by Blackorby *et al.* (2005), who examine how CLU can be extended to the case with multi-species. They consider the possibility of having different critical levels across different species. Note that Anonymity is violated when different critical levels apply. Recently, the multi-species extension of population ethics has gained much attention. For instance, Williamson (2021) and Cato and Ishida (2026) examine the Animal Repugnant Conclusion and Animal Sadistic Conclusion and how these conclusions are related

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to species-relative critical levels. Moreover, Zuber *et al.* (2024) examine how Anonymity can apply within/cross species under the axiom of separability.

The rest of this paper is organized as follows. Section 2 introduces our framework and basic axioms. Section 3 offers an intuitive dilemma for a general class of utilitarian population principles. Section 4 establishes our main impossibility theorem. Section 5 considers extensions of the dilemma. Section 6 discusses the weakening of axioms that our main result allows and compares it to similar results in Blackorby *et al.* (2005) and Spears and Budolfson (2021). Section 7 concludes this paper. An appendix contains some proofs.

2. Framework, axioms and undesirable conditions

Blackorby *et al.* (1998) make an important contribution by extending population ethics beyond anonymous frameworks. Their analysis shows that fundamental impossibility results can emerge even when individuals' identities matter. Following the non-anonymous framework introduced by Blackorby *et al.* (1998) and applied by Cato and Harada (2023), we consider a general setting with variable populations. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{R}$ the set of real numbers. Let $\mathcal{N}$ be the collection of non-empty and finite subsets of $\mathbb{N}$. Each element N of $\mathcal{N}$ represents a population (a set of individuals). For each $i \in N$, $u_i \in \mathbb{R}$ represents individual i's utility level. For each $N \in \mathcal{N}$, $\mathbb{R}^N$ denotes the subspace of $\mathbb{R}^{\mathbb{N}}$ for which the coordinate labels correspond to the elements in N. The collection of distributions is given as follows:

$$\Omega = \bigcup_{N \in \mathcal{N}} \mathbb{R}^N.$$

For each element of Ω, there exists the corresponding set N of individuals and a profile of their utility levels, $(u_i)_{i \in N}$. For simplicity, $(u_i)_{i \in N}$ is written as u_N. A utility level of 0 corresponds to neutrality, which is equivalent to a life with no experience.

A *population principle* R is a reflexive, complete and transitive binary relation over Ω. We note that $u_M R v_N$ means 'u_M is at least as good as v_N'. The asymmetric and symmetric parts of R are denoted by P and I, respectively.

Throughout this paper, we consider two axioms as fundamental properties of the population principle. The first axiom, known as Existence Independence, states that the ranking between two distributions, u_M and v_N, remains unchanged if a population L with distribution w_L, which has no effect on either, is added to both. In other words, the ranking between (u_M, w_L) and (v_N, w_L) mirrors that between u_M and v_N. The converse also holds: if a subpopulation common to both u_M and v_N has no effect, then removing it leaves their relative ranking unchanged.

Axiom (Existence Independence) For all $M, N, L \in \mathcal{N}$ such that $(M \cup N) \cap L = \varnothing$, all $u_M \in \mathbb{R}^M$, all $v_N \in \mathbb{R}^N$ and all $w_L \in \mathbb{R}^L$, $u_M R v_N$ if and only if $(u_M, w_L) R (v_N, w_L)$.

The following axiom is a version of the Archimedean property. Consider two distributions, u_N and v_N, with the same population N. It may be the case that v_N is a distribution with extremely high utility levels, while u_N is a distribution with extremely low utility levels, so that v_N is ranked far above u_N. Now suppose we add

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a large number of individuals to both populations: each added individual receives utility μ in one distribution and utility ν in the other, where $\mu > \nu$. The axiom requires that the distribution $(\mu 1_M, u_N)$ be ranked above $(\nu 1_M, v_N)$, provided that the number of individuals added is sufficiently large, regardless of how much better v_N was than u_N to begin with. Intuitively, this expresses a principle of aggregation: even a small advantage per person becomes decisive when it is shared by a sufficiently large number of individuals. For the statement of this axiom, we define

$$\mathcal{N}_{-N} = \{M \in \mathcal{N} | M \cap N = \varnothing\},$$

being the collection of elements M of $\mathcal{N}$ such that each set M does not include any element of N.

Axiom (Strict Limit Property) For all $N \in \mathcal{N}$, all $u_N, v_N \in \mathbb{R}^N$ and all $\mu, \nu \in \mathbb{R}$ with $\mu > \nu$, there exists $M^* \in \mathcal{N}_{-N}$ such that, for all $M \in \mathcal{N}_{-N}$ with $M \supseteq M^*$, $(\mu \mathbb{1}_M, u_N) P (\nu \mathbb{1}_M, v_N)$.

This axiom is related to the Positive-Limit and Negative-Limit Properties in Blackorby *et al.* (1998), and it strengthens the Limit Property axiom of Cato and Harada (2023). With Anonymity, Spears and Budolfson (2021), Bossert *et al.* (2023) and Cato *et al.* (2024) offer impossibility results with versions of limit properties. A related Archimedean condition is also employed by Rusonik *et al.* (2026) in their position-based approach to population ethics.

We now define the Repugnant and Very Sadistic Conclusions. The Repugnant Conclusion states that even for a population with very high utility levels, there exists a much larger population with small but positive utility levels that is better (Parfit 1984). As alluded to above, this conclusion necessarily holds for total utilitarianism. The Repugnant Conclusion is widely considered something that any plausible population principle should avoid, and it has been extensively examined by many since its introduction by Parfit (1984). It is formulated as follows:

The **Repugnant Conclusion** holds for the population principle R if, for all $\xi \in \mathbb{R}_{++}$, all $N \in \mathcal{N}$ and all $\varepsilon \in (0, \xi)$, there exists $M \supsetneq N$ such that $\varepsilon \mathbb{1}_M P \xi \mathbb{1}_N$.

The Very Sadistic Conclusion was introduced by Arrhenius (2000*a*, 73; forthcoming, Section 5.1)[1] and states that, for any population consisting solely of negative utility levels, there exists a population consisting solely of positive utility levels that is nonetheless worse. As we will see below, there are several variants of the sadistic conclusion, but there is a special reason to avoid this 'very' version: even if there were an extremely large number of people living in hellish conditions, there would still exist an alternative consisting entirely of positive utility that is, implausibly, worse. It is formulated as follows:

The **Very Sadistic Conclusion** holds for the population principle R if, for all $N \in \mathcal{N}$ and all $v_N \in \mathbb{R}^N_{--}$, there exists $M \in \mathcal{N}$ and $u_M \in \mathbb{R}^M_{++}$ such that $v_N P u_M$.

[1]This concept of the Very Sadistic Conclusion is also introduced in Arrhenius (2000b), 256, but there referred to as the *Strong Sadistic Conclusion*. Below, we use this latter term for a different concept.

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In our formulations above, these conclusions need not presume that population principles respect Anonymity. We say that a population principle *raises a dilemma between repugnancy and sadism* if it implies that either the Repugnant or the Very Sadistic Conclusion holds. Next, we introduce the standard Sadistic Conclusion, which was also proposed by Arrhenius (2000*a*,b, forthcoming). According to this conclusion, it can be the case that adding a population with negative utility levels is better than adding a population with positive utility levels. This does not imply that adding any hellish distribution is better than adding some population consisting solely of positive utility levels. Formally, the Sadistic Conclusion is defined as follows.

The **Sadistic Conclusion** holds for R if there exist M, N, $L \in \mathcal{N}$, $u_M \in \mathbb{R}^M_{++}$, $v_N \in \mathbb{R}^N_{--}$ and $w_L \in \mathbb{R}^L$ such that $L \cap M = \varnothing$, $L \cap N = \varnothing$ and $(v_N, w_L)\ P(u_M, w_L)$.

Another version, which can be called the Strong Sadistic Conclusion, states that there exists a population with positive utility levels and another with negative utility levels such that the former is worse than the latter. The contrast between the Strong Sadistic Conclusion and the Very Sadistic Conclusion is comparatively straightforward to understand. Under the Strong Sadistic Conclusion, the negative utility of the population that is better than one with positive utility may be extremely close to zero – that is, it may not be all that bad. It remains possible, for example, that a hellish distribution is worse than any population with solely positive utility levels. The Very Sadistic Conclusion is particularly extreme in that it rules out even this possibility.

The **Strong Sadistic Conclusion** holds for R if there exist M, $N \in \mathcal{N}$, $u_M \in \mathbb{R}^M_{++}$ and $v_N \in \mathbb{R}^N_{--}$ such that $v_N P u_M$.

Note that the Very Sadistic Conclusion logically implies the Strong Sadistic Conclusion. On the other hand, in general, the Sadistic Conclusion is logically independent from the Very Sadistic Conclusion. However, as observed by Blackorby *et al.* (2004), the Very Sadistic Conclusion implies the Sadistic Conclusion if R satisfies the Strong Pareto principle, which is normally considered uncontroversial. This means that the Very Sadistic Conclusion is essentially more problematic than the Sadistic Conclusion. Notably, we do not impose the Strong Pareto principle. In our framework, which includes not only humans but also various things in the universe, the Strong Pareto principle may not be very plausible. This is because the Strong Pareto principle requires that any improvement of a single individual's utility is good. We consider a non-anonymous setting, which may include 'individuals' that should be ignored in welfare evaluations. However, also in our setting, the Very Sadistic Conclusion is more problematic than the Sadistic Conclusion, as we impose Existence Independence, under which the Strong Sadistic Conclusion implies the Sadistic Conclusion. Thus, the Sadistic Conclusion is implied by the Very Sadistic Conclusion even without the Strong Pareto principle.

3. A utilitarian condition for the dilemma between repugnancy and sadism

We start by formally introducing CLU. A population principle R is critical-level generalized utilitarian (CLGU) if there exists a continuous and strictly increasing

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transformation function $g : \mathbb{R} \to \mathbb{R}$ such that $0 \in g(\mathbb{R})$ and, for all $M, N \in \mathcal{N}$, all $u_M \in \mathbb{R}^M$ and all $v_N \in \mathbb{R}^N$,[2]

$$u_M R v_N \quad \text{if and only if} \quad \sum_{i \in M} g(u_i) \geq \sum_{i \in N} g(v_i).$$

Note that $\alpha = g^{-1}(0)$ is the critical level. That is, adding a person with this level is always indifferent. Axiomatic characterizations of this class of principles have been established by a series of works of Blackorby and Donaldson (1984), Blackorby *et al.* (1995) and Blackorby *et al.* (2005). Existence Independence is a key axiom for their characterizations.

We now show that the following generalized utilitarian condition is sufficient for the dilemma.

Condition. There exists an infinite subset $\mathbb{M}$ of $\mathbb{N}$ and, for each $i \in \mathbb{M}$, an increasing and continuous function $g_i : \mathbb{R} \to \mathbb{R}$, with the properties that

(i) for all $\mu, \nu \in \mathbb{R}$ with $\mu > \nu$, $\sum_{i \in \mathbb{M}} (g_i(\mu) - g_i(\nu)) = +\infty$, and
(ii) for all $M, N \in \mathcal{N}$, all $u_M \in \mathbb{R}^M$ and all $v_N \in \mathbb{R}^N$,

$$u_M P v_N \quad \text{if} \quad \sum_{i \in M \cap \mathbb{M}} g_i(u_i) > \sum_{i \in N \cap \mathbb{M}} g_i(v_i).$$

It is easy to see that the class of critical-level utilitarian principles falls into a subclass of orderings that satisfy this Condition.

The following result establishes that the generalized utilitarian condition is sufficient for the dilemma.

Proposition. *If the population principle R satisfies the Condition, then R raises a dilemma between repugnancy and sadism.*

Proof. Case 1: $\sum_{i \in \mathbb{M}} g_i(\mu) > 0$ *for all* $\mu \in \mathbb{R}_{++}$. Fix $\xi \in \mathbb{R}_{++}$ and $N \in \mathcal{N}$. Then, by the Condition, $\sum_{i \in \mathbb{M}} g_i(\varepsilon) = +\infty$ for all $\varepsilon \in (0, \xi)$ since, for any $\varepsilon \in \mathbb{R}_{++}$, there exists $\nu \in (0, \varepsilon)$ with $\sum_{i \in \mathbb{M}} g_i(\nu) > 0$. Fix $\varepsilon \in (0, \xi)$. By letting $M \supsetneq N$ be a sufficiently large subset of $\mathbb{M} \cup N$, it follows that $\sum_{i \in M \cap \mathbb{M}} g_i(\varepsilon) > \sum_{i \in N \cap \mathbb{M}} g_i(\xi)$, implying that $\varepsilon 1_M P \xi 1_N$. Hence, the Repugnant Conclusion holds for R.

Case 2: *There exists* $\mu \in \mathbb{R}_{++}$ *such that* $\sum_{i \in \mathbb{M}} g_i(\mu) \leq 0$. By the Condition, $\sum_{i \in \mathbb{M}} g_i(\varepsilon) = -\infty$ for all $\varepsilon \in (0, \mu)$. Fix $\varepsilon \in (0, \mu)$ and consider any $v_N \in \mathbb{R}^N_{--}$. By letting $M \in \mathcal{N}$ be a sufficiently large subset of $\mathbb{M}$, it follows that $\sum_{i \in M \cap \mathbb{M}} g_i(\varepsilon) < \sum_{i \in N \cap \mathbb{M}} g_i(v_i)$, implying that $v_N P \varepsilon 1_M$. Hence, the Very Sadistic Conclusion holds for R.

The Proposition follows since Cases 1 and 2 are exhaustive. □

We then provide two examples of generalized utilitarian criteria that satisfy the Condition.

[2]This is also called generalized CLU. However, we follow the terminology of Blackorby *et al.* (2005) who highlight this principle as a variable-population extension based on the critical-level approach for generalized utilitarianism, a principle used for same-number comparisons.

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Example 1 *(Some people do not matter).* There exists an infinite subset $\mathbb{M}$ *of* $\mathbb{N}$ and an increasing and continuous function $g : \mathbb{R} \to \mathbb{R}$ such that, for all $M, N \in \mathcal{N}$, all $u_M \in \mathbb{R}^M$ and all $v_N \in \mathbb{R}^N$,

$$u_M R v_N \text{ if and only if } \sum\nolimits_{i \in M \cap \mathbb{M}} g(u_i) \geq \sum\nolimits_{i \in N \cap \mathbb{M}} g(v_i).$$

If $\mathbb{M} = \mathbb{N}$, then we obtain CLGU with critical level $g^{-1}(0)$, presuming that $0 \in g(\mathbb{R})$. If $\mathbb{M} \subsetneq \mathbb{N}$, then people in $\mathbb{N} \backslash \mathbb{M}$ do not matter. In the latter case, the Strong Pareto principle is not satisfied.

Example 2 *(Some people matter more).* There exists a partition $\{\mathbb{M}^0, \mathbb{M}^1, \ldots, \mathbb{M}^{m-1}, \mathbb{M}^m\}$ *of* $\mathbb{N}$ *with* $\mathbb{M}^0$ being infinite, and, for each $i \in \mathbb{N} \backslash \mathbb{M}^m$, an increasing and continuous function $g_i : \mathbb{R} \to \mathbb{R}$, with the properties that

(i) for all $\mu, \nu \in \mathbb{R}$ with $\mu > \nu$, $\sum_{i \in \mathbb{M}^0} \big(g_i(\mu) - g_i(\nu)\big) = +\infty$, and
(ii) for all $M, N \in \mathcal{N}$, all $u_M \in \mathbb{R}^M$ and all $v_N \in \mathbb{R}^N$, $u_M P v_N$ if and only if there exists $k \in \{0, 1, \ldots, m-1\}$ such that $\sum_{i \in M \cap \mathbb{M}^k} g_i(u_i) > \sum_{i \in N \cap \mathbb{M}^k} g_i(v_i)$ and $\sum_{i \in M \cap \mathbb{M}^l} g_i(u_i) = \sum_{i \in N \cap \mathbb{M}^l} g_i(v_i)$ for all $\ell \in \{0, 1, \ldots, k-1\}$.

In such a lexicographic population principle, people in $\mathbb{M}^l$ matter infinitely more than people in $\mathbb{M}^k$ if $0 \leq \ell < k \leq m-1$, while people in $\mathbb{M}^m$ do not matter at all. The Strong Pareto principle is satisfied if and only if $\mathbb{M}^m = \varnothing$.

It is straightforward to show that all population principles in the two classes considered in Examples 1 and 2 satisfy our two axioms, Existence Independence and the Strict Limit Property.

We note that a population principle that satisfies Existence Independence and Strict Limit Property does not necessarily satisfy the Condition.

Example 3 *(Non-utilitarian principle).* Let $\alpha \in \mathbb{R}$ be the critical level. Compare u_M with v_N. Let $r = \max M \cup N$ in the sense that $r \geq i$ for all $i \in M \cup N$. Define $u_M^* \in \mathbb{R}^r$ by letting

$$u_i^* = u_i \text{ for all } i \in M$$

and

$$u_i^* = \alpha \text{ for all } i \in \{1, \ldots, r\} \backslash M.$$

Analogically, define $v_N^* \in \mathbb{R}^r$ by letting

$$v_i^* = v_i \text{ for all } i \in N$$

and

$$v_i^* = \alpha \text{ for all } i \in \{1, \ldots, r\} \backslash N.$$

The set of individuals is $\{1, \ldots, r\}$ for u_M^* and v_N^*. Let R_T^r be a lexicographic ordering that compares the r-dimensional distributions: for all $u', v' \in \mathbb{R}^r$, $u' R_T^r v'$ if and only if

$$u_i' = v_i' \text{ for all } i \in \{1, \ldots, r\},$$

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or

$$\exists k \in \{1, \ldots, r\} \text{ such that } u'_i = v'_i \text{ for all } i \in \{k+1, \ldots, r\} \text{ and } u'_k > v'_k.$$

Define R by letting: for all $u_M \in \mathbb{R}^M$ and all $v_N \in \mathbb{R}^N$,

$$u_M R v_N \iff u^*_M R^r_T v^*_N.$$

This R is a population principle satisfying Existence Independence and the Strict Limit Property without being in the class that satisfies the Condition.

4. General repugnancy–sadism dilemma

As shown by the Proposition, the class of generalized utilitarian orderings cannot avoid the repugnancy–sadism dilemma, even in a non-anonymous setting. The following result shows that any population principle satisfying the axioms of Existence Independence and the Strict Limit Property raises the dilemma between repugnancy and sadism. Hence, this dilemma holds for a larger class of population principles, including non-utilitarian principles, such as the one in Example 3.

Theorem. *If the population principle R satisfies Existence Independence and the Strict Limit Property, then R raises a dilemma between repugnancy and sadism.*

Proof. Assume that R satisfies Existence Independence and the Strict Limit Property and, furthermore, avoids the Very Sadistic Conclusion. It is sufficient to show that the Repugnant Conclusion holds for R. Take any $\xi > 0$ and $N \in \mathcal{N}$, and let $\varepsilon \in (0, \xi)$. By the avoidance of the Very Sadistic Conclusion, there exist $N^* \in \mathcal{N}$ and $v_{N^*} \in \mathbb{R}^{N^*}_{--}$ such that, for all $M \in \mathcal{N}$ and all $u_M \in \mathbb{R}^M_{++}$,

$$u_M R v_{N^*}.$$

We distinguish two cases.

Case 1: $N \cap N^* = \varnothing$. Let $\varepsilon^- \in (0, \varepsilon)$. By the Strict Limit Property, there exists a non-empty set $M^* \subseteq \mathbb{N} \backslash (N \cup N^*)$ such that, for all $M \subseteq \mathbb{N} \backslash (N \cup N^*)$ with $M \supseteq M^*$,

$$(\varepsilon \mathbb{1}_M, v_{N^*}, \varepsilon \mathbb{1}_N)\, P(\varepsilon^- \mathbb{1}_M, \varepsilon^- \mathbb{1}_{N^*}, \xi \mathbb{1}_N).$$

Note that $(\varepsilon^- \mathbb{1}_M, \varepsilon^- \mathbb{1}_{N^*}) \in \Omega_{++}$. Thus, since R avoids the Very Sadistic Conclusion,

$$(\varepsilon^- \mathbb{1}_M, \varepsilon^- \mathbb{1}_{N^*})\, R\, v_{N^*}.$$

Existence Independence implies that

$$(\varepsilon^- \mathbb{1}_M, \varepsilon^- \mathbb{1}_{N^*}, \xi \mathbb{1}_N)\, R\, (v_{N^*}, \xi \mathbb{1}_N).$$

Hence, by transitivity we obtain

$$(\varepsilon \mathbb{1}_M, v_{N^*}, \varepsilon \mathbb{1}_N)\, P\, (v_{N^*}, \xi \mathbb{1}_N). \tag{1}$$

This holds true for all $M \subseteq \mathbb{N} \backslash (N \cup N^*)$ with $M \supseteq M^*$.

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Existence Independence implies

$$(\varepsilon \mathbb{1}_M, \varepsilon \mathbb{1}_N)\, P\, (\xi \mathbb{1}_N).$$

The Repugnant Conclusion holds.

Case 2: $N \cap N^* \neq \emptyset$. Let $K = N \cap N^*$. Now, we take $N^{**} \in \mathcal{N}$ such that $N^* \cap N^{**} = \emptyset$ and $N \backslash K \subseteq N^{**}$. Let $\varepsilon^* \in (0, \varepsilon)$. By applying the argument in Case 1, (1) holds for ε^* and N^{**}. That is, there exists a non-empty set $M^* \subseteq \mathbb{N} \backslash (N^* \cup N^{**})$ such that

$$(\varepsilon^* \mathbb{1}_M, v_{N^*}, \varepsilon^* \mathbb{1}_{N^{**}})\, P\, (v_{N^*}, \xi \mathbb{1}_{N^{**}})$$

for all $M \subseteq \mathbb{N} \backslash (N^{**} \cup N^*)$ with $M \supseteq M^*$. By applying Existence Independence, it follows that $(\varepsilon^* \mathbb{1}_M, \varepsilon^* \mathbb{1}_{N^{**}}) P \xi \mathbb{1}_{N^{**}}$ and therefore, by another application of Existence Independence,

$$(\varepsilon^* \mathbb{1}_M, \xi \mathbb{1}_{N^*}, \varepsilon^* \mathbb{1}_{N^{**}})\, P\, (\xi \mathbb{1}_{N^*}, \xi \mathbb{1}_{N^{**}}). \tag{2}$$

This holds for all $M \subseteq \mathbb{N} \backslash (N^{**} \cup N^*)$ with $M \supseteq M^*$.

Note that $N \subseteq N^* \cup N^{**}$. Let $L = (N^* \cup N^{**}) \backslash N$. Thus,

$$N^* \cup N^{**} = L \cup N.$$

By the Strict Limit Property, there exists a non-empty set $M^{**} \subseteq \mathbb{N} \backslash (N^* \cup N^{**})$ such that, for all $M \subseteq \mathbb{N} \backslash (N^* \cup N^{**})$ with $M \supseteq M^{**}$,

$$(\varepsilon \mathbb{1}_M, \xi \mathbb{1}_L, \varepsilon \mathbb{1}_N)\, P\, (\varepsilon^* \mathbb{1}_M, \xi \mathbb{1}_{N^*}, \varepsilon^* \mathbb{1}_{N^{**}}) \tag{3}$$

because $\varepsilon > \varepsilon^*$. Note that (2) and (3) hold for all $M \subseteq \mathbb{N} \backslash (N^* \cup N^{**})$ with $M \supseteq M^* \cup M^{**}$. Thus, by transitivity we obtain

$$(\varepsilon \mathbb{1}_M, \xi \mathbb{1}_L, \varepsilon \mathbb{1}_N)\, P\, (\xi \mathbb{1}_{N^*}, \xi \mathbb{1}_{N^{**}}).$$

This implies

$$(\varepsilon \mathbb{1}_M, \xi \mathbb{1}_L, \varepsilon \mathbb{1}_N)\, P\, (\xi \mathbb{1}_L, \xi \mathbb{1}_N)$$

since $N^* \cup N^{**} = L \cup N$. Existence Independence implies that

$$(\varepsilon \mathbb{1}_M, \varepsilon \mathbb{1}_N)\, P\, \xi \mathbb{1}_N.$$

The Repugnant Conclusion holds. □

We note that this dilemma is not raised if either Existence Independence or the Strict Limit Property is dropped. The following does not raise the dilemma; it satisfies the Strict Limit Property and violates Existence Independence.

Example 4 *(Variable-Value principle).* There exists an infinite subset $\mathbb{M}$ *of* $\mathbb{N}$ and a non-decreasing and bounded function $f : \mathbb{N} \to \mathbb{R}$ such that, for all $M, N \in \mathcal{N}$, all $u_M \in \mathbb{R}^M$ and all $v_N \in \mathbb{R}^N$,

$$u_M R v_N \text{ if and only if } \frac{f(|M \cap \mathbb{M}|)}{|M \cap \mathbb{M}|} \cdot \sum\nolimits_{i \in M \cap \mathbb{M}} u_i \geq \frac{f(|N \cap \mathbb{M}|)}{|N \cap \mathbb{M}|} \cdot \sum\nolimits_{i \in N \cap \mathbb{M}} v_i.$$

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If $\mathbb{M} = \mathbb{N}$ and f is constant, then we obtain the standard Average Utilitarian principle. If $\mathbb{M} = \mathbb{N}$ and f is increasing, then we obtain the standard Variable-Value principle, proposed by Hurka (1983) and Ng (1986, 1989).

The next example, which satisfies Existence Independence and violates the Strict Limit Property, does not raise the dilemma if the critical level α is positive.

Example 5 *(Critical-Level Leximin principle).* For each $m \in \mathbb{N}$, we first define the m-dimensional lexicographic ordering R_L^m over populations with m individuals. Given $M \in \mathcal{N}$ with $|M| = m$, let

$$u_{(M)} = (u_{(1)}, \ldots, u_{(m)}) \in \mathbb{R}^m$$

be a rank-ordered permutation of u_M such that $u_{(1)} \leq u_{(2)} \leq \ldots \leq u_{(m)}$, with ties being broken arbitrarily. For all M, $M' \in \mathcal{N}$ with $|M| = |M'| = m$, and all $v_{M'} \in \mathbb{R}^{M'}$,

$$u_M R_L^m v_{M'}$$

if and only if

$$u_{(i)} = v_{(i)} \text{ for all } i \in \{1, \ldots, m\},$$

or

$$(\exists k \in \{1, \ldots, m\} \text{ such that } u_{(i)} = v_{(i)} \text{ for all } i < k \text{ and } u_{(k)} > v_{(k)}).$$

Let $\alpha \in \mathbb{R}$ be the critical level. For all M, $N \in \mathcal{N}$, all $u_M \in \mathbb{R}^M$ and all $v_N \in \mathbb{R}^N$,

$$u_M R v_N \iff \left(u_M, \alpha \mathbb{1}_{M'}\right) R_L^n v_N \text{ if } |N| = n > m = |M|,$$

$$u_M R v_N \iff u_M R_L^m \left(v_N, \alpha \mathbb{1}_{N'}\right) \text{ if } |N| = n < m = |M|,$$

$$u_M R v_N \iff u_M R_L^m v_N \text{ if } |N| = n = m = |M|,$$

where $|M'| = n - m$, $M' \cap M = \varnothing$, $|N'| = m - n$ and $N' \cap N = \varnothing$. This is an extension of the Leximin principle proposed by Blackorby *et al.* (1996).

Finally, we consider Rank-Discounted Utilitarianism introduced by Asheim and Zuber (2014); see also Asheim and Zuber (2022). This principle applies rank discounting to welfare evaluations, where an individual's welfare contribution is weighted according to their relative position in the welfare ranking of the total population, with higher weights assigned to individuals with lower welfare. To be precise, R is rank-discounted generalized utilitarian if there exist $\beta \in (0, 1)$ and a continuous and strictly increasing transformation function $g : \mathbb{R} \to \mathbb{R}$ with $0 \in g(\mathbb{R})$ such that, for all M, $N \in \mathcal{N}$, all $u_M \in \mathbb{R}^M$ and all $v_N \in \mathbb{R}^N$,

$$u_M R v_N \quad \text{if and only if} \quad \sum\nolimits_{r=1}^{|M|} \beta^r g\left(u_{(r)}\right) \geq \sum\nolimits_{r=1}^{|N|} \beta^r g\left(v_{(r)}\right).$$

This population principle might be considered to be positioned between utilitarianism and lexicographic preferences. As shown by Asheim and Zuber (2014), Rank-Discounted Utilitarianism orderings avoid both the Repugnant

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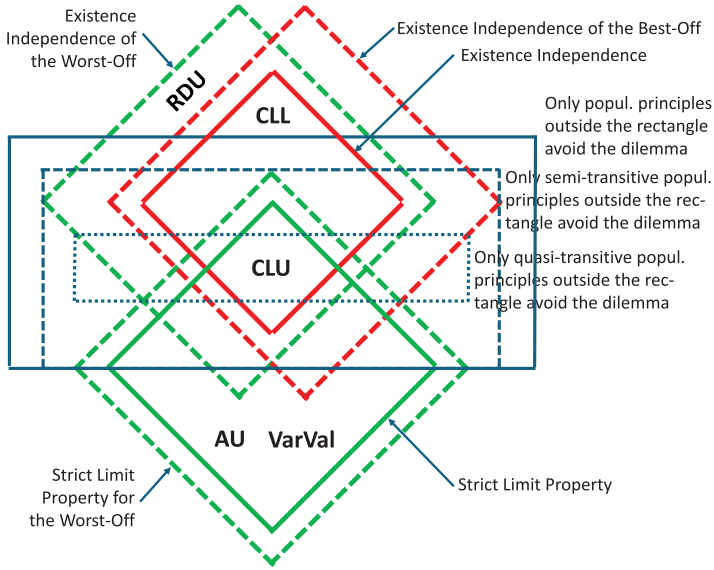


Figure 1. Graphical illustration of the main result and its extensions.

Conclusion and Very Sadistic Conclusion, but at the cost of satisfying neither Existence Independence nor the Strict Limit Property.[3]

5. Extensions of the dilemma

In this section, we present six observations that extend our main result. The first one follows from the proof of the Theorem, while the proofs of the five others are contained in the appendix. Figure 1 illustrates graphically our main result and the extensions provided in the present section.

5.1. Strengthening the Very Sadistic Conclusion

Consider the following strengthening of the Very Sadistic Conclusion, where the distribution with positive utility levels is constrained to be egalitarian.

The **Very Sadistic Conclusion*** holds for the population principle R if, for all $N \in \mathcal{N}$ and all $v_N \in \mathbb{R}^N_{--}$, there exists $M \in \mathcal{N}$ and $\mu > 0$ such that $v_N P \mu \mathbb{1}_M$.

Since the proof of the Theorem only uses the avoidance of the Very Sadistic Conclusion*, we can state the following strengthened version of the Theorem.

Observation 1. *If the population principle R satisfies Existence Independence and the Strict Limit Property, then either the Repugnant Conclusion or the Very Sadistic Conclusion* holds.*

[3]We note that the Sadistic Conclusion still holds for Rank-Discounted Utilitarianism.

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This version of our main result will be used in Section 6.3 to discuss a similar result in Spears and Budolfson (2021).

5.2. Relaxing transitivity and completeness

In the previous arguments, we assumed the population principle to be reflexive, complete and transitive. However, both completeness and transitivity have each been criticized. For example, Sen (2004) argues that it is not plausible to force all states of affairs to be comparable, and it is rather natural to assume that there are some states that cannot be ranked ('assertive incompleteness'). A powerful challenge to the transitivity of indifference is illustrated by the classic coffee–sugar example (Raiffa 1968).[4] Suppose that adding 0.1 grams of sugar to a cup of coffee is too small a difference to affect taste and is therefore considered indifferent. Then, by transitivity, adding $k \cdot 0.1$ grams and adding $(k+1) \cdot 0.1$ grams must also be indifferent for all $k \in \mathbb{N}$. This would imply, absurdly, that zero grams and 200 grams of sugar are indifferent. The core issue revealed by this example is the assumption that the symmetric part I of R is transitive.

Armstrong (1939) offers an alternative explanation: indifference occurs when the difference between utilities is below a certain threshold. Hence, if the difference in the value between A and B is $a > 0$, and between B and C is $b > 0$, where both a and b are below the threshold $\theta > 0$ that triggers preference, the difference between A and C would be $a + b$, which could exceed θ and thus trigger a preference for A over C. This violates the transitivity of the symmetric part I of R.

It does not, however, violate the transitivity of the asymmetric part P of R. Hence, we can still impose *quasi-transitivity* of R in the sense that for all $M, N, K \in \mathcal{N}$, all $u_M \in \mathbb{R}^M$, all $v_N \in \mathbb{R}^N$ and all $w_K \in \mathbb{R}^K$,

$$u_M P\, v_N \text{ and } v_N P\, w_K \Rightarrow u_M P\, w_K.$$

It turns out that there is no dilemma between repugnancy and sadism if the transitivity of R is weakened to quasi-transitivity of R, as the following example shows.

Example 6 *(Same-People Utilitarian Extension principle).* For all $M, N \in \mathcal{N}$, all $u_M \in \mathbb{R}^M$ and all $v_N \in \mathbb{R}^N$,

$$u_M R v_N \iff \left[\left(M = N \text{ and } \sum\nolimits_{i\in M} u_i \geq \sum\nolimits_{i\in N} v_i\right) \text{ or } N \neq M\right].$$

The relation R of Example 6 satisfies Existence Independence and the Strict Limit Property, while avoiding the dilemma between repugnancy and sadism. It satisfies quasi-transitivity but violates the following weakening of transitivity, which has been widely examined; see, for instance, Luce (1956): *Semi-transitivity* of R requires that, for all $M, N, K, L \in \mathcal{N}$, all $u_M \in \mathbb{R}^M$, all $v_N \in \mathbb{R}^N$, all $w_K \in \mathbb{R}^K$ and all $s_L \in \mathbb{R}^L$,

$$u_M P\, v_N \text{ and } v_N P\, w_K \Rightarrow s_L P\, w_K \text{ or } u_M P\, s_L.$$

[4]See Temkin (1987) for a related criticism of transitivity in the context of population ethics.

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Any reflexive, complete and transitive binary relation R satisfies this property. On the other hand, any reflexive and semi-transitive relation R is quasi-transitive. Hence, in these senses, semi-transitivity is a weakening of transitivity but a strengthening of quasi-transitivity. Armstrong's approach fails transitivity but is compatible with both semi-transitivity and quasi-transitivity.

We make the observation that the dilemma between repugnancy and sadism, while non-existent under quasi-transitivity, remains under semi-transitivity. For the purpose of this observation, say that a relation R is a *semi-transitive population principle* if R satisfies semi-transitivity and reflexivity. Note that the semi-transitive population principle R is not necessarily complete. Hence, both completeness and transitivity are relaxed to some degree.

Observation 2. *If the semi-transitive population principle R satisfies Existence Independence and the Strict Limit Property, then R raises a dilemma between repugnancy and sadism.*

This follows from Proposition A in the appendix.

5.3. Limit properties for the worst-off

The Strict Limit Property is critical to our main theorem. The essence of this axiom lies in the idea of value aggregation. However, aggregation is often a source of philosophical debate. For instance, if a certain choice results in severe suffering for one person, can that harm be ethically outweighed by small benefits to a large number of others? Some philosophers argue that such aggregation is morally problematic and cannot always be permissible. While we will not delve into this ethical debate here, a natural question arises: what kind of axiom concerning aggregation – logically weaker than the Strict Limit Property – would still suffice to establish the impossibility theorem?

Under the following axiom, the limit property applies only when the utility levels of individuals added to the original distributions are lower than those of the existing people in v_N.

Axiom (Strict Limit Property for the Worst-Off) For all $N \in \mathcal{N}$, all $u_N, v_N \in \mathbb{R}^N$ and all $\mu, \nu \in \mathbb{R}$ with $\mu > \nu$, if $\min v_N \geq \nu$, then there exists $M^* \in \mathcal{N}_{-N}$ such that, for all $M \in \mathcal{N}_{-N}$ with $M \supseteq M^*$, $(\mu \mathbb{1}_M, u_N)\, P\, (\nu \mathbb{1}_M, v_N)$.

Neither Rank-Discounted Utilitarianism nor Critical-Level Leximin satisfies the Strict Limit Property for the Worst-Off.

We have the following extension of our main result.

Observation 3. *If the population principle R satisfies Existence Independence and the Strict Limit Property for the Worst-Off, then R raises a dilemma between repugnancy and sadism.*

This follows from Proposition A in the appendix. The proof follows similar lines to our main theorem, with adjustments to account for the additional constraint on the welfare of the worst-off individuals. This observation further strengthens our main

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finding by showing that even with this more restricted version of the Strict Limit Property, the dilemma cannot be avoided while maintaining Existence Independence.

In order to show how this limit property works (even if it is a restricted version), we consider the mere addition paradox, which has been widely examined in the literature of population ethics.

The following axiom states that adding a person with positive welfare to an existing population does not make the overall welfare worse.

Axiom (Mere Addition) For all $N \in \mathcal{N}$, all $u_N \in \mathbb{R}^N$, for all $k \in \mathbb{N}\backslash N$, if $v_k > 0$, then $(u_N, v_k)Ru_N$.

The following result shows that even with the modified limit property that respects the priority of the worst-off, any population principle that also satisfies the intuition that adding lives worth living improves welfare will inevitably lead to the Repugnant Conclusion.

Observation 4. *If the population principle R satisfies Mere Addition and the Strict Limit Property for the Worst-Off, then R entails the Repugnant Conclusion.*

Note that this result holds also for the following version, which is logically weaker than the Strict Limit Property for the Worst-Off.

Axiom (Strict Limit Property for the Worst-Off*) For all $N \in \mathcal{N}$, all $u_N, v_N \in \mathbb{R}^N$ and all $\mu, \nu \in \mathbb{R}$ with $\mu > \nu$, if $\min u_N \geq \nu$ and $\min v_N \geq \nu$, then there exists $M^* \in \mathcal{N}_{-N}$ such that, for all $M \in \mathcal{N}_{-N}$ with $M \supseteq M^*$, $(\mu \mathbb{1}_M, u_N)P(\nu \mathbb{1}_M, v_N)$.

The Strict Limit Property for the Worst-Off* is satisfied by Rank-Discounted Utilitarianism and its variations. Moreover, for any critical level α, Critical-Level Leximin satisfies this property. As a result, our observation here significantly extends the theorems of Blackorby *et al.* (1998) and Cato and Harada (2023), which establish the incompatibility of Mere Addition and the Repugnant Conclusion under the standard limit properties.

We also note that the dilemma is not necessarily raised under the Strict Limit Property for the Worst-Off*. More precisely, there is a population principle that satisfies Existence Independence and the Strict Limit Property for the Worst-Off* and does not raise a dilemma between repugnancy and sadism. Critical-Level Leximin with a positive critical level α is an example for this case.

5.4. Weaker Existence Independence axioms

As alluded to above, Existence Independence is violated by rank-discounted utilitarian population principles, which avoid the dilemma between repugnancy and sadism. However, Rank-Discounted Utilitarianism satisfies some types of independence, and thus, it is of interest to explore weaker versions of Existence Independence.

First, we examine a way of violating Existence Independence under Rank-Discounted Utilitarianism. Consider the following two populations:

$$u_N = (0, 4) \text{ and } v_N = (1, 1)$$

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where $|N| = 2$. Now, assume that $g(x) = x$ and $\beta = 0.5$. Note that the corresponding values under Rank-Discounted Utilitarianism are

$$\sum_{r=1}^{|N|} \beta^r u_{(r)} = 4\beta^2 = 1 \quad \text{and} \quad \sum_{r=1}^{|N|} \beta^r v_{(r)} = \beta(1+\beta) = 0.75.$$

Hence, the former is better than the latter. Let us add a singleton, M, to both populations, where $N \cap M = \varnothing$ and the population size of M is the same as N. Consider the following distributions:

$$\hat{u}_{N\cup M} = (u_N, 1 \cdot \mathbb{1}_M) \text{ and } \hat{v}_{N\cup M} = (v_N, 1 \cdot \mathbb{1}_M).$$

The total number of individuals is 3. Now, the corresponding values under Rank-Discounted Utilitarianism are

$$\sum_{r=1}^{|N|+m} \beta^r \hat{u}_{(r)} = \beta^2 + 4\beta^3 = 0.75 \quad \text{and} \quad \sum_{r=1}^{|N|} \beta^r \hat{v}_{(r)} = (\beta + \beta^2 + \beta^3) = 0.875.$$

The latter is better than the former. Thus, Existence Independence fails. This occurs because the newly added individuals belong to the middle class, in contrast to those who already exist, especially for u_N.[5]

Therefore, it is of interest to consider restricted versions of Existence Independence that apply only to cases involving the addition or removal of individuals who would be the best-off or worst-off. The following two axioms are non-anonymous versions of axioms offered by Asheim and Zuber (2014).

Axiom (Existence Independence of the Best-Off*) For all M and $L \in \mathcal{N}$ such that $M \cap L = \varnothing$, all u_M, $v_M \in \mathbb{R}^M$ and all $w_L \in \mathbb{R}^L$ satisfying $\min w_L \geq \max\{\max u_M, \max v_M\}$, $u_M R v_M$ if and only if $(u_M, w_L)R(v_M, w_L)$.

Axiom (Existence Independence of the Worst-Off) For all $M, N, L \in \mathcal{N}$ such that $(M \cup N) \cap L = \varnothing$, all $u_M \in \mathbb{R}^M$, all $v_N \in \mathbb{R}^N$ and all $w_L \in \mathbb{R}^L$ satisfying $\max w_L \leq \min\{\min u_M, \min v_N\}$, $u_M R v_N$ if and only if $(u_M, w_L)R(v_N, w_L)$.

It is easy to confirm that these axioms are satisfied by Rank-Discounted Utilitarianism. However, we have not been able to determine whether any population principle R satisfying Existence Independence of the Best-Off*, Existence Independence of the Worst-Off and the Strict Limit Property raises a dilemma between repugnancy and sadism. We cannot prove this, and we have no counterexample to offer. However, the following strengthening of Existence Independence of the Best-Off* is considered, where the population M in u_M is allowed to differ from the population N in v_N, leading to the following Existence Independence axiom.

Axiom (Existence Independence of the Best-Off) For all $M, N, L \in \mathcal{N}$ such that $(M \cup N) \cap L = \varnothing$, all $u_M \in \mathbb{R}^M$, all $v_N \in \mathbb{R}^N$ and all $w_L \in \mathbb{R}^L$ satisfying $\min w_L \geq \max\{\max u_M, \max v_N\}$, $u_M R v_N$ if and only if $(u_M, w_L)R(v_N, w_L)$.

We can establish the following result, which demonstrates that Existence Independence of the Best-Off and Existence Independence of the Worst-Off,

[5]This demonstration assumes that the transformation function g is the identity function. However, the argument holds for any continuous, strictly increasing transformation function $g : \mathbb{R} \to \mathbb{R}$.

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combined with the Strict Limit Property, are sufficient for raising the dilemma. Note that Existence Independence of the Best-Off and Existence Independence of the Worst-Off are logically implied by Existence Independence, which applies regardless of the relative positions of the individuals being added or removed.

Observation 5. *If the population principle R satisfies Existence Independence of the Best-Off, Existence Independence of the Worst-Off and the Strict Limit Property, then R raises a dilemma between repugnancy and sadism.*

This follows from Proposition A in the appendix. There is actually little need to modify the proof of the original theorem, since it only relies on Existence Independence in cases involving the best-off and worst-off individuals.

One might argue that this observation suggests that a violation of the Strict Limit Property is a more essential reason why the dilemma is not raised by Rank-Discounted Utilitarianism, since this class of population principles satisfies Existence Independence of the Worst-Off and Existence Independence of the Best-Off*. We warn against this conclusion as the weakening of Existence Independence of the Worst-Off to Existence Independence of the Worst-Off* is not innocent; note in particular that the class of Variable-Value principles considered in Example 4 satisfy Existence Independence of the Worst-Off* but not Existence Independence of the Worst-Off.

5.5. Weak Limit Property and potential dominance

Blackorby *et al.* (1998), Bossert *et al.* (2023) and Cato and Harada (2023) impose another weaker version of the limit property, replacing strict betterness with the weaker notion of at-least-as-good-as.

Axiom (Weak Limit Property) For all $N \in \mathcal{N}$, all $u_N, v_N \in \mathbb{R}^N$ and all $\mu, \nu \in \mathbb{R}$ with $\mu > \nu$, there exists $M^* \in \mathcal{N}_{-N}$ such that, for all $M \in \mathcal{N}_{-N}$ with $M \supseteq M^*$, $(\mu \mathbb{1}_M, u_N) R (\nu \mathbb{1}_M, v_N)$.

If the Strict Limit Property is replaced with the Weak Limit Property, our main theorem is no longer valid. As an example of this, we can take the population principle that judges all populations to be indifferent (the all-indifference principle).

The following axiom ensures that there are infinitely many individuals whose welfare matters positively.

Axiom (Potential Dominance) There exists an infinite subset $\mathbb{M}$ of $\mathbb{N}$ such that for all $i \in \mathbb{M}$, if $u_i > v_i$, then $u_i P v_i$.

Note that all population principles introduced in this paper, except for the all-indifference principle, satisfy this axiom. As we prove in the appendix, under Existence Independence and Potential Dominance, the Weak Limit Property implies the Strict Limit Property.

Lemma *If a population principle R satisfies the Weak Limit Property, Existence Independence and Potential Dominance, then it satisfies the Strict Limit Property.*

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The following observation is obtained from this lemma and our main theorem.

Observation 6. *If the population principle R satisfies Existence Independence, the Weak Limit Property and Potential Dominance, then R raises a dilemma between repugnancy and sadism.*

6. Discussion

In this section, we offer discussions on related works.

6.1. Weakening of Anonymity, Strong Pareto, transitivity and completeness

Notably, our results make it possible to drop or relax several axioms that are commonly imposed. In this section, we examine the implications of these findings.

One of the central features of our analysis is the abandonment of the axiom of Anonymity in favour of an explicit and possibly unequal treatment of identities. Anonymity requires that any distribution obtained through a permutation of individuals be treated as equivalent to the original distribution. In other words, only the shape of the utility distribution is relevant. This condition, also known as impartiality, is commonly regarded as a fundamental requirement of any plausible population principle.

What is the significance of our non-Anonymity result, which does not impose Anonymity? First, there may be cases in which Anonymity is not self-evidently compelling (see, e.g., the recent critical examination of the plausibility of Anonymity by Muñoz 2025). For example, when the entire population includes not only necessary people but also possible people, we must ask whether Anonymity should be imposed across the boundary between these two groups of people. Similarly, when different species are involved – some possessing memories of their lives, while others experience only moment-to-moment existence without memory – the plausibility of Anonymity may be different from the case of comparisons within a single species. Moreover, in the presence of temporal distance, discounting is accepted especially in many fields of economics. This implies that different weights apply to individuals at different times. The discounting approach violates Anonymity and can indeed be regarded as a solution to the Repugnant Conclusion. The result of this paper shows that the fundamental dilemma of population ethics persists even in such contexts where Anonymity is not accepted, provided that the Strict Limit Property limits the unequal treatment of possible people.

There are many cases in which Anonymity is plausible. Even in such cases, this paper still has its significance. Even if Anonymity is imposed, our theorem still holds. Moreover, the Strict Limit Property and Existence Independence are not implied by Anonymity; more precisely, the three conditions are independent of each other. Furthermore, arguably, their plausibility is not necessarily diminished under Anonymity. Therefore, our result can be said to retain its novelty even under Anonymity.

We highlight the significance of the fact that our analysis does not rely on Strong Pareto (or even Weak Pareto). Pareto is often regarded as a relatively uncontroversial requirement. However, not all ethical theories satisfy Strong Pareto. For instance, in

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views such as telic egalitarianism, the levelling-down objection arises, and Strong Pareto is violated (Parfit 1997; Hirose 2014). Similarly, some versions of sufficientarianism do not satisfy Strong Pareto (Frankfurt 1987). Moreover, once one adopts a deontological perspective, there is no clear reason to accept Pareto, which is closely associated with welfarism (Cato *et al.* 2024). The dilemma identified in this paper applies to a broad class of ethical theories: it persists not only for theories that accept Pareto but also for those that reject it, provided that the extent of insensitivity is limited by the Strict Limit Property.

Finally, we note that our analysis continues to hold even when transitivity and completeness are relaxed to some extent. Completeness is often regarded as a rather controversial requirement (Chang 1997, 2002). While many endorse the transitivity of betterness (Broome 1991), some have rejected it (Temkin 1987; Rachels 1998). In Section 5.2, we drop completeness and replace full transitivity with semi-transitivity, yet still obtain an impossibility result. In this sense, our work can be seen as a contribution to intransitive and incomplete population ethics.[6]

6.2. Comparison to Blackorby et al. (2005)

In this subsection, we compare our main result to similar results presented in Blackorby *et al.* (2005), ch. 6. Two results are pertinent.

Theorem 6.8. *There exists no population principle R that satisfies Anonymity, Continuity, Strong Pareto, Intermediate Existence of Critical Levels, Utility Independence, Avoidance of the Repugnant Conclusion and Priority for Lives Worth Living.*

Theorem 6.12. *There exists no population principle R that satisfies Anonymity, Continuity, Strong Pareto, Weak Existence of Critical Levels, Existence Independence, Avoidance of the Repugnant Conclusion and Priority for Lives Worth Living.*

A key axiom, transcribed to our non-anonymous setting, in these results is the following.

Axiom (Priority for Lives Worth Living) For all $M \in \mathcal{N}$, all $u_M \in \mathbb{R}^M_{++}$, all $N \in \mathcal{N}$ and all $v_N \in \mathbb{R}^N_{--}$, $u_M P v_N$.

To facilitate comparison between the two theorems of Blackorby *et al.* (2005) and our main result, we rewrite our theorem.

Reformulated theorem. *There exists no population principle R that satisfies the Strict Limit Property, Existence Independence, Avoidance of the Repugnant Conclusion and Avoidance of the Very Sadistic Conclusion.*

Also, it is helpful to spell out the avoidance of the Very Sadistic Conclusion as a separate axiom.

[6]However, semi-transitivity remains a relatively strong condition. It is thus an important task for future research to examine whether similar results can be obtained under other conditions, such as the interval order property.

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Axiom (Avoidance of the Very Sadistic Conclusion*) There exists $N \in \mathcal{N}$ and $v_N \in \mathbb{R}^N_{--}$ such that, for all $M \in \mathcal{N}$ and all $u_M \in \mathbb{R}^M_{++}$, $u_M R v_N$.

Thereby, it becomes evident that Priority for Lives Worth Living is stronger than Avoidance of the Very Sadistic Conclusion.

This observation implies that Theorem 6.8 is unrelated to our theorem. The reason is that Theorem 6.8 involves Priority for Lives Worth Living, which is not implied by Avoidance of the Very Sadistic Conclusion or the other axioms of our theorem, while our theorem invokes Existence Independence, which is not implied by the axioms of Theorem 6.8.

On the other hand, by Blackorby *et al.* (2005), Theorem 6.10, Anonymity, Continuity, Strong Pareto, Weak Existence of Critical Levels and Existence Independence characterize CLGU. Hence, these axioms imply the Strict Limit Property and Existence Independence, as well as raising the dilemma between repugnancy and sadism. Thereby, since its axioms are stronger, we can conclude that Theorem 6.12 is weaker than our theorem.

6.3. Comparison to Spears and Budolfson (2021)

In this subsection, we compare our main result to the relevant part of Theorem 1 in Spears and Budolfson (2021).

Part of Spears and Budolfson (2021, Theorem 1). *There exists no population principle R that satisfies Anonymity, Aggregation, Avoidance of the Very Repugnant Conclusion and Egalitarian Priority for Lives Worth Living.*

Egalitarian Priority for Lives Worth Living is the following weakening of Blackorby *et al.*'s (2005) Priority for Lives Worth Living.

Axiom (Egalitarian Priority for Lives Worth Living) For all $M \in \mathcal{N}$, all $\mu > 0$, all $N \in \mathcal{N}$ and all $\nu < 0$, $\mu \mathbb{1}_M P \nu \mathbb{1}_N$.

This axiom is stronger than avoiding the Very Sadistic Conclusion*, as introduced in Section 5.1.

Axiom (Avoidance of the Very Sadistic Conclusion*) There exists $N \in \mathcal{N}$ and $v_N \in \mathbb{R}^N_{--}$ such that, for all $M \in \mathcal{N}$ and all $\mu > 0$, $\mu \mathbb{1}_M R v_N$.

On the other hand, Spears and Budolfson (2021), Theorem 1 invokes the following weakening of the Strict Limit Property.

Axiom (Aggregation) For all $N \in \mathcal{N}$, all $u_N \in \mathbb{R}^N$, all $\mu \in \mathbb{R}$ and all $\varepsilon \in \mathbb{R}_{++}$, there exists $M^* \in \mathcal{N}_{-N}$ such that, for all $M \in \mathcal{N}_{-N}$ with $M \supseteq M^*$, $(\mu + \varepsilon)\mathbb{1}_{M \cup N} P(\mu \mathbb{1}_M, u_N) P(\mu - \varepsilon)\mathbb{1}_{M \cup N}$.

Therefore, it follows that the relevant part of Spears and Budolfson (2021), Theorem 1 is unrelated to our main theorem, even in its strengthened version, as introduced in Observation 1.

Reformulated strengthened theorem. *There exists no population principle R that satisfies the Strict Limit Property, Existence Independence, Avoidance of the Repugnant Conclusion and Avoidance of the Very Sadistic Conclusion*.*

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The reason is that the relevant part of Spears and Budolfson (2021), Theorem 1 invokes Anonymity and strengthens Avoidance of the Very Sadistic Conclusion* to Egalitarian Priority for Lives Worth Living, while weakening the Strict Limit Property to Aggregation.

7. Concluding remarks

Our analysis demonstrates that the fundamental dilemma associated with the Repugnant Conclusion and the Very Sadistic Conclusion extends far beyond CLU to encompass a broad class of population principles. Any population principle that satisfies the fundamental axioms of Existence Independence and the Strict Limit Property inevitably faces this dilemma. This result holds without requiring Anonymity, which has significant implications for evaluating populations that differ in temporal, spatial or probabilistic dimensions, or when including non-human populations. The non-anonymous nature of our framework allows for greater flexibility in addressing ethical problems where the identity of individuals may matter, such as intergenerational justice or multi-species welfare considerations.

The unavoidable nature of this dilemma suggests that population ethics may require reconsidering some deeply held intuitions. While we have shown that abandoning either Existence Independence or the Strict Limit Property can help avoid the dilemma, as demonstrated by the Variable-Value principle and the Critical-Level Leximin principle, respectively, these alternatives come with their own limitations. Future research might develop hybrid approaches that develop new axioms that better capture our moral intuitions while avoiding the repugnancy–sadism dilemma. Otherwise, it strengthens the position that avoiding the Repugnant Conclusion should not be a prerequisite for any acceptable population principle (Zuber *et al.* 2021).

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Appendix

Proposition A. *If the semi-transitive population principle R satisfies Existence Independence of the Best-Off, Existence Independence of the Worst-Off and the Strict Limit Property for the Worst-Off, then R raises a dilemma between repugnancy and sadism.*

Proof. Assume that the semi-transitive population principle R satisfies Existence Independence of the Best-Off, Existence Independence of the Worst-Off and the Strict Limit Property for the Worst-Off, and, furthermore, avoids the Very Sadistic Conclusion. It is sufficient to show that the Repugnant Conclusion holds for R. Take any $\xi > 0$ and $N \in \mathcal{N}$, and let $\varepsilon \in (0, \xi)$. By the avoidance of the Very Sadistic Conclusion, there exist $N^* \in \mathcal{N}$ and $v_{N^*} \in \mathbb{R}^{N^*}_{--}$ such that, for all $M \in \mathcal{N}$ and all $u_M \in \mathbb{R}^M_{++}$,

$$u_M R\, v_{N^*}.$$

We distinguish two cases.

Case 1: $N \cap N^* = \varnothing$. Let $\varepsilon^- \in (0, \varepsilon)$. By the Strict Limit Property for the Worst-Off, there exists a non-empty set $M^* \subseteq \mathbb{N}\backslash(N \cup N^*)$ such that, for all $M \subseteq \mathbb{N}\backslash(N \cup N^*)$ with $M \supseteq M^*$,

$$(\varepsilon \mathbb{1}_M, v_{N^*}, \varepsilon \mathbb{1}_N)\, P\, (\varepsilon^- \mathbb{1}_M, \varepsilon^- \mathbb{1}_{N^*}, \xi \mathbb{1}_N).$$

Again, the Strict Limit Property for the Worst-Off implies that there exists a non-empty set $\hat{M} \subseteq \mathbb{N}\backslash(N \cup N^*)$ such that, for all $M \subseteq \mathbb{N}\backslash(N \cup N^*)$ with $M \supseteq \hat{M}$,

$$(\varepsilon^- \mathbb{1}_M, \varepsilon^- \mathbb{1}_{N^*}, \xi \mathbb{1}_N)\ P\ \left(\frac{\varepsilon^-}{2} \mathbb{1}_M, \frac{\varepsilon^-}{2} \mathbb{1}_{N^*}, \xi \mathbb{1}_N\right).$$

For $M \supseteq M^* \cup \hat{M}$, semi-transitivity implies that

$$(v_{N^*}, \xi \mathbb{1}_N)\ P\ \left(\frac{\varepsilon^-}{2} \mathbb{1}_M, \frac{\varepsilon^-}{2} \mathbb{1}_{N^*}, \xi \mathbb{1}_N\right)$$

or

$$(\varepsilon \mathbb{1}_M, v_{N^*}, \varepsilon \mathbb{1}_N)\, P\, (v_{N^*}, \xi \mathbb{1}_N).$$

Note that $\left(\frac{\varepsilon^-}{2} \mathbb{1}_M, \frac{\varepsilon^-}{2} \mathbb{1}_{N^*}\right) \in \Omega_{++}$. Thus, since R avoids the Very Sadistic Conclusion,

$$\left(\frac{\varepsilon^-}{2} \mathbb{1}_M, \frac{\varepsilon^-}{2} \mathbb{1}_{N^*}\right) R\, v_{N^*}.$$

Existence Independence of the Best-Off implies that

$$\left(\frac{\varepsilon^-}{2} \mathbb{1}_M, \frac{\varepsilon^-}{2} \mathbb{1}_{N^*}, \xi 1_N\right) R\, (v_{N^*}, \xi \mathbb{1}_N).$$

Hence,

$$(\varepsilon \mathbb{1}_M, v_{N^*}, \varepsilon \mathbb{1}_N)\, P\, (v_{N^*}, \xi \mathbb{1}_N). \tag{A1}$$

Existence Independence of the Worst-Off implies

$$(\varepsilon \mathbb{1}_M, \varepsilon \mathbb{1}_N)\, P\, (\xi \mathbb{1}_N).$$

The Repugnant Conclusion holds.

Case 2: $N \cap N^* \neq \varnothing$. Let $K = N \cap N^*$. Now, we take $N^{**} \in \mathcal{N}$ such that $N^* \cap N^{**} = \varnothing$ and $N\backslash K \subseteq N^{**}$. Let $\varepsilon^* \in (0, \varepsilon)$. By applying the argument in Case 1, (A1) holds for ε^* and N^{**}. That is, there exists a non-empty set $M^* \subseteq \mathbb{N}\backslash(N^* \cup N^{**})$ such that,

$$(\varepsilon^* \mathbb{1}_M, v_{N^*}, \varepsilon^* \mathbb{1}_{N^{**}})\, P\, (v_{N^*}, \xi \mathbb{1}_{N^{**}})$$

for all $M \subseteq \mathbb{N}\backslash(N^{**} \cup N^*)$ with $M \supseteq M^*$. By applying Existence Independence of the Worst-Off, it follows that $(\varepsilon^* \mathbb{1}_M, \varepsilon^* \mathbb{1}_{N^{**}}) P \xi \mathbb{1}_{N^{**}}$ and therefore, by Existence Independence of the Best-Off,

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$$(\varepsilon^* \mathbb{1}_M, \xi \mathbb{1}_{N^*}, \varepsilon^* \mathbb{1}_{N^{**}})\, P\, (\xi \mathbb{1}_{N^*}, \xi \mathbb{1}_{N^{**}}). \tag{A2}$$

This holds for all $M \subseteq \mathbb{N} \backslash (N^{**} \cup N^*)$ with $M \supseteq M^*$.

Note that $N \subseteq N^* \cup N^{**}$. Let $L = (N^* \cup N^{**}) \backslash N$. Thus,

$$N^* \cup N^{**} = L \cup N.$$

By the Strict Limit Property for the Worst-Off, there exists a non-empty set $M^{**} \subseteq \mathbb{N} \backslash (N^* \cup N^{**})$ such that, for all $M \subseteq \mathbb{N} \backslash (N^* \cup N^{**})$ with $M \supseteq M^{**}$,

$$(\varepsilon \mathbb{1}_M, \xi \mathbb{1}_L, \varepsilon \mathbb{1}_N)\, P\, (\varepsilon^* \mathbb{1}_M, \xi \mathbb{1}_{N^*}, \varepsilon^* \mathbb{1}_{N^{**}}) \tag{A3}$$

because $\xi > \varepsilon > \varepsilon^*$. Note that (A2) and (A3) hold for all $M \subseteq \mathbb{N} \backslash (N^* \cup N^{**})$ with $M \supseteq M^* \cup M^{**}$. As noted in Section 5.2, any reflexive and semi-transitive relation is quasi-transitive. Thus, (A2) and (A3) imply that

$$(\varepsilon \mathbb{1}_M, \xi \mathbb{1}_L, \varepsilon \mathbb{1}_N)\, P\, (\xi \mathbb{1}_{N^*}, \xi \mathbb{1}_{N^{**}}).$$

This implies

$$(\varepsilon \mathbb{1}_M, \xi \mathbb{1}_L, \varepsilon \mathbb{1}_N)\, P\, (\xi \mathbb{1}_L, \xi \mathbb{1}_N)$$

since $N^* \cup N^{**} = L \cup N$. Existence Independence of the Best-Off implies that

$$(\varepsilon \mathbb{1}_M, \varepsilon \mathbb{1}_N)\, P\, \xi \mathbb{1}_N.$$

The Repugnant Conclusion holds. □

Corollary *Observations 2, 3 and 5 are true.*

Proof. The Corollary follows from Proposition A since

- any population principle R (being reflexive, complete and transitive) is also a semi-transitive population principle R (being only reflexive and semi-transitive),
- any semi-transitive population principle R satisfying Existence Independence satisfies also Existence Independence of the Best-Off and Existence Independence of the Worst-Off and
- any semi-transitive population principle R satisfying the Strict Limit Property satisfies also the Strict Limit Property for the Worst-Off.

Proof of Observation 4. We assume that R satisfies Mere Addition and the Strict Limit Property for the Worst-Off. Take any $\xi > 0$ and $N \in \mathcal{N}$, and let $\varepsilon \in (0, \xi)$. By applying Mere Addition repeatedly, we have the following: for any $M \subseteq \mathbb{N} \backslash N$,

$$(\varepsilon^- \mathbb{1}_M, \xi \mathbb{1}_N)\, R\, (\xi \mathbb{1}_N),$$

where $\varepsilon^- \in (0, \varepsilon)$. Note that

$$\varepsilon^- = \min(\varepsilon^- \mathbb{1}_M, \xi \mathbb{1}_N).$$

Thus, the Strict Limit Property for the Worst-Off implies that there exists $M^* \in \mathcal{N}_{-N}$ such that, for all $M \in \mathcal{N}_{-N}$ with $M \supseteq M^*$,

$$(\varepsilon \mathbb{1}_M, \varepsilon 1_N) P (\varepsilon^- \mathbb{1}_M, \xi \mathbb{1}_N).$$

Thus, transitivity implies for all $M \in \mathcal{N}_{-N}$ with $M \supseteq M^*$, $(\varepsilon \mathbb{1}_M, \varepsilon \mathbb{1}_N) P (\xi \mathbb{1}_N)$. The Repugnant Conclusion holds.

Proof of the Lemma. Assume that a population principle R satisfies the Weak Limit Property, Existence Independence and Potential Dominance. Let $N \in \mathcal{N}$, u_N, $v_N \in \mathbb{R}^N$ and μ, $\nu \in \mathbb{R}$ with $\mu > \nu$. By the Weak Limit Property, there exists $M^* \in \mathcal{N}_{-N}$ such that, for all $M \in \mathcal{N}_{-N}$ with $M \supseteq M^*$, $(\mu \mathbb{1}_M, u_N) R (\nu \mathbb{1}_M, v_N)$. By Potential Dominance, there exists an infinite subset $\mathbb{M}$ of $\mathbb{N}$ such that for all $i \in \mathbb{M}$, if $u_i > v_i$, then $u_i P v_i$. Take $k \in \mathbb{M} \backslash (M^* \cup N)$. Define

$$\hat{M} = M^* \cup \{k\}.$$

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Take any $M \supseteq \hat{M}$. Let $M_{-k} = M \backslash \{k\}$. Note that $\mu \mathbb{1}_{\{k\}} P \nu \mathbb{1}_{\{k\}}$. Existence Independence implies that

$$(\nu \mathbb{1}_{M_{-k}}, \mu \mathbb{1}_{\{k\}}, \nu_N)\, P\, (\nu \mathbb{1}_M, \nu_N).$$

Since $M_{-k} \supseteq M^*$, we have $(\mu \mathbb{1}_{M_{-k}}, u_N) R (\nu \mathbb{1}_{M_{-k}}, \nu_N)$. Existence Independence implies that

$$(\mu \mathbb{1}_M, u_N)\, R\, (\nu \mathbb{1}_{M_{-k}}, \mu \mathbb{1}_{\{k\}}, \nu_N).$$

Transitivity implies that $(\mu \mathbb{1}_M, u_N)\, P\, (\nu \mathbb{1}_{M_{-k}}, \nu_N)$. This establishes the Strict Limit Property. □

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