A Representation Theorem for Absolute Confirmation

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Proposals for rigorously explicating the concept of confirmation in probabilistic terms abound. To foster discussions on the formal properties of the proposed measures, recent years have seen the upshot of a number of representation theorems that uniquely determine a confirmation measure (up to ordinal equivalence) based on a number of desiderata. However, the results that have been presented so far focus exclusively on the concept of incremental confirmation. This leaves open the question whether similar results can be obtained for the concept of absolute confirmation. This article closes the gap by providing a representation theorem for absolute confirmation measures.

1. Introduction. Gathering evidence to scrutinize the adequacy of scientific hypotheses is the hallmark of scientific enterprise. If the evidence speaks 'in favor of' a hypothesis, the hypothesis is said to be *confirmed* by the evidence; otherwise it is *disconfirmed* (or the evidence is irrelevant to the hypothesis). Much of twentieth-century philosophy of science has been concerned with the project of explicating this idea of confirmation with the aim of replacing the pretheoretic concept by a formally precise and fruitful scientific concept. In this regard, some scholars have focused on the qualitative aspects of confirmation, aiming for an explication of what it means for pieces of evidence to confirm a scientific hypothesis without going into the details of the comparative or quantitative aspects. Two of the adequacy conditions for any such account that have been discussed in this context, most notably by Hempel (1945), are as follows:

> (**SCC**) (*Special consequence condition*) If evidence e confirms some hypothesis h, and if another hypothesis h' is deductively entailed by h, then e confirms h'.

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(**CCC**) (*Converse consequence condition*) If evidence e confirms some hypothesis h, and if h is deductively entailed by another hypothesis h', then e confirms h'.

If h is a conjunctive hypothesis, then from the fact that some piece of evidence e confirms h we can infer via SCC that it also confirms h's conjuncts; this seems indeed reasonable. Similarly, CCC "has had an undying popularity" (Glymour 1980, 30). However, as Hempel has shown, together with the following seemingly innocuous principle they lead to the absurd consequence that each piece of evidence confirms every hypothesis whatsoever. The required addition is the following principle relating logical entailment and confirmation:

(**EC**) If evidence e logically entails the hypothesis h, then e confirms h.

As a consequence, Hempel rejected the converse consequence condition.[1] In his discussion of Hempel's adequacy conditions, Carnap (1962) argues that these are based on different and even conflicting concepts of confirmation that he labels confirmation$_1$ and confirmation$_2$.[2] Keeping an eye especially on the quantitative side of confirmation, Carnap presents a formal reconstruction of these concepts by means of probability theory. In this sense, e confirms$_1$ h if and only if (iff) $p(h \mid e) > p(h \mid \overline{e})$ (or equivalently $p(h \mid e) > p(h)$). This is the concept of *incremental confirmation*, or confirmation as *increase in firmness*. But, e confirms$_2$ h iff $p(h \mid e) > p(\overline{h} \mid e)$.[3] This concept of confirmation is nowadays called *absolute confirmation*, or confirmation as *firmness*. Incremental confirmation violates Hempel's special consequence condition that is satisfied by absolute confirmation. But, Hempel also discusses a condition that makes sense only for incremental confirmation:

1. It can be shown that EC and CCC alone suffice for deriving the paradoxical consequence (cf. Le Morvan 1999; Moretti 2003). Note, however, that the well-known account of HD confirmation satisfies CCC. As a result, it is faced by the so-called tacking paradoxes that highlight the untoward consequence that if h is confirmed by e, then so is $h \wedge x$ for any arbitrary x. For refined accounts to HD confirmation that mitigate this (and related) paradoxes, see Schurz (1991), Gemes (1993), and, more recently, Sprenger (2013).

2. For a critical assessment of Carnap's rendition of Hempel and an alternative interpretation, see Huber (2007).

3. Sometimes a slightly more general characterization of confirmation$_1$ is given along the following lines: e confirms$_1$ h iff $p(h \mid e) > t$ for some threshold t such that $1/2 \leq t < 1$. Here we stick to the above characterization, which can be obtained for $t = 1/2$. Given that our focus in the remainder of the article will be on the quantitative concept of confirmation, this choice is inconsequential.

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(**CE**) (*Converse entailment condition*) If evidence e is logically entailed by the hypothesis h, then e confirms h.

It is easily shown that this condition holds true only for confirmation$_1$. Most analyses in Hempel's wake have focused on the incremental concept, and it is not an exaggeration to say that nowadays Bayesian confirmation theory is dominated by incremental confirmation-based approaches. Particularly, there is a long-standing debate on quantitative measures of the degree of incremental confirmation that e provides for h (among others, see Fitelson 1999; Eells and Fitelson 2002; Crupi, Tentori, and Gonzalez 2007; Zalabardo 2009; Festa 2012; Brössel 2013). Nonetheless, the concept of absolute confirmation plays an important role both in scientific reasoning and in other domains of philosophical inquiry like rational decision making or inductive logic. What is more, in Bayesian confirmation theory recent discussions on the *(in)transitivity* of the confirmation relation and on various conditions that establish transitivity have incorporated a number of different probabilistic explications of confirmation, among which are both absolute and incremental confirmation (and combinations thereof; see Douven 2011; Roche 2012a, 2012b, 2016; Roche and Shogenji 2014). Finally, probabilistic measures based on the absolute concept of confirmation have proved useful for the purpose of measuring *coherence*, often with a better performance in test cases than their counterparts based on the concept of incremental confirmation (see Schippers 2014; Koscholke 2016; Koscholke and Jekel 2016).

Now we turn to the quantitative counterparts of both these concepts of confirmation. Let $c(h,e)$ denote the degree of confirmation that e provides for h, then the relation between the qualitative concept of (incremental) confirmation and the quantitative measures of the degree of (incremental) confirmation is given by the following qualitative criterion of incremental confirmation (where θ denotes the measure-relative threshold separating confirmation and disconfirmation):

(**QIC**) $c(h,e) \gtreqless \theta$ iff $p(h \mid e) \gtreqless p(h \mid \bar{e})$.

The counterpart of QIC for the concept of absolute confirmation reads as follows:

(**QAC**) $c(h,e) \gtreqless \theta$ iff $p(h \mid e) \gtreqless p(\bar{h} \mid e)$.

Strong results have been shown by, among others, Milne (1996), Huber (2008), Crupi, Chater, and Tentori (2013), and Crupi and Tentori (2013, 2015). All of these papers contain representation theorems that determine (up to ordinal equivalence) a class of measures as the best explication of (incremental) confirmation based on a small number of adequacy constraints.

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This raises the question whether similar results can be achieved for the absolute concept of confirmation.

2. A New Representation Theorem. To set the stage, let $\mathcal{L}$ be a finite propositional language and denote the set of *contingent* formulas by $\mathcal{L}_c$. That is, $\mathcal{L}_c$ is the subset of formulas of $\mathcal{L}$ that are neither tautologous nor contradictory. Furthermore, let **P** denote the set of *regular* probability measures, so that for each $p \in \mathbf{P}$ and each $e, h \in L_c$, $0 < p(h), p(e) < 1$. Formally, a confirmation measure is a four-place function $c : L_c^3 \times \mathbf{P} \rightarrow \mathbb{R}$ such that $c(h, e, b, p)$ denotes the degree of confirmation that $e \in L_c$ provides for $h \in L_c$ against the background of $b \in L_c$ under probability distribution $p \in \mathbf{P}$.[4] As usual, in what follows we omit reference to the background knowledge; furthermore, we will write $c_p(h, e)$ instead of $c(h, e, p)$. The following desideratum assures that all explications of confirmation use probability theory as their main building block (cf. Crupi and Tentori 2013, 2015).

> ($\mathbf{AC}_0$) (*Formality*) There exists a function g such that for any $h, e \in L_c$ and any $p \in \mathbf{P}$, $c_p(h, e) = g[p(h \wedge e), p(h), p(e)]$.

Because $p(h \wedge e)$, $p(h)$, and $p(e)$ completely determine the probability distribution over the algebra generated by h and e, AC_0 simply states that the function value of $c(h, e)$ is exclusively based on this probabilistic information. The next constraint to be considered has gained widespread acceptance among Bayesian confirmation theorists. For occurrences, see Fitelson (2006, 506) and Hájek and Joyce (2008, 122), where it is called the 'law of conditional probability'.[5]

> ($\mathbf{AC}_1$) (*Final probability incrementality*) For any $h, e, e' \in L_c$ and any $p \in \mathbf{P}$, $c_p(h, e) \gtrless c_p(h, e')$ iff $p(h \mid e) \gtrless p(h \mid e')$.

According to AC_1, the degree of confirmation that two different pieces of evidence e and e' provide for a hypothesis h covaries with h's conditional probability. One-half of the condition (from right to left) is crucially involved in Bayesian analyses of confirmation paradoxes like the ravens paradox (cf. Horwich 1982, 54–63); Eells and Fitelson even surmise that "it is not an exaggeration to say that most Bayesian confirmation theorists [would

4. These constraints on formulas and probability distributions assure mathematical definiteness.

5. Note furthermore that AC_0 and AC_1 form the common core of a couple of recent representation theorems for incremental confirmation measures (Crupi et al. 2013; Crupi and Tentori 2013, 2015).

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accept it] as a desideratum for Bayesian measures of confirmation" (2000, 670).[6]

Now we turn to the third adequacy condition that is needed in order to set up the desired representation theorem. According to this condition, the degree of confirmation that e provides for h is the same as the degree of confirmation it provides for the conjunctive hypothesis $h \wedge e$. This condition obviously makes no sense for incremental confirmation measures: although it should be the case that if e confirms h then so it confirms $h \wedge e$, the degrees of confirmation it bestows on these propositions might reasonably be held to be different. Remember that we compare the increment of probability between h's unconditional and conditional probability on the one hand; and on the other hand, the increment between $p(h \wedge e \mid e)$ and $p(h \wedge e)$. However, as we turn to absolute confirmation no such discrimination is appropriate. For the question we are up to answer by absolute confirmation is whether h is plausible in the light of e. But, if we assign h a certain degree of plausibility in the light of e, then we should reasonably assign $h \wedge e$ the same degree of plausibility. Hence, we add the following adequacy condition:[7]

> ($\mathbf{AC}_2$) (*Plateau condition*) For any $e, h \in L_c$ and any $p \in \mathbf{P}$, $c_p(h \wedge e, e) = c_p(h, e)$.

Given this condition we can establish the following representation theorem:

> **Theorem 1.** AC_0, AC_1, and AC_2 hold iff there is a strictly increasing function f such that $c_p(h, e) = f[p(h \mid e)]$.
>
> *Proof.* See the appendix.

The plateau condition is violated by all incremental confirmation measures and so is the following generalization of logical entailment. All confirmation measures based on probability theory are extensional in the sense that the degree of confirmation is not affected if we replace either h or e by a logically equivalent proposition. But absolute confirmation measures also satisfy the following condition on local equivalence replacement (cf. Törnebohm 1966; Levi 1986).[8]

6. However, note that AC_1 is also not entirely innocent as it is not satisfied by all Bayesian confirmation measures. Exceptions are, e.g., the measures due to Nozick (1981) and Christensen (1999). See also the discussions on the problem of measure sensitivity by Fitelson (1999) and Brössel (2013).

7. For occurrences, see Törnebohm (1966) and Crupi and Tentori (2015).

8. In Huber (2007) this condition is called the 'Levi principle'. Here, we stick to the terminology in Crupi (2014).

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(**AC**$_3$) (*Local equivalence*) For any $e, h, h' \in L_c$ such that h and h' are logically equivalent given e and any $p \in \mathbf{P}$, $c_p(h, e) = c_p(h', e)$.

According to AC_3, we can also replace h by another hypothesis h' that is not logically equivalent to h stricto sensu but only modulo e, that is, only when e is taken for granted. What is more, it is not only true that this condition is satisfied by the absolute concept of confirmation, but AC_2 can be replaced in the above representation theorem for AC_3. These two conditions are, in fact, equivalent given the basic condition AC_0. That is,

Observation 1. Given AC_0, a measure c satisfies AC_2 iff it satisfies AC_3.

Proof. If h and h' are logically equivalent modulo e, then $h \wedge e$ and $h' \wedge e$ are plainly logically equivalent. Thus, if c satisfies AC_0 and AC_2, we get $c_p(h, e) = c_p(h \wedge e, e) = c_p(h' \wedge e, e) = c_p(h', e)$ for all $p \in \mathbf{P}$. But, $h \wedge e$ and h are logically equivalent given e. Hence, if c satisfies AC_0 and AC_3, then $c_p(h, e) = c_p(h \wedge e, e)$ for all $p \in \mathbf{P}$. QED

Thus, AC_2 and AC_3 are the characteristic conditions to keep apart absolute and incremental concepts of confirmation. Hence, none of the incremental confirmation measures satisfies either of these two conditions.

3. Conclusion. This article bridges a gap in recent discussions on confirmation measures. While so far only incremental confirmation measures have been axiomatized by means of representation theorems, this article provides a corresponding theorem for the absolute concept of confirmation.

Appendix

Proof of Theorem 1

Right-to-Left Implication.

(**AC**$_0$) Assuming that there exists a strictly increasing function f such that $c_p(h, e, p) = f[p(h \mid e)]$, AC_0 is trivially satisfied.

(**AC**$_1$) Let $h, e, e' \in \mathcal{L}_c$ and a regular probability p be given; assume furthermore that there is a strictly increasing function f such that $c_p(h, e, p) = f[p(h \mid e)]$, then $f[p(h \mid e)] \gtrless f[p(h \mid e')]$ iff $p(h \mid e) \gtrless p(h \mid e')$. By ordinal equivalence, if there exists a strictly increasing function f such that $c_p(h, e, p) = f[p(h, e)]$, then AC_1 follows.

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($\mathbf{AC}_2$) Let $h, e \in \mathcal{L}_c$ and a regular probability p be given, then by definition it follows that $p(h \mid e) = p(h \wedge e \mid e)$. By ordinal equivalence, if there is a strictly increasing function f such that $c_p(h, e, p) = f[p(h \mid e)]$, then AC_2 follows.

Left-to-Right Implication. Note that $p(h \wedge e) = p(h \mid e) \cdot p(e)$. Consequently, by AC_0, there exists a function g such that, for any $h, e \in \mathcal{L}_c$ and any regular probability p, $c_p(h, e, p) = g[p(h \mid e), p(h), p(e)]$. With no loss of generality, we assume probabilistic coherence and regularity by constraining the domain of g to include triples of values (x, y, z) such that the following conditions are jointly satisfied:

- $0 < y, z < 1$.
- $x \geq 0$, so that $p(h \mid e) = p(h \wedge e)/p(e) \geq 0$; hence, $p(h \wedge e) \geq 0$.
- $x \leq 1$, so that $p(h \mid e) = p(h \wedge e)/p(e) \leq 1$; hence, $p(h \wedge e) \leq p(e)$ and thus $p(\overline{h} \wedge e) \geq 0$.
- $x \leq y/z$, so that $xz/y = p(e \mid h) \leq 1$; hence, $p(h \wedge e) \leq p(h)$ and therefore $p(h \wedge \overline{e}) \geq 0$.
- $x \geq (y + z - 1)/z$, so that $xz = p(h \wedge e) \geq p(h) + p(e) - 1 = y + z - 1$. By the law of total probability we infer $p(h \wedge e) + p(h \wedge \overline{e}) + p(\overline{h} \wedge e) \leq 1$.

Thus, we posit $g : \{(x, y, z) \in [0, 1] \times (0, 1)^2 : (y + z - 1)/z \leq x \leq y/z\} \rightarrow \mathbb{R}$. The domain of g will be denoted by D_g. The following lemma will be useful for the subsequent proof steps.

Lemma 1. For any $(x, y, z_1), (x, y, z_2) \in D_g$ there exist $h, e, e' \in \mathcal{L}_c$ and a regular probability p such that $p(h \mid e) = p(h \mid e') = x, p(h) = y, p(e) = z_1$, and $p(e') = z_2$.

Proof. The equalities in lemma 1 are the result of the following scheme of probability assignments:

$$
\begin{array}{ll}
p(h \wedge e \wedge e') = \frac{(xz_1)(xz_2)}{y} & p(\overline{h} \wedge e \wedge e') = \frac{(1-x)^2 z_1 z_2}{(1-y)} \\
p(h \wedge e \wedge \overline{e'}) = (xz_1)\left[1 - \frac{(xz_2)}{y}\right] & p(\overline{h} \wedge e \wedge \overline{e'}) = (1-x)z_1\left[1 - \frac{(1-x)z_2}{(1-y)}\right] \\
p(h \wedge \overline{e} \wedge e') = \left[1 - \frac{(xz_1)}{y}\right](xz_2) & p(\overline{h} \wedge \overline{e} \wedge e') = \left[1 - \frac{(1-x)z_1}{(1-y)}\right](1-x)z_2 \\
p(h \wedge \overline{e} \wedge \overline{e'}) = \left[1 - \frac{(xz_1)}{y}\right]\left[1 - \frac{(xz_2)}{y}\right]y & p(\overline{h} \wedge \overline{e} \wedge \overline{e'}) = \left[1 - \frac{(1-x)z_1}{(1-y)}\right]\left[1 - \frac{(1-x)z_2}{(1-y)}\right](1-y).
\end{array}
$$

Suppose there exist $(x, y, z_1), (x, y, z_2) \in D_g$ such that $g(x, y, z_1) \neq g(x, y, z_2)$. Then, by lemma 1 and the definition of D_g, there exist $h, e, e' \in \mathcal{L}_c$ and a regular probability p such that $p(h \mid e) = p(h \mid e') = x$, $p(h) = y, p(e) = z_1$,

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and $p(e') = z_2$. Thus, there exist $h, e, e' \in \mathcal{L}_c$ and a regular probability p such that $c_p(h, e, p) = g(x, y, z_1) \neq g(x, y, z_2) = c_p(h, e', p)$ even though $p(h \mid e) = p(h \mid e')$, contradicting AC_1.

Conversely, AC_1 implies that for any $(x, y, z_1), (x, y, z_2) \in D_g g(x, y, z_1) = g(x, y, z_2)$. So, for AC_1 to hold, there must exist a function k such that for any $h, e, e' \in \mathcal{L}_c$ and any regular probability p, $c_p(h, e, p) = k[p(h \mid e), p(h)]$ and $k(x, y) = g(x, y, z)$. We thus posit $k : \{(x, y) \in [0, 1] \times (0, 1)\} \rightarrow \mathbb{R}$ and denote the domain of k by D_k.

Lemma 2. For any $x \in (0, 1], y_1, y_2 \in (0, 1)$, there exist $h, h', e \in \mathcal{L}_c$ and a regular probability p such that $p(h \mid e) = p(h' \mid e) = x$, $p(h) = y_1$, $p(h') = y_2$, and $p(h \wedge e) = p(h' \wedge e)$.

Proof. Let $z \in (0, 1)$ be given, so that $z \leq x, y_1, y_2, (1 - y_1)x/(1 - x), (1 - y2)x/(1 - x)$ (as the latter quantities must all be positive, z exists). The equalities in lemma 2 arise from the following scheme of probability assignments:[9]

Suppose there exist $(x, y_1), (x, y_2) \in D_k$ such that $k(x, y_1) \neq k(x, y_2)$. Then, by lemma 2 and the definition of D_k, there exist $h, h', e \in \mathcal{L}_c$ and a regular p such that $p(h \mid e) = p(h \wedge e \mid e) = p(h' \mid e) = p(h' \wedge e \mid e) = xp(h) = y_1$, $p(h') = y_2$, and $p(h \wedge e) = p(h' \wedge e) = z$, and

$$p(h \wedge h' \wedge e) = xz \qquad p(\overline{h} \wedge h' \wedge e) = (1 - x)z$$

$$p(h \wedge h' \wedge \overline{e}) = \tfrac{(y_1 - z)(y_2 - z)x}{(x - z)} \qquad p(\overline{h} \wedge h' \wedge \overline{e}) = \left[1 - \tfrac{(y_1 - z)x}{(x - z)}\right](y_2 - z)$$

$$p(h \wedge \overline{h'} \wedge e) = (1 - x)z \qquad p(\overline{h} \wedge \overline{h'} \wedge e) = \tfrac{(1 - x)^2 z}{x}$$

$$p(h \wedge \overline{h'} \wedge \overline{e}) = (y_1 - z)\left[1 - \tfrac{(y_2 - z)x}{x - z}\right] \qquad p(\overline{h} \wedge \overline{h'} \wedge \overline{e}) = \left[1 - \tfrac{(y - 1 - z)x}{x - z}\right]\left[1 - \tfrac{(y_2 - z)x}{x - z}\right]\left(\tfrac{x - z}{x}\right),$$

with $z \in (0, 1)$. Thus, there exist $h, h', e \in \mathcal{L}_c$ and a regular p such that either $c_p(h, e) = k(x, y_1) \neq k(x, z) = c_p(h \wedge e, e)$ or $c_p(h', e) = k(x, y_2) \neq k(x, z) = c_p(h' \wedge e, e)$, contradicting AC_2. Conversely, AC_2 implies that, for any $(x, y_1), (x, y_2) \in D_k$, $k(x, y_1) = k(x, y_2)$. So, for AC_2 to hold, there must exist a function m such that, for any $h, e \in \mathcal{L}_c$ and any regular probability p, $c_p(h, e) = m[p(h \mid e)]$ and $m(x) = k(x, y)$.

Note that the case $x = 0$ is missing in lemma 2. Consequently, the following lemma closes this gap.

Lemma 3. For $x = 0$ and any $y_1, y_2 \in (0, 1)$ there exist $h^*, h', e \in \mathcal{L}_c$, where $h^* = \overline{h}$ for some $h \in \mathcal{L}_c$, and a regular probability p such that $p(h^* \mid e) = p(h' \mid e) = x^*$, $p(h^*) = y_1$, $p(h') = y_2$, and $p(h^* \wedge e) = p(h' \wedge e)$.

9. I thank an anonymous reviewer for spotting a mistake in an earlier proof and also for providing me with a detailed advice for how to fix it.

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Proof. Analogous to the former proof for lemma 2.

Lemma 4. For any $x_1, x_2 \in [0, 1]$, there exist $h, e, e' \in \mathcal{L}_c$ and a regular probability p such that $p(h \mid e) = x_1$ and $p(h \mid e') = x_2$.

Proof. Let $y, z_1, z_2 \in (0, 1)$ be given, so that $z_1 \leq (1 - y)/(1 - x_1), y/x_1$ and $z_2 \leq (1 - y)/(1 - x_2), y/x_2$ (as the latter quantities must be positive [if defined], there will always be such z_i. Otherwise, if $x_1 = 1$ or $x_2 = 1$, $(1 - x_i)z_i$ will always be smaller than $(1 - y)$ for all y.) The equalities in lemma 4 arise from the following scheme of probability assignments:

Suppose there exist $x_1, x_2 \in D_m$ such that $x_1 > x_2$ and $m(x_1) \leq m(x_2)$. Then, by lemma 4 and the definition of D_m, there exist $h, e, e' \in \mathcal{L}_c$ and a regular probability p such that $p(h \mid e) = x_1$ and $p(h \mid e') = x_2$. Clearly, if the latter equalities hold, then $p(h \mid e) > p(h \mid e')$. Thus, there exist $h, e, e' \in \mathcal{L}_c$ and a regular probability p such that $c_p(h, e, p) = m(x_1) \leq m(x_2) = c_p(h, e', p)$ even if

$$p(h \wedge e \wedge e') = \tfrac{(x_1 z_1)(x_2 z_2)}{y} \qquad p(\overline{h} \wedge e \wedge e') = \tfrac{(1-x_1)z_1(1-x_2)z_2}{(1-y)}$$

$$p(h \wedge e \wedge \overline{e'}) = (x_1 z_1)\left[1 - \tfrac{(x_2 z_2)}{y}\right] \qquad p(\overline{h} \wedge e \wedge \overline{e'}) = (1 - x_1)z_1\left[1 - \tfrac{(1-x_2)z_2}{(1-y)}\right]$$

$$p(h \wedge \overline{e} \wedge e') = \left[1 - \tfrac{(x_1 z_1)}{y}\right](x_2 z_2) \qquad p(\overline{h} \wedge \overline{e} \wedge e') = \left[1 - \tfrac{(1-x_1)z_1}{(1-y)}\right](1 - x_2)z_2$$

$$p(h \wedge \overline{e} \wedge \overline{e'}) = \left[1 - \tfrac{(x_1 z_1)}{y}\right]\left[1 - \tfrac{(x_2 z_2)}{y}\right]y \qquad p(\overline{h} \wedge \overline{e} \wedge \overline{e'}) = \left[1 - \tfrac{(1-x_1)z_1}{(1-y)}\right]\left[1 - \tfrac{(1-x_2)z_2}{(1-y)}\right](1 - y).$$

Thus, $p(h \mid e) > p(h \mid e')$, contradicting AC_1. Conversely, AC_1 implies that for any $x_1, x_2 \in D_m$, if $x_1 > x_2$, then $m(x_1) > m(x_2)$. By a similar argument, AC_1 also implies that for any $x_1, x_2 \in D_m$, if $x_1 = x_2$, then $m(x_1) = m(x_2)$. So, for AC_1 to hold, it must be the case that for any $h, e \in \mathcal{L}_c$ and any regular p, $c_p(h, e, p) = m(p(h \mid e))$ and m is a strictly increasing function. QED

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