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Kinematic dexterity of mobile manipulators: an endogenous configuration space approach

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SUMMARY

A mobile manipulator is treated as a robotic system composed of a non-holonomic mobile platform and a holonomic manipulator mounted on the platform. The kinematics of the mobile manipulator can be represented as a driftless control system with outputs. By adopting the endogenous configuration space approach we propose two kinematic dexterity measures, called local and global dexterity. The local dexterity, modeled upon the manipulability of stationary manipulators, indicates how infinitesimal motions in the configuration space propagate to the taskspace of the mobile manipulator. The global dexterity corresponds to L_2-norm of the local dexterity over a prescribed region of the configuration space. Advantages of the endogenous dexterity measures over traditional performance measures of mobile manipulators known from the literature are described. Both the dexterities are employed for determining optimal configurations and optimal geometries of an exemplary mobile manipulator.

KEYWORDS: Mobile manipulator; Kinematics; Endogenous configuration; Dexterity; Optimal design.

1. INTRODUCTION

The construction of performance measures for stationary robotic manipulators belongs to a well established domain of robotics. Local performance measures usually assess fine, local motion accomplished by the end effector of a manipulator whose joints stay around a given configuration. A desirable control and design objective is achieving maximal dexterity or mobility of the end effector in a small region of the taskspace, in response to small changes of the underlying configuration. Global performance measures are most often defined by averaging local measures over a fixed region of the configuration space. Performance measures restricting to kinematic properties of a manipulator are called kinematic. When the dynamics are taken into account, the measures become dynamic. The choice of a particular performance measure depends on the task that should be executed by the manipulator. Let us recall that, generally, performance measures serve two purposes: they are used as additional criteria in inverse kinematics algorithms (a homogeneous part of the solution) or as a means for performance evaluation and optimization of the manipulator. In the second case, local measures detect optimal configurations, whereas global performance measures facilitate the manipulator design.

Looking through the robotic literature concerned with stationary manipulators we find a local kinematic performance measure, called joint range availability, introduced by Ligeois.[1] The determinant of manipulability matrix, called manipulability, was proposed as another local performance measure by Yoshikawa.[2] This measure was further exploited by Doty et al.[3] Klein and Blaho[4] invented the condition number measuring the degree of anisotropy of a configuration, and the minimal singular value of the analytic Jacobian. These authors also used performance measures in order to define optimal configurations and geometries of a manipulator executing a given task. Van den Doel and Pai[5] set forth a differential geometric framework for establishing local, kinematic or dynamic, performance measures for stationary manipulators. Global performance measures, obtained by averaging the local measures, were described by Gosselin and Angeles.[6] Ma and Angeles[7] used the dynamic isotropy measure in optimal design of manipulators. A global kinematic measure, called distortion, expressing a distance of the kinematics from an isometry between a configuration and a task manifold, was introduced by Park and Brockett,[8] and applied in optimal design of manipulator kinematics.

The question of providing a mobile manipulator with dexterity facilitating the accomplishment of its task was addressed by Yamamoto and Yun[9] as a part of the motion coordination problem. The mentioned paper exemplifies a manipulator-oriented approach to the motion coordination problem according to which the motion coordination results in delivering the aboard manipulator to a location where it achieves a high manipulability. An attempt at extending the concept of manipulability from stationary to mobile manipulators and making it more specific was undertaken by Yamamoto and Yun,[10] Foulon et al.,[11] and Bayle et al.[12] The first work provides not only kinematic, but also dynamic performance measures for mobile manipulators. Their extension to multiple mobile manipulators has been proposed in reference [13]. Gardner and Velinsky presented in reference [14] an application of kinematic manipulability ellipsoids to optimal design of mobile manipulators. Integrated mobility and manipulation capabilities of mobile manipulators with holonomic mobile platform were dealt with by Khatib and collaborators.[15,16] A constrained optimization approach to the motion planning of mobile manipulators was developed by Desai and Kumar.[17]

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A dominating approach toward performance measures in mobile manipulators can be summarized in the following way. Suppose that the platform kinematics and the end effector location of the manipulator mounted on the platform are described as a control system with outputs of the form

$$\dot{q}=G(q)u, \quad y=k(q,x),$$

where vectors q, x, y represent, respectively, the platform posture, the manipulator joint position, and the end effector position and orientation. After a time differentiation of the output function

$$\dot{y}=\frac{\partial k}{\partial q}(q,x)\dot{q}+\frac{\partial k}{\partial x}(q,x)\dot{x}=\left[\frac{\partial k}{\partial q}(q,x)G(q) \quad \frac{\partial k}{\partial x}(q,x)\right]\begin{pmatrix}u\\ \dot{x}\end{pmatrix}$$

we obtain a Jacobian

$$J(q,x)=\left[\frac{\partial k}{\partial q}(q,x)G(q) \quad \frac{\partial k}{\partial x}(q,x)\right],$$

and arrive at a sort of manipulability matrix

$$M(q,x)=\frac{\partial k}{\partial x}(q,x)\left(\frac{\partial k}{\partial x}\right)^{T}(q,x)$$

$$+\frac{\partial k}{\partial q}(q,x)G(q)G^{T}(q)\left(\frac{\partial k}{\partial q}\right)^{T}(q,x). \qquad (1)$$

that may serve as a starting point for defining performance measures in a way analogous as for stationary manipulators.[10,12,14] However, it turns out that a necessary condition to pursue this route of introducing performance measures is that the taskspace dimension (the number of components in y vector) must not exceed the number of platform controls plus the number of degrees of freedom of the aboard manipulator (otherwise all configurations (q,x) would be singular). Such a requirement will not be satisfied, e.g. when the mobile manipulator consists of the platform exclusively or when the manipulator mounted on the platform has not enough degrees of freedom (e.g. it has been partially immobilized). To be more specific, let us consider a simple device that might be called a wheeled jack, consisting of a unicycle-type mobile platform equipped with a 1 degree of freedom jack able to move up and down in the vertical direction. A straightforward analysis of the kinematics of wheeled jack shows that the matrix (1) is always singular, therefore corresponding performance measures become void. On the other hand, with suitable control the wheeled jack can reach any point in R^3 (at least above the platform), so its performance measures (manipulability or dexterity) should be non-trivial. It is easily checked that according to the endogenous space approach they are indeed so.

In an attempt to remove this disadvantage of the traditional approach and to establish dexterity concepts that would be well defined in all cases when a mobile manipulator remains controllable we proposed in reference [18] a control-centered approach to kinematic modeling of mobile manipulators. This approach takes as the fundamental a concept of endogenous configuration $(u(\cdot),x)$, composed of a control function of the platform and a joint position of the manipulator. Within such a framework, the pair (q,x) may be called an exogenous configuration, as it is a manifestation of the underlying endogenous configuration at a certain time instant. Detailed account of this approach was made in references [18–21]. It has been demonstrated that within control theoretic approach a kinematic theory of mobile manipulators can be reconstructed in full compliance with the existing theory of stationary manipulators.

The present paper belongs entirely to the endogenous configuration space approach. Its objective is to introduce performance measures for mobile manipulators in a way alternative to the traditional, and to examine usability of these measures. We shall concentrate on kinematic performance measures called kinematic dexterity. Our local dexterity measure corresponds to the manipulability concept of stationary manipulators. According to this measure an endogenous configuration of a mobile manipulator is regarded as dexterous, if the motion abilities of the end effector, obtained by varying this configuration slightly, are high. Since the endogenous configuration incorporates a control function of the platform as well as a joint position of the manipulator on board, by using configurations close to a dexterous configuration, the end effector can reach effectively all directions in the taskspace. As for stationary manipulators, dexterity is always non-negative; at singular configurations it falls down to zero. Our global dexterity exploits the L_2-norm of the local dexterity over a prescribed region of endogenous configuration space. Depending on a representation of control functions of the platform (we use truncated Fourier series), one can construct a collection of local and global dexterity measures. In the sequel these measures will be applied to the configuration optimization and to the optimal design of an exemplary mobile manipulator consisting of a platform equipped with two independently driven wheels and an RTR (Rotation-Translation-Rotation) manipulator. It should be noted that results obtained within control theoretic approach specialize in a natural way to the limit cases of a mobile manipulator composed of a platform only, and of a mobile manipulator reduced to a stationary manipulator (e.g. with the platform held up). In the first case we arrive at the concept of mobility introduced in reference [22], while the second case returns the classic manipulability. Preliminary results included in this paper have been presented in reference [23].

The paper is composed as follows. Section 2 introduces basic concepts of endogenous configuration, kinematics, analytic Jacobian, and Jacobian equation. Section 3 defines dexterity ellipsoids and dexterity measures. Section 4 presents applications of local and global dexterity measures to configuration and geometry optimization of exemplary mobile manipulator. The paper is concluded with Section 5. As far as possible, the exposition emphasizes similarities between stationary and mobile manipulators.

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2. BASIC CONCEPTS

In this section we collect definitions of basic kinematic concepts for mobile manipulators. As mentioned above, our presentation will be double-track, i.e. definitions for mobile manipulators will be preceded by their well known analogues for stationary manipulators.

2.1. Kinematics

2.1.1. Stationary manipulator For a stationary manipulator with n degrees of freedom and r-dimensional taskspace (end effector positions and orientations) the coordinate representation of the (forward) kinematics is an analytic map

$$k:\mathbb{R}^n \longrightarrow \mathbb{R}^r,\ \ y=k(x). \tag{2}$$

from jointspace coordinates into taskspace coordinates. The variable $x\in\mathbb{R}^n$ is often called a configuration of the stationary manipulator. The configuration plays the role of a controlled variable in the kinematics.

2.1.2. Mobile manipulator. A mobile manipulator consists of a non-holonomic mobile platform and a stationary manipulator fixed to the platform. Let $q\in\mathbb{R}^p$ denote a generalized position (a posture) of the platform. Suppose that the platform motion is subject to $l\leq p$ independent velocity constraints in the form of Pfaff

$$A(q)\dot{q}=0, \tag{3}$$

where $A(q)$ is an $l\times p$ full rank matrix with analytic entries. Given (3), the platform kinematics can be represented as a driftless control system[24]

$$\dot{q}=\sum_{i=1}^{m} g_i(q)u_i=G(q)u, \tag{4}$$

where $g_1(q),\ldots,g_m(q)$ span the null space $KerA(q)$, and $m=p-l$. Let $x\in\mathbb{R}^n$ be the joint position vector of the stationary manipulator mounted on the platform, and let $y\in\mathbb{R}^r$ denote end effector position and orientation coordinates computed with respect to an external coordinate frame. The space $\mathbb{R}^r$ will be referred to as the taskspace of the mobile manipulator. Actual task coordinates depend on the platform posture q and the joint position of the manipulator, i.e.

$$y=k(q,x). \tag{5}$$

The control system (4) with the output map (5) provides a control system representation of kinematics of the mobile manipulator. It should be observed that this representation is driven by control functions $u(\cdot)$ of the platform and joint positions x of the manipulator. Pairs $(u(\cdot),x)$ constitute the endogenous configuration space X of the mobile manipulator. Admissible control functions $u(\cdot)$ are assumed Lebesgue square integrable on a fixed time interval [0,T], whereas x are constant over $[0,T]$, so the endogenous configuration space

$$X=L_m^2[0,T]\times\mathbb{R}^n \tag{6}$$

becomes a Hilbert space with the inner product

$$\langle(u_1(\cdot),x_1),(u_2(\cdot),x_2)\rangle=\int_0^T u_1^T(t)u_2(t)dt+x_1^Tx_2 \tag{7}$$

and the induced norm

$$\|(u(\cdot),x)\|^2=\int_0^T u^T(t)u(t)dt+x^Tx. \tag{8}$$

Suppose that for every initial posture q_0 of the platform and every $u(\cdot)\in L_m^2[0,T]$ the trajectory

$$q(t)=\varphi_{q_0,T}(u(\cdot)) \tag{9}$$

of (4) exists for every $t\in[0,T]$. Then the output reachability map at time T

$$K_{q_0,T}:X\longrightarrow\mathbb{R}^r,\ \ y=K_{q_0,T}(u(\cdot),x)=k(\varphi_{q_0,T}(u(\cdot)),x) \tag{10}$$

of the system (4), (5) will be referred to as the instantaneous kinematics of the mobile manipulator.[18] The kinematics (10) determine the end effector position and orientation at time T on condition that the platform initialized at q_0 is driven by $u(\cdot)$ and that manipulator joints assume the position x.

2.2. Analytic Jacobian

2.2.1. Stationary manipulator Given the kinematics (2), we define the analytic Jacobian of the stationary manipulator at the configuration $x\in\mathbb{R}^n$ as a linear map

$$J(x):\mathbb{R}^n\longrightarrow\mathbb{R}^r$$

between tangent spaces to the jointspace at x (identified with $\mathbb{R}^n$) and to the taskspace at $k(x)$ (identified with $\mathbb{R}^r$), defined by the differentiation

$$\eta=J(x)\mathrm{v}=\frac{d}{d\alpha}\Big|_{\alpha=0}k(x+\alpha\mathrm{v}). \tag{11}$$

The Jacobian $J(x)$ indicates how infinitesimal changes of the configuration x translate into infinitesimal end effector motions in a vicinity of $k(x)$.

2.2.2. Mobile manipulator. For the kinematics (10) the analytic Jacobian at configuration $(u(\cdot),x)\in X$ determines a linear transformation

$$J_{q_0,T}(u(\cdot),x):X\longrightarrow\mathbb{R}^r$$

of the tangent space to the configuration space at $(u(\cdot),x)$ (identified with the ambient space X) into the tangent space

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to the taskspace at $K_{q_0,T}(u(\cdot),x)$ (identical with $\mathbb{R}^r$), defined through the differentiation

$$\eta = J_{q_0,T}(u(\cdot),x)\mathrm{v}(\cdot),\mathrm{w}) = \frac{d}{d\alpha}\Big|_{\alpha=0} K_{q_0,T}(u(\cdot) + \alpha\mathrm{v}(\cdot),x+\alpha\mathrm{w}). \tag{12}$$

The result of this differentiation is suitably described in terms of the variational system

$$\dot{\xi} = A(t)\xi + B(t)\mathrm{v}, \quad \eta = C(t,x)\xi + D(t,x)\mathrm{w}, \tag{13}$$

associated with (4), (5),[18,19] where

$$A(t) = \frac{\partial(G(q(t))u(t))}{\partial q}, \quad B(t) = G(q(t)),$$

$$C(t,x) = \frac{\partial k(q(t),x)}{\partial q}, \quad D(t,x) = \frac{\partial k(q(t),x)}{\partial x}, \tag{14}$$

and $q(t)$ denotes the trajectory (9) corresponding to $u(\cdot)$. Given matrices (14) we deduce from (12) the following form of the analytic Jacobian[18]

$$\eta = J_{q_0,T}(u(\cdot),x)(\mathrm{v}(\cdot),\mathrm{w}) = C(T,x)\int_0^T \Phi(T,t)B(t)\mathrm{v}(t)dt + D(T,x)\mathrm{w}, \tag{15}$$

where the fundamental matrix $\Phi(t,s)$ satisfies the differential equation

$$\frac{\partial}{\partial t}\Phi(t,s) = A(t)\Phi(t,s), \quad \Phi(s,s) = I_p. \tag{16}$$

2.3. Jacobian equation

2.3.1. Stationary manipulator Suppose that $k(x)$ represents the kinematics of a redundant manipulator whose configuration space and taskspace dimensions satisfy the inequality $n \geq r$. Choose a configuration $x \in \mathbb{R}^n$, and for a fixed $\eta \in \mathbb{R}^r$ consider a Jacobian equation

$$\eta = J(x)\mathrm{v}, \tag{17}$$

that should be solved for $\mathrm{v} \in \mathbb{R}^n$. We recall that the configuration x is called regular, if the analytic Jacobian has rank r, i.e. the Jacobian equation is solvable for every η. Otherwise x is singular. Furthermore, if x is regular then (17) can be solved by the least squares method that amounts to minimizing the squared norm

$$\min \frac{1}{2}\mathrm{v}^T\mathrm{v}$$

under the equality constraint (17). This requirement leads to the notion of the (right) Jacobian pseudoinverse

$$J^{\#}(x) : \mathbb{R}^r \to \mathbb{R}^n, \quad J^{\#}(x) = J^T(x)M^{-1}(x), \tag{18}$$

where $M(x) = J(x)J^T(x)$ is known as the manipulability matrix, such that

$$\mathrm{v} = J^{\#}(x)\eta \tag{19}$$

solves (17). It is easily seen that the configuration x is regular, if and only if rank $M(x) = r$. The Jacobian pseudoinverse yields the following iterative inverse kinematics algorithm

$$\dot{x} = -\gamma J^{\#}(x)(k(x) - y_d) \tag{20}$$

where y_d is a desirable taskspace vector, and $\gamma > 0$.

2.3.2. Mobile manipulator. For the instantaneous kinematics $K_{q_0,T}(u(\cdot),x)$ the Jacobian equation takes the form

$$\eta = J_{q_0,T}(u(\cdot),x)(\mathrm{v}(\cdot),\mathrm{w}). \tag{21}$$

An endogenous configuration $(u(\cdot),x) \in X$ will be called regular, if the analytic Jacobian $J_{q_0,T}(u(\cdot),x)$ is surjective or, equivalently, the Jacobian equation is solvable with respect to $(\mathrm{v}(\cdot),\mathrm{w})$ for any η. Observe that from a control theoretic viewpoint at regular configurations the variational system (13) is output controllable. Endogenous configurations that are not regular are referred to as singular.[18,19] At regular configurations a solution to the Jacobian equation can be sought by minimizing the squared norm

$$\min \frac{1}{2}\left(\int_0^T \mathrm{v}^T(t)\mathrm{v}(t)dt + \mathrm{w}^T\mathrm{w}\right)$$

under the constraint (21). An application of the Lagrange multipliers technique results in introducing a concept of the (right) Jacobian pseudoinverse[19]

$$J^{\#}_{q_0,T}(u(\cdot),x) : \mathbb{R}^r \to X,$$

defined as

$$(J^{\#}_{q_0,T}(u(\cdot),x)\eta)(t) = \begin{bmatrix} B^T(t)\Phi^T(T,t)C^T(T,x) \\ D^T(T,x) \end{bmatrix} \mathcal{D}^{-1}_{q_0,T}(u(\cdot),x)\eta, \tag{22}$$

where the matrix

$$\mathcal{D}_{q_0,T}(u(\cdot),x) = C(T,x)\mathcal{M}_{q_0,T}(u(\cdot))C^T(T,x) + D(T,x)D^T(T,x) \tag{23}$$

is called the dexterity matrix of the mobile manipulator.[18] It follows that the endogenous configuration

$$\begin{pmatrix} \mathrm{v}(t) \\ \mathrm{w} \end{pmatrix} = (J^{\#}_{q_0,T}(u(\cdot),x)\eta)(t), \tag{24}$$

solves the Jacobian equation. In control theoretic terminology the dexterity matrix is identified as the output reachability Gramian of the variational system (13). It includes the mobility matrix of the platform.[22]

$$\mathcal{M}_{q_0,T}(u(\cdot)) = \int_0^T \Phi(T,t)B(t)B^T(t)\Phi^T(T,t)dt. \tag{25}$$

3. DEXTERITY

Relying on the concepts of the analytic Jacobian and of the Jacobian pseudoinverse we shall introduce a kinematic

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dexterity ellipsoid as well as a local and a global kinematic dexterity measure for mobile manipulators. As in the previous section, mobile manipulators are considered by analogy with stationary manipulators.

3.1. Dexterity ellipsoid

3.1.1. Stationary manipulator Suppose that $x \in \mathbb{R}^n$ denotes a regular configuration of a stationary manipulator with kinematics $k(x)$. We recall that analytic Jacobian $J(x)$ at this configuration transforms tangent vectors to the jointspace at x into tangent vectors to the taskspace at $k(x)$. Let

$$S(x) = \{ \mathrm{v} \in \mathbb{R}^n \,|\, \|\mathrm{v}\|^2 = \mathrm{v}^T \mathrm{v} = 1 \} \tag{26}$$

be the unit sphere in the tangent space at x to the jointspace. The image of this sphere by the analytic Jacobian

$$E(x) = J(x)S(x) = \{ \eta \in \mathbb{R}^r \,|\, \eta^T M^{-1}(x) \eta = 1 \}, \tag{27}$$

computed using the Jacobian pseudoinverse (19), is called the manipulability ellipsoid at the configuration x. It is easily seen that the maximal and minimal half-axes of $E(x)$ are respectively equal to $\sqrt{\bar{\lambda}_{M(x)}}$ and $\sqrt{\underline{\lambda}_{M(x)}}$, where $\bar{\lambda}_{M(x)}$, $\underline{\lambda}_{M(x)}$, denote the maximal and the minimal eigenvalue of the manipulability matrix $M(x)$. Obviously, at singular configurations the manipulability ellipsoid collapses.

3.1.2. Mobile manipulator. Similarly to the stationary case, let $(u(\,\cdot\,), x) \in X$ denote a regular endogenous configuration of a mobile manipulator whose instantaneous kinematics $K_{q_0,T}(u(\,\cdot\,), x)$ are defined by (10). The analytic Jacobian $J_{q_0,T}(u(\,\cdot\,), x)$ transforms the tangent space at $(u(\,\cdot\,), x)$ to the configuration space into the tangent space to the taskspace at $K_{q_0,T}(u(\,\cdot\,), x)$. We choose the unit sphere

$$S_{q_0,T}(u(\,\cdot\,), x) = \{ (\mathrm{v}(\,\cdot\,), \mathrm{w}) \in X \,|\, \|(\mathrm{v}(\,\cdot\,), \mathrm{w})\|^2 = 1 \}, \tag{28}$$

and compute its image by the analytic Jacobian employing the Jacobian pseudoinverse (22)

$$\begin{aligned} E_{q_0,T}(u(\,\cdot\,), x) &= J_{q_0,T}(u(\,\cdot\,), x) S_{q_0,T}(u(\,\cdot\,), x) \\ &= \{ \eta \in \mathbb{R}^r \,|\, \eta^T \mathcal{D}_{q_0,T}^{-1}(u(\,\cdot\,), x) \eta = 1 \}. \end{aligned} \tag{29}$$

The object $E_{q_0,T}(u(\,\cdot\,), x)$ lives in the tangent space at $K_{q_0,T}(u(\,\cdot\,), x)$ to the taskspace, and is called the kinematic dexterity ellipsoid of the mobile manipulator at configuration $(u(\,\cdot\,), x)$.[18] The matrix $\mathcal{D}_{q_0,T}(u(\,\cdot\,), x)$ is the kinematic dexterity matrix defined by (23). It is straightforward to observe that the maximal and minimal half-axes of the dexterity ellipsoid equal, respectively, square roots of the maximal and the minimal eigenvalue of the dexterity matrix $\mathcal{D}_{q_0,T}(u(\,\cdot\,), x)$, i.e. $\sqrt{\bar{\lambda}_{\mathcal{D}_{q_0,T}}(u(\,\cdot\,), x)}$ and $\sqrt{\underline{\lambda}_{\mathcal{D}_{q_0,T}}(u(\,\cdot\,), x)}$. At singular endogenous configurations the kinematic dexterity ellipsoid flattens.

3.2. Dexterity measures

3.2.1. Stationary manipulator A volume measure of the manipulability ellipsoid for a stationary manipulator,

$$m(x) = \sqrt{\det M(x)} = \sqrt{\prod_{i=1}^{r} \lambda_{iM(x)}}, \tag{30}$$

defines the well known manipulability of the configuration x[2]. Since $m(x)$ refers to a particular configuration, it is often referred to as a local manipulability. By definition, manipulability of singular configurations is zero. Let $X \subset \mathbb{R}^n$ denote a compact region of the jointspace. The mean value over X of the squared local manipulability

$$\mathrm{m}^2(k(\,\cdot\,)) = \frac{1}{vol(X)} \int_X m^2(x) dx. \tag{31}$$

characterizes a global manipulability $\mathrm{m}(k(\,\cdot\,))$ of the kinematics. It is easily seen that the global manipulability is proportional to L_2-norm of the local manipulability.

3.2.2. Mobile manipulator. Proceeding in the same way as for stationary manipulators we define a local kinematic dexterity of the mobile manipulator at the configuration $(u(\,\cdot\,), x) \in X$ as[18,23]

$$\mathrm{d}_{q_0,T}(u(\,\cdot\,), x) = \sqrt{\det \mathcal{D}_{q_0,T}(u(\,\cdot\,), x)} = \sqrt{\prod_{i=1}^{r} \lambda_{i\,\mathcal{D}_{q_0,T}(u(\,\cdot\,), x)}}. \tag{32}$$

From a control theoretic perspective, the kinematic dexterity (32) assesses a degree of output controllability of the variational system (13).

A special instance of the kinematic dexterity, appearing when the output map (5) assumes the form

$$y = k(q, x) = q, \tag{33}$$

(e.g. when there is no manipulator mounted on the platform), so when the dexterity matrix coincides with the mobility matrix (25), will be referred to as the local kinematic mobility of the platform. Observe that the local kinematic mobility depends on the control function exclusively, i.e.

$$\begin{aligned} \mathrm{m}_{q_0,T}(u(\,\cdot\,)) &= \sqrt{\det \mathcal{M}_{q_0,T}(u(\,\cdot\,))} \\ &= \sqrt{\prod_{i=1}^{n} \lambda_{i,\mathcal{M}_{q_0,T}} X_u M_{q_0,T(u(\,\cdot\,))}}. \end{aligned} \tag{34}$$

Also, if the mobile manipulator has a platform that is unable to move ($u(\,\cdot\,) \equiv 0$, $q(t) = q_0$, $y = k(q, x) = k(x)$), we deduce an equality of dexterity and manipulability matrices

$$\mathcal{D}_{q_0,T}(u(\,\cdot\,), x) = M(x), \tag{35}$$

resulting in the identity of the kinematic dexterity (32) and the manipulability (30).

In order to effectively compute the local dexterity of a mobile manipulator we need a finite parameterization of platform control functions. This may be accomplished by choosing a truncated Fourier series representation, of the form

$$u(\lambda, t) = \lambda_0 + \sum_{j=1}^{k} \lambda_{2j-1} \sin j\omega t + \lambda_{2j} \cos j\omega t, \tag{36}$$

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for a given k, where $\omega=\frac{2\pi}{T}$, $\lambda=(\lambda_0, \lambda_1, \ldots, \lambda_{2k})$, $\lambda_j=(\lambda_{1j}, \ldots, \lambda_{mj})^T, j=0, 1, \ldots, 2k$. Elements of (36) will be called harmonics; we say that (36) contains harmonics up to order k. Given a finite parameterization of controls of order k, we replace the endogenous configuration space X by its finite-dimensional counterpart $\tilde{X}^k=\mathbb{R}^{(2k+1)m+n}$. A replacement of $(u(\cdot),x)\in X$ by $(\lambda,x)\in\tilde{X}^k$ yields a finitely parameterized kinematic dexterity matrix $\tilde{\mathcal{D}}^{\,k}_{q_0,T}(\lambda,x)$, and a corresponding local kinematic dexterity $\tilde{\mathrm{d}}^k_{q_0,T}(\lambda,x)$.

Now, letting $X^k\subset\tilde{X}^k$, denote a compact region of the finitely parameterized endogenous configuration space, we compute the mean value of the squared local kinematic dexterity

$$(\tilde{\mathrm{d}}^k_{q_0,T})^2\,(K_{q_0,T})=\frac{1}{vol(X^k)}\int_{X^k}(\tilde{\mathrm{d}}^k_{q_0,T})^2(\lambda,x)d(\lambda,x) \qquad (37)$$

and define $\tilde{\mathrm{d}}^k_{q_0,T}(K_{q_0,T})$ as a global kinematic dexterity of the mobile manipulator. As in the case of stationary manipulators, the global kinematic dexterity is proportional to L_2-norm of the local dexterity.

4. OPTIMAL DESIGN

Local and global kinematic dexterity measures introduced in the previous section can be exploited in order to determine optimal configurations or optimal geometries of mobile manipulators. Below we shall illustrate this possibility using a mobile manipulator consisting of a 2-wheel platform with independently driven wheels, endowed with an RTR manipulator fixed to the platform.[23] The mobile manipulator has been portrayed in Figure 1 that explains the meaning of all variables and parameters characterizing the kinematics of the mobile manipulator. In agreement with a general philosophy of our presentation, we shall begin with optimizing configuration and geometry of the stationary manipulator mounted on the platform.

4.1. Stationary manipulator

The kinematics of RTR manipulator shown in Figure 1, expressed in local Cartesian coordinates, are represented by the map

$$y=k(x)=((l_2+l_3\cos x_3)\cos x_1,\ (l_2+l_3\cos x_3)\sin x_1,\ x_2+l_3\sin x_3), \qquad (38)$$

where $x=(x_1, x_2, x_3)\in\mathbb{R}^3$. Singular configurations of this manipulator are discovered at $x_3=0,\pi, \arccos\left(-\frac{l_2}{l_3}\right)$. It turns out that the squared manipulability of the configuration x amounts to

$$m^2(x)=l_3^2\sin^2x_3(l_2+l_3\cos x_3)^2, \qquad (39)$$

so the maximal manipulability will be reached at configurations

$$x_3=\arccos\left(\frac{-l_2\mp\sqrt{l_2^2+8l_3^2}}{4l_3}\right).$$

Now, let us choose $X=[0,2\pi]\times[0,x_{2max}]\times[0,2\pi]\subset\mathbb{R}^3$, and consider the squared global manipulability

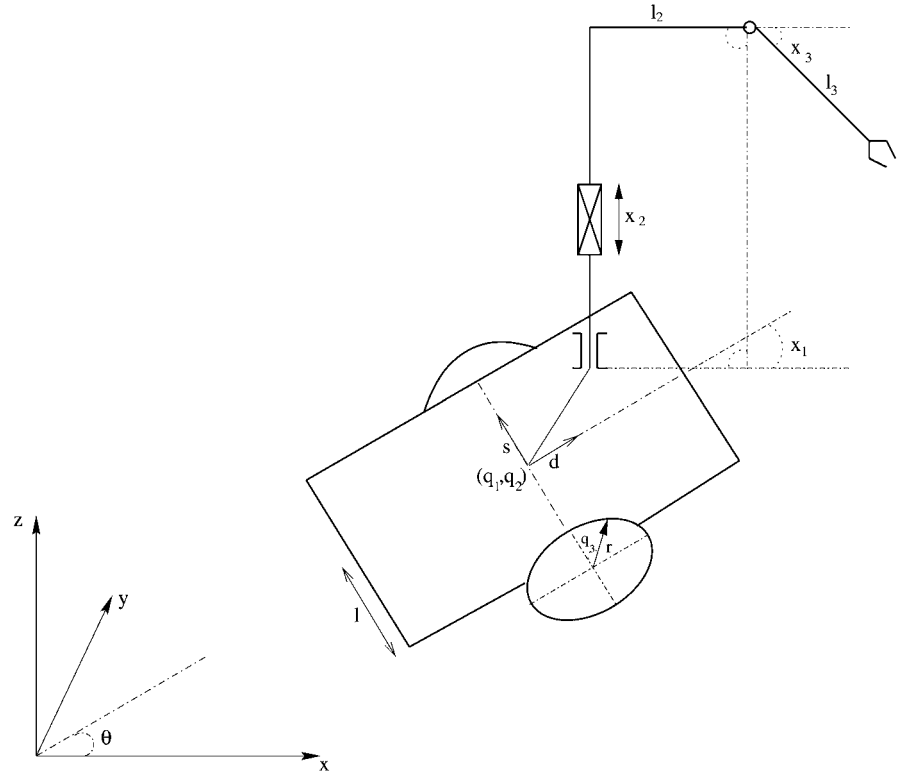


Fig. 1. Schematic of the mobile manipulator.

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$$\mathrm{m}^2(k(\cdot))=\frac{1}{vol(X)}\int_X m^2(x)dx=\frac{1}{8}l_3^2(4l_2^2+l_3^2). \qquad (40)$$

It is easily computed that under conditions $l_2+l_3=L$ (L − fixed) and $l_2, l_3\geq 0$, the maximal global manipulability is achieved for $l_2=0$, $l_3=L$.

4.2. Mobile manipulator

Assuming non-slipping motion of the platform wheels, the control system representation of kinematics of the mobile manipulator from Figure 1 takes the following form[23]

$$\begin{cases} \dot{q}_1=(u_1+u_2)l\cos\dfrac{r}{2l}(q_3-q_4) \\ \dot{q}_2=(u_1+u_2)l\sin\dfrac{r}{2l}(q_3-q_4) \\ \dot{q}_3=\dfrac{2l}{r}u_1 \\ \dot{q}_4=\dfrac{2l}{r}u_2, \end{cases} \qquad (41)$$

$$y=k(q,x)=\begin{pmatrix} q_1+d\cos\theta-s\sin\theta+(l_2+l_3\cos x_3)\cos(\theta+x_1) \\ q_2+d\sin\theta+s\cos\theta+(l_2+l_3\cos x_3)\sin(\theta+x_1) \\ x_2+l_3\sin x_3 \end{pmatrix}$$

where q_1, q_2 are position coordinates of the center of wheel axle, q_3, q_4 are rotation angles of the wheels, and $\theta=\dfrac{r}{2l}(q_3-q_4)$ denotes orientation angle of the platform. Controls u_1, u_2 driving the system (41) determine angular velocities of the wheels.

For fixed values of geometric parameters $l=d=l_2=1$, $l_3=2$, $r=0.5$, $s=0$, initial posture of the platform $q_0=(0,0,\dfrac{\pi}{4},\dfrac{\pi}{4})$, and control time horizon $T=\pi$, we have chosen control functions in the form (36) with either $k=0$ (constant controls, $\tilde{X}^k=\mathbb{R}^5$), $k=1$ (first order harmonics, $\tilde{X}^k=\mathbb{R}^9$) or $k=2$ (second order harmonics, $\tilde{X}^k=\mathbb{R}^{13}$), and computed optimal configurations of the mobile manipulator providing the maximal kinematic dexterity $(\tilde{\mathrm{d}}^k_{q_0,T})^2(\lambda,x)$ (see Section 3.2.2). Constraints imposed on admissible configurations are the following: $x_1,\ x_3\in[0,2\pi]$, $x_2\in[0,x_{2max}]$, $|\lambda_{ij}|\leq 3$, $i=1,2$, $j=0,1,2,3,4$. Numerical values of optimal (λ,x) are collected in Table I. It turns out that the maximal dexterity is achieved by a pair of (symmetric) configurations. In neither case the height (x_2) of the end effector above the platform has affected the dexterity.

Figures 2–7 display taskspace manifestations of optimal configurations of the mobile manipulator for $k=0$, 1, 2 (i.e. the final positioning of the mobile manipulator as well as paths of the platform and of the end effector) along with dexterity ellipsoids corresponding to these configurations.

Table I. Optimal configurations of the mobile manipulator.

Constraints	Optimal (λ, x)	Optimal (λ, x)
$k=0$	$x_1=5.69$	$x_1=0.59$
$0\leq x_1\leq 2\pi$,	$x_3=0.00$	$x_3=0.00$
$0\leq x_3\leq 2\pi$,	$\lambda_{10}=2.39$	$\lambda_{10}=3.00$
$-3\leq\lambda_{10}\leq 3$,	$\lambda_{20}=3.00$	$\lambda_{20}=2.39$
$-3\leq\lambda_{20}\leq 3$.	$\tilde{\mathrm{d}}^0_{q_0,T}(\lambda,x)=460.63$	$\tilde{\mathrm{d}}^0_{q_0,T}(\lambda,x)=460.63$
$k=1$	$x_1=5.93$	$x_1=0.36$
$0\leq x_1\leq 2\pi$,	$x_3=0.00$	$x_3=0.00$
$0\leq x_3\leq 2\pi$,	$\lambda_{10}=3.000$	$\lambda_{10}=3.00$
$-3\leq\lambda_{10}\leq 3$,	$\lambda_{11}=-1.38$	$\lambda_{11}=-3.00$
$-3\leq\lambda_{11}\leq 3$,	$\lambda_{12}=-2.39$	$\lambda_{12}=-3.00$
$-3\leq\lambda_{12}\leq 3$,	$\lambda_{20}=3.00$	$\lambda_{20}=3.00$
$-3\leq\lambda_{20}\leq 3$,	$\lambda_{21}=-3.00$	$\lambda_{21}=-1.38$
$-3\leq\lambda_{21}\leq 3$,	$\lambda_{22}=-3.00$	$\lambda_{22}=-2.39$
$-3\leq\lambda_{22}\leq 3$.	$\tilde{\mathrm{d}}^1_{q_0,T}(\lambda,x)=669.89$	$\tilde{\mathrm{d}}^1_{q_0,T}(\lambda,x)=669.89$
$k=2$	$x_1=0.19$	$x_1=6.09$
$0\leq x_1\leq 2\pi$,	$x_3=0.00$	$x_3=0.00$
$0\leq x_3\leq 2\pi$,	$\lambda_{10}=3.00$	$\lambda_{10}=3.00$
$-3\leq\lambda_{10}\leq 3$,	$\lambda_{11}=-1.10$	$\lambda_{11}=-3.00$
$-3\leq\lambda_{11}\leq 3$,	$\lambda_{12}=-3.00$	$\lambda_{12}=-3.00$
$-3\leq\lambda_{12}\leq 3$,	$\lambda_{13}=-3.00$	$\lambda_{13}=-3.00$
$-3\leq\lambda_{13}\leq 3$,	$\lambda_{14}=1.63$	$\lambda_{14}=3.00$
$-3\leq\lambda_{14}\leq 3$,	$\lambda_{20}=3.00$	$\lambda_{20}=3.00$
$-3\leq\lambda_{20}\leq 3$,	$\lambda_{21}=-3.00$	$\lambda_{21}=-1.10$
$-3\leq\lambda_{21}\leq 3$,	$\lambda_{22}=-3.00$	$\lambda_{22}=-3.00$
$-3\leq\lambda_{22}\leq 3$,	$\lambda_{23}=-3.00$	$\lambda_{23}=-3.00$
$-3\leq\lambda_{23}\leq 3$,	$\lambda_{24}=3.00$	$\lambda_{24}=1.63$
$-3\leq\lambda_{24}\leq 3$.	$\tilde{\mathrm{d}}^2_{q_0,T}(\lambda,x)=954.76$	$\tilde{\mathrm{d}}^2_{q_0,T}(\lambda,x)=954.76$

As the next step, we shall examine the following optimization problem of geometric parameters of the mobile manipulator (41) (an optimal design problem): find a mounting point coordinates (d,s) of the manipulator on the platform and link lengths (l_2,l_3) of this manipulator such that the global dexterity (37) achieves its maximal value. This problem has been solved under various constraints on decision variables (d,s,l_2,l_3), first order harmonics in the

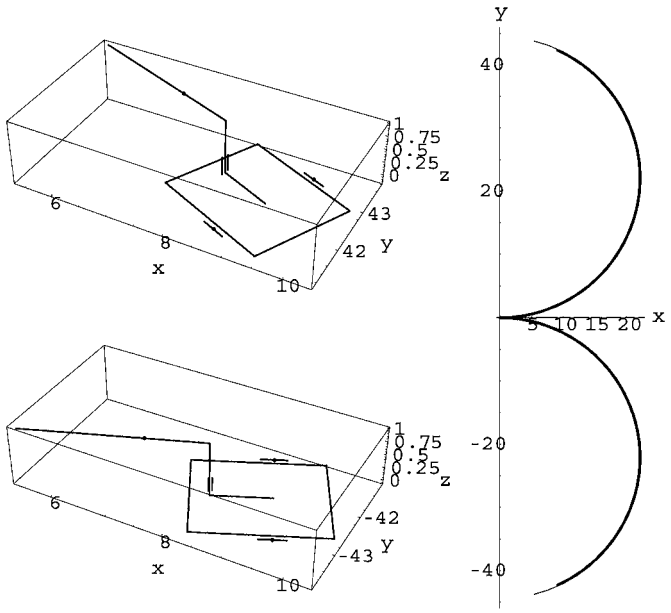


Fig. 2. Taskspace manifestations of optimal configurations for $k=0$ (bold line – the platform path).

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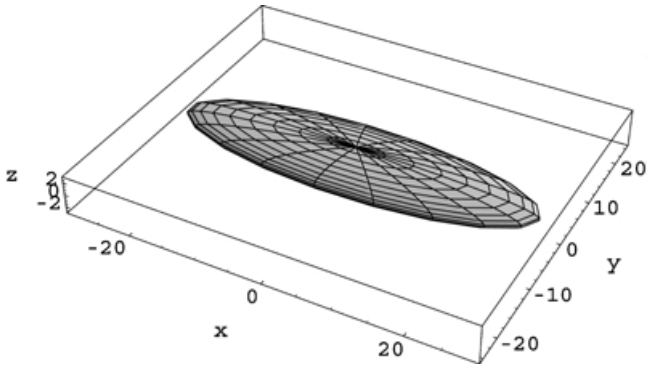


Fig. 3. Optimal dexterity ellipsoid for $k=0$.

control $(k=1)$, $|\lambda_{ij}| \leq 1$, $i=1,2$, $j=0,1$, and fixed $l=1, r=0.5$, fixed initial platform posture $q_0=(0,0,\frac{\pi}{4}, \frac{\pi}{4})$, and $T=\pi$. Results are shown in Table II. It follows that optimal mounting points of the manipulator lie always at a corner of the platform, most distant from its center.

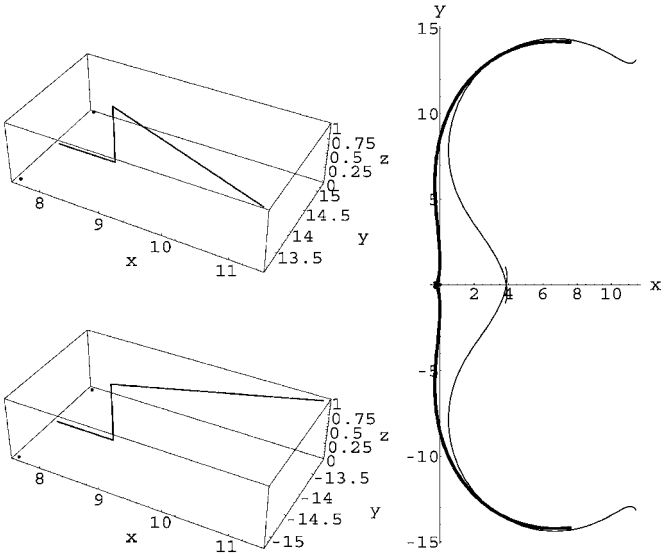


Fig. 4. Taskspace manifestations of optimal configurations for $k=1$ (bold line – the platform path).

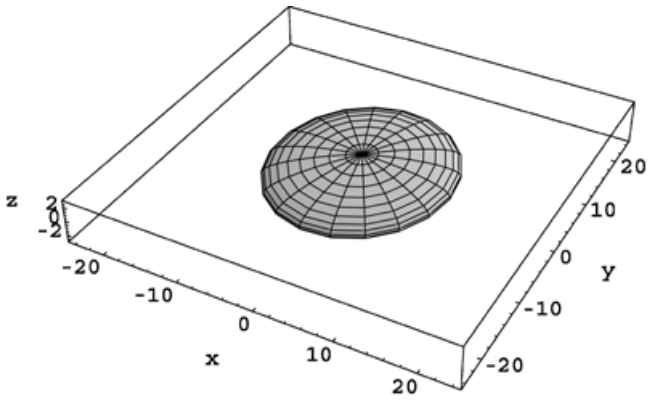


Fig. 5. Optimal dexterity ellipsoid for $k=1$.

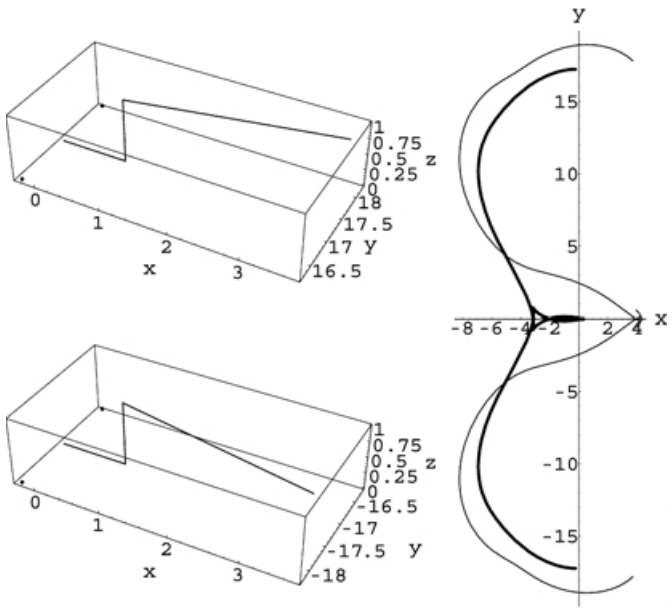


Fig. 6. Taskspace manifestations of optimal configurations for $k=2$ (bold line – the platform path).

5. CONCLUSION

We have proposed local and global kinematic dexterity measures for mobile manipulators, based on a control theoretic representation of kinematics and a control theoretic concept of the endogenous configuration. Our measures refer to a propagation of fine changes of endogenous configurations into fine motions of the end effector in the taskspace. This feature has been reflected by the size of the dexterity ellipsoid. Our measures generalize in a natural way the classic concept of manipulability for stationary manipulators. We have also demonstrated that the approach presented in this paper specifies immediately either to the kinematics of mobile platform, resulting in the concept of mobility or to the manipulability of the stationary manipulator alone. In particular we are in a position to adequately evaluate the performance of the wheeled jack

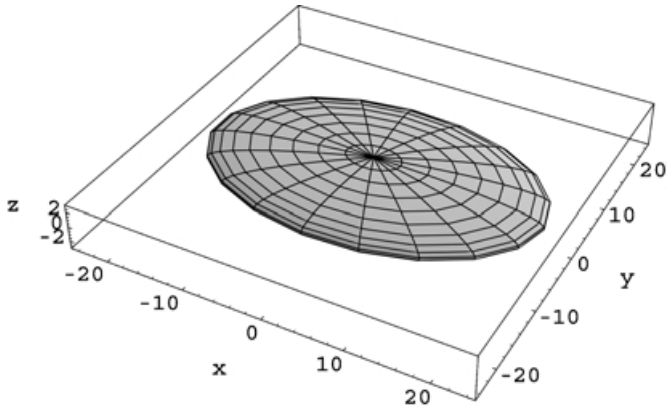


Fig. 7. ptimal dexterity ellipsoid for $k=2$.

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Table II. Optimal geometry of the mobile manipulator.

Constraints	Optimal geometry	Global dexterity
$l_2+l_3=8$, $l_2\geq 0$, $l_3\geq 0$, $-1\leq s\leq 1$, $-3\leq d\leq 3$.	$l_2=0$, $l_3=8$, $(s,d)=\{(\pm 1, \pm 3)\}$	$(\tilde{\mathrm{d}}^1_{q_0,T})^2\,(K_{q_0T})=5.84*10^4$
$l_2+l_3=8$, $l_2\geq 0$, $l_3\geq 0$, $-1\leq s\leq 1$, $-6\leq d\leq 3$.	$l_2=0$, $l_3=8$, $(s,d)=\{(\pm 1, -6)\}$	$(\tilde{\mathrm{d}}^1_{q_0,T})^2\,(K_{q_0T})=1.32*10^5$
$l_2+l_3=8$, $l_2\geq 0$, $l_3\geq 0$, $-0.5\leq s\leq 1$, $-3\leq d\leq 3$.	$l_2=0$, $l_3=8$, $(s,d)=\{(1, \pm 3)\}$	$(\tilde{\mathrm{d}}^1_{q_0,T})^2\,(K_{q_0T})=5.84*10^4$
$0\leq l_2\leq 5$, $0\leq l_3\leq 3$, $-1\leq s\leq 1$, $-3\leq d\leq 3$.	$l_2=5$, $l_3=3$, $(s,d)=\{(\pm 1, \pm 3)\}$	$(\tilde{\mathrm{d}}^1_{q_0,T})^2\,(K_{q_0T})=8.95*10^3$

described in Introduction. By its very nature the performance measures defined in the endogenous configuration space are qualitatively different from the measures used so far in the literature. To make this difference more transparent, let us write down the dexterity matrix (23) in the form

$$\mathcal{D}_{q_0,T}(u(\cdot),x)=\frac{\partial k}{\partial x}(q(T),x)\left(\frac{\partial k}{\partial x}\right)^T(q(T),x)$$

$$+\frac{\partial k}{\partial q}(q(T),x)\int_0^T\Phi(T,t)B(t)B^T(t)\Phi^T(T,t)dt\left(\frac{\partial k}{\partial q}\right)^T(q(T),x) \tag{42}$$

and compare it with the matrix (1). In both formulas the first term refers to the aboard manipulator, while the second one describes properties of the mobile platform. Now, taking into account expressions (14), (16) and knowing that the platform trajectory $q(t)=\varphi_{t,q_0}(u(\cdot))$, one may notice that matrices (1) and (42) coincide provided that the mobile platform is immobilized, i.e. when $u(\cdot)$ vanishes identically. When the platform is in motion, the "endogenous" dexterity matrix depends on the whole history of platform motion over the time horizon T, reflects local controllability properties of the mobile manipulator in the taskspace, and so differs substantially from the traditional concept. Furthermore, as we have already said, for non-holonomic mobile platforms without any manipulator aboard or devices like the wheeled jack the traditional performance measures are not applicable.

Using the local kinematic dexterity we have studied the problem of detecting optimal configurations of a mobile manipulator. The global dexterity has provided us with a tool of optimizing the geometry of a mobile manipulator, aiding in this way design procedures of mobile manipulators. Computations accomplished for an exemplary mobile manipulator proved that new dexterity measures can be used efficiently, and that design recommendations based on these measures are rational.

Obviously, both local and global kinematic dexterities depend on the control time horizon T. In order to free the dexterities from this dependence one might introduce corresponding averaged measures over admissible time horizons $0\leq T\leq T^*$. In this way we obtain the time-averaged local kinematic dexterity

$$\mathrm{d}_{q_0}(u(\cdot),x)=\frac{1}{T^*}\int_0^{T^*}\mathrm{d}_{q_0,T}(u(\cdot),x)dT,$$

and the time-averaged global kinematic dexterity

$$\tilde{\mathrm{d}}^k_{q_0}(K_{q_0,T})=\frac{1}{T^*}\int_0^{T^*}\tilde{\mathrm{d}}^k_{q_0,T}(K_{q_0,T})dT.$$

Some preliminary computations suggest that design recommendations based on both time-dependent and time-averaged global dexterities remain the same.

As may be expected, by exploiting further the analogy between mobile and stationary manipulators, one can define other local kinematic performance measures for mobile manipulators, referring not only to the size, but also to the shape of the dexterity ellipsoid, like the minimal eigenvalue of the kinematic dexterity matrix or the condition number of an endogenous configuration. The latter index leads naturally to a concept of isotropic endogenous configurations. It is clear that all these measures can be given their global counterparts. Also, a generalization of the endogenous space approach to encompass a dynamic dexterity looks encouraging; the first step toward this objective has already been made.[25]

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