Time as Relative

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Abstract
Philosophical development of Leibniz's view that time is merely earlier–later order is necessary because neither Leibniz nor modern followers sufficiently answered the Newtonian charge that order does not give quantity. Logically, order is transitive, quantity, as in distance, is not. Quantity, as well as order, is naturally assumed in Newton's absolute time, so that to declare the mere relative order sufficient is to have to show how quantity can arise for it. The modern theory of the continuum, perfectly applicable to Newton's absolute, does not show this but assumes quantity. The development given here shows how interval, instant and simultaneity can be logically developed from Leibniz's insight.

St. Augustine's famous answer to the question 'What is time?' is still with us: 'If nobody asks me, I know; if I want to explain it to someone asking, I don't know'.[1] Even yet, in our science–informed modern world, we may agree with Augustine that what we may call our intuitive notions of time come easily to us: the notion of when, those of earlier and later, and indeed those of past, present and future, and we are at least comfortable with that of the temporal interval, with its extension and continuity, that of the extensionless instant and that of simultaneity, of two occurrences at the same *when*. It is on the philosophical level of explanation, ontological, logical, epistemological, that difficulties arise. We find ourselves wondering whether past, present and future are real, whether the instant can be a real existent as opposed to a mathematical construction, and, on the other hand, whether there can be anything but instants, since we have to wonder whether the temporal interval, of which *when* one end exists the other does not, can have extension at all let alone continuity. Such problems, some of them going back to Augustine and beyond him to Aristotle, keep cropping up because, it seems, of the linking of the notions of existence and nonexistence with that of when. We tend to be more comfortable with the concept of space where such a linking does not occur, or not obviously.

It should be emphasized here that such problems are not mathematical or scientific but properly philosophical. Mathematics and science, as it is commonly regarded, start not from the individually

[1] St. Augustine, *Confessions* 11. 14 (17).

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primitive level of intuition as philosophy must but from a more common level where the fundamental propositions, for example Newton's laws or Euclid's definitions, are agreed upon by all. These sciences can then advance progressively. The mathematicians and the scientists may be entitled, for the purposes of the progressive development of their disciplines, to think of time, along with number, as analogous to space, with which they share the more abstract logic of precedes–succeeds, time here being considered as like the line; and space has been fruitfully studied ever since the Greeks. But philosophy, despite perhaps the science-emulative wishes of some modern philosophers, is not so progressive. It tends to look back sceptically, worry about the assumptions and the logic which they seem to certify rather than constantly pushing forward in the manner of a science. One of the things we shall have to worry about, we shall find as we pursue the philosophical path to the nature of time, is the question whether time can simply be taken as analogous to the line, whether the question of when must be settled before that of where.

My own attempt, like that of others, to address Augustine's question, will of course be an attempt to remove the second part of his answer and show that we can have a consistent explanation of the nature of time, one which manages not to violate our fundamental, seemingly non-problematic, assumptions. This will be done here in terms of a relativist view of time. I will first simply state my proposals without explanation, though with some mention of what needs explaining and why, then proceed first to show that the common view of time as relative rather than absolute is not in general well developed philosophically, and finally to the explanations of my proposals, so far as the very constricted space of a paper allows. This will all be an attempt to set the relativist view of time on a sound philosophical basis.

1. Statement of Proposals

First then, and logically, time is the earlier–later relation, to which past, present and future can, by suitable definitions, be reduced. Obviously such reduction must be justified, but I shall not discuss it here since I have already dealt with it in some detail.[2] More fundamentally, we must explain how time as such a relation,

[2] Denis Corish, 'Earlier and Later If and Only If Past, Present and Future', *Philosophy* **8** (2011), 41–58.

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a mere order, can produce the mathematical ideals of interval, with its extension and continuity, non-extended instant and simultaneity. This requirement, the demand for which exists from the time of Newton, has been almost totally ignored. And it should be said here, with as much emphasis as possible, that, whatever may be thought of my efforts at providing a philosophical account of such ideals, somebody must attempt to provide one according to the claim that time is relative. So far as I can see, the claim that time is relative has in many cases, including, we shall see, that of Leibniz, been accepted with no such account, though it seems to be demanded by the logic of the claim. People have been content to say that time is relative, and then carry on as though the mathematical ideals were already automatically provided without any need of showing how they conformed logically to that claim. The main first task of this essay will be to persuade philosophers and others that the claim that time is relative logically requires such a showing, as the claim that time is absolute does not. This is a very important point.

Secondly, though in the explanatory section we shall treat it first, the ontological foundation of time is the existence of change, which gives the fundamental meaning to 'earlier' and 'later'. There is a third, epistemological problem: time is change observed, and the very possibility of such observation should, and I believe can, be examined, though mere lack of space will prevent that examination here. Our efforts here will go into the logical and ontological considerations.

2. The Relativist View and the Newtonian Challenge

Of course the suggestion that, logically, time is the earlier–later relation is not original. In modern times it goes back to Gottfried Wilhelm Leibniz who put it forward against Sir Isaac Newton's notion of time as absolute, and has had many adherents since. As expressed in Andrew Motte's standard translation of the third edition of Newton's *Philosophiae Naturalis Principia Mathematica*, Newton said:

> Absolute, True, and Mathematical Time, of it self, and from its own nature flows equably without regard to any thing external, and by another name is called Duration: Relative, Apparent, and Common Time is some sensible and external (whether accurate or unequable) measure of Duration by means of

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> motion, which is commonly used instead of True time; such as an Hour, a Day, a Month, a Year.[3]

What should for our purposes be particularly noted about this famous passage is that absolute, truly mathematical time, existing entirely by itself alone and dependent upon nothing else, especially not on anything physical, must necessarily exist if relative time, which is so dependent, is to exist and be measured by clock or calendar. Relative time is merely 'some sensible and external (whether accurate or unequable) measure' of such absolute time. Newton, as a physicist, is naturally interested in measurement as it may be useful for the mathematical treatment of the physical world, but for him the mathematical itself in its pure form is not at all physical, and if physical time is to be useful mathematically, it must depend upon, and be in some sense derivable from that pure form. What it is very important for us to observe here is that in that pure form, in absolute time, intervals and instants must be expected, assumed, naturally to exist; otherwise there could be no meaning given to saying that time as relative 'is some sensible and external … measure' of such absolute time. We might possibly devise, or simply decree, some system of physical measurement, but what it was that was being so measured, that the physical had some semblance to, would be quite unknown without the assumption of exact, non-physical quantities.

It is commonly said that Newton's relative measurements, in time, space and motion, (though only the first is our particular interest here) approximate the absolute. Approximation is of course satisfactorily handled wholly within the idealized domain of mathematics, but if Newton insists upon separating the ideal mathematical from the non-ideal relative, as he does, a proper *philosophical* question to ask is what such approximation of the not strictly mathematical to the mathematical is supposed to mean. It appears that Newton falls back on some intuitive notion such as that relative time, as also relative space and motion, occurs within what might be called the footprint of the absolute, and so approximates the latter. The absolute is like a box, totally independent of its contents, into which, at a particular and truly mathematical time and place, physical, not strictly mathematical, entities occur. Their physical presence then is approximately at the real, mathematical time and place at which they occur.

[3] I. Bernard Cohen (ed.), *The Mathematical Principles of Natural Philosophy by Sir Isaac Newton: Translated into English by Andrew Motte, 1729* (London: Dawsons of Pall Mall, 1968), Vol. I, 9.

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This intuitive approximation allows the physical measurements to be converted into the ideals necessary for manipulations such as those of Euclidean geometry and Newtonian dynamics, and the results to be converted back again into the physically observable, relative measurements. This system, in its nineteenth-century form, was good enough to predict and verify the existence of the new planet, Neptune, because of observable anomalies in the orbit of the already known Uranus. It was, of course, a scientific triumph.

But there were still philosophical problems. The major one was that the absolute, true and mathematical, as so set up by Newton, appears to be quite unobservable in itself, which is not a good situation for the physical science that Newton wanted to establish. This led him to claim that the famous bucket experiment (roughly, that water in a vessel hung upon a twisted rope that was allowed to untwist showed, in climbing in the rotation the sides of the vessel, an absolute effect not visible if the vessel were still and the surrounding area rotated to the same relative extent) – the bucket experiment showed that the motion, made up of time and space, was absolute, not relative. Ernst Mach however argued to the contrary that the climbing effect was produced by a rotation relative to 'the mass of the earth and other celestial bodies'.[4] We shall not go into this well-discussed question here.

Back in his own day Newton's absolutes were objected to as unnecessary, by Leibniz, who believed instead that the relative was quite sufficient for the scientific study of the physical world. Newton himself said of absolute time, 'It may be, that there is no such thing as an equable motion, whereby time may be accurately measured', and we must put up with such time being 'distinguish'd from Relative by the Equation or correction of the vulgar time'.[5] This, as H. G. Alexander points out, is done by an averaging out of the sidereal day, with some help from the pendulum and the moons of Jupiter,[6] but of course it comes under the philosophical caution about approximation that we saw earlier. And again the important point must be made that such 'Equation or correction of the vulgar time' must presuppose that the interval to be so measured must already exist in absolute time.

[4] E. Mach, *The Science of Mechanics*, tr. T. J. McCormack (Chicago: 1902), 232.

[5] Cohen, op. cit., 11.

[6] H. G. Alexander (ed.), *The Leibniz-Clarke Correspondence* (Manchester: Manchester University Press, 1956), xxxv–xxxvi.

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In his attack upon Newton's absolutes Leibniz calls time 'an order of successions'.[7] This may be taken as a simple report of observation rather than as a definition which would have to be justified by a reference back to time, so that Leibniz need not be accused here of circularity. On the same stipulation of observation time may be called the relation of earlier–later, which relation of course does not exist unless there are things that can be observed to be earlier and later.

Leibniz's so regarding time (and space and motion) as relative led him to assert that 'time does only co-exist with creatures, and is only conceived by the order and quantity of their changes',[8] and, as he says of space, 'we have no need to fancy any absolute reality out of the things whose situation we consider'.[9] In fact, as we said before of Newton's view that the absolute is a kind of independent box in which there are occurrences at particular times and places, we may now say of Leibniz's view that the occurrences, in virtue of their relations to each other, *constitute* the box, the times and places. That is the essence of the relativist, as opposed to the absolutist, view. The earlier–later relation as such, of course, occurs in the absolute as well as in the relative; Newton's instants, for example, are earlier or later one than another. The relativist view is that time *consists* in that relation rather than in something, quantities, as we shall see, possessing that relation.

This stance prompted an implicit but extremely important challenge from Samuel Clarke, who was answering Leibniz's attack on Newton, and who was presumably in so doing in touch with Newton himself.[10] This challenge, which is fundamentally important philosophically, has been almost totally ignored in the literature. The challenge was not, as we shall see, fully realized either by Leibniz himself or by modern supporters who tend to fall back on modern mathematical theories, taking them for granted without showing how such theories are consistent with time regarded as relative in precisely Leibniz's sense, that is, as a mere order of things. We saw that the mathematical ideals for Newton's account must be assumed for an existent ideal mathematical time, but it is another matter to deny that existence but then simply take for granted those ideals. This, though not generally recognized, simply because of that taking for granted, is the fundamental philosophical problem for the relativist view as

[7] Alexander, op. cit., 26.
[8] Ibid., 75.
[9] Ibid., 69.
[10] Ezio Vailati, *Leibniz and Clarke: A Study Of Their Correspondence* (Oxford: Oxford University Press, 1997), 4.

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generally accepted. The challenge, when put explicitly rather than in the implicit way that Clarke puts it, is this: *Show how time regarded as the mere, relative order of succession of things, the mere relation of earlier–later* (*or space regarded as 'an order of coexistences'*[11]), *can support the mathematical ideals necessary for the scientific study of the physical world*. For the Newtonians, to assert, as Leibniz does, that time is relative rather than absolute is to be deprived of the 'Absolute, True, and Mathematical' without which the 'Relative, Apparent, and Common' cannot exist as 'some sensible and external (whether accurate or unequable) measure' of that absolute time. The actual, implicit, way in which Clarke puts his challenge is this: 'Space and time are quantities; which situation and order are not'.[12] How then to get quantity from relation, rather than merely assuming quantity, though the latter may be justified in a purely absolutist theory, is the problem.

This challenge of Clarke's constitutes, philosophically, a very radical criticism not only of Leibniz's but of any theory of time, or space, as relative rather than absolute, and, as I have said, it has been almost totally ignored. An exception is H. G. Alexander, the editor of *The Leibniz–Clarke Correspondence*, who in his Introduction points to Clarke's challenge and says: 'In fact the objection shows that any relational theory of space and time is radically incomplete unless supplemented by an analysis of the process of measurement. Leibniz never provides such an analysis'.[13] Leibniz himself sets the mood for such ignoring, by so failing to take Clarke's challenge into consideration that Clarke, in his fourth reply challenged him again on it. Finally Leibniz, in his last letter, and, as it happens, shortly before he died, did attempt to answer:

> As for the objection that space and time are quantities, or rather things endowed with quantity; and that situation and order are not so: I answer, that order also has its quantity; there is in it, that which goes before, and that which follows; there is distance or interval.[14]

This certainly does not tell us *how* the relations of temporal and spatial order are supposed to exhibit quantity, and Clarke answers it easily:

[11] Alexander, op. cit., 25.
[12] Ibid., 32.
[13] Ibid., xxvii.
[14] Ibid., 75.

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> Going before, and following, constitutes situation or order; but the distance, interval, or quantity of time or space, wherein one thing follows another, is entirely a distinct thing from the situation or order, and does not constitute any quantity of situation or order: the situation or order may be the same, when the quantity of time or space intervening is very different.[15]

That is, M may come three seconds or three centuries before N, in which case, though the order and *relative* situation of M and N is precisely the same, M is earlier than N, the distances are entirely different.

Earlier on in his fifth and last letter Leibniz, describing the human perception of the spatial 'order of coexistence' of things said: 'This order is their *situation* or distance'.[16] So we see him here again identifying situation, which is a matter of order, with quantity, which is a matter of distance. Of course distance involves order, but it is too complex to be simply identified with it. Order is transitive, distance is not. If M is earlier than N and N is earlier than O, then M is earlier than O, but if M is one hour earlier than N and N is one hour earlier than O, it does not follow that M is one hour earlier than O. Logically, to insist that the relative situation of order is distance is to violate the transitivity of order. It appears that Leibniz, who, according to Ezio Vailati, came to the notion that space and time are relations relatively late,[17] seems not to have worked out the logic. Clarke in his fourth reply had said that 'the order of things succeeding each other in time is not time itself: for they may succeed each other faster or slower in the same order of succession, but not in the same time'.[18] Leibniz, replying to this in his last letter denied it with a bold 'this is not so', and went on: 'for if the time is greater, there will be more successive and like states interposed; and if it will be less, there will be fewer; seeing there is no vacuum, nor condensation, or penetration (if I may so speak), in times any more than in places'.[19] Leibniz's denial of vacuums did not do well historically, and does not help here. What he is denying is that the order remains the same if the quantity is changed but the evidence he offers is in terms of quantity rather than in terms of order as such. Clarke is challenging Leibniz to show just how quantity is supposed to come from order, but Leibniz appears to be merely supposing it rather than showing it. And this

[15] Alexander, op. cit., 105.
[16] Ibid., 69, italics in text.
[17] Vailati, op. cit., 112.
[18] Alexander, op. cit., 52.
[19] Ibid., 89–90.

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will not be an unusual response, or rather an unusual attitude, since Clarke's challenge has hardly been recognized by those who favour a relativist view.

There is one place where Leibniz does indicate what should be required of such a view. In his third letter, arguing that even God could have no reason to distinguish one empty pre-creative instant from another, as He would in Clarke's Newtonian, absolutist view, says 'that ['mere instants as such being indistinguishable from each other'] instants, consider'd without the things are nothing at all; and that they consist only in the successive order of things'.[20] But he does not say here or elsewhere just how this is so, as Clarke's challenge, taken explicitly, wants him to do, that is, does not say just how instants are to be derived from 'the successive order of things'. Instants as well as intervals, we saw, are given as naturally assumed in Newton's absolutist view, but if Newtonian absolute time is denied, as by Leibniz, then it must be shown how, and not merely assumed that, ideal mathematical instants and intervals can appear.

The final evidence, in Leibniz's final letter, that he does not realize the full seriousness of Clarke's challenge is that he seems content, as it appears others will be also, with the notion that time, and space, are 'merely ideal'. He even offers a decidedly odd 'proof' that this is so:

> Whatever exists of time and of duration, [being successive] perishes continually: and how [as in Newton's view] can a thing exist eternally, which (to speak exactly) does never exist at all? For how can a thing exist, whereof no part does ever exist? Nothing of time does ever exist, but instants; and an instant is not even itself a part of time. Whoever considers these observations, will easily apprehend that time can only be an ideal thing. And the analogy between time and space, will easily make it appear, that the one is as merely ideal as the other.[21]

(There is also a separate odd argument, based on the Aristotelian notion of attributes not existing outside of substances, for the 'mere' ideality of space, which we shall not consider here).

This old and good argument for the non-extended nature of instants, going back to Augustine and Aristotle, is used very oddly here: time cannot exist and therefore is 'merely ideal'. If 'merely ideal' meant simply that it cannot exist, there would be nothing logically wrong with that, but it would be redundant. But 'merely ideal'

[20] Alexander, op. cit., 27.

[21] Ibid., 72–73 – brackets, with the exception of those around 'as in Newton's view', are in the text.

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somehow sloughs off the 'merely' and becomes simply 'ideal', that is, it seems, with all the required mathematical, ideal properties. So the time which cannot exist has all the mathematical properties required for the scientific examination of nature. Leibniz, though denying Newton's absolute time, with its natural mathematical properties, simply assumes those properties, 'without regard to any thing external' for relative time. The quoted words are of course Newton's, not Leibniz's, and are for absolute, not relative, time.

Hans Reichenbach says:

> [Clarke] attempts to point out contradictions in Leibniz' views; he thinks it is a contradiction to say that an order of situations is a quantity. Leibniz had previously explained that the serial order, the existence of preceding and succeeding elements, provides the conditions of measurability. Clarke evidently holds the view that quantity is an absolute concept.[22]

He does indeed. But from that point of view, with its naturally assumed mathematical properties, he is able to challenge the existence of such properties in the relativist view and demand an account of how they can be provided. Reichenbach does not here explain how this is possible or how Leibniz's serial order 'provides the conditions of measurability'. He would surely not subscribe to Leibniz's denial of Clarke's claim that the quantity of temporal distance may be greater or less but the order be the same or embrace Leibniz's notion about temporal vacuums. He probably takes the modern mathematical 'existence of preceding and succeeding elements' for granted, which, it is our argument here, should not be done. We need to say a little more about this to prevent the simply taking it for granted that it can be done.

It is claimed for the modern continuum theory that it 'is based solely on the relation of order',[23] but that does not mean it is just a matter of Leibniz's simple precedes–succeeds relation, for that simple relation holds for all series, from simple to linear continuous, and so would not discriminate between them. So 'based solely on the relation of order' means something more like saying for time that it has within the earlier–later order its instants, as 'elements', arranged in the same manner as the various kinds of real numbers in their usual

[22] Hans Reichenbach, *Modern Philosophy of Science: Selected Essays*, translated and edited by Maria Reichenbach (London: Routledge and Kegan Paul, 1959), 56.

[23] Edward V. Huntington, The Continuum *and Other Types of Serial Order*, 2nd Ed (Cambridge Mass: Harvard University Press, 1917), 1.

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order. It is rather about the things 'quantities' in Clarke's term, 'elements' in the modern view, in that relation and their arrangements within it. In this sense the modern continuum applies to the instants or points within the Newtonian absolute system and is more like the Newtonian than like Leibniz's mere-order system. It appears to me that it is Alexander rather who is correct in saying that 'any relational theory of space and time is radically incomplete unless supplemented by an analysis of the process of measurement.' And what such an analysis requires first is an account of how the mathematical properties, naturally assumed in the absolutist view, can be supplied when that view is denied, to be replaced with the notion that time is a mere order of things. Either it must be *shown* how those ideal properties can be assumed or it must be shown how those properties themselves can be produced from the relativist mere-order substitute for the absolutist view.

3. Explanations of Proposals

Any philosopher who believes, as I do, that Leibniz was intuitively right in proposing that time (also space) is relative rather than absolute must try to answer Clarke's challenge rather than merely taking for granted the existence of the mathematical ideals for time as relative. But again it should be emphasized that the problem is a strictly philosophical one. The mathematicians and the physicists may be justified in not worrying about the ultimate origin of their mathematical ideals so long as their systems work, but if there is a philosophical problem about that origin, there is an explanatory gap which is ultimately insupportable. This was something recognized by Plato in his discussion of the Divided Line in Book VI of the Republic.[24] It is not right for philosophy simply to accept its premises from mathematics or physics, no matter how strong the emulative urge for progress is; philosophy is not the handmaid of the sciences and must insist on trying to answer the ultimate questions without dictation from the premise-unquestioning sciences.

All this being said, it appears to me that there is no obvious way in which the mathematical ideals necessary for time can be taken for granted in the relativist view and so they must somehow be derived in that view, taking as the first insight that time is the relation of earlier and later between things, and indeed quite physical things.

[24] *Republic* vi. 510b–511b.

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And the first requirement here is to ask what the ontology behind earlier and later must be.

It appears to be the fact of change, which is something we observe. Aristotle made that point in his typically empirical fashion; '[Time] does not exist without change [*metabolē*]. For when we do not change with respect to thought, or miss the changing, it does not seem to us time has passed.'[25] Sydney Shoemaker in an influential article has argued the possibility of time without change, but I have argued to the contrary that he begs the question 'by assuming an, in effect, Newtonian absolute time, that "flows equably" in a region in which there is no change and in one in which there is',[26] so I shall not address Shoemaker's argument here. But, assuming that there is no time without change, what is change? We shall not here try to follow Aristotle's complex definition of change in *Physics* IV, where he tries to fit it, scientifically, we might say, into his wider explanatory scheme of potentiality and actuality; rather again we shall follow his more empirical account, under the name 'coming to be' (*gignesthai*) in *Physics* II.7–10. As Ursula Coope summarizes it in her book, *Time for Aristotle*, 'change always involves an underlying thing and two contraries. The underlying thing persists through the change, losing one contrary and gaining the other'.[27] A similar description is to be found in Leibniz, in a paper 'On Substance, Change, Time, and Extension': 'Next we observe also novelty or change, that is, contradictory attributes of the same thing'.[28] He almost immediately adds:

> But since it is impossible for two contradictory things to be said about the same thing, it follows that the only difference that occurs when everything else remains the same, and that brings it about that there is no contradiction of any kind … is the difference of time.

Kant too gives a similar account of change, under the name 'alteration', *Veränderung*: 'a combination of contradictorily opposed predicates in

[25] *Physics* IV.11.218b21–23.

[26] Denis Corish, 'Could Time be Change?' *Philosophy* **84** (2009), Abstract, 219.

[27] Ursula Coope, *Time for Aristotle:* Physics IV 10–14 (Oxford: Oxford University Press, 2005), 6.

[28] G. W. Leibniz, *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672–1686*, translated, edited, and with an introduction by Richard T. W. Arthur (New Haven and London: Yale University Press, 2001), 267.

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one and the same object'. He adds 'Only in time can two contradictorily opposed predicates meet in one and the same object, namely, *one after the other*'.[29]

We shall of course be coming back to the question of time here, but first let us look at change itself. The empirically expressed views of Aristotle, Leibniz and Kant here point to our intuitive grasp of change. It seems to be an appreciation of our noticing the replacement of one thing by something else which is not it. There is a negativity about it, well expressed by the words above, 'contrary' and 'contradictory'. From a certain point of view it might be argued that 'replacement' here is simply a synonym for 'change'. If so, it is, at such a fundamental, intuitive level, the best we can do, though I think 'replacement' does point to something that we can easily not notice in using the familiar 'change'. One thing I am going to assume here, since I know of no instance to the contrary, is that change is never totally reversible.

Put symbolically then, change may be said to be the replacement of A by not-A. Given this, we may even say, as an echo of Leibniz and Kant, that change is both A and not-A, the negative not-A being equally important as the positive A. This will be true of a change situation of any kind, substantial (the living cat is replaced by the corpse), accidental (the small kitten is replaced by the big cat) or whatever, even, should it be possible, creation, where nothing is replaced by something. And such replacement should seem to give the fundamental meaning to the temporal terms 'earlier' and 'later'. What is replaced is earlier, what replaces is later. There will of course be later things that do not seem to be, in any obvious sense, replacements of earlier ones (Leibniz is later than Aristotle but not obviously a replacement of Aristotle, as the corpse of Aristotle is), but, as we shall see, given simultaneity, which for now we may take intuitively, something simultaneous with a replacement will have the same later position as that replacement, so that it seems possible, at least in principle, to have a single earlier–later series from this time of ours back to Big Bang, or to creation. And if there should be creation we can now see that the replaced nothing, though it has or had no properties of its own to make it earlier, was really earlier simply because it was replaced by something. That is all that 'earlier' need mean. There is then no mystery about what existed or happened before time; even, given creation, the original nothing was the first moment of time, because it was replaced. And all this without the need of Newton's

[29] *Immanuel Kant's Critique of Pure Reason*, translated by Norman Kemp Smith (New York: St. Martin's Press, 1965), 76.

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moments succeeding each other within, or along with, such a nothing.

Empirically expressed views of the type of Aristotle, Leibniz and Kant, and our own intuitive views, seem to take it for granted that change occurs *within* time. Does this require that time is ontologically more fundamental than change, as it would be for Newton? Must time exist first for change to exist within it? In fact, no. From the relativist point of view something such as this desk is in space if space is an abstraction from this desk and other such objects. Einstein says: 'as regards the concept of space: this seems to presuppose the concept of the solid body,[30] and that for Euclid 'the point, the plane, the straight line, the segment are solid objects idealized'.[31] And as for how such idealization takes place, again for space, we are helped not by Leibniz but by Aristotle:

> The mathematician [like the astronomer] is concerned with [things such as shape], but not as the limits of the natural body. Nor does he regard their properties as going with such bodies. On that account he separates them; for by thought they are capable of being separated from motion [of the natural bodies, motion viewed here as a sign of the physical]; and the separating makes no difference, and no error results[32]

This 'separating' by thought is obviously abstraction. We shall not here go into the troubled question of just how Aristotle thought abstraction worked in the mind. It seems, intuitively, to be a power of the mind to ignore certain aspects of something, so putting other aspects to the fore, ignoring, for example, the material from which the body is made. It seems clear from the quoted passage that however it was done the edge (of, say, a table) was the physical basis for the straight line, the corner, where two edges meet, for the point and the surface for the area. And we see how, such abstraction being by whatever means possible, the mathematical ideals can exist alongside of their physical counterparts, so that 'no error results', without having to suppose a Newtonian absolute to be ontologically necessary. Leibniz's intuition was correct, however inadequate his arguments.

Things are in time as things are in space, that is, the time or the space is an abstraction from the things. (The individual human is

[30] Albert Einstein, *Ideas and Opinions* (New translations and revisions by Sonja Bargmann), (New York: Crown Publishers, Inc., 1954), 278.
[31] Einstein, op. cit., 279.
[32] Aristotle, *Physics* II.2.193b31–35.

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also within humanity, which is an abstraction from such individuals). But how does this work for time, as, with the help of Aristotle, we have seen it work for space? Could we simply say that, assuming it works for space, we can take time as, intuitively, analogous to space, so that the when of time is founded on the where of space? This might seem to be in accord with the opinion of many philosophers and others. The difficulty about it is that it does not give an adequate account of the troubling question of when plus existence, or rather nonexistence (just as important, as we said, as existence) which is the cause of so much that is problematical in the history of the philosophy of time.

It is of course clear that 'when' is a positional term for the earlier–later relation and that 'where' is a positional term for the spatial relation, however expressed. *When* one temporal position is another is not just as *where* one spatial position is another is not. But could time be based upon the line? Rather, it appears, to the contrary: to think in terms of the line is to take time for granted. Consider a chalk line from A to B on the blackboard. Both A and B and every point on the line must exist together for the line to exist. That is, they must all exist at the same time, or eternally. Eternity is not the condition of such a chalk line, so the ends and the parts and the points, if such points exist, must exist at the same time. If when A existed B did not, and vice versa, then the line could not exist; if no two points of it existed together, at the same time, then the line could not exist. For the line to exist time, at least to the extent of the same time, must exist. So, unless some other meaning outside of time can be given to 'at the same time', then to speak in terms of the line is already to take time for granted. There may still be an analogy between the when and the where, but it is based upon the when rather than the where. Time, it appears, is a more fundamental consideration than space, and so, *a fortiori*, than space–time.

But how then should we consider time to have intervals, as the spatial line does? For to consider the spatial interval, the line segment, is to take time, at least as the same time, for granted, and so to try to derive the temporal interval from the spatial would be to beg the temporal question. We must ask whether merely by itself, entirely without reference to the line, we can discover a time interval. Here logic, rather than picture-thinking based upon the line, comes to the rescue. All that is required, even if we take picture-thinking, as comfortably familiar, into account, is that the logical pattern be the same. A line segment is, say, from A to C. Similarly, a time interval is, say, from M to O. So far there seems to be no problem. Again, the line AC is divisible at B, whatever

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B is, and so the time interval is divisible at N. But, to revert to the old, good argument of Aristotle and Augustine, used in that odd way we saw by Leibniz, how can the time interval even exist in order to be divided, since, purely in time, when M exists O, or N, does not, and when O exists M, or N, does not, M being totally replaced by N and *then*, another word of temporal position, N by O?

Again, logic will supply the answer, and it should be unproblematic if we are really willing to replace picture-thinking by logic. As the line segment AC is divided by B and at B, so the time interval MO is divided by N and at N. Again, as the line segment AC is exactly equal to the sum of the segments AB and BC, so the time interval MO is exactly equal to the sum of the intervals MN and NO. And this is true even though there is not in time, though there is in the line, any *pre-existing* interval to be divided. All we are saying, logically, using the even more abstract relation precedes–succeeds, applicable to both space and time (and to number), is that as A precedes B which in turn precedes C, so M precedes N which in turn precedes O. The logic is sufficient and 'provides the conditions of measurability', to use Reichenbach's phrase. And that is without the necessity of picture-thinking based upon the visible spatial line, but with full respect for the nonexistence constitutive of change and time. It is a direct answer to Clarke's challenge to show how the relation of order provides for the mathematical ideals of time.

It is an answer so far only for the time interval. What about instants, which, as Leibniz said, 'consist only in the successive order of things'? Again the logic, rather than picture-thinking will be sufficient to provide them. All that is required, logically, of the instant, or point, is that it is a position and not an extension. Now the extension of time or of space is from something preceding to something succeeding, from the right to the left, say, in the case of the line, and from the earlier to the later in the case of the time interval. What of these positions themselves, the preceding one or the succeeding one? It should be clear that, *as such*, they are not extensions. The logical conditions are then fulfilled in their case: the single position, *as such*, is not an extension, which is *from* one position *to* another. The single position is then instantaneous in the case of time, point-like in the case of space. Clarke's challenge is again directly answered in their regard.

Now of course it will occur to the reader that an interval, of time or space, may in fact occupy a position preceding or succeeding another. The time of Aristotle, stretching over the interval of Aristotle's life, is earlier than the corresponding time of Leibniz; the first inch of the line is, in a long enough line, to the left, say, of the last inch. It

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must follow then from the logic of what I have been saying that that time stretch is instantaneous, is an instant, that line segment a point. And is it not a contradiction that something should be both extended and not?

It is a contradiction only if the extension and the non-extension are taken in exactly the same respect, just as it would be a contradiction to say 'both A and not-A' only if they are understood to obtain at the same time. In respect of mere single relative position, earlier or later, right or left, preceding or succeeding, anything, including an interval, is, *in that respect merely*, without extension because it is not from … to but only an *a from* or an *a to*. In the quite different respect of stretching, in time or in space, from a preceding position to a succeeding position, it will, and *without violating its position as a whole*, be also an interval. What this means is that the instant or the point is relative. Relative, that is, to its function as mere position, or to its function as extension holding that position.

We shall speak of the relativity of instantaneousness again. But for now what that relativity of instant or point means is that instants or points can be said really to exist and be observed, without our having to hunt for them in the picture-direction of the small. They exist and are observed merely as positions which, *as such, and merely in that respect*, have no extension, whatever quite other respects they may also be taken in. Instants, and points, have no size, because size is a property of extensions, and instants and points, as such, are not extensions. What has no size is not small any more than large. 'Little or none' is a practical, not a logical, description, and to say that an instant or a point must be small is a contradiction in terms because what has no size cannot have a small size, or a smaller than small size. Again, the logic must win over the picture-thinking.

Perhaps a somewhat fanciful thought experiment can make clear this important point. Assume that we could fashion a computer to register nothing else either internal or external to itself but a series of lights occurring one after the other with no overlap, reacting only, say, to a given wavelength, though with some memory of previous such occurrences, that is, of the succession. How long, purely in terms of that computer, not in our terms outside of it or our analysis of its working, does a light occur for? From its beginning to its end, one is inclined to say. But note what is being said here. The beginning occurs no earlier than the light itself, the end occurs no later, though the beginning is earlier than the end. Speaking somewhat colloquially, we may say that the succession of the beginning and the end goes along with the light. It is precisely in terms of such a temporal succession going along with something that that something has temporal

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extension. Anything compared only with what is entirely earlier than it or entirely later has in that comparison no extension but only position. It is, in that comparison, instantaneous. And this is as true, logically, of, say, the Great Pyramid, or the earth, as it is of this flash of light. The flash of light, however brief, will have an extension provided only that it goes along with even briefer events that are one altogether earlier than the other; the earth will be instantaneous, will mark, so to speak, its own instant provided only that it is compared with nothing that goes along with it but only with what is entirely earlier or later than it. There is in that respect no way of telling that it has temporal extension. It is a matter of rather simple logic. But, since there are many things and many successions of things going along with the earth, it is also a matter of simple logic that it has temporal extension. The points of view are different and do not contradict each other. What we are left with then from these two different, non-conflicting points of view is that the instant and instantaneousness are relative. And similar logical considerations will be true of the point.

We have then both the time interval and the instant established logically in terms of time as relative, as coming by abstraction from things as earlier and later, and Clarke's challenge is answered in their regard. Order is seen to provide quantity. What now of simultaneity? Intuitively, simultaneity means that two things occur at the same time, that is, the one neither earlier nor later than the other and each equally earlier and later than the same other things. If our computer could be programmed to register a single sound together with a light, neither earlier nor later, that sound would be simultaneous with that light, holding either by itself or in combination with the light exactly the same temporal position in the succession of lights as the light itself. And that simultaneity would be instantaneous. If it were not, it would not really be simultaneous. Our intuition tells us that much.

If the computer could also determine two successive sounds, one simultaneous with the beginning of the light, the other simultaneous with its end, that succession could, without contradiction, be said to be simultaneous with the instant of the light. Without contradiction because that succession too has a position. What we, unlike Newton, may call a duration is the simultaneity of an instantaneous position with a succession, the simultaneity of an instant with a temporal extension. It is this consideration which complicates the question of succession, and makes distance, though it involves succession, not transitive like succession. This in some sense Clarke, or Newton, grasped whereas Leibniz did not. And yet the whole thing is

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explainable, as we can now see, in terms of the earlier–later order and does not require a Newtonian, underlying absolute quantity.

But now a problem arises because we intuitively recognize also that simultaneity, in the simple sense of it, is transitive. If X is simultaneous with Y and Y is simultaneous with Z, then X is simultaneous with Z. The light and the sound and the combination of light and sound are all at the same time, the same earlier–later position in the succession. So if we were to take the instant of, say, the Great Pyramid and observe that Alexander the Great exists at that instant and so do I, does that make me simultaneous with Alexander? It does, but only in terms of that instant of the Great Pyramid itself. In terms of my instant as such and Alexander's, I am later than Alexander, and the pyramid lasts or endures at least from one to the other. Given the different respects, so important in logic, there is no real problem.

The relativity of instantaneousness has one other consideration that is at least interesting: the relativity of simultaneity that we know from Einstein calls for it. For, since simultaneity has to be instantaneous or it is not really simultaneity, the relativity of simultaneity requires the relativity of instantaneousness. All that the relativity of simultaneity seems to mean, logically, is that what is simultaneous from one point of view is not from another; it does not do away with simultaneity altogether, or there would be no use in the concept of simultaneity.[33] Similarly, the relativity of instantaneousness not only does not do away with the instant but indeed makes it real and observable, and so our account of the temporal interval and instant allows for the relativity of Einstein at least to that extent, though our interest here is not physics or mathematics but philosophy.

Our interest in the continuity of the temporal interval will also be philosophical and will follow naturally from what we have seen about that interval. Again it is a matter of simple logic: if the interval of time is as we have represented it, the mere earlier-later relation between two occurrences, and if, as it appears, continuity is to be found, intuitively, in that interval, then continuity is to be found in that relation. This is a natural consequence of the relativist view.

Does this violate the distinction between the relation and the interval, the distance: that the one is transitive, the other not? For if the

[33] Max Jammer remarks that ordinary, local simultaneity 'posed no physical problem' for Einstein, though Jammer argues that Einstein 'argued for the priority of the notions of simultaneity over that of time'. Max Jammer, *Concepts of Simultaneity: From Antiquity to Einstein and Beyond* (Baltimore, MD: Johns Hopkins University Press, 2006), 110.

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interval is the relation, the mere order between the two end points of the interval, M and O, say, it appears that the interval should also be transitive, which it clearly is not. But there is in fact no violation. For the interval MO is not only continuous in itself, it is also continuous with the time outside it, as the line segment is also continuous with the line outside it, whereas the interval or segment begins and ends at its end points. The continuity, the unbrokenness, carries over, so that the time interval or the line segment owes its continuity to what reaches beyond it, that is, the unbroken relation. This is so different from any view that would try to build up continuity from the indivisible positions within that relation that any such view, of time or the line, must be labelled non-relativist, whatever general claims of relativism may be made in its regard.

To make this clear, let us reiterate that the interval MO, say, or AC, sufficiently fulfills the logic of divisibility. Let us take MO, the time interval, for the pattern of all, since it dispenses with picture-thinking, the basing everything on the appearance of the line, in favour of the mere logic of the relation, as the relativist view properly requires. So, N divides the interval, the earlier–later relation, MO, N being a position without extension in that relation, that succession. The interval is divided into the sub-intervals MN and NO, which are continuous at N. As M is earlier than O so M is earlier than N and N is earlier than O. Whatever positions, N or others in succession with N, may be found in the interval MO may be said to be *between* M and N or between N and O, as they are, also and naturally, between M and O. Such *betweenness* is transitive: if N is between M and O and X is between N and O, then X is between M and O. And each such position, as it will be, from mere succession, mere comparison with what is totally earlier or later than it, indivisible, instantaneous, will divide the interval MO into two continuous sub-intervals, the first beginning with M, the second ending with O. I should more properly say beginning *in* M and ending *in* O, for M and not anything later than M, or earlier, is the beginning of that interval, and O and not anything earlier than O, or later, is the end. The original interval, MO, is the relation *between* M and O; it is the interval *between* them. These two *betweens* are exactly the same. But they are different from the *between* of, say, N. It may be said to be *in-between*, and that, it appears, is subsidiary to the original *between* of the relation, of the interval. Though there have been attempts to do it, the interval cannot be made up of indivisible positions in-between; the continuous, it is known from the time of Zeno, cannot be made up of indivisibles. Nor can it be made up of intervals in-between. Our, relativist, account of that is as follows.

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The interval MO, the interval between M an O, begins precisely in M and ends precisely in O. It is the earlier–later relation between those two occurrences. Any position or positions, any interval or intervals in-between M and O, will begin later than M and end earlier than O. Consequently the sum of such intervals, and the sum of whatever such positions are the beginnings and endings of such intervals will never amount precisely to the interval MO; there will always be another interval between M and the beginning of such intervals, and yet another between the end of such intervals and O. Therefore no adding up of anything in-between M and O will make up the interval MO. If mere positions are added up, they will not amount to any interval; if in-between intervals are added up, they will amount to an interval but it will be less than MO. It will be *discontinuous* with MO. (It should be said, to avoid confusion, that if N is in-between M and O, the interval MN is not an interval in-between M and O, since it begins exactly in M, and similarly for the interval NO, which ends exactly in O. Subdivisions beginning or ending thus are not in-between intervals). Continuity is a property of the whole interval and cannot, as a whole, be made up from lesser in-between intervals or from whatever infinity of in-between instantaneous positions.

It is well known from antiquity that no mere addition of indivisibles, however infinite in number they may be, will make up the divisible continuum. However, from the relativist point of view just two such indivisibles, provided only that they are in successive relation, will suffice. Indeed just one will, in that relation. For if something is, say, later, it cannot be so unless there is also a correlative earlier, and the two together, in that relation, provide the continuous interval between them, which, we have seen, is just that relation itself. Of course there is nothing here to prevent an infinite number of positions in-between those two things, or an infinite number in-between any two of such positons, nothing, that is to say to prevent them from being densely packed. But it is the betweenness of the original two, the successive relation between them, that is fundamental and that is continuous. Continuity, on this view, comes down from above not up from below.

An effort then such as J. W. R. Dedekind's to find continuity in the dividing of the interval at any indivisible point within it, in the in-between sense of 'within',[34] or Aristotle's finding continuity as the unifying in a single indivisible position of the adjoining limits

[34] Richard Dedekind, *Essays on the Theory of Numbers*, authorized translation by Wooster Woodruff Beman (New York: Dover Publications, Inc., 1963), 8–12.

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of contiguous intervals,[35] may indicate continuity at an indivisible position, the intervals MN and NO being continuous at N, but cannot be a full account of continuity. In Aristotle, for example, the two contiguous parts must first be each continuous in itself for both parts to be continuous at the point in question. Continuity, as our intuition of it as having no gaps or breaks indicates, comes down from the full interval or relation between and not up from positions or intervals in-between. As regards Dedekind, it is of course no business of philosophy to dictate to mathematics, but it is, fully equally, no business of philosophy to be dictated to by mathematics. Philosophy must try to solve its own more fundamental problems in its own way. Like mathematics, it must of course be logical, but it must also try to respect as far as possible our seemingly non-puzzling intuitions. As Einstein put it: 'Pure logical thinking cannot yield us any knowledge of the empirical world; all knowledge of reality starts from experience and ends in it.'[36]

Such then is what time is (and analogously space) from the relativist view initiated for modern times by Leibniz but not adequately worked out by him, or by anyone who, supposing time to be relative, then merely without further ado assumes that the mathematical ideals of interval, instant and simultaneity are available to it. They were, it seems, available, simply by assumption, to the absolutist view of Newton, but if that view is denied, then just how those ideals are provided by the view which says time is merely the earlier–later order of things is a challenge. This I have identified as Clarke's challenge to Leibniz. I believe that the suggestions I have made here answer that challenge logically and without the pictorial thinking based upon the line. I may of course have made mistakes, logically or otherwise. If so, and if this essay is not totally ignored (ignoring being, in philosophy as elsewhere, the first defense against radical innovation), then somebody will usefully move to correct it. Or perhaps somebody will try to show that the relativist view can after all successfully simply assume the mathematical ideals. Either way, it will be something that does need to be shown, not *merely* taken for granted without any further justification. The philosophical, as opposed to the mathematical or physical, quest demands no less.

It all comes down to the simple logical proposition: if, as Leibniz sensibly proposes, time is a mere order of things, then we must try to understand how, and not merely assume that, such a mere order can provide the quantity that we intuitively know time to have, can

[35] *Physics* V.3.227a10–12.
[36] Einstein, op. cit., 271.

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provide the mathematical ideals necessary for the scientific treatment of the physical world.[37]

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[37] I should like to thank Anthony O'Hear for allowing the publication of these and other not commonly accepted notions of mine about time. We must somehow preserve the possibility of radical, non–bespoke innovation.

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