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A note on ion–dust streaming instability in a collisional dusty plasma

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Abstract. This note investigates an ion–dust streaming instability with frequency ω less than the dust collision frequency ν_d, in an unmagnetized collisional dusty plasma. Under certain conditions, a resistive instability can be excited by an ion drift on the order of the ion thermal speed, even when the dust acoustic wave is heavily damped. The effect of weak collisions on the usual dust acoustic instability in the regime $\omega > \nu_d$ is also considered. Applications to experimental observations of low-frequency fluctuations in laboratory d.c. glow discharge dusty plasmas are discussed.

1. Introduction

Dust acoustic waves, which are very low-frequency modes associated with the dynamics of charged dust (Rao et al. 1990; D'Angelo 1990), have been observed in several laboratory dusty plasma experiments (e.g. Barkan et al. 1995; Thompson et al. 1997). In laboratory dusty plasmas where neutral densities are relatively high, collisions of charged particles with neutrals can affect the properties of such modes (e.g. Shukla and Rosenberg 1999). The instability of dust acoustic waves driven by streaming ions in collisional dusty plasmas has been investigated theoretically in various regimes by several authors (e.g. Rosenberg 1996; D'Angelo and Merlino 1996; Kaw and Singh 1997; Mamun and Shukla 2000; Fortov et al. 2000). Recently, observations of dust wave fluctuations with frequency ω less than the dust–neutral collision frequency ν_d have been reported in several experiments (e.g. Fortov et al. 2000; Praburam and Goree 1996). In this note, we investigate a resistive instability with $\omega < \nu_d$, driven by ions streaming with speed of the order of their thermal speed relative to the dust, in an unmagnetized collisional dusty plasma. We also consider the effect of weak collisions on the usual dust acoustic instability in the frequency range $\omega > \nu_d$. The results are applied to experimental observations of low-frequency fluctuations in laboratory d.c. glow discharge dusty plasmas.

2. Analysis

We consider a plasma with three charged species: electrons, ions, and negatively charged dust of uniform size, in a background neutral gas. The condition of charge neutrality is

$$n_e + Z_d n_d = n_i, \tag{1}$$

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where the subscript $\alpha = e, i$, and d refers to electrons, ions, and dust, and Z_d is the dust charge state. In the presence of an external electric field $\boldsymbol{E}_0$ (e.g. a discharge electric field in a glow discharge, or a sheath field in a radiofrequency discharge) the charged species can acquire drifts. In the frame in which the dust is stationary, we assume that the electron and ion drifts are given by (see D'Angelo and Merlino 1996)

$$\begin{aligned} \boldsymbol{u}_{0e} &= \frac{-e\boldsymbol{E}_0}{m_e\nu_e}, \\ \boldsymbol{u}_{0i} &= \frac{e\boldsymbol{E}_0}{m_i\nu_i}, \end{aligned} \tag{2}$$

where ν_e and ν_i are the electron and ion collision frequencies, which we assume are due primarily to collisions with neutrals. The model system considered is similar to that considered by D'Angelo and Merlino (1996).

In the following kinetic analysis, we consider the case when the magnitude of the ion drift speed u_{0i} is of the order of or less than the ion thermal speed and when the electron drift speed u_{0e} is much less than the electron thermal speed. Using drifting Maxwellians for the electron and ion velocity distributions, and a Maxwellian for the dust, and taking collisions into account, the linear dispersion relation for electrostatic waves with $\boldsymbol{k}$ in the $\boldsymbol{E}_0$ direction is given by (e.g. Miyamoto 1989; Ossakow et al. 1975)

$$1 + \sum_{\alpha} \frac{1}{k^2\lambda_{D\alpha}^2} \frac{1 + \zeta_\alpha Z(\zeta_\alpha)}{1 + (i\nu_\alpha/\sqrt{2}kv_\alpha)Z(\zeta_\alpha)} = 0, \tag{3}$$

where

$$\left.\begin{aligned} \zeta_e &= \frac{\omega + ku_{0e} + i\nu_e}{\sqrt{2}kv_e}, \\ \zeta_i &= \frac{\omega - ku_{0i} + i\nu_i}{\sqrt{2}kv_i}, \\ \zeta_d &= \frac{\omega + i\nu_d}{\sqrt{2}kv_d}. \end{aligned}\right\} \tag{4}$$

Here $\lambda_{D\alpha} = (T_\alpha/4\pi n_\alpha Z_\alpha^2 e^2)^{1/2}$ is the Debye wavelength for each species, $v_\alpha = (T_\alpha/m_\alpha)^{1/2}$ is the thermal speed, and $Z(\zeta)$ is the plasma dispersion function (Fried and Conte 1971). The collision frequency of each species ν_α is assumed to be due primarily to collisions with neutrals. For the electron–neutral and ion–neutral collision rates ν_e and ν_i, we have (Book 1990)

$$\begin{aligned} \nu_e &\approx \sigma_{en} n_n v_e, \\ \nu_i &\approx \sigma_{in} n_n v_i, \end{aligned} \tag{5}$$

where n_n is the neutral density, σ_{in} (σ_{en}) is the cross-section for collisions between ions (electrons) and neutrals, and we have assumed that the ion (electron) drift speed is of the order of or less than its thermal speed. For the dust–neutral collision rate ν_d, we use the hard-sphere rate (Baines et al. 1965)

$$\nu_d \sim \tfrac{4}{3}\pi a^2 n_n v_n (m_n/m_d), \tag{6}$$

where m_n is the mass of the neutrals.

We analyze (3) in the limits $\zeta_e \ll 1$ for the electrons and $\zeta_d \gg 1$ for the dust. For

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the ions, first we consider the case of strong ion collisions, with $\zeta_i \gg 1$ (i.e. where the wavelength λ is much greater than the ion mean free path, v_i/ν_i), Note that in laboratory dusty plasmas, where the electron temperature can be of the order of $T_e \sim 2\,\mathrm{eV}$, the electron–neutral collision cross section σ_{en} can be low (e.g. the Ramsauer minimum occcuring at $T_e < 1\,\mathrm{eV}$) for inert gases such as argon or neon (Brown 1967). Thus it may be possible to satisfy both the conditions $\nu_i/kv_i \gg 1$ and $\nu_e/kv_e \ll 1$. In these limits, (3) becomes

$$1 + \frac{1}{k^2\lambda_{De}^2} - \frac{\omega_{pi}^2}{A_i} - \frac{\omega_{pd}^2}{A_d} \approx 0, \tag{7}$$

where

$$A_i \approx \Omega_i(\Omega_i + i\nu_i) - \frac{i\nu_i k^2 v_i^2}{\Omega_i + i\nu_i}, \tag{8}$$

$$A_d \approx \omega(\omega + i\nu_d) - \frac{i\nu_d k^2 v_d^2}{\omega + i\nu_d}, \tag{9}$$

where $\Omega_i = \omega - ku_{0i}$.

For the case of high collisionality where $\nu_i \gg |\Omega_i|$, we obtain

$$1 - \frac{\omega_{pi}^2}{B(\Omega_i i\nu_i + \Omega_i^2 - k^2 v_i^2)} - \frac{\omega_{pd}^2}{BA_d} \approx 0, \tag{10}$$

where $B = 1 + (k\lambda_{De})^{-2}$. We further assume that $BA_d \ll \omega_{pd}^2$, and consider the frequency regime where $\omega < \nu_d \ll \nu_i$. Using $\omega = \omega_r + i\gamma$, with $|\gamma| \ll \omega_r$ for weak growth or damping, we obtain for the real and imaginary parts of the frequency for this resistive instability

$$\omega_r \approx \frac{ku_{0i}}{1 + \Psi}, \tag{11}$$

$$\gamma \approx \frac{\Psi}{1 + \Psi}\frac{1}{\nu_d}\left[\omega_r^2 + \frac{\omega_{pd}^2}{\omega_{pi}^2}(k^2u_{0i}^2 - k^2v_i^2) - k^2v_d^2\right], \tag{12}$$

where

$$\Psi = \frac{\omega_{pi}^2}{\omega_{pd}^2}\frac{\nu_d}{\nu_i}.$$

To obtain (12), we have in addition assumed that that $\Psi \gg 1$ so that $\Omega_i^2 \approx k^2u_{0i}^2$. We note that D'Angelo and Merlino (1996) have also considered dust acoustic instability in higher-collisionality regimes where $k < \nu_d/c_d$, where c_d is the dust acoustic speed, using a fluid analysis and assuming cold dust with $T_d = 0$.

The critical drift can be obtained by setting $\gamma \approx 0$ in (12). For cold dust and $\Psi \gg 1$, we obtain

$$\frac{u_{0i}}{v_i} \approx \left[1 + \left(\frac{\nu_i}{\omega_{pi}}\frac{\omega_{pd}}{\nu_d}\right)^2\right]^{-1/2}. \tag{13}$$

It appears that critical drifts $u_{0i} \sim v_i$ should be sufficient for wave excitation. This instability can persist even when the dust acoustic wave is damped, that is, when $c_d \lesssim v_d$, as long as $\nu_d/kv_d \gg 1$, as will be shown in Sec. 3.

The physical nature of the instability can perhaps be understood by analogy with the Farley–Buneman instability, which is a resistive instability with $\omega \lesssim \nu_i$, driven by electron Hall currents in a collisional electron–ion plasma (e.g. Kelley 1989). To

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see why there is a mode with the phase velocity $v_{\rm ph} = u_{0i}/(1+\Psi)$ (see (11)), consider the case where $T_e \gg T_i$ and $k^2\lambda_{De}^2 \gg 1$, so the electron screening contribution to the mode is small (see (7)). We then consider the approximate model of an ion–negative-dust plasma. Following the analysis of Dimant and Sudan (1995) for the Farley–Buneman mode, for this case, in the frame of reference of the wave (co-moving frame), the ions and dust move in opposite directions with speeds $u_{0i} - v_{\rm ph}$ and $-v_{\rm ph}$, respectively. The perturbed ion and dust flux densities are given by

$$\tilde{\Gamma}_i \approx \tilde{n}_i(u_{0i} - v_{\rm ph}) + n_i\mu_i\tilde{E},$$

$$\tilde{\Gamma}_d \approx -\tilde{n}_d v_{\rm ph} - n_d\mu_d\tilde{E},$$

where perturbed quantities are denoted with a tilde, and where the ion and dust mobilities are given by $\mu_i = e/m_i\nu_i$ and $\mu_d = eZ_d/m_d\nu_d$, respectively. Setting the perturbed flux densities to zero for this quasistationary wave, and using quasineutrality $\tilde{n}_i \approx Z_d\tilde{n}_d$, we obtain

$$v_{\rm ph} \approx \frac{u_{0i}}{1 + (n_i\mu_i/Z_d n_d\mu_d)} = \frac{u_{0i}}{1+\Psi}.$$

The instability arises when the dust inertia force $n_d m_d(\partial\tilde{v}_d/\partial t)$ is large enough to overcome the pressure gradient force (where the pressure is the dust pressure combined with the ion pressure via the ambipolar diffusion effect), which tries to smooth out the density perturbation by diffusion (see Kelley (1989), p. 171, for discussion of the Farley–Buneman instability).

Next we consider the case where $\zeta_i \ll 1$, corresponding to weak ion collisions with $\nu_i \ll kv_i$ (i.e. where the wavelength λ is much less than the ion mean free path, v_i/ν_i), and weak ion drifts $u_{0i}/v_i \ll 1$. In this case, and in the limits $\zeta_e \ll 1$ and $\zeta_d \gg 1$ as above, the dispersion relation (3) becomes

$$1 + \frac{1}{k^2\lambda_{Di}^2}\left[1 + i\sqrt{\frac{\pi}{2}}\frac{\Omega_i}{kv_i}e^{-\zeta_i^2}\left(1 + \sqrt{\frac{\pi}{2}}\frac{\nu_i}{kv_i}e^{-\zeta_i^2}\right)\right] - \frac{\omega_{pd}^2}{\omega(\omega + i\nu_d)}$$
$$+\frac{1}{k^2\lambda_{De}^2}\left[1 + i\sqrt{\frac{\pi}{2}}\frac{\Omega_e}{kv_e}e^{-\zeta_e^2}\left(1 + \sqrt{\frac{\pi}{2}}\frac{\nu_e}{kv_e}e^{-\zeta_e^2}\right)\right] \approx 0, \qquad (14)$$

where $\Omega_e = \omega + ku_{0e}$, and it has been assumed that $\nu_i < |\Omega_i|$ and $\nu_e < |\Omega_e|$. We consider the case where the parameters are such that $u_{0e}/v_e < u_{0i}/v_i$ (i.e. roughly $\sigma_{in}T_i < \sigma_{en}T_e$) and $\lambda_{De}^2 \gg \lambda_{Di}^2$ (i.e. $T_e \gg T_i$). Assuming the frequency range $\omega \gg \nu_d$, and using $\omega = \omega_r + i\gamma$, with $|\gamma| \ll \omega_r$ for weak growth or damping, we obtain for the real and imaginary parts of the frequency

$$\omega_r \approx \frac{k\lambda_{Di}\omega_{pd}}{(1 + k^2\lambda_{Di}^2)^{1/2}}, \qquad (15)$$

$$\frac{\gamma}{\omega_r} \approx -\frac{\omega_r^2}{2\omega_{pd}^2}\frac{1}{k^2\lambda_{Di}^2}\sqrt{\frac{\pi}{2}}\frac{\Omega_i}{kv_i}e^{-\zeta_i^2}\left(1 + \sqrt{\frac{\pi}{2}}\frac{\nu_i}{kv_i}e^{-\zeta_i^2}\right) - \frac{\nu_d}{2\omega_r}. \qquad (16)$$

In this case, weak ion collisions have a destabilizing effect on the dust acoustic instability (e.g. Rosenberg 1993).

3. Application

Observations of low-frequency fluctuations, with inferred frequencies smaller than the dust–neutral collision frequency, have been reported for a dusty plasma in a d.c.

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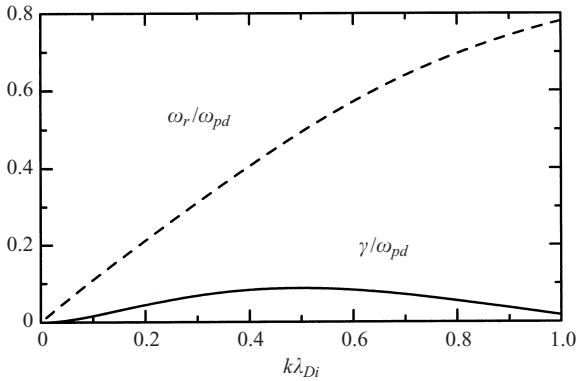


Figure 1. Solutions of (3) for the following parameters: $Z_d = 1800$, $n_d/n_i = 5 \times 10^{-4}$, $T_e/T_i = T_e/T_d = 100$, $m_i/m_p = 20$, $m_d/m_p = 3 \times 10^{12}$, $u_{0i}/v_i = 0.8$, $u_{0e}/v_i = 260$, $\nu_i/\omega_{pi} = 0.5$, $\nu_e/\omega_{pi} = 50$, and $\nu_d/\omega_{pi} = 3.7 \times 10^{-5}$.

glow discharge by Fortov et al. (2000). In the latter paper, a mechanism for wave excitation involving both ion streaming and dust-charge variations in the presence of external charge-dependent forces was considered.

Here we point out that waves with frequency less than the dust–neutral collision frequency might be excited via the resistive instability discussed above, (11) and (12), for the parameters given in Fortov et al. (2000). We use the following representative parameters from Fortov et al. (2000): $T_e \sim 3\,\mathrm{eV}$, $T_i \sim T_n \sim 0.03\,\mathrm{eV}$, $n_i = 10^8\,\mathrm{cm}^{-3}$, $m_i/m_p \sim 20$ (so that $\omega_{pi} \sim 3 \times 10^6\,\mathrm{s}^{-1}$ and $\lambda_{Di} \sim 0.013\,\mathrm{cm}$), $a \sim 1\,\mu\mathrm{m}$, $m_d/m_p \sim 3 \times 10^{12}$, $n_d/n_i = 5 \times 10^{-4}$, and $Z_d \sim 1800$ (giving $\omega_{pd} \sim 300\,\mathrm{s}^{-1}$). We use $\sigma_{en} \sim 2 \times 10^{-16}\,\mathrm{cm}^2$ for the electron–neon scattering cross-section (e.g. Mitchner and Kruger 1973, p. 103) and estimate $\sigma_{in} \sim 4 \times 10^{-15}\,\mathrm{cm}^2$ for the ion–neutral (charge-transfer) cross-section (Brown 1967, p. 64). The solution of (3) for two different neutral pressures P are given in Figs 1 ($P \sim 0.3\,\mathrm{Torr}$) and 2 ($P \sim 0.2\,\mathrm{Torr}$). It is assumed that $T_d = T_i$. The collisional parameters in Fig. 1 are $\nu_e/\omega_{pi} = 50$, $\nu_i/\omega_{pi} = 0.5$, and $\nu_d/\omega_{pi} = 3.7 \times 10^{-5}$ (corresponding to $\nu_d/\omega_{pd} \sim 0.37$), and the drift speeds are $u_{0i}/v_i = 0.8$ and $u_{0e}/v_i = 260$ (corresponding to $E_0 \sim 1\,\mathrm{V\,cm}^{-1}$). The collisional parameters in Fig. 2 are $\nu_e/\omega_{pi} = 33$, $\nu_i/\omega_{pi} = 0.33$, and $\nu_d/\omega_{pi} = 2.5 \times 10^{-5}$ (corresponding to $\nu_d/\omega_{pd} \sim 0.25$), and the drift speeds are again $u_{0i}/v_i = 0.8$ and $u_{0e}/v_i = 260$ (corresponding to $E_0 \sim 0.67\,\mathrm{V\,cm}^{-1}$).

From Fig. 1, there is a resistive instability in the range of approximately $0.02 < k\lambda_{Di} < 0.3$, with real frequency $\omega_r < \nu_d$. At $k\lambda_{Di} = 0.25$, $\omega_r \sim 0.25 \sim \omega_{pd} \sim 75\,\mathrm{s}^{-1}$. This is of the order of the inferred frequency of the observed modes reported in Fortov et al. (2000). However, the unstable wavenumber, $k \sim 20\,\mathrm{cm}^{-1}$, is different from the observed value $k \sim 60\,\mathrm{cm}^{-1}$. In Fig. 2, resistive instability occurs in the range of approximately $0.01 < k\lambda_{Di} < 0.2$. At $k\lambda_{Di} \sim 0.2$ (corresponding to $k \sim 16\,\mathrm{cm}^{-1}$), the real part of the frequency is of the order of $\omega_r \sim 0.2\omega_{pd} \sim 60\,\mathrm{s}^{-1}$.

From Figs 1 and 2, it can be seen that growth rates for these sets of parameters maximize in the smaller-wavelength regime, where the ions are weakly collisional. In this case, wavenumbers $k \sim 60\,\mathrm{cm}^{-1}$ ($k\lambda_{Di} \sim 0.8$) can be unstable, but this corresponds to real frequencies of the order of $0.7\omega_{pd} \sim 200\,\mathrm{s}^{-1}$, which is larger than the frequencies inferred in the experiment of Fortov et al. (2000).

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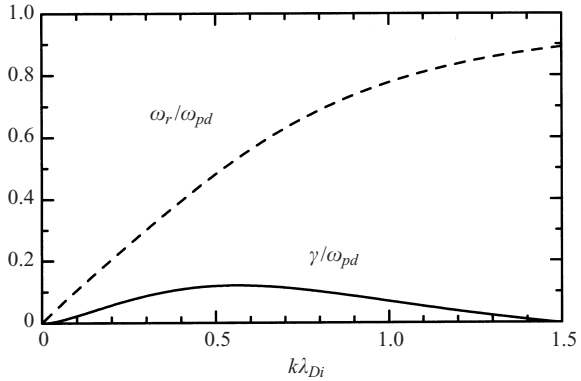


Figure 2. Solutions of (3) for the following parameters: $Z_d = 1800$, $n_d/n_i = 5 \times 10^{-4}$, $T_e/T_i = T_e/T_d = 100$, $m_i/m_p = 20$, $m_d/m_p = 3 \times 10^{12}$, $u_{0i}/v_i = 0.8$, $u_{0e}/v_i = 260$, $\nu_i/\omega_{pi} = 0.33$, $\nu_e/\omega_{pi} = 33$, and $\nu_d/\omega_{pi} = 2.5 \times 10^{-5}$.

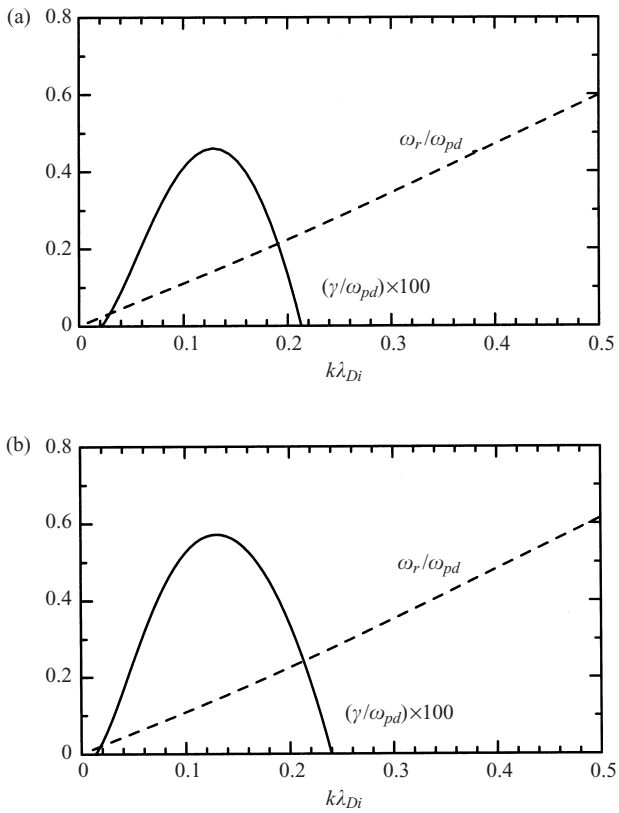


Figure 3. Solutions of (3). (a) Parameters same as in Fig. 1, but with $T_e/T_d = 0.15$. (b) Parameters same as in Fig. 2, but with $T_e/T_d = 0.15$.

The effect of increasing dust temperature (i.e. dust kinetic energy) T_d is shown in Fig. 3. In Fig. 3(a), the parameters are the same as in Fig. 1 for the higher-pressure case, but with $T_e/T_d = 0.15$ (corresponding to a dust thermal speed $v_d \sim 2{-}3\,\mathrm{cm\,s^{-1}}$ for the above parameters). In Fig. 3(b), the parameters are the same as in Fig. 2,

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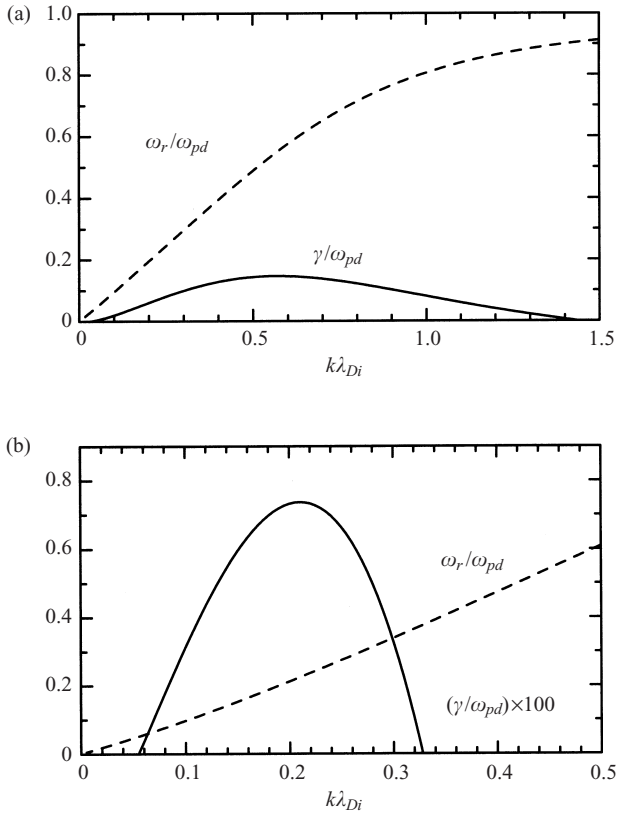


Figure 4. Solutions of (3) for the following parameters: $Z_d = 5 \times 10^4$, $n_d/n_i = 5 \times 10^{-6}$, $T_e/T_i = 67$, $m_i/m_p = 40$, $m_d/m_p = 4 \times 10^{16}$, $u_{0i}/v_i = 1$, $u_{0e}/v_i = 450$, $\nu_i/\omega_{pi} = 0.25$, $\nu_e/\omega_{pi} = 45$, $\nu_d/\omega_{pi} = 1.0 \times 10^{-6}$, and (a) $T_e/T_d = 67$, (b) $T_e/T_d = 0.012$.

but with $T_e/T_d = 0.15$. As can be seen, a resistive instability persists, while the dust acoustic instability with $\omega > \nu_d$ is suppressed.

It seems that either the unstable frequencies or the unstable wavenumbers – but not both together – may be consistent with the experimental observations reported in Fortov et al. (2000). It is possible that the estimated dust charge and density values may be somewhat different, and this may affect estimates of the unstable spectrum. Another possibility is that boundary effects, such as reflections from the end of the cloud, can affect the unstable spectrum: analysis of such effects are left for further work. It is also possible that a different instability mechanism, discussed in Fortov et al. (2000), is responsible for wave excitation in that experiment.

Experimental observations of very low-frequency modes in a d.c. glow discharge dusty plasma experiment were reported by Thomas and Watson (1999). Although the observed waves appeared to have $\omega > \nu_d$, here we consider consider conditions for exciting waves with $\omega < \nu_d$ via the resistive instability discussed in this paper. We use the following representative parameters from Thomas and Watson (1999): $T_e \sim 2\,\text{eV}$, $n_i \sim 10^9\,\text{cm}^{-3}$, $m_i/m_p \sim 40$ (so that $\omega_{pi} \sim 6 \times 10^6\,\text{s}^{-1}$), $a \sim 20\,\mu\text{m}$, $m_d/m_p \sim 4 \times 10^{16}$, with $n_d \sim 10^3 - 10^4\,\text{cm}^{-3}$, and $P \sim 90 - 300\,\text{mTorr}$. We assume $T_i \sim T_n \sim 0.03\,\text{eV}$, which gives an ion Debye length $\lambda_{Di} \sim 0.005\,\text{cm}$. Assuming $P \sim 300\,\text{mTorr}$ (corresponding to $n_n \sim 10^{16}\,\text{cm}^{-3}$), and estimating $\sigma_{en} \sim 4 \times 10^{-16}\,\text{cm}^2$

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for the electron–argon collision cross-section (e.g. Mitchner and Kruger 1973, p. 103) and $\sigma_{in} \sim 5 \times 10^{-15}\,\mathrm{cm}^2$ for the ion–neutral (charge-transfer) cross-section (Brown 1967, p. 72), we obtain $\nu_e/\omega_{pi} \sim 45$ and $\nu_i/\omega_{pi} \sim 0.25$. Estimating the dust-charge state using $Z_d \approx a|\phi_s|/e$ with the grain surface potential $\phi_s \approx -2T_e/e$, we obtain $Z_d \sim 5 \times 10^4$. Assuming $n_d \sim 5 \times 10^3\,\mathrm{cm}^{-3}$, we obtain $\omega_{pd} \sim 23\,\mathrm{s}^{-1}$.

The solution of (3) for $u_{0i}/v_i = 1$ and $u_{0e}/v_i = 450$ (corresponding to $E_0 \sim 2\,\mathrm{V\,cm}^{-1}$) is given in Fig. 4. The collisional parameters are $\nu_e/\omega_{pi} = 45$, $\nu_i/\omega_{pi} = 0.25$, and $\nu_d/\omega_{pi} = 1 \times 10^{-6}$ (so that $\nu_d/\omega_{pd} \sim 0.3$). In Fig. 4(a), where $T_d = T_i$, resistive instability can occur in the range of approximately $0.05 < k\lambda_{Di} < 0.2$, with real frequency $\omega_r < \nu_d$, but maximum growth occurs in the dust acoustic regime where $\omega > \nu_d$. In Fig. 4(b), the dust temperature is increased to $T_e/T_d = 0.012$ (corresponding to a dust thermal speed $v_d \sim 0.7\,\mathrm{mm\,s}^{-1}$ for the above parameters), and it can be seen that only the resistive instability persists.

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References

Baines, M. F., Williams, I. P. and Asebimo, A. S. 1965 *Mon. Not. R. Astron. Soc.* **130**, 63.
Barkan, A., Merlino, R. L. and D'Angelo, N. 1995 *Phys. Plasmas* **2**, 3563.
Book, D. L. 1990 *NRL Plasma Formulary*, NRL.
Brown, S. C. 1967 *Basic Data of Plasma Physics, 1966*. Cambridge, MA: MIT Press.
D'Angelo, N. 1990 *Planet. Space Sci.* **38**, 1143.
D'Angelo, N. and Merlino, R. L. 1996 *Planet. Space Sci.* **44**, 1593.
Dimant, Y. S. and Sudan, R. N. 1995 *J. Geophys. Res.* **100**, 14 605.
Fortov, V. E., Khrapak, A. G., Khrapak, S. A., Molotkov, V. I., Nefedov, A. P., Petrov, O. F. and Torchinsky, V. M. 2000 *Phys. Plasmas* **7**, 1374.
Fried, B. D. and Conte, S. D. 1961 *The Plasma Dispersion Function*. New York: Academic Press.
Kaw, P. and Singh, R. 1997 *Phys. Rev. Lett.* **79**, 423.
Kelley, M. C. 1989 *The Earth's Ionosphere: Plasma Physics and Electrodynamics*. San Diego: Academic Press.
Mamun, A. A. and Shukla, P. K. 2000 *Phys. Plasmas* **7**, 4412.
Mitchner, M. and Kruger, C. H. Jr 1973 *Partially Ionized Gases*. New York: Wiley-Interscience.
Miyamoto, K. 1989 *Plasma Physics for Nuclear Fusion*. Cambridge, MA: MIT Press.
Ossakow, S. L., Papadopoulos, K., Orens, J. and Coffey, T. 1975 *J. Geophys. Res.* **80**, 141.
Praburam, G. and Goree, J. 1996 *Phys. Plasmas* **3**, 1212.
Rao, N. N., Shukla, P. K. and Yu, M. Y. 1990 *Planet. Space Sci.* **38**, 543.
Rosenberg, M. 1993 *Planet. Space Sci.* **41**, 229.
Rosenberg, M. 1996 *J. Vac. Sci. Technol. A* **14**, 631.
Shukla, P. K. and Rosenberg, M. 1999 *Phys. Plasmas* **6**, 1038.
Thomas, E. Jr and Watson, M. 1999 *Phys. Plasmas* **6**, 4111.
Thompson, C., Barkan, A., D'Angelo, N. and Merlino, R. L. 1997 *Phys. Plasmas* **4**, 2331.

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