J. Plasma Physics (1997), *vol.* 57, *part* 4, *pp.* 741–751. Printed in the United Kingdom
© 1997 Cambridge University Press

Solutions of the hydromagnetic equation with magnetic null points: uniform-resistivity case

F. JAMITZKY and M. SCHOLER
Max-Planck-Institut für extraterrestrische Physik, PO Box 1603,
85740 Garching, Germany

(Received 29 January 1996)

A general class of solutions is found for the resistive hydromagnetic equation. The results are applied to the case of a prescribed stagnation point flow. It is shown that general solutions with magnetic null points exist. Scaling laws for the length and the width of the current sheet of the solutions are given for the general case. It is shown that the length of the current sheet increases with magnetic Reynolds number unless the outflow boundary conditions prevent the sheet length from growing. The latter would result in the appearance of boundary layers in the outflow region.

1. Introduction

Beginning with Parker (1963) and Sweet (1958), a number of steady-state magnetohydrodynamic models have been proposed in order to describe magnetic field reconnection, a process that has been recognized to be of fundamental importance in cosmic plasmas. Because of the small reconnection rate in the Sweet–Parker model, Petschek (1964) introduced the idea that the current sheet is considerably smaller than the overall system size and that the outer region contains two pairs of standing slow-mode shocks. These shocks deflect and accelerate the incoming plasma into two exit jets wedged between the shocks. However, numerical simulations (see e.g. Biskamp 1986; Scholer 1989) have not been able to produce Petschek-type reconnection in the case of spatially constant resistivity, since the current sheet in these simulations expands until it reaches the boundaries; Forbes (1995) has suggested that this is because the boundary conditions are underspecified. In order to obtain a finite diffusion-region length in such experiments, it was necessary to constrain the resistivity to a small region. For such a spatially localized resistivity, which is often the relevant case in physical applications, Petschek reconnection has been obtained (see e.g. Scholer 1989; Yan *et al.* 1992). For high magnetic Reynolds numbers, in regimes where the diffusion region is too long, it becomes unstable to tearing, and so reconnection can become highly non-stationary (Forbes and Priest 1987).

The models of Sweet–Parker and Petschek were further refined, and it was attempted to achieve analytical solutions for the MHD equations, which is rather difficult because of the inherent nonlinearity of the equations. For this purpose it was necessary to simplify the nonlinear resistive MHD equations. This can be achieved by the use of constraining symmetries, as was done by Parker (1973), Sonnerup and Priest (1975) and Phan and Sonnerup (1990). A translational symmetry along the current sheet reduces the MHD equations to ordinary differential equations that

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press

can easily be solved. The constraining symmetry, however, involves a current sheet of infinite length. The plasma transported into the current sheet is never ejected. Furthermore, it is not possible to have a magnetic field component perpendicular to the current sheet. Solutions based on such a symmetry thus describe magnetic annihilation rather than magnetic reconnection, and so may be regarded as modelling the central part of a current sheet before the ends are reached. On the other hand, it is possible to simplify the MHD equations and solve them in the external region by neglecting the dissipative terms. A solution can then be obtained by matching of simple waves. This was done by Priest and Forbes (1986) and Heyn *et al.* (1988) by matching the external solutions to a Sweet–Parker layer in an integral sense rather than by a formal two-dimensional matching, which is inordinately difficult and has not yet been accomplished. As far as the external solution is concerned, the current sheet degenerates to a singular point or line, and no detailed solutions are given for the diffusion region where the magnetic field is reconnected. Resistivity and, correspondingly, magnetic Reynolds number are not included from the external solutions, but are present in the internal (diffusion) region, which is treated only in an integral sense.

Recently Jamitzky and Scholer (1995) have performed a boundary-layer analysis of two-dimensional current sheets appearing in magnetic reconnection. The coordinate normal to the current sheet is rescaled by a power of the (spatially constant) resistivity in order to obtain inner limit equations that satisfy the symmetry conditions and represent magnetic annihilation to lowest order. They can be matched to the outer expansion of the incompressible resistive MHD equations with a domain of validity smaller than the current sheet length. The characteristic length for growth of the magnetic field component normal to the current sheet increases with magnetic Reynolds number, and a Sweet–Parker scaling is found for the external reconnection rate. Although not a proof, this result suggests that in the case of constant spatial resistivity the current sheet length increases with increasing resistivity.

The mathematical difficulties involved in solving the full set of resistive MHD equations for the problem at hand are formidable. Therefore in this paper we shall approach the physics occurring near magnetic null points from a different point of view: instead of solving the full set of MHD equations self-consistently, we shall present a general kinematic study of the magnetic field near null points. A kinematic study of the magnetic field behaviour near magnetic null points by specifying the velocity field and constructing solutions of the hydromagnetic equation was first given by Parker (1973). By choosing a stream function of the form $\phi(x, y) \sim xy$, Parker (1973) constructed magnetic field solutions and concluded that the reconnection rate is controlled by the diffusion in the neighbourhood of the neutral point.

In this paper we shall generalize the investigation of Parker (1973). We first present a new solution method leading to a general class of solutions of the resistive hydromagnetic equation. This solution method is applied to the class of curl-free stagnation-point flows. It is shown that the current sheet length of a solution with a magnetic null increases with increasing magnetic Reynolds number unless the outflow boundary conditions prevent the sheet length from growing. In the case of constraining boundary conditions boundary layers appear in the outflow region, connecting the central current sheet to the prescribed boundary.

The outline of the paper is as follows. After deriving the hydromagnetic equation

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press

for the case of a two-dimensional incompressible plasma, we shall describe in Sec. 2 a general solution method for incompressible irrotational plasma flows. In Sec. 4 a WKB-expansion is given for the case of high magnetic Reynolds number, and in Sec. 5 the WKB expansion is applied to general stagnation-point flows with magnetic null points.

2. The hydromagnetic equation

The system under consideration is a two-dimensional current sheet in the (x, z) plane with a given plasma flow in the (x, y) plane. Let the system be translationally invariant in the z direction, i.e. all derivatives in the z direction are zero. The magnetic field $\mathbf{b}$ and the plasma velocity $\mathbf{v}$ are supposed to have only components in the (x, y) plane. For a given plasma velocity $\mathbf{v}$ the magnetic field $\mathbf{b}$ is given by

$$\dot{\mathbf{b}} + \nabla \times \mathbf{E} = 0, \tag{1}$$

where $\mathbf{E}$ is the electric field, which is given by Ohm's law

$$\mathbf{E} + \mathbf{v} \times \mathbf{b} = \eta \mathbf{j}, \tag{2}$$

with η denoting the resistivity of the plasma. The current density $\mathbf{j}$ and the magnetic field are related by

$$\mathbf{j} = \nabla \times \mathbf{b}. \tag{3}$$

It follows from $\nabla \cdot \mathbf{b} = 0$ that one can define a magnetic vector potential $\mathbf{A}$ such that

$$\mathbf{b} = \nabla \times \mathbf{A}. \tag{4}$$

By using the translational invariance in the z direction, the magnetic field $\mathbf{b}$ in the (x, y) plane can be written as

$$\mathbf{b} = \nabla a \times \mathbf{z}, \tag{5}$$

where a denotes the z component of the magnetic vector potential (flux function). The current density in the z direction is given by

$$j = -\nabla^2 a. \tag{6}$$

It follows further from the two-dimensionality of the system that the electric field has only a z component. Ohm's law can then be written as

$$-E + \mathbf{v} \cdot \nabla a = \eta \nabla^2 a, \tag{7}$$

where E is the z component of the electric field. We now focus on incompressible irrotational plasma flows. The incompressibility condition $\nabla \cdot \mathbf{v} = 0$ allows the introduction of a potential ϕ such that

$$\mathbf{v} = \nabla \phi \times \mathbf{z}. \tag{8}$$

The potential ϕ is also called the stream function. Ohm's law can then be written as

$$-E + [a, \phi] = \eta \nabla^2 a, \tag{9}$$

where the bracket is defined by the Jacobian:

$$[u, v] = \frac{\partial (u, v)}{\partial (x, y)} = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}. \tag{10}$$

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press

This equation is called the hydromagnetic equation. It describes the behaviour of the magnetic field under a given incompressible plasma flow. The equation is a linear inhomogeneous partial differential equation of elliptic type. In the following we shall investigate solutions of (9) for special plasma flows, i.e. for prescribed $\phi(x, y)$. We first show that (9) can be solved explicitly for general incompressible and irrotational plasma flows.

3. Incompressible irrotational flows

A irrotational and incompressible flow is given by a stream function ϕ with vanishing vorticity. The vorticity of a plasma flow is defined by

$$\mathbf{\Omega} = \nabla \times \mathbf{v}. \tag{11}$$

For a two-dimensional system the only nonvanishing component is the z-component which is given by

$$\Omega = -\nabla^2 \phi. \tag{12}$$

In the case of an irrotational flow the stream function therefore has to be a potential function, and there exists a complex function $\chi(z)$ of the complex variable $z = x+iy$ such that

$$\chi(x + iy) = \phi(x, y) + i\psi(x, y). \tag{13}$$

The bracket $[\chi, \bar{\chi}]$ can be written as $-2i\,[\phi, \psi] = -2i|\chi'|^2$, where $\bar{\chi}$ is the complex conjugate of χ and the prime denotes the complex derivative: $\chi' = d\chi/dz$. The Laplace operator is then given by

$$\nabla^2 = 4\partial_z \partial_{\bar{z}} = 4\left|\frac{d\chi}{dz}\right|^2 \partial_\chi \partial_{\bar{\chi}} = \frac{1}{\Gamma(\phi, \psi)}(\partial_\phi^2 + \partial_\psi^2), \tag{14}$$

where $\Gamma(\phi(x, y), \psi(x, y)) = |dz/d\chi|^2$. Therefore one can write the hydromagnetic equation in the coordinates (ϕ, ψ) as

$$\eta(\partial_\phi^2 + \partial_\psi^2)a + \partial_\psi a = -E\Gamma(\phi, \psi). \tag{15}$$

This equation is an inhomogeneous linear partial differential equation with constant coefficients, and can be solved by standard methods. In the following we shall analyse the hydromagnetic equation in the coordinate system (ϕ, ψ).

The general solution is given by

$$\begin{aligned} a(\phi, \psi) = &-\frac{Ee^{\psi/2\eta}}{8\pi\eta} \int\int \Gamma(\phi_1, \psi_1) K_0\left(\frac{[(\psi_1 - \psi)^2 + (\phi_1 - \phi)^2]^{1/2}}{2\eta}\right) d\psi_1\, d\phi_1 \\ &+ e^{\psi/2\eta} \int_{-\infty}^{\infty} \gamma_1(\alpha) e^{\alpha\phi + \beta\psi}\, d\alpha + e^{\psi/2\eta} \int_{-\infty}^{\infty} \gamma_2(\alpha) e^{\alpha\phi - \beta\psi}\, d\alpha, \end{aligned} \tag{16}$$

with

$$\alpha^2 + \beta^2 = \left(\frac{1}{2\eta}\right)^2. \tag{17}$$

Here the functions γ_1 and γ_2 are defined by the boundary conditions, and K_0 denotes the zeroth order modified Bessel function of the second kind. Although a general solution exists, this solution is too complicated to obtain solutions for

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press

explicit boundary conditions. In the next section a WKB approximation is derived by which it is possible to solve for arbitrary boundary conditions.

4. WKB approximation

The transition from ideal to resistive MHD is singular owing to the change of order in the differential equation. In ideal MHD the magnetic field lines are bound to the plasma, while in resistive MHD the field lines can slip through the plasma owing to the diffusion term. The topology of the field lines can change even in the case of small resistivity. In the case $\eta = 0$ the hydromagnetic equation degenerates to

$$[a, \phi] = E. \tag{18}$$

This equation is, in contrast to the original resistive hydromagnetic equation, of first order. By using the coordinates y and ϕ as independent variables, one obtains

$$E = \frac{\partial (a, \phi)}{\partial (x, y)} = \left(\frac{\partial a}{\partial y}\right)_{\phi} [y, \phi], \tag{19}$$

$$E = -E[y, x] = -E[y, X(y, \phi)] = -E[y, \phi] \left(\frac{\partial x}{\partial \phi}\right)_{y}. \tag{20}$$

Therefore

$$\left(\frac{\partial a}{\partial y}\right)_{\phi} = -E \left(\frac{\partial x}{\partial \phi}\right)_{y}, \tag{21}$$

which gives after integration

$$a(\phi, y) = -E \int \partial_{\phi} X(y, \phi)\, dy + F(\phi), \tag{22}$$

where $X(y, \phi)$ is given by $x = X(y, \phi(x, y))$ and F is defined by the boundary conditions. This solution has only one free function F. This has to be contrasted to (16), where two functions can be chosen. There exists no regular expansion of the hydromagnetic equation for small η. However, there exists a WKB expansion, which has a singular part for $\eta \to 0$. In the following we determine the kind of singularity for the homogeneous hydromagnetic equation and then give the WKB expansion of the hydromagnetic equation for a general stagnation-point flow.

We use the fact that the homogeneous hydromagnetic equation

$$\nabla\psi \cdot \nabla a_H = \eta \nabla^2 a_H \tag{23}$$

can be transformed to

$$\nabla\psi \cdot \nabla a_H = -\eta \nabla^2 a_H \tag{24}$$

by $a_H \to e^{\psi/\eta} a_H$. Therefore the hydromagnetic equation is invariant under the transformation $a_H(x, y, \eta) \to e^{\psi/\eta} a_H(x, y, -\eta)$. The lowest-order term of the general solution for the homogeneous hydromagnetic equation is therefore given by

$$a_H = F(\phi) + e^{\psi/\eta} G(\phi) + O(\eta), \tag{25}$$

where the functions F and G are determined by the boundary conditions. This

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press

solution is a uniform approximation for small η of the homogeneous part of the flux function. The expansion (25) can also be obtained from (16) by expanding for small η . From the exact solution

$$a_H(x,y) = \int_{-\infty}^{\infty} \gamma_1(\alpha)\, \exp\left\{\alpha\phi + \left[\frac{1}{2\eta} + \left(\frac{1}{4\eta^2} - \alpha^2\right)^{1/2}\right]\psi\right\} d\alpha + \int_{-\infty}^{\infty} \gamma_2(\alpha)\, \exp\left\{\alpha\phi + \left[\frac{1}{2\eta} - \left(\frac{1}{4\eta^2} - \alpha^2\right)^{1/2}\right]\psi\right\} d\alpha, \tag{26}$$

one obtains

$$a_H(x,y) = \int_{-\infty}^{\infty} \gamma_1(\alpha)\, \exp\left(\alpha\phi + \frac{1}{\eta}\psi\right) d\alpha + \int_{-\infty}^{\infty} \gamma_2(\alpha)\, \exp(\alpha\phi)\, d\alpha + O(\eta), \tag{27}$$

where the functions γ_1 and γ_2 are given by the boundary conditions. Therefore the functions F and G from (25) are given by

$$F(\phi) = \int_{-\infty}^{\infty} \gamma_2(\alpha)\, \exp(\alpha\phi)\, d\alpha, \tag{28}$$

$$G(\phi) = \int_{-\infty}^{\infty} \gamma_1(\alpha)\, \exp(\alpha\phi)\, d\alpha. \tag{29}$$

It can be seen from (25) that the expansion of the homogeneous solution has an *essential singularity* for $\eta \to 0$, owing to the appearance of the term $e^{\psi/\eta}$, which has no power-series expansion in the case of small η. This singularity is due to the change of order of the differential equation (23) for vanishing η, and corresponds to the appearance of boundary layers near the outflow boundary.

Another singularity appears in the inhomogeneous part of the flux function at positions where $\mathbf{v}^2 = 0$. At these locations the magnetic field has to approach infinity in order to result in a finite electric field $\mathbf{v} \times \mathbf{b} = -E\hat{\mathbf{z}}$. In order to obtain a spatially uniform approximation for small η of the inhomogeneous part of the flux function that is also valid near the current sheet centre where the resistivity is important, we solve the hydromagnetic equation in (ϕ, ψ) coordinates:

$$\eta(\partial_\phi^2 + \partial_\psi^2)a + \partial_\psi . a = -E\Gamma(\phi,\psi). \tag{30}$$

A boundary layer appears near the stagnation point where $\mathbf{v}^2 = 0$. By a rescaling of the coordinates (ϕ, ψ), one obtains an equation that shows no change of order in the case of vanishing η. One uses the form $\chi = \eta\bar{\chi}$, and finally obtains

$$(\partial_{\bar\phi}^2 + \partial_{\bar\psi}^2)a + \partial_{\bar\psi} a = -E\bar\Gamma(\bar\phi,\bar\psi), \tag{31}$$

with $\bar\Gamma(\bar\phi,\bar\psi) = \eta\Gamma(\eta\bar\phi,\eta\bar\psi)$. For a general stagnation-point flow the function χ behaves near the current-sheet centre like

$$\chi = \frac{v_0}{L_e}\frac{z^2}{2} + O(z^4). \tag{32}$$

Therefore one obtains for small η

$$\bar\Gamma(\bar\phi,\bar\psi) = \frac{L_e}{2v_0}\frac{1}{|\bar\chi|} + O(\eta). \tag{33}$$

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press

Equation (9) can thus be written as

$$(\partial_{\bar{\phi}}^2 + \partial_{\bar{\psi}}^2)a + \partial_{\bar{\psi}}a = -\frac{EL_e}{2v_0}\frac{1}{(\bar{\phi}^2 + \bar{\psi}^2)^{1/2}} + O(\eta). \tag{34}$$

This equation is the WKB approximation of (9) in the case of small η, and by solving this equation one obtains spatially uniform approximations of the original equation (9). An inhomogeneous solution of (34) is given by

$$a_I = \frac{EL_e}{v_0}\mathrm{E_I}([\bar{\psi} + (\bar{\phi}^2 + \bar{\psi}^2)^{1/2}]^{1/2}), \tag{35}$$

where the function $\mathrm{E_I}$ is defined by

$$\mathrm{E_I}(y) = \int_0^y e^{-y_1^2/2}\int_0^{y_1} e^{y_2^2/2}\,dy_2\,dy_1. \tag{36}$$

By combining the asymptotic homogeneous solution (25) and the asymptotic inhomogeneous solution (35), one obtains a spatially uniform asymptotic solution of (9) for small η

$$a = \frac{EL_e}{v_0}\mathrm{E_I}\left(\left(\frac{\psi + |\chi|}{\eta}\right)^{1/2}\right) + F(\phi) + e^{\psi/\eta}G(\phi) + O(\eta). \tag{37}$$

For small η two boundary layers appear: one due to the vanishing of the plasma velocity near the origin, and another due to the reduction of degree of freedom at the outflow boundary.

The transformation $(x, y) \to (\phi, \psi)$ maps the x and y axes to the axis $\phi = 0$, since the stream function has to vanish on the x and y axes owing to symmetry; the x and y axes are streamlines of the stagnation-point flow. The x axis corresponds to $\psi > 0$ while the y axis corresponds to $\psi < 0$. By expanding the solution (37) for small ψ at the location $\phi = 0$, one obtains

$$a(0, \psi) = \frac{EL_e}{v_0}\frac{\psi}{\eta} + O(\eta, \psi) \tag{38}$$

for $\psi > 0$ and

$$a(0, \psi) = \tfrac{1}{2}\gamma_2''(0)\eta\psi + O(\eta, \psi) \tag{39}$$

for $\psi < 0$. From the expansion of a general stagnation-point flow near the origin (32), one obtains for the function ψ

$$\psi(x, y) = \frac{v_0}{L_e}\frac{x^2 - y^2}{2} + O(x^4, y^4), \tag{40}$$

and therefore

$$a(0, y) \sim -\frac{y^2}{\eta} + O(y^4), \tag{41}$$

$$a(x, 0) \sim \eta x^2 + O(x^4). \tag{42}$$

The current-sheet thickness d therefore scales like $d \sim \eta^{1/2}$ while the current sheet length L scales like $L \sim \eta^{-1/2}$. In the next section the general results are applied to a specific example in order to obtain the response of the system to well-defined external parameters.

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press

5. A specific example

The solution method obtained in the last two sections is now applied to a specific stagnation point flow. A specific flow that describes a converging or diverging stagnation point at the origin is given by the following two solutions of the Laplace equation:

$$\chi_1(z) = \frac{v_0}{L_e} l^2 \left(\cosh \frac{z}{l} - 1 \right), \tag{43}$$

$$\chi_2(z) = \frac{v_0}{L_e} l^2 \left(\cos \frac{z}{l} - 1 \right), \tag{44}$$

where the parameter l describes the convergence of streamlines in (43) and the divergence of streamlines in (44). These two solutions can be unified into one solution:

$$\chi(z) = \frac{v_0}{L_e} l^2 \left(\cosh \frac{z}{l} - 1 \right) \tag{45}$$

where the parameter l can now be real or imaginary and describes the convergence ($l^2 > 0$) or divergence ($l^2 < 0$) of the flow lines respectively. In the case of large l the stream function $\phi(x, y)$ turns into the stagnation-point flow $\phi \sim xy$ used by Parker (1973). From (45), one obtains the stream function

$$\phi(x, y) = \frac{v_0}{L_e} l^2 \sinh \frac{x}{l} \sin \frac{y}{l}. \tag{46}$$

The inflow velocity v_e is therefore given by $\partial_x \phi(0, L_e)$:

$$v_e = \frac{v_0 l}{L_e} \sin \frac{L_e}{l}. \tag{47}$$

By expanding for small z, one obtains (32). The approximation (37) can therefore be applied. This can also be seen from the expansion of the function Γ, which is given by

$$\eta \Gamma(\eta \bar{\phi}, \eta \bar{\psi}) = \frac{L_e}{2 v_0} \frac{1}{|\chi|} + O(\eta), \tag{48}$$

where the function Γ is given by

$$\Gamma(\chi) = \left| 2 \frac{v_0}{L_e} \chi + \frac{\chi^2}{l^2} \right|^{-1}. \tag{49}$$

A uniform approximation can thus be written as

$$a = \frac{E L_e}{v_0} \mathrm{E_I} \left(\left(\frac{\psi + |\chi|}{\eta} \right)^{1/2} \right) + F(\phi) + e^{\psi/\eta} G(\phi) + O(\eta). \tag{50}$$

Contour plots of the flux and stream function for the cases (from top to bottom) $l/L_e = \infty, 0.5i$ and 0.5, i.e. for the stagnation point flow $\phi \sim xy$ and a diverging and converging plasma flow, are shown in Fig. 1. The flux function is plotted in the panels to the left and the stream function to the right. In all cases the inflow boundary condition $a(x, L_e) \sim x$ is used. A converging plasma flow results in a shortening while a diverging flow results in a stretching of the current sheet. No change in the thickness of the current sheet is observed.

From the solution (50), one obtains the following scaling laws for the length L

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press

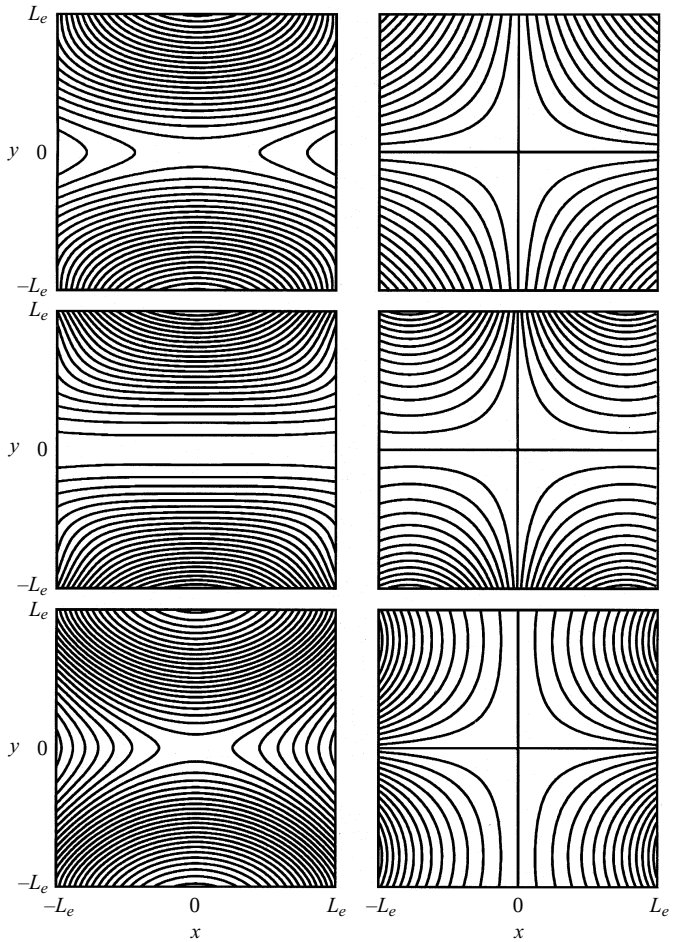


Figure 1. Contour plots of the flux (left) and stream function (right) for the cases $l/L_e = \infty, 0.5i$ and 0.5 (from top to bottom), i.e. for the stagnation-point flow $\phi \sim xy$ and a diverging and converging plasma flow. A converging plasma flow results in a shortening, while a diverging flow results in a stretching of the current sheet. No change in the thickness of the current sheet is observed.

and width d of the current sheet:

$$\left(\frac{L}{L_e}\right)^2 = R_m M_e \frac{l}{L_e} \sin \frac{L_e}{l}, \tag{51}$$

$$\left(\frac{d}{L_e}\right)^2 = \frac{1}{M_e R_m}. \tag{52}$$

The current-sheet length increases for increasing magnetic Reynolds number unless the outflow boundary conditions constrain the current sheet to a finite length. For large l, (51) can be expanded in $1/l$, and one obtains

$$\frac{L}{L_e} = (R_m M_e)^{1/2} \left(1 - \frac{1}{12} \frac{L_e^2}{l^2}\right) + O\left(\frac{1}{l^4}\right). \tag{53}$$

For converging streamlines ($l^2 > 0$) the current sheet length decreases, while for

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press

diverging stream lines ($l^2 < 0$) it increases. As has been shown in the last section, the term $e^{\psi/\eta}G(\phi)$ in (50) corresponds to a boundary layer in the outflow region because of the essential singularity of the function $e^{\psi/\eta}$. This term generates a boundary layer in the outflow region, which prevents the current sheet from growing. For an open outflow region this term will not appear, and the current sheet can grow indefinitely in length.

The dependence on the resistivity η of the scaling laws (51) and (52) has already been obtained in the last section. The additional dependence on the reconnection rate M_e and the convergence/divergence of the plasma flow l produces merely a relative change in the exact length and width of the current sheet, but the general behaviour for large magnetic Reynolds numbers is similar in each case.

6. Conclusions

In this paper a general solution method has been given for the hydromagnetic equation in the case of an incompressible irrotational two-dimensional plasma flow. The method has been applied to stagnation-point flows as appear in magnetic reconnection near magnetic null points. It has been shown that the current-sheet length grows for increasing magnetic Reynolds number R_m with the square root of R_m, independently of the specific stagnation-point flow chosen. This may have implications concerning the process of fast magnetic reconnection in the case of constant resistivity η. In the Petschek model the size of the diffusion region is small for large magnetic Reynolds number. This fact is crucial for the logarithmic dependence of the reconnection rate on the magnetic Reynolds number. In the case of a growing current sheet the reconnection rate is limited by the Sweet–Parker rate, which is much smaller for large magnetic Reynolds numbers than the Petschek value. Numerical simulations have shown that the current sheet length can be held fixed by spatially localizing the resistivity (Scholer 1989; Yan *et al.* 1992). A similar kinematic analysis as done here for spatially uniform resistivity has been done for the inhomogeneous resistivity case (Jamitzky and Scholer, submitted). The current-sheet length is then indeed limited by the spatially distribution of the resistivity.

Acknowledgements

Part of this work was performed during a visit of one of the authors, F. Jamitzky, to the Institute of Space and Astronautical Sciences (ISAS), Japan. We are grateful to M. Hoshino for helpful discussions.

References

Biskamp, D. 1986 Magnetic reconnection via current sheets. *Phys. Fluids* **29**, 1520.

Forbes, T. G. and Priest, E. R. 1987 A comparison of analytical and numerical models for steadily-driven magnetic reconnection. *Rev. Geophys.* **25**, 1583.

Forbes, T. G. 1995 How does fast reconnection work? *Small Scale Structures in Three-Dimensional Hydro and Magnetohydrodynamic Turbulence* (ed. A. Pouquet and P.L. Sulem). Springer-Verlag, Berlin.

Heyn, M. F., Biernat, H. K., Rijnbeek, R. P. and Semenov, V. S. 1988 The structure of reconnection layers. *J. Plasma Phys.* **40**, 235.

Jamitzky, F. and Scholer, M. 1995 Steady state magnetic reconnection at high magnetic Reynolds number: a boundary layer analysis. *J. Geophys. Res.* **100**, 19 277.

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press

Parker, E. N. 1963 The solar flare phenomenon and the theory of reconnection and annihilation of magnetic fields. *Astrophys. J. Suppl.* **8**, 177.

Parker, E. N. 1973 Comments on the reconnexion rate of magnetic fields. *J. Plasma Phys.* **9**, 49.

Petschek, H. E. 1964 Magnetic field annihilation. *Proceedings of AAS–NASA Symposium on Physics of Solar Flares. NASA* SP-50, p. 425.

Phan, T. D. and Sonnerup, B. U. O. 1990 MHD stagnation-point flows at a current sheet including viscous and resistive effects: general two-dimensional solutions. *J. Plasma Phys.* **44**, 525.

Priest, E. R. and Forbes, T. G. 1986 New models for fast steady state magnetic reconnection. *J. Geophys. Res.* **91**, 5579.

Scholer, M. 1989 Undriven reconnection in an isolated current sheet. *J. Geophys. Res.* **94**, 8805.

Sonnerup, B. U. Ö. and Priest, E. R. 1975 Resistive MHD stagnation point flows at a current sheet. *J. Plasma Phys.* **14**, 283.

Sweet, P. A. 1958 The neutral point theory of solar flares. *IAU Symp.* **6**, 123.

Yan, M., Lee, L. C. and Priest, E. R. 1992 Fast magnetic reconnection with small shock angles. *J. Geophys. Res.* **97**, 8277.

https://doi.org/10.1017/S0022377897005473 Published online by Cambridge University Press