FRICTIONLESS CONTACT PROBLEM FOR A FUNCTIONALLY GRADED LAYER LOADED THROUGH TWO RIGID PUNCHES USING FINITE ELEMENT METHOD

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ABSTRACT

In this study, continuous contact problem in the functionally graded (FG) layer loaded with two rigid flat blocks resting on the elastic semi-infinite plane was analyzed by the finite element method. The two-dimensional numerical model of the FG layer was made with the software added to the ANSYS program. This software can be adapted to all contact problem types by making minor changes. The accuracy check of the program was performed by comparing with the analytical solution of the problem by homogeneous layer and its solution by the finite element method. So, fast and practical solutions can be obtained by the developed finite element method on many applications such as; automotive, aviation and space industry applications. The comparisons made showed that the proposed solution gave good results at acceptable levels. In the problem, it was thought that all surfaces were frictionless. The external loads P and Q were transmitted to the FG layer via two flat rigid blocks. Normal stresses between the FG layer and the elastic plane, initial separation loads, initial separation distances and contact stresses under the blocks were investigated for various dimensionless quantities.

Keywords: Finite element method, Functionally graded layer, Contact problem, Elasticity, Homogeneous layer

1. INTRODUCTION

Contact problems have frequently been encountered in many field of engineering such as roads, airfields, foundations, cylindrical mills, silos, and fuel tanks. With the progress in technology in various industries such as aerospace, automotive, space and defense industry, the need for new materials that provide thermal resistance and

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DOI : 10.1017/jmech.2018.55
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https://doi.org/10.1017/jmech.2018.55 Published online by Cambridge University Press

high strength has been increased and enforced the researchers to develop innovative ideas and solutions. For this reason, composite materials that provide superior mechanical properties to metals have begun to be produced and used in various industrial areas. Functionally graded material (FGM) is a special type of composite in which the material properties change gradually and continuously within the solid from one surface to the other. The continuous variation of the constituents provides a sophisticated solution to the problems of the traditional composite materials [1]. Contact between the parts connecting the structural elements or mechanical systems has been the focus of considerable research for many years [2-8,30-32]. The analytical solution of the problem is time-consuming due to the large number of mathematical operations. Depending on developments in computer technology, studies on the contact problems have been centered upon the use of different numerical solution methods. The finite element method has frequently been used in the contact problems with friction [9-11]. Moreover, some research papers dealt with the comparison of analytical solution to that of the finite element method in contact problems [12,13,28,29].

With increasing demand for new and advanced materials such as functionally graded (FG) layers which provide high strength instead of traditional homogeneous materials, have been resolved by various methods and geometries [14-24]. Consequently, in the literature, there are different studies presenting the solutions to contact problems with the analytical and finite element method (FEM). Especially with FEM-based programs, the solution of homogeneous layers can now be done easily. However, in FG layers, special software should be added to the program since the material properties of the layer cannot be identified, assigned and meshed with the standard menus of these programs. For this reason, the problems involving the FEM solutions of the FG layers are very few in the literature. Furthermore, the analytical and FEM solutions of continuous contact problem in the FG layer loaded with two flat rigid blocks resting on the elastic plane were not found. It is possible to encounter the type of problem presented in this study in fields such as machine elements, aircraft and spacecraft coatings, and thermal barriers. This problem provides practical solutions for the contact analysis of FG materials, which are a type of material needed in developing and changing industrial application fields. To know the stress, deformation and displacement expressions that may occur as a result of contact is very important during the safe dimensioning of the systems. The purpose of this study is to perform the solution of the FG layer loaded with two flat rigid blocks resting on the elastic plane, the analytical solution of which is not present in the literature, with FEM. Processing time of analytical solution of contact problems involving FG material is quite long due to the fact that FG material consists of long and complicated mathematical expressions. For this reason, alternative methods have been studied by scientists for a long time. There are no commands available for analysis of FG materials in the ANSYS package program. The software was added to the program for the solution. This software also allows the identification of all kinds of geometry and material properties. The material properties of the FG layer are a function showing exponential variation, and its layer depth is $\mu(y)=\mu_0 e^{\beta y}$ along (h). Where, μ is the shear modulus and is defined as follows:

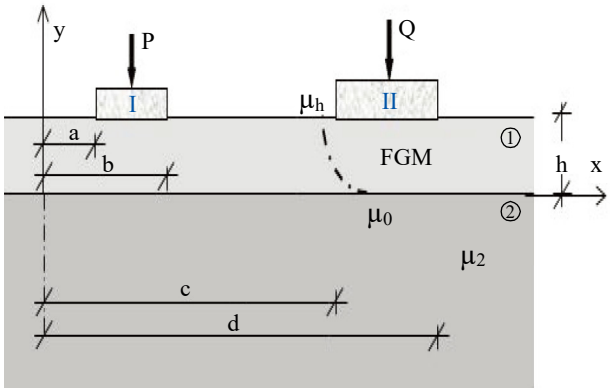


Fig. 1 FG layer loaded with two rigid flat blocks resting on the elastic semi-infinite plane

$$\mu=\frac{E}{2(1+\upsilon)}$$

Also E is Young Modulus and β refers to the inhomogeneity parameter in the direction of the depth, and the layer shows homogeneous behaviour if it is taken very close to zero. The FG layer loaded with two rigid blocks resting on the elastic semi-infinite plane is presented in figure 1. There, μ_h is the shear modulus of the upper surface of FG layer and μ_0 is the shear modulus of bottom surface of FG layer. The solution of the study with the finite element method was performed using the ANSYS Mechanical APDL Product Launcher (2015) [25] and the results obtained from the solution performed by taking β very close to zero in the macro added to program could be compared with the analytical and FEM solutions of the problem performed according to the homogeneous layer. Therefore, three different solutions were proposed in the study, and it was observed that the results obtained were very compatible with each other.

2. ELASTICITY SOLUTION

In this section, the frictionless contact problem of a homogeneous layer of height h loaded with two singular loads such as P and Q on two flat rigid blocks resting on the elastic semi-infinite plane with block widths of $(b-a)$ and $(d-c)$ was solved based on the theory of elasticity. Displacements of each elastic media expressed as the Fourier transforms of the unknown functions. Necessary derivatives of these functions are substituted into Navier equations and an ordinary differential equation system is obtained. This equation system is solved and general expressions for displacements and

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stresses are determined with regard to unknown constants. They are determined from the 1(a-f) boundary conditions;

$$\sigma_{y_1}(x,h)=\begin{cases}-P(x) & a<x<b\\ -Q(x) & c<x<d\\ & -\infty<x<a,\\ 0 & b<x<c,\\ & d<x<\infty\end{cases} \tag{1a}$$

$$\tau_{xy_1}(x,h)=0, \qquad -\infty<x<\infty, \tag{1b}$$

$$\tau_{xy_1}(x,0)=0, \qquad -\infty<x<\infty, \tag{1c}$$

$$\tau_{xy_2}(x,0)=0, \qquad -\infty<x<\infty, \tag{1d}$$

$$\sigma_{y_1}(x,0)=\sigma_{y_2}(x,0), \qquad -\infty<x<\infty \tag{1e}$$

$$\frac{\partial}{\partial x}\left[v_2(x,0)-v_1(x,0)\right]=0, \qquad -\infty<x<\infty, \tag{1f}$$

$$\frac{\partial}{\partial x}\left[v_1(x,h)\right]=0 \qquad a<x<b \tag{1g}$$

$$\frac{\partial}{\partial x}\left[v_1(x,h)\right]=0, \qquad d<x<c \tag{1h}$$

where $P(x)$ and $Q(x)$ are the unknown contact pressures under punch I and punch II. Subscript 1 and 2 indicate the elastic layer and the elastic half plane respectively. v is refer to displacement in the y direction. Equilibrium conditions of the problem may be expressed as

$$\int_a^b P(x)dx=P, \tag{2a}$$

$$\int_c^d Q(x)dx=Q, \tag{2b}$$

Boundary and continuity conditions (1a-f) with stress and displacement expressions give the unknown constants in terms of unknown functions $P(x)$ and $Q(x)$ and by substituting values of these constants into equations (1g,h), after necessary manipulations, for $P(x)$ and $Q(x)$ singular integral equation may found.

$$-\frac{1}{\pi\mu_1}\int_a^b\left[k_a(x,t)+\frac{(1+\kappa_1)}{4}\frac{1}{t-x}\right]P(t)dt$$

$$-\frac{1}{\pi\mu_1}\int_c^d\left[k_a(x,t)+\frac{(1+\kappa_1)}{4}\frac{1}{t-x}\right]Q(t)dt=0, \qquad a<x<b, \tag{3a}$$

$$-\frac{1}{\pi\mu_1}\int_a^b\left[k_a(x,t)+\frac{(1+\kappa_1)}{4}\frac{1}{t-x}\right]P(t)dt$$

$$-\frac{1}{\pi\mu_1}\int_c^d\left[k_a(x,t)+\frac{(1+\kappa_1)}{4}\frac{1}{t-x}\right]Q(t)dt=0, \qquad c<x<d, \tag{3b}$$

where,

$$k_a(x,t)=\int_0^\infty\left\{\left\{4\alpha^2(1+\kappa_1)\begin{bmatrix}(1+\kappa_2)\left[-e^{-4\alpha h}+4\alpha he^{-2\alpha h}+1\right]\\ +\mu_2/\mu_1(1+\kappa_1)\left[e^{-4\alpha h}-2e^{-2\alpha h}+1\right]\end{bmatrix}\right\}\Big/\Delta \quad -(1+\kappa_1)/4\right\}\{\sin\alpha(t-x)\}d\alpha \tag{4}$$

in which,

$$\Delta=16\alpha^2\begin{Bmatrix}(1+\kappa_2)\left[e^{-4\alpha h}+e^{-2\alpha h}(-2-4\alpha^2h^2)+1\right]\\ +\mu_2/\mu_1(1+\kappa_1)\left[-e^{-4\alpha h}+4\alpha he^{-2\alpha h}+1\right]\end{Bmatrix} \tag{5}$$

where $k_a(x,t)$ is the Fredholm kernel of singular integral equations. μ_i and κ_i $(i=1,2)$ are the elastic constants, $\kappa_i=3-4\upsilon_i$ for plane strain, $\kappa_i=(3-4\upsilon_i)/(1+\upsilon_i)$ for plane stress, υ_i is the Poisson's ratio. υ_i is taken 0.25 in this study.
Also contact stress may be;

$$\sigma_{y_1}(x,0)=-\rho gh-\frac{1}{\pi}\int_a^b k_b(x,t)P(t)dt \quad -\frac{1}{\pi}\int_c^d k_b(x,t)Q(t)dt, \quad -\infty<x<\infty \tag{6}$$

in which ρ is mass density and g is gravity acceleration, where,

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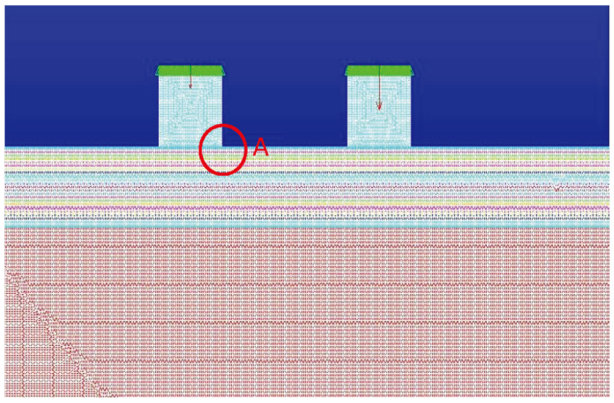


Fig. 2 FEM modeling of the FG problem



Fig. 3 Free-Triangular Mesh (Section A)

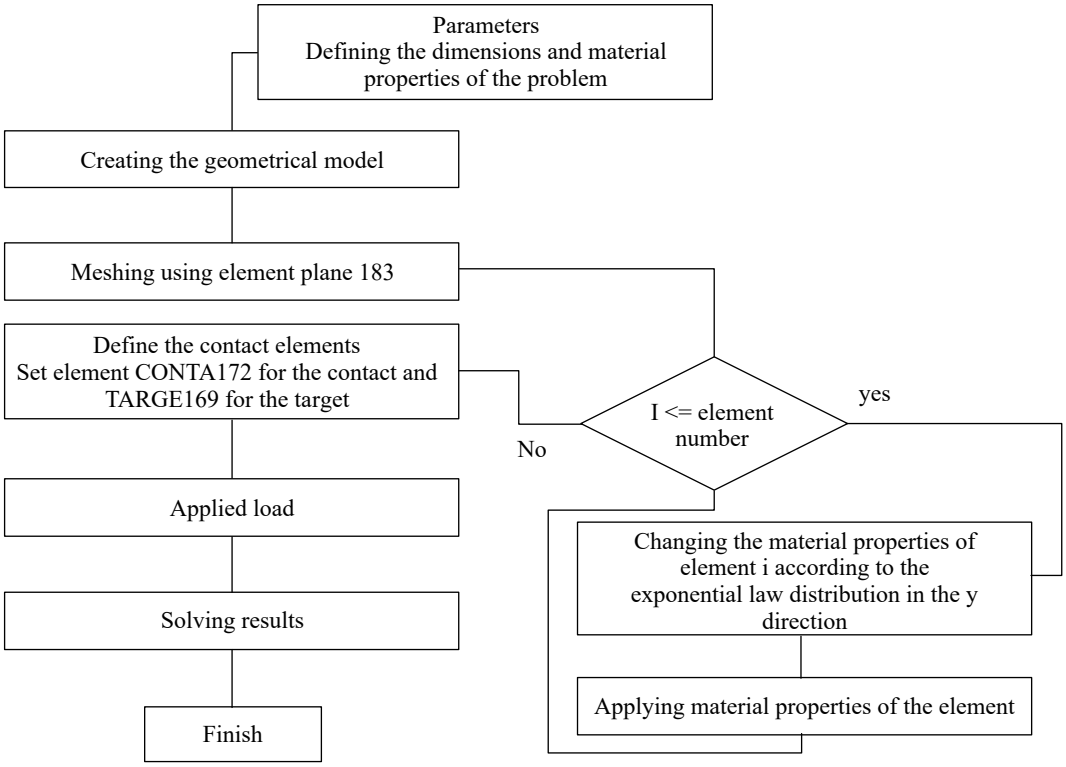


Fig.4 The details of the FEM procedure.

$$k_b(x,t)=\int_0^{\infty}\left\{\begin{matrix}32\alpha^2\mu_2/\mu_1(1+\kappa_1)\\ \left[e^{-3\alpha h}(-1+\alpha h)+e^{-\alpha h}(1+\alpha h)\right]\end{matrix}\right\}\Big/\Delta \qquad \{\cos\alpha(t-x)\}d\alpha \tag{7}$$

Other equations can be seen in the ref [7].

3. FINITE ELEMENTS SOLUTION

The finite element method (FEM) has been frequently used in solution of complex engineering problems. Especially, it is very effective method to find approximate solution to partial differential and integral equations. In recent years, various FEM-based programs have been generated in parallel with the developments in computer technology. These programs have become the centre of interest of researchers with their rapid and effective solutions. In this study, the ANSYS Mechanical APDL Product Launcher (2015) program was used in the solution with FEM. ANSYS finite element package is a great, multi-purpose analysis tool effectively used in extensive diversity of engineering applications, and allows users to perform analysis in different physical fields.

In this section, two different FEM solutions were

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performed. Firstly, the problem was solved by being modelled based on the homogeneity of the layer. In this model, the geometry of the problem, element type, material properties, separation to the finite elements, contact pairs were determined, and the displacement and loads were defined using the standard menus of ANSYS program, and the problem was solved. In the second part, the finite element model of the FG layer was made. Here, similarly, the geometry and element type were created using the standard menus of the program. However, the definition of the material properties, their assignment to the layer and separation to the finite elements were performed with the software added to the log file. Mesh processing is important in terms of precision of the results. When mesh ranges are reduced, the results are fairly convergent, but the processing time increases. For this reason it would be appropriate to select a slightly larger mesh in other areas, especially in contact areas. In this study, the lines forming the geometry were divided into 0.03 parts in the contact areas and 0.05 parts in the other areas to obtain the net areas. This software is general and can be applied to all contact problem types with minor changes regardless of geometry, material properties, and mesh types. The finite element model of the program according to the functionally graded layer is presented in figure 2 and 3.The solution steps of ANSYS FE modeling for FG layer are depicted in the flowchart in Fig.4

The code developed starts by defining the problem parameters such as geometrical and material properties using *set command. Due to the simple geometry of the problem, a two-dimensional (2D) plain strain geometrical model for the FG layer and the homogeneous plane is used and created by BLC4 command. The material properties of the homogeneous elastic semi-infinite plane and the blocks are chosen as linear-elastic-isotropic. The Poisson's ratio is assumed to be constant for each layer and blocks. The mesh process is achieved using the element PLANE183 which have eight nodes with two degrees of freedom at each node. It must be noted that the material set in this stage is only used to create the mesh, and later will be changed using a *doloop and a macro contains the exponential law of FGM properties. Free triangular meshing with 418860 nodes is used since the problems can be modeled regardless of geometry (see Fig.3). The following macro [27] is used to create the FG layer:

```
*create, fgm_material
/nopr
 y_coord = arg1
 E_fgm=E_t*EXP(B*y_coord)
 nu_fgm=nu_t*EXP(B*y_coord)
 den_fgm     = dens_t*EXP(B*y_coord)
     /gopr
*end
```

The last step is creation of the contact region. CONTA172 element is used to represent contact between the 2-D "target" surfaces which are defined by TARGE169. The Augmented Lagrangian Method using the total potential energy theorem is chosen as the contact algorithm. A good and fast convergence was provided to the results with this algorithm. Interaction between the contact surfaces is modelled with frictionless surface-to-surface elements. Contact pairs are defined as CONTA172 and TARGE169 at the block-FG layer interface and the FGM layer- elastic half-plane interface. For the functionally graded layer considered here, it is assumed that the shear modulus is represented by an exponential function of $\mu(y) = \mu_0 e^{\beta y}$ along the depth. Where μ is the shear modulus in the homogeneous condition, and β is the inhomogeneity parameter. In case of dimensionless $\beta h = 0.001$, the layer exhibits approximately homogeneous behavior. This is clear when comparing the results obtained from ANSYS for the homogeneous layer with that in ref. [7] in which $\beta h = 0.001$.

4. NUMERICAL RESULTS AND DISCUSSION

In this study, normal and contact stress distributions, initial separation loads and distances are presented for various values of μ_0/μ_h , μ_2/μ_h , and P/Q for FG layer resting on an elastic half plane which is not investigates before. Here, μ_0 and μ_h refer to the shear modulus below and above the FG layer, respectively, μ_2 refers to the shear modulus of the elastic semi-infinite plane. The change in the material properties of the FG layer is as follows:

(a) $\mu_0/\mu_h > 1$ shows that the layer's stiffness increases along its depth and βh takes positive values,

(b) $\mu_0/\mu_h = 1$ there is no change in y direction for the layer's stiffness. The layer shows homogeneous behavior. ($\beta h = 0.001$),

(c) $\mu_0/\mu_h < 1$ demonstrates that the layer's stiffness decreases through the depth, and βh takes negative values.

In this section, the results of three different solutions of the problem are presented in tables and figure. The analytical and FEM solutions of the homogeneous layer are compared in Table 1. In figure 5 and 6, the software for the FG layer was compared with the homogeneous results, and the program was checked.

Where x_{cr} is refer initial separation dictance and λ_{cr} is initial separation load. The initial separation load that causes the initial separation is given as follows:

$$\lambda_{cr} = \frac{P_{cr}}{\rho_0 g h^2}$$

Fig. 5 $\sigma_y(x,0)$ Comparison of stress distributions between layer and elastic half plane (a/h=3, b/h=4, c/h=5, d/h=6, Q=2P, h=1,κ_1= κ_2=2, μ_2/μ_h=2)

The comparison of analytical results of the homogeneous layer and the results of finite element solution for the functionally graded layer with $\beta h = 0.001$ is given in Table 1. It is seen that the results for the initial separation load and distances obtained

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Table 1 Comparison of the initial separation loads and distance between the homogeneous layer and the elastic half plane (a/h=3, b/h=4, c/h=5, d/h=6, h=1, $\kappa_1=\kappa_2=2$, $\mu_2/\mu_h=2$)

Method	Q = P		Q = 2P		Q = 3P	
	x_{cr}/h	λ_{cr}/h	x_{cr}/h	λ_{cr}/h	x_{cr}/h	λ_{cr}/h
Analytical	8.068	106.79	8.077	56.82	8.080	38.7
FEM	8.050	103.95	8.100	55.11	8.100	37.5

Table 2 Change of the initial separation loads and distances between the FG layer and the elastic plane depending on βh (a/h=3, b/h=4, c/h=5, d/h=6, h=1, $\kappa_1=\kappa_2=2$, $\mu_2/\mu_h=1$)

Values of μ_0/μ_h	Q = P		Q = 2P		Q = 3P	
	x_{cr}/h	λ_{cr}	x_{cr}/h	λ_{cr}	x_{cr}/h	λ_{cr}
βh (0.001)	8.55	129.0	8.55	70.58	8.55	48.68
βh (0.693)	8.90	157.5	8.95	88.25	8.95	61.25
βh (1.386)	9.30	171.8	9.35	98.01	9.35	68.64
βh (1.792)	9.55	174.6	9.55	100.8	9.60	70.78
βh (2.079)	9.70	174.9	9.75	101.8	9.75	71.62

according to various loading conditions are in good agreement. In figure 5, comparisons between the analytical and FEM solutions for the stress distributions are presented. It is seen that, the results are in good agreement. So it is verified that code written in ANSYS works successfully.

The initial separation loads and distances according to various inhomogeneity parameters (βh) of the functionally graded layer are given in Table 2. According to these results, both the initial separation loads and the initial separation distances are increased when (βh) increases, that means the layer is getting stiffer from its top to bottom. There is no change in the initial separation distance when increasing the external load, however, the initial separation load decreases.

As it seen in Table 3, when βh decreases, means the layer is getting stiffer from its bottom to top, the initial separation loads and distances decrease. Here, as can be seen again, there no change in the initial separation distance but the initial separation load decreases with increasing the external load. When Tables 2 and 3 were examined together, it was observed that the increase in the second block load did not change the initial separation distances while initial separation loads were reduced. So, it can be said that the layer separates more easily from the elastic semi-infinite plane.

In FG layer, the normalized contact stresses *P(x)/P/h* under the blocks are given in figure 6. As it seen, the contact stresses slightly increase when βh changes from negative to positive values.

Figure 7 shows the FEM solution of the stresses between block 1 and FG layer when two block loads are taken equal. While stress concentrations occurred in the corners of the block, there was more stress on the right side of the 1st block due to the inter-block interaction. If

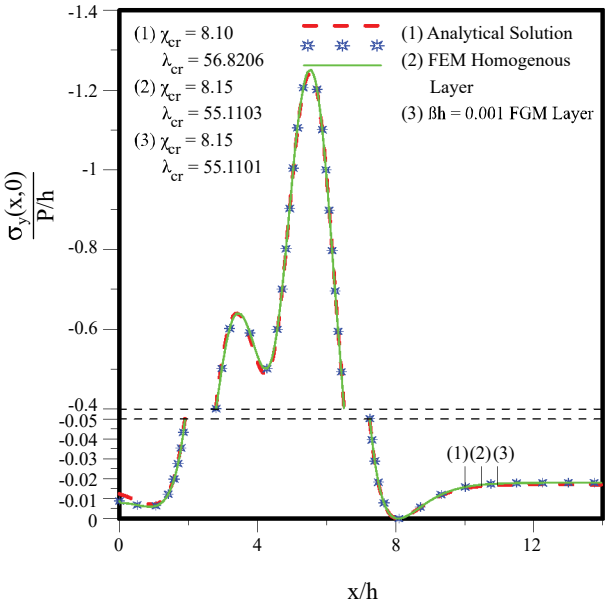


Fig. 5 $\sigma_y(x,0)$ Comparison of stress distributions between layer and elastic half plane (a/h=3, b/h=4, c/h=5, d/h=6, Q=2P, h=1,$\kappa_1=\kappa_2=2$, $\mu_2/\mu_h=2$)

the blocks are distant enough, the same stress values are obtained at both corners.

In figure 8, normal stresses $\sigma_y(x,0)$ at FG layer-half-plane interface for different inhomogeneity parameters βh are given. As it seen, when βh changes from negative to positive values, maximum values of σ_y decrease. The initial separation load and distances were reduced for all βh values. Acquired results show coherence with the results of Öner *et al.*'s [33] study

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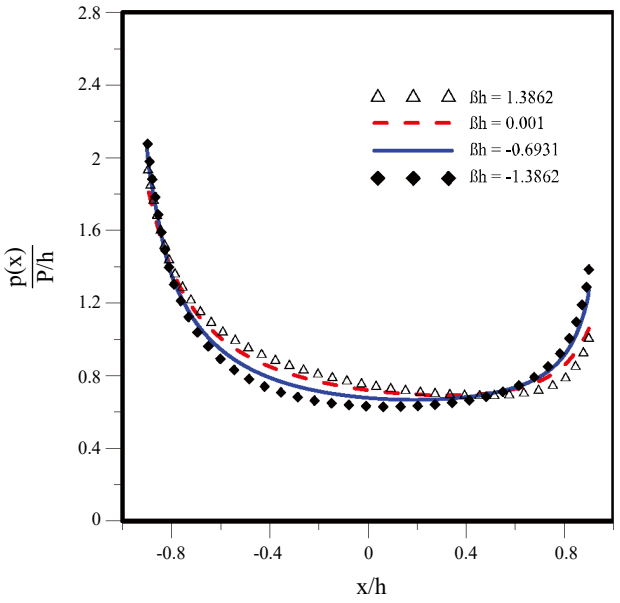


Fig. 6 Stress distributions under the first block for different βh values in FG layer (a/h=3, b/h=4, c/h=5, d/h=6, Q=2P, h=1,μ_2/μ_h=2, κ_1= κ_2=2)

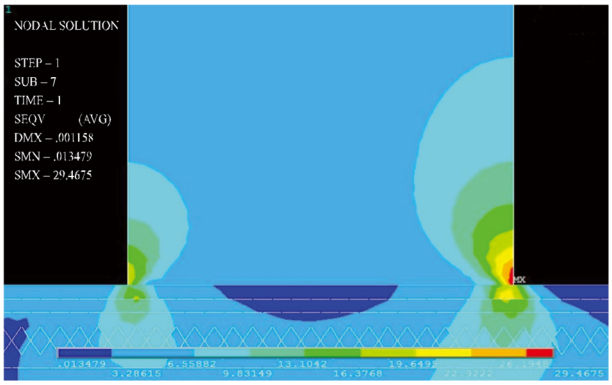


Fig. 7 Stress distributions under the first block of FG layer for βh =2.071 (a/h=3, b/h=4, c/h=5, d/h=6, Q=P, h=1,κ_1= κ_2=2, μ_2/μ_h=2)

which is about the contact problem of the FG layer. Similarly, the $\sigma_x(x,0)$ stress distribution occurring between the layer and the plane was examined in fig. 9. However, reduced rigidity throughout the depth of the layer has resulted in an increase in $\sigma_x(x,0)$) stresses.

In fig. 10 shows the stress distribution of the FEM solution is given in case of the second block load is twice as much as the first block load. It is seen that there is more stress concentration at the edges of the second block. Similarly, there is an increase in stress values in the region under the second block and along the layer and semi-infinite plane

In figure 11, $\sigma_y(4.5,y)$ normal stress distributions along the layer depth are given . As it seen, when βh changes from negative to positive, normal stresses slightly increase.

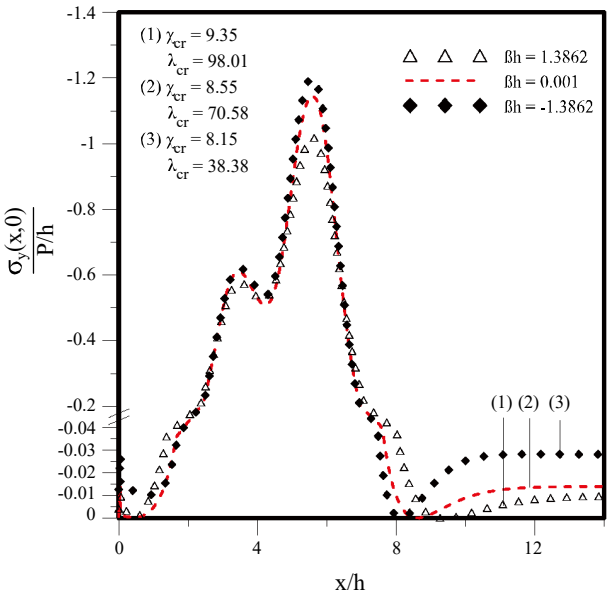


Fig. 8 $\sigma_y(x,0)$ Stress distribution for different βh values between FG layer and elastic plane for μ_2/μ_h=1 (a/h=3, b/h=4, c/h=5, d/h=6, Q=2P, h=1, κ_1= κ_2=2)

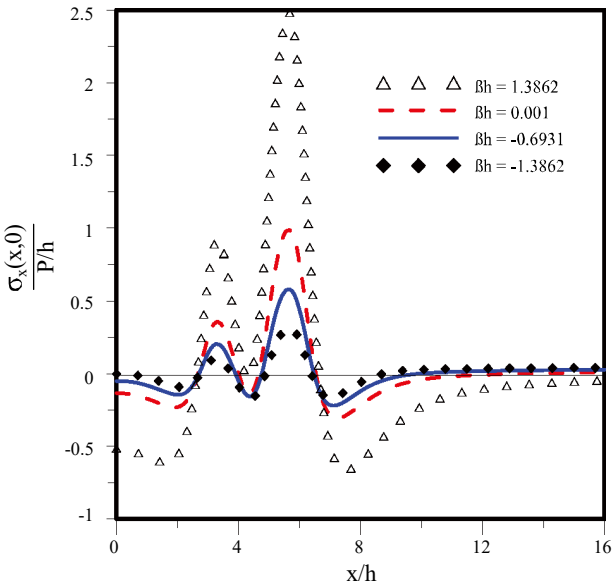


Fig. 9 $\sigma_x(x,0)$ Stress distribution for different βh values between FG layer and elastic plane for μ_2/μ_h=1 (a/h=3, b/h=4, c/h=5, d/h=6, Q=2P, h=1, κ_1= κ_2=2)

the stress values according to the change of the inhomogeneity parameter showed differences in the center of the layer, there were no significant differences under the layer. Compared with the homogeneous solution, stress values sligthly decrease when the stiffness increases under the layer.

In figure 11, the change of $\sigma_y(4.5,y)$ stresses occurring in the middle of the rigid blocks (x = 4.5) and along the layer depth with βh was examined. When the solution is examined according to the homogeneous plane βh = 0.001 it can be observed that both tensile and compression

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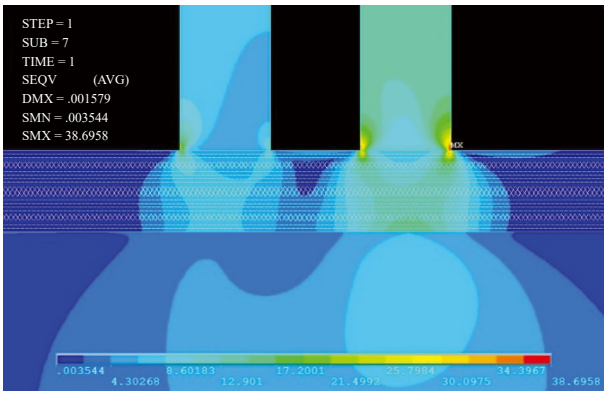


Fig. 10 Von Misses stress distributions for $\beta h = 2.071$ (a/h=3, b/h=4, c/h=5, d/h=6, Q=2P, h=1,κ_1= κ_2=2, μ_2/μ_h=1)

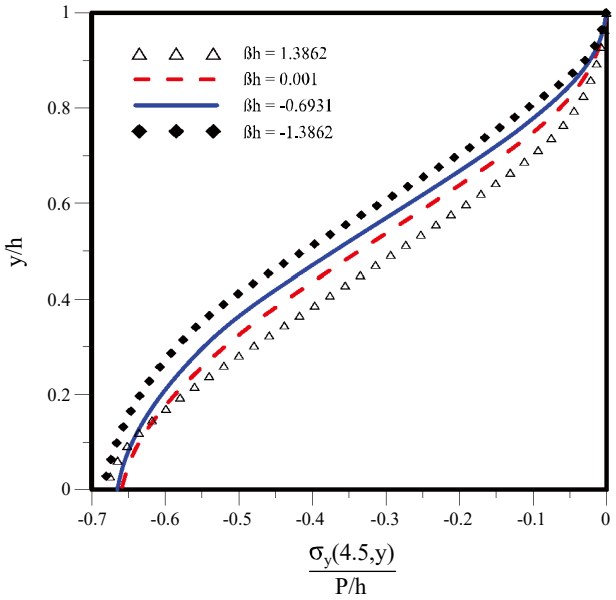


Fig. 11 $\sigma_y(4.5, y)$ Stress distribution along depth of FG layer for μ_2/μ_h=1 (a/h=3, b/h=4, c/h=5, d/h=6, Q=2P, h=1, κ_1= κ_2=2)

stresses are evenly distributed along the depth. However, the fact that the top of the layer was more rigid in the FG layer significantly influenced the σ_x values. Here, greater tensile stresses occurred on the layer while the compression stresses under the layer were about 3 times less. The σ_x stress values variation of a point away from the second block $\sigma_x(7, y)$ along the depth is presented in figure 12.

5.CONCLUSION

In this study, the solution of the FG layer loaded with two flat rigid blocks resting on the elastic plane, the analytical solution of which is not present in the literature, with FEM was performed. The special software, which was added to the ANSYS package program, was used for

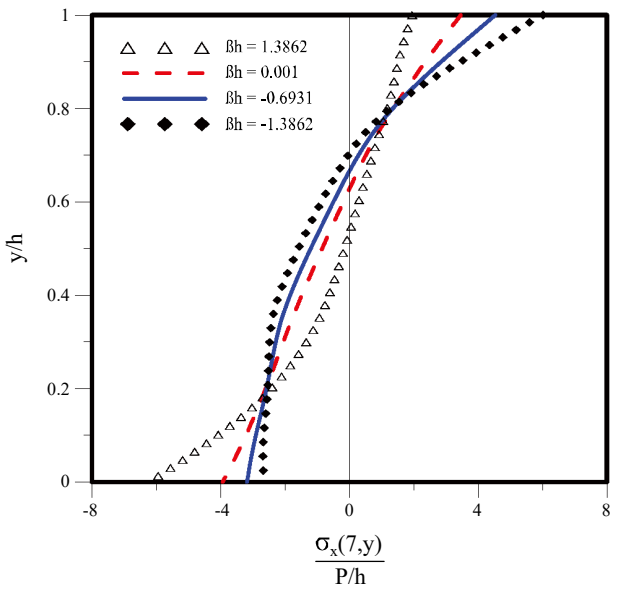


Fig. 12 $\sigma_x(7, y)$ Stress distribution along depth of FG layer (a/h=3, b/h=4, c/h=5, d/h=6, Q=2P, h=1, κ_1= κ_2=2, μ_2/μ_h=1)

the solution. The results obtained from the solution were compared with the analytical and FEM solutions of the problem with respect to the homogeneous layer. It was observed that the results were very consistent, and thus the software was proven to work well. This software is also general-purpose and can be used in all geometry, material properties and mesh types with minor changes. The developed software for finite element method can be used for practical solution of complex problems on many industries using functionally graded materials such as automotive, aviation and space industry. It will also be possible to achieve acceptable results in a relatively short period of time, without the need for long and complex mathematical expressions of theoretical solutions. With this software, it was observed that the analytic solution of the problems with or without solutions in the literature could be performed with FEM. When the results obtained from the analysis of the problem were summarized;

- It was observed that the separation between layer-elastic plane became difficult if the rigidity of the FG layer increased along the depth, and the separation was easier if rigidity decreased.
- The increase in the second block load facilitated the separation for each situation.
- When the ratio of the shear modulus of the elastic plane to the shear modulus at the top of the FG layer (μ_2/μ_h) increased, σ_x stresses significantly decreased while no significant change was observed in σ_y normal stresses.
- The increase in rigidity on the FG layer also increased the stress values.

REFERENCES

1. Miyamoto, Y., Kaysser, W. A., Rabin, B. H., Kawasaki, A., Ford, R. G., *Functionally Graded Materials Design, Processing and Applications*,

https://doi.org/10.1017/jmech.2018.55 Published online by Cambridge University Press

Kluwer Academic Publishers, Boston, (1999).

2. Hertz, H., "On The Contact of Elastic Solids," *Journal für die Reine und Angewandte Mathematik*, **92**, pp.156-171, (1982).
3. Dhaliwal, R.S., Rau, I.S., "Axisymmetric Boussinesq Problem for A Thick Elastic Layer Under A Punch of Arbitrary Profile," *International Journal of Engineering Science*, **8**, pp. 843-856, (1970).
4. Uflyand, I. S., *Survey of Articles on The Applications of Integral Transforms In The Theory of Elasticity*, Raleigh N.C.S.,North Carolina State College Translation Series, (1965).
5. Adams, G.G., Bogy, D.B., "The Plane Solution for The Elastic Contact Problem of A Semi-Infinite Strip and Half Plane," *Journal of Applied Mechanics*, **43** ,pp. 603-607, (1976).
6. Galin, L.A., *Contact Problems in The Theory of Elasticity*, North Carolina State College Translation Series,Raleigh,North Carolina ,(1961).
7. Özşahin, T. Ş., "Frictionless Contact Problem for A Layer on An Elastic Half Plane Loaded By Means of Two Dissimilar Rigid Punches," *Structural Engineering and Mechanics*, **25 (4),** pp. 383-403, (2007).
8. Kahya, V., Birinci, A., Erdöl, R., "Frictionless Contact Problem An Elastic Layer Bonded to Rigid Support and A Rigid Stamp," *Mathematical and Computational Applications*, **6**, 1, pp.13-22, (2001).
9. Dolbow, J., Moës, N., Belytschko, T., "An Extended Finite Element Method for Modeling Crack Growth with Frictional Contact," *Computer Methods in Applied Mechanics and Engineering*, **190**, (51–52),pp. 6825–6846, (2001).
10. Chan, S.K., Tuba, I.S., "A Finite Element Method for Contact Problems of Solid Bodies- Part I. Theory and Validation," *International Journal of Mechanical Sciences*, **13**,pp.519-530, (1971).
11. Tsuta, T., Yamaji, S., *Theory and Practice in Finite Element Structural Analysis*, University of Tokyo Press Japan,pp.10-50, (1973).
12. Öner, E., Yaylacı, M., Birinci, A., "Analytical Solution of A Contact Problem and Comparison with The Results from FEM," *Structural Engineering and Mechanics*,**54 (2)**,pp. 607-622, (2015).
13. Yaylacı, M., Öner, E., Birinci, A., "Comparison between Analytical and ANSYS Calculations for A Receding Contact Problem," *Journal of Engineering Mechanics*, **140 (9)**, (2014).
14. Güler, M.A., Erdoğan, F., "The Frictional Sliding Contact Problems of Rigid Parabolic and Cylindrical Stamps on Graded Coatings," *International Journal of Mechanical Sciences*, **49 (2)**, pp. 161-182, (2007).
15. Yang, J., Ke, L. L., "Two-Dimensional Contact Problem for A Coating– Graded Layer– Substrate Structure under A Rigid Cylindrical Punch," *International Journal of Mechanical Sciences*, **50**, pp. 985-994, (2008).
16. Liu, T. J., Zhang, C., Swang, Y., Xing, Y. M., " The Axisymmetric Stress Analysis of Double Contact Problem for Functionally Graded Materials Layer with Arbitrary Graded Materials Properties," *International Journal of Solids And Structures*, **96**, pp. 229-239,(2016).
17. El-Borgi, S., Abdelmoula, R., Keer, L., "A Receding Contact Plane Problem Between A Functionally Graded Layer and A Homogeneous Substrate", *International Journal of Solids and Structures*, **43** ,pp. 658-674, (2006).
18. Çömez, İ.,"Contact Problem of A Functionally Graded Layer Resting on A Winkler Foundation," *Acta Mechanica*, **224 (11)**, pp.2833-2843, (2013).
19. Chidlow, S. J., Chong, W. F., Teodorescu, M., "On The Two-Dimensional Solution of Both Adhesive and Non-Adhesive Contact Problems Involving Functionally Graded Materials," *European Journal of Mechanics*, **39** ,pp.86-103, (2013).
20. Yan, J., Li, X., "Double Receding Contact Plane Problem Between A Functionally Graded Layer and An Elastic Layer," *European Journal of Mechanics A-Solids*, **53**,pp. 143-150, (2015).
21. Abhilash, M. N., Murty, H., "Finite Element Analysis of 2-D Elastic Contacts Involving Fgms," *International Journal for Computational Methods in Engineering Science and Mechanics*, pp. 253-257, (2014).
22. Demirhan, N., Kanber, B., "Finite Element Analysis of Frictional Contacts of FGM Coated Elastic Members," *Mechanics Based Design of Structures and Machines*, **41**, pp. 383-398, (2013).
23. Güler, M. A., Küçüksucu, A., Yılmaz, K. B., Yıldırım, B., "On The Analytical and Finite Element Solution of Plane Contact Problem of A Rigid Cylindrical Punch Sliding over A Functionally Graded Orthotropic Medium," *International Journal of Mechanical Sciences*, **120**, pp. 12-29, (2016).
24. Çömez, İ., Birinci, A., Erdol, R., "Double Receding Contact Problem for A Rigid Stamp and Two Elastic Layers," *European Journal of Mechanics A-Solids*, **23(2)** , pp.301-309, (2004).
25. ANSYS Software, Houston PA, Swanson Analysis System.
26. Erdogan, F., Gupta, G., "On the Numerical Solutions of Singular Integral Equations," *The Quarterly Journal of Mechanics and Applied Mathematic,* **29**, pp.525-534, (1972).
27. Kouider, B., Boukhoulda, B.F., Nouari, M., Satla, Z., "Structural Modeling and Active Vibration Control of Smart FGM Plate through ANSYS," *International Journal of Computational Methods*, **14(2)**, pp. 9-20, (2016).
28. Polat, A., Kaya, Y., Özşahin, T.Ş., "Analytical solution to continuous contact problem for a

https://doi.org/10.1017/jmech.2018.55 Published online by Cambridge University Press

functionally graded layer loaded through two dissimilar rigid punches", *Meccanica,* **53(14),** pp. 3565-3577, (2018).

29. Kaya, Y., Polat, A., Özşahin, T.Ş., "Comparison of FEM Solution with Analytical Solution of Continuous and Discontinuous Contact Problem", *Sigma Journal of Engineering and Natural Sciences,* **36(4),** 977-992, (2018).
30. Hu, Z., "Contact around a Sharp Corner with Small Scale Plasticity", *Advances in Materials*, **6(1-1)**, pp. 10-17, (2017).
31. Hu, Z., Lu, W., Thouless, M.D., Barber, J.R., "Effect of plastic deformation on the evolution of wear and local stress fields in fretting", *International Journal of Solids and Structures*, **82,** pp. 1-8, (2016).
32. Hu, Z., Lu, W., Thouless, M.D., "Slip and wear at a corner with Coulomb friction and an interfacial strength", *Wear*, **338-339,** pp. 242-251, (2015).
33. Öner, E., Adiyaman, G., Birinci, A., "Continuous contact problem of a functionally graded layer resting on an elastic half-plane", *Archives of Mechanics* , **69(1),** pp. 53-73, (2017).

(Manuscript received May 30, 2018,
accepted for publication September 25, 2018.)

https://doi.org/10.1017/jmech.2018.55 Published online by Cambridge University Press