Improved sliding mode guidance law based on fuzzy variable coefficients strategy

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ABSTRACT

An adaptive sliding mode guidance law guiding the line-of-sight angular rate to converge to zero was highly appraised. However, compared with the conventional proportional navigation law, adaptive sliding mode guidance law leads to large acceleration demand of the interceptor before the line-of-sight angular rate converging to zero, especially in a situation where the target has strong manoeuvrability as well as high velocity. In this paper, a strategy making the coefficients of the guidance law vary according to a fuzzy rule is proposed. Smaller guidance coefficients are selected at the beginning of the terminal guidance. Therefore, the guidance command is reduced and a smaller acceleration of the interceptor is incurred. As the coefficients grow to the fixed and desired values, the line-of-sight angular rate converges to zero rapidly, so that the convergence to the sliding surface is guaranteed. It is concluded that the fuzzy variable coefficients strategy is highly effective for tail-chase, head-on interception and head pursuit engagements as shown in the simulations.

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NOMENCLATURE

f	frequency
G_0	constants of the filter
n	maneuvering overload of interceptor
n_T	maneuvering overload of target
N	navigation ratio
k	fixed coefficient of guidance law
q	the line-of-sight (LOS) angle
$\dot{q}$	derivation of q with respect to time
$\dot{q}_s$	measurement value of $\dot{q}$
$\ddot{q}$	second-order derivation of q with respect to time
r	distance from interceptor to target
$\dot{r}$	derivation of r with respect to time
t,t_s,t_f	time, the initial time of guidance, the final time of guidance
u_q	acceleration of interceptor normal to LOS
V	vertical distance
V_T	velocity of target
w_q	acceleration of target along LOS
X	lateral distance
Y	velocity of interceptor
δ	tiny constant greater than zero
δt	short duration
δt_1, δt_2	short duration derived from strategy
ε	fixed coefficient of guidance law
η_T	heading angle of target
η	heading angle of interceptor
ρ	sliding surface of guidance law
$\dot{\rho}$	converging rate of LOS angular rate
σ_T	flight-path angle of target
σ	flight-path angle of interceptor
$\dot{\sigma}_T$	derivation of σ_T with respect to time
$\dot{\sigma}$	derivation of σ with respect to time
τ	time-to-go
χ	tiny constant
ω_c	frequency of the filter

1.0 INTRODUCTION

The conventional proportional navigation (PN) law has been fully developed and is easy to implement. However, the required acceleration of the interceptor is largely influenced by the maneuvering acceleration of the target[1]. Thus, the PN law nearly fails to satisfy the requirement of high accuracy against maneuvering targets, especially for the hypersonic maneuvering targets. In the early 1990s, the variable structure control was applied to design a guidance law, proving the validity of the new method[2]. Then, plenty of studies were conducted in an attempt to render further improvements. In recent years, the sliding mode guidance law has gained significant attention due to its impressive advantage of strong robustness to disturbance and ambient noise[3-6]. By using the fuzzy strategy or the adaptive rules, the chattering of the sliding mode guidance law is alleviated[7-9].

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Guiding the line-of-sight (LOS) angular rate to converge to zero is a recognised way to design a sliding mode guidance law and is effective in improving the guidance accuracy[10-12]. The guidance command can be analysed as a superposition of the navigation part and the variable structure part. The navigation part is similar to proportional navigation law with a coefficient (always named the navigation ratio) always set to be a value from two to six[1,7], while the variable structure part is the key aspect to improve the guidance accuracy. Adaptive rules are applied on the coefficient of the variable structure part to ensure the robustness of variable structure guidance law[7-8]. Thus, the sliding mode guidance law causes a greater guidance command than PN law due to introduce of the extra variable structure part, which is not expected to decrease the acceleration demand. Especially, an acceleration demand larger than the maximum acceleration allowed for the interceptor may be needed before the LOS angular rate converging to zero. This becomes even worse if the closing speed turns out to be hypersonic[7,13]. To solve the problem, based on the existing researches and results, a variable coefficients strategy is initiated.

Based on the experiences and expectations, fuzzy control is efficient to implement strategy that lack of the accurate mathematical model[14-15]. By adjusting the coefficient of the variable structure part according to the disturbance, the fuzzy controller plays an important role in alleviating the chattering of variable structure guidance system[8-9]. The fuzzy system is also used to improve autopilot dynamics, so that the robustness to the noises and uncertainties of nonlinear system is obtained[4].

In order to reduce the acceleration demand at the beginning of the terminal guidance, a strategy making the coefficients of the sliding mode guidance law vary according to a fuzzy rule is proposed in this paper. Smaller guidance coefficients are generated at the beginning of the terminal guidance. Therefore, the guidance command is reduced and a smaller acceleration demand is incurred. As the coefficients grow to the fixed and desired values rapidly, the convergence to the sliding surface is achieved. The variable coefficients strategy is implemented by a fuzzy controller, which is defined as a single-input and single-output system. To verify the effectiveness of the fuzzy variable coefficients strategy, the proposed guidance law is tested in the three types of engagements, including the tail-chase, the head-on interception and the head pursuit[16-17].

In the next section, the equations of motion are presented. Then, the adaptive sliding mode guidance law is introduced. The fuzzy variable coefficients strategy is initiated and implemented in Section 4. In Section 5, the results and analysis of the guidance laws in different engagements are provided, followed by the concluding remarks in the last section.

2.0 EQUATIONS OF MOTION

The kinematics model of interception is denoted as three-dimensional nonlinear differential Equations (18). To simplify the research model for convenience, longitudinal planar guidance law is studied as an example. The hypothesis used is as follows:

(i) The interceptor and the target are regarded as mass points.

(ii) The speeds of the interceptor and the target are uniform in the terminal guidance.

(iii) The coupling of latex demand is ignored in longitudinal guidance law design.

There are three types of engagement, including tail-chase, head-on interception and head pursuit[19]. Tail-chase engagement is the most familiar engagement because most interceptors have the velocity advantage to the targets. Head-on interception has been used frequently to intercept reentry missiles. However, it turns out to be the most difficult method to apply due to the high closing speed. Head pursuit is a new method proposed recently, providing another method to intercept high-speed targets[16]. Considering the two-dimensional engagement shown in Fig. 1,

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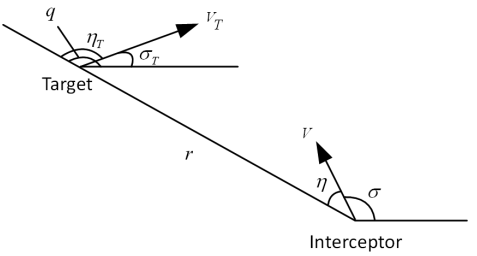


Figure 1. Geometry of interceptor-target engagement.

where r is the relative distance from the interceptor to the target, V_T and V are the velocity of the target and the interceptor respectively, q is the LOS angle, η and η_T are the heading angles of the interceptor and the target respectively, σ and σ_T are the flight-path angles of the interceptor and the target respectively.

The corresponding equations of relative motion are given as:

$$\left.\begin{aligned} \dot{r} &= V_T \operatorname{Cos}\eta_T - V \operatorname{Cos}\eta \\ r\dot{q} &= V \operatorname{Sin}\eta - V_T \operatorname{Sin}\eta_T \\ q &= \eta + \sigma \\ q &= \eta_T + \sigma_T \end{aligned}\right\} \quad \ldots (1)$$

Where $\dot{r}$ is the derivation of r with respect to time, which denotes the closing speed. $\dot{q}$ is the derivation of q with respect to time, which denotes the LOS angular rate.

Differentiating of Equation (1) with respect to time, yields[10]:

$$\ddot{q} = -\frac{2\dot{r}}{r}\dot{q} - \frac{1}{r}u_q + \frac{1}{r}w_q \quad \ldots (2)$$

$$\left.\begin{aligned} w_q &= -\dot{V}_T \operatorname{Sin}\eta_T + V_T\dot{\sigma}_T \operatorname{Cos}\eta_T \\ u_q &= -\dot{V} \operatorname{Sin}\eta + V\dot{\sigma} \operatorname{Cos}\eta \end{aligned}\right\} \quad \ldots (3)$$

where $\ddot{q}$ is the second-order derivation of q with respect to time. w_q is the acceleration of target along the LOS, and u_q is the acceleration of the interceptor normal to the LOS.

3.0 GUIDANCE LAW GUIDING LOS ANGULAR RATE TO CONVERGE TO ZERO

Considering guiding the LOS angular rate to converge to zero[10-12], the sliding surface ρ is given as:

$$\rho = r\dot{q} \quad \ldots (4)$$

A significant reaching law proposed in the literature, which indicates that the converging rate $\dot{\rho}$ is adjusted to the relative distance[10], is given as:

$$\dot{\rho} = -k\frac{|\dot{r}|}{r}\rho - \varepsilon \operatorname{sgn}(\rho) \quad \ldots (5)$$

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where k and ε are constants greater than zero, and $|\dot{r}|$ is the absolute value of ($\dot{r}$) in this article. Substituting Equation (2) and Equation (4) into Equation (5), with the acceleration of the target along LOS w_q ignored, the adaptive sliding mode guidance law (named ASMG in this article) is obtained as:

$$u_q = (k+1)|\dot{r}|\dot{q} + \varepsilon \operatorname{sgn}(\dot{q}) \quad \ldots (6)$$

where u_q represents the value of maneuvering acceleration of the interceptor. To be noted, the guidance laws discussed in this paper are obtained under the assumption that $\dot{r}$ is always negative, written as:

$$\dot{r} < 0 \quad \ldots (7)$$

The guidance law is proposed without consideration of autopilot dynamics, otherwise, $\varepsilon \operatorname{sgn}(\dot{q})$ becomes $\varepsilon\operatorname{sgn}(\dot{q})\operatorname{sgn}(f(\tau))$, where τ is time-to-go and ($f(\tau)$) is related to the rate of change in miss due to the latax demand.

The ASMG can be considered as the superposition of navigation part ($(k+1)|\dot{r}|\dot{q}$) and variable structure part ($\varepsilon\operatorname{sgn}(\dot{q})$). ($k+1$) is similar to the navigation ratio of the traditional PN law, which is usually created from 2 to 6. If the navigation ratio is set to be large, the larger acceleration demand may be incurred and the noisy effects of the disturbances will be enlarged; if the navigation ratio is set to be small, the PN law would have similar performances to that of the tracking guidance law[1]. So, k usually take a value from 1 to 5. The variable structure coefficient ε ought to take a value greater than the acceleration of the target according to the adaptive rules to ensure the convergence to the sliding surface[7]. Obviously, the ASMG causes a larger guidance command compared with the PN law due to introduce of the variable structure part. The acceleration demand will be too large if $\dot{r}$ turns out to be hypersonic, especially in a short period of time before $\dot{q}$ converging to the sliding surface.

If the velocity variation of the interceptor is ignored, the acceleration of the interceptor normal to LOS can be rewritten as:

$$u_q = V\dot{\sigma}\operatorname{Cos}\eta \quad \ldots (8)$$

The required overload n of the interceptor is equal to:

$$n = V\dot{\sigma}/g \quad \ldots (9)$$

Substituting Equation (8) into Equation (9), yields:

$$n = \frac{(k+1)|\dot{r}|\dot{q} + \varepsilon \operatorname{sgn}(\dot{q})}{g\operatorname{Cos}\eta} \quad \ldots (10)$$

Obviously, influenced parameters of required acceleration include k, $|\dot{r}|$, $\dot{q}$, ε and η. $|\dot{r}|$ denotes the closing speed, which largely depends on the target velocity, and it is not easy to be controlled. η is a random initial value dependant on the initial positions of the interceptor and the target. Moreover, the small variation allowed of η can slightly change the u_q value, so it is not a good adjustable parameter either. Thus, the guidance coefficients k and ε are the main factors adjusted to decrease the guidance command.

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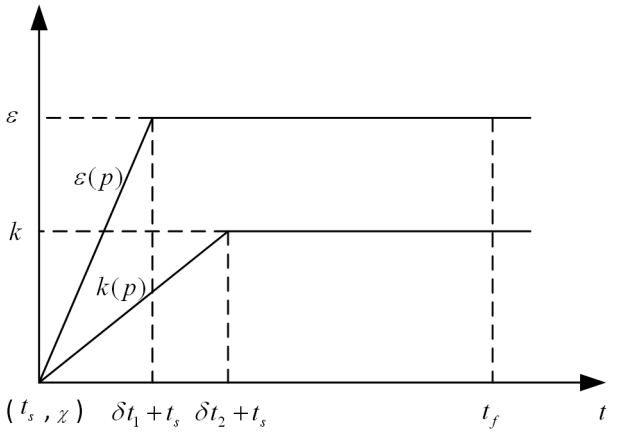


Figure 2. The variation of $k(p)$ and $\varepsilon(p)$.

4.0 FUZZY VARIABLE COEFFICIENTS STRATEGY

4.1 Variable coefficients strategy

Rather than the existing methods in the literature of selecting fixed values of k and ε[5-7], the variable coefficients strategy is proposed to achieve overload distribution idea, so that the acceleration demand at the beginning of the guidance will be reduced. Taking ASMG into account, the fuzzy variable coefficients adaptive sliding mode guidance law (named FASMG in this article) is given as:

$$u_q = [k(p)+1]|\dot{r}|\dot{q} + \varepsilon(p)\,\mathrm{sgn}(\dot{q}) \qquad \ldots (11)$$

where $k(p)$ and $\varepsilon(p)$ are functions with respect to the system parameters. The focus of this strategy is to adjust the coefficients at the beginning of the guidance to values smaller than the fixed values. After a short period of time, the coefficients grow to the fixed values to meet the system requirements. An available approach in this paper is to make $k(p)$ and $\varepsilon(p)$ linear-vary with guidance time until they grow to the fixed value. The variations of $k(p)$ and $\varepsilon(p)$ are shown in Fig. 2, and the functions of $k(p)$ and $\varepsilon(p)$ are defined as:

$$\varepsilon(p) = \begin{cases} \dfrac{\varepsilon-\chi}{\delta t_1}(t-t_s)+\chi & t \le \delta t_1 + t_s \\ \varepsilon & t > \delta t_1 + t_s \end{cases} \qquad \ldots (12)$$

$$k(p) = \begin{cases} \dfrac{k-\chi}{\delta t_2}(t-t_s)+\chi & t \le \delta t_2 + t_s \\ k & t > \delta t_2 + t_s \end{cases} \qquad \ldots (13)$$

δt_1 and δt_2 are short durations for the linear variation of $\varepsilon(p)$ and $k(p)$. t_s is the initial time of the terminal guidance, t_f is the final time, χ is a constant, and it is used to be selected as zero in the simulation process.

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The key to the variable coefficients strategy is how to obtain δt_1 and δt_2. If the values are set to be too small, the strategy will not decrease the acceleration efficiently. On the other hand, if large values are selected, the time for ($\dot{q}$) to converge to zero is significantly increased, or even worse, if values greater than the terminal guidance time ($t_f - t_s$) are selected, the convergence to the sliding surface may be damaged.

4.2 Preparation for the fuzzy controller

Before determining the proper values of δt_1 and δt_2, the whole guidance time ($t_f - t_s$) should be estimated. Because the terminal guidance time is short, the variation of the closing speed is minute, and there is no need to obtain the precise value. The finite time ($t_f - t_s$) can be estimated as:

$$t_f - t_s = -\frac{r(0)}{\dot{r}(0)} \quad \ldots (14)$$

where $r(0)$ and $\dot{r}(0)$ denote the approximate distance and the approximate closing speed when the terminal guidance begins respectively. Obviously, δt_1 and δt_2 should meet:

$$\left.\begin{aligned} 0 < \delta t_1 < t_f - t_s \\ 0 < \delta t_2 < t_f - t_s \end{aligned}\right\} \quad \ldots (15)$$

Simply, the way to derive δt_1 or δt_2 can be defined as a single-input and single-output system, where the input is the guidance time ($t_f - t_s$), and the output is $\delta t_{1,2}$. In fact, the selection of values of δt_1 and δt_2 largely depends on experiences and skills achieved from the ASMG.

As stated previously, the terminal guidance time ($t_f - t_s$) is essential to generate $\delta t_{1,2}$, meanwhile, the time for $\dot{q}$ to converge to zero is also tested here. The illustration of terminal guidance time and time for $\dot{q}$ to converge to zero are shown in Fig. 3. In the simulations of the ASMG, the target performs a sinusoidal trace with the fixed acceleration set to 100ms^{-1}. Briefly, the numerical details and results are presented in Table 1, where $\bullet(0)$ denotes the initial value of ($\bullet$) (more details and figures are provided in Section 5).

The results of the ASMG shows that ($t_f - t_s$) is almost not influenced by the variation of k or ε, while the time for $\dot{q}$ to converge to zero is increased when k or ε is decreased. Moreover, no matter in tail-chase, head-on interception or head pursuit, the time for $\dot{q}$ to converge to zero is very short compared with ($t_f - t_s$) as shown in Table 1. Thus, the short durations (δt_1 and δt_1) would perform a delay effect instead of a damage on the convergence to the sliding surface.

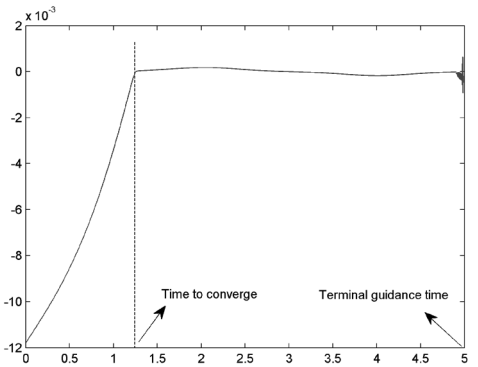


Figure 3. Illustration of the time to converge and the terminal guidance time.

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Table 1
Numerical details and results of the ASMG

Settings	k, ε	$t_f - t_s\,(s)$	Time to converge(s)
Tail-chase $V = 1020m/s$ $V_T = 272m/s$ $r(0) = 15000m$ $q(0) = 170\,\text{deg}$ $\sigma(0) = 0\,\text{deg}$ $\sigma_T(0) = 0\,\text{deg}$	$k = 2$ $\varepsilon = 150$	12.87	2.23
	$k = 2$ $\varepsilon = 100$	12.89	2.80
	$k = 1.2$ $\varepsilon = 150$	12.87	2.48
	$k = 1.2$ $\varepsilon = 100$	12.90	3.04
Head-on $V = 1360m/s$ $V_T = 1700m/s$ $r(0) = 15000m$ $q(0) = 170\,\text{deg}$ $\sigma(0) = 165\,\text{deg}$ $\sigma_T(0) = 0\,\text{deg}$	$k = 2$ $\varepsilon = 150$	4.99	1.24
	$k = 2$ $\varepsilon = 100$	4.99	1.50
	$k = 1.2$ $\varepsilon = 150$	4.99	1.29
	$k = 1.2$ $\varepsilon = 100$	4.99	1.65
Head pursuit $V = 1360m/s$ $V_T = 1700m/s$ $r(0) = 5000m$ $q(0) = 160\,\text{deg}$ $\sigma(0) = 10\,\text{deg}$ $\sigma_T(0) = 0\,\text{deg}$	$k = 2$ $\varepsilon = 150$	13.42	0.41
	$k = 2$ $\varepsilon = 100$	13.42	0.50
	$k = 1.2$ $\varepsilon = 150$	13.42	0.43
	$k = 1.2$ $\varepsilon = 100$	13.42	0.56

Further, according to the analysis, details and results of ASMG, the experiences on how to select δt_1 and δt_2 are concluded as follows:

(i) The converging rate of $\dot{q}$ varies inversely with both δt_1 or δt_2, meaning that the converging rate increases as the value δt_1 or δt_2 decreases.

(ii) In most engagements, the time for $\dot{q}$ to converge to zero of ASMG is less than four seconds, depending on the converging rate.

(iii) The values of δt_1 or δt_2 are selected according to $(t_f - t_s)$. The bigger the values of δt_1 or δt_2 are selected, the greater the delay effect that will be obtained; however, the more time would be needed for the convergence to the sliding surface.

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4.3 The fuzzy controller

According to the preparations and statements above, the fuzzy control settings can be defined. By multiplying the terminal guidance time $(t_f - t_s)$ with a quantisation factor, the fuzzy input variables defined as $(t_f - t_s)$* are obtained. Then, the linguistic input variables are defined as {ST MS ME ML LO}, where ST means short, and in an increasing order, LO means long. In this fuzzy strategy, $(t_f - t_s)$ is considered less than 18 seconds, and a fixed output value of five is selected if $(t_f - t_s)$ is greater than 18 seconds. For the convenience of defining the membership functions, the quantisation factor of input is selected as ⅓. Thus, $(t_f - t_s)$* ranges from 0 to 6. δt_1 and δt_2 are the output variables needed, and the fuzzy output variables are defined as δt_1* and δt_2*. The linguistic output variables are defined as {ST MS ME ML}, with the quantisation factor selected as 1.

The fuzzy strategy is implemented by Mamdani controller in Simulink/Matlab. The input variables are described by five levels (ST, MS, ME, ML and LO), while the output variables are described by four levels (ST, MS, ME and ML). In fact, the definition of the membership functions largely depends on the experiences and the expectations based on the ASMG. Here, more attention is paid on the engagements of guidance time ranging from 5 to 15 seconds, so the MS, ME and ML functions in fuzzy input ranges wider than the ST and the LO functions. Simply, triangular membership functions are used in this paper, and the parameters of input membership functions are set as: ST(0,0,1) MS(0,1,5,3), ME(2,3,5,5), ML(4,5,6), LO(5,6,6). Similarly, the MS and the ME functions in fuzzy output ranges wider than the ST and the ML functions, and the parameters of output membership functions are set as: ST(0,0,0,5), MS(0,2,5,3), ME(2,4,5), ML(4,5,5). The membership functions of the fuzzy input and the fuzzy output are shown in Fig. 4, and the fuzzy rules are defined as follows:

If $(t_f - t_s)$* is ST, δt_1* or δt_2* is ST;

If $(t_f - t_s)$* is MS, δt_1* or δt_2* is MS;

If $(t_f - t_s)$* is ME, δt_1* or δt_2* is ME,

If $(t_f - t_s)$* is ML, δt_1* or δt_2* is ME,

If $(t_f - t_s)$* is LO, δt_1* or δt_2* is ML.

According to the rules above, the relationship of fuzzy input and fuzzy output is obtained, as shown in Fig. 5.

Simply, assuming δt_1 and δt_2 to be equal to δt in this paper, gives:

$$\delta t_1 = \delta t_2 = \delta t \qquad \ldots (16)$$

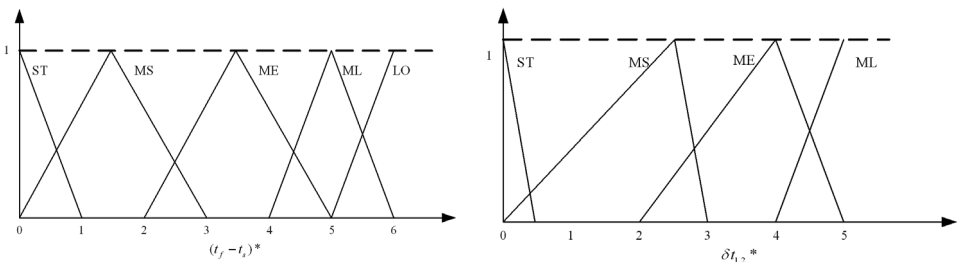


Figure 4. Membership functions of the fuzzy input and fuzzy output.

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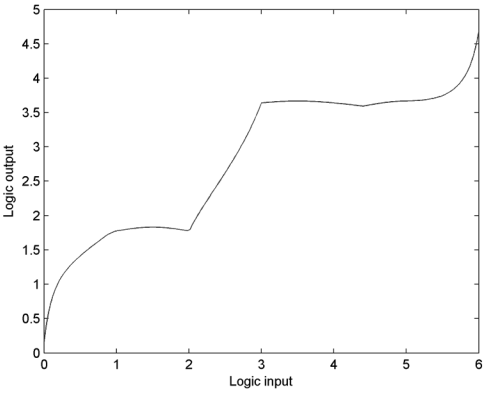


Figure 5. Relationship of fuzzy input and fuzzy output.

By dividing the fuzzy output $\delta t_{1,2}$* derived from the fuzzy strategy by the quantisation factor of the output, the values of δt_1 and δt_2 are available for Equation (12) and Equation (13), making the FASMG obtained as:

$$u_q = \begin{cases} [\frac{k-\chi}{\delta t}(t-t_s)+\chi+1]|\dot{r}|\dot{q}+[\frac{\varepsilon-\chi}{\delta t}(t-t_s)+\chi]\mathrm{sgn}(\dot{q}) & t \le \delta t + t_s \\ [k+1]|\dot{r}|\dot{q}+\varepsilon\,\mathrm{sgn}(\dot{q}) & t > \delta t + t_s \end{cases} \quad \ldots (17)$$

The chattering of the variable structure control system is a disadvantage to the engagement. Based on the estimation of the disturbances such as the acceleration of the target, both fuzzy controller and adaptive rules can alleviate the chattering. Simply, a continuous function given as $\dot{q}/(|\dot{q}| + \delta)$ is selected to replace $\mathrm{sgn}(\dot{q})$ to alleviate the chattering here, where δ is a tiny constant greater than zero[10].

Besides the acceleration of the target, another important disturbance is the noise of $\dot{q}$. The disturbances including the acceleration of the target and the noise of $\dot{q}$ will be tested in the simulations. The simplified measurements of $\dot{q}$ is shown in Fig. 6, where $\dot{q}$ is the input, G_0 and ω_c are constants of the first-order filter system, and $\dot{q}$ is the practical input of the guidance law.

4.4 Convergence to the sliding surface

Generally, a well developed guidance system includes the guidance law, the autopilot dynamics, the missile dynamics and the sensor dynamics, and each part of the system may influence the guidance loop stability. In this paper, the main attention is paid on how to improve the existed

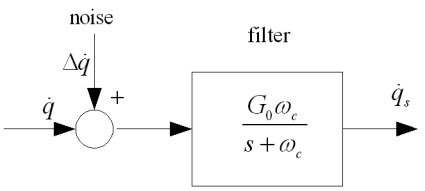


Figure 6. Measurement model of $\dot{q}$ considering noise and filtering.

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ASMG by the fuzzy coefficients strategy, so the missile dynamics and the autopilot dynamics is not considered. The invariance of sliding mode of the ASMG to disturbances and the parameter perturbations was proved in literature[10]. Thus, the key point of the sliding mode guidance law is to guidance $\dot{q}$ to converge to the sliding surface as discussed in literatures[9,10,20]. The convergence to the sliding surface is analysed by the second method of Lyapunov, which is given as:

$$V = \frac{1}{2}\rho^2 \qquad \ldots (18)$$

As proved in (10), if $\dot{V} < 0$ is satisfied, the convergence to the sliding surface is guaranteed. Differentiating of Equation (18) with respect to time, gets:

$$\dot{V} = \rho\dot{\rho} \qquad \ldots (19)$$

When $t > \delta t + t_s$, considering Equation (2), Equation (4) and Equation (18), gets:

$$\dot{V} = (k-1)r\dot{r}\dot{q}^2 - r\dot{q}[\varepsilon \operatorname{sgn}(\dot{q}) - w_q] \qquad \ldots (20)$$

Considering $r > 0$, $\dot{r} < 0$, and k is always set to a value bigger than 2 ($k > 1$), so $(k-1)r\dot{r}\dot{q}^2 < 0$.

(i) If $\dot{q} > 0$ and $\varepsilon > w_q - r\dot{q}[\varepsilon \operatorname{sgn}(\dot{q}) - w_q] < 0$ is satisfied, so that $\dot{V} < 0$ is satisfied.

(ii) If $\dot{q} > 0$, $-r\dot{q}[\varepsilon \operatorname{sgn}(\dot{q}) - w_q] < 0$ is satisfied when $\varepsilon > -w_q$, then $\dot{V} < 0$ is satisfied.

So $\varepsilon > |w_q|$ and $k > 1$ are the requirements to the convergence to the sliding mode.

Similarly, when $t \le \delta t + t_s$, $\dot{V}$, is obtained as:

$$\dot{V} = [\frac{k-\chi}{\delta t}(t-t_s) + \chi - 1]r\dot{r}\dot{q}^2 - r\dot{q}\left\{[\frac{\varepsilon-\chi}{\delta t}(t-t_s) + \chi]\operatorname{sgn}(\dot{q}) - w_q\right\} \qquad \ldots (21)$$

By similar analysis, requirements of $\dot{V} < 0$ are obtained as:

$$\begin{cases} \dfrac{k-\chi}{\delta t}(t-t_s) + \chi > 1 \\ \dfrac{\varepsilon-\chi}{\delta t}(t-t_s) + \chi > |w_q| \end{cases} \qquad \ldots (22)$$

If δt is big, while $(t - t_s)$ and χ are small, Equation (22) may not be satisfied, meaning that the variable coefficients will cause a delay of the convergence to the sliding surface. That's why the fuzzy controller is proposed to generate $\delta t_{1,2}$ according to $(t - t_s)$. To be mentioned, the completed guidance system including autopilot dynamics, missile dynamics, filtering and angle sensor should be developed in future, and the guidance system stability needs to discuss then.

5.0 SIMULATION AND ANALYSIS

The simulations are carried out upon the tail-chase, the head-on interception and the head pursuit engagements. The ASMG given as Equation (6) with the fixed coefficients k and ε, and the FASMG with the fuzzy variable coefficients strategy proposed in this article are simulated respectively. Moreover, the PN law given as Equation (23) is also considered in the head-on pursuit engagement

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so that more characteristics can be compared with:

$$\dot{\sigma} = N\dot{q} \quad \ldots (23)$$

where N is the navigation ratio, and it is selected as $N = k + 1$ in the simulation.
In the three types of engagements, the target performs a sinusoidal trace with the fixed overload n_T as the terminal guidance, and n_T is set to 10. The initial guidance time t_s and constant χ are assumed to be zero. $\dot{\sigma}_T$ is given as:

$$\dot{\sigma}_T = \frac{n_T g}{V_T} \mathrm{Cos}(2\pi f t) \quad \ldots (24)$$

where f is frequency of maneuvering. In this simulation, f is selected as 0·25, and the fixed coefficient k of the navigation part is set to be 2. The fixed coefficient ε can be derived from the adaptive rules, and it is set to be 150 on the condition that it is greater than the acceleration of the target. δ is selected as 0·001 to alleviate the chattering. The noise of $\dot{q}(\Delta\dot{q})$ is considered as a Gaussian distributed random signal with the variance set to 10^{-7} and the frequency set to 50Hz, and the constants of filter are selected as: $G_0 = 1$, $\omega_c = 5$.

4.5.1 Tail-chase engagement

The velocity of the target and the interceptor is set to be 0·8Ma and 3Ma and the initial distance is 10,000m. The initial σ is 0°, the initial q is 30°, and the initial σ_T is 0°.

The trajectory, the LOS angular rate and the acceleration demand of the ASMG and the FASMG are shown in Fig.7.

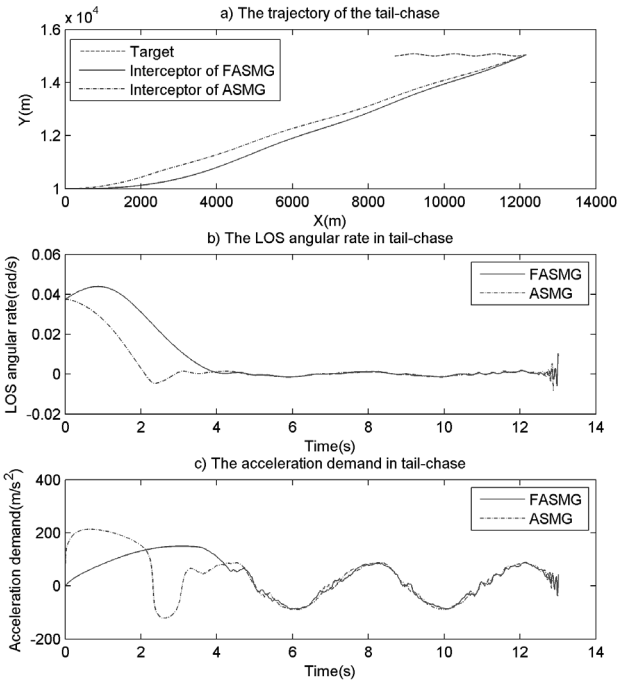


Figure 7. The trajectory, LOS angular rate, and acceleration demand of the tail-chase.

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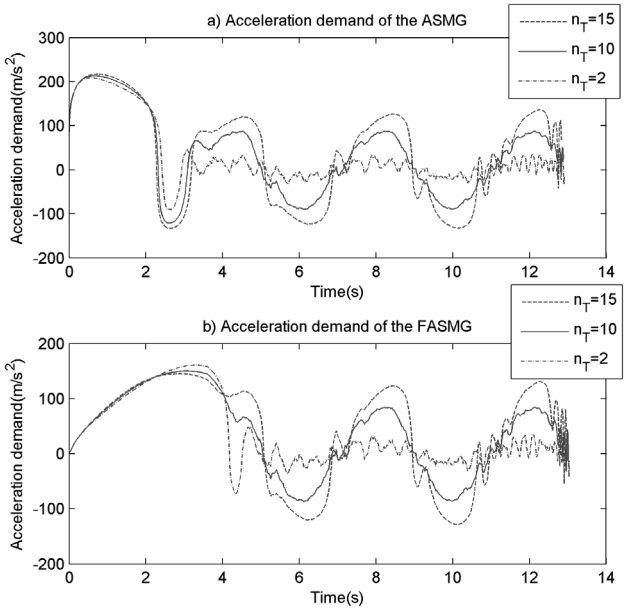


Figure 8. The effects of the target maneuvor on the acceleration demand in the tail-chase.

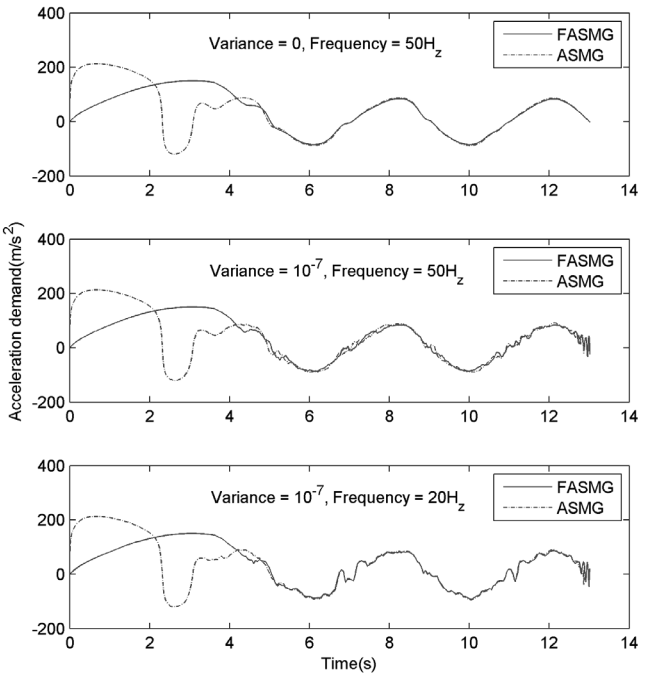


Figure 9. The effects of the noise of $\dot{q}$ on the acceleration demand in the tail-chase.

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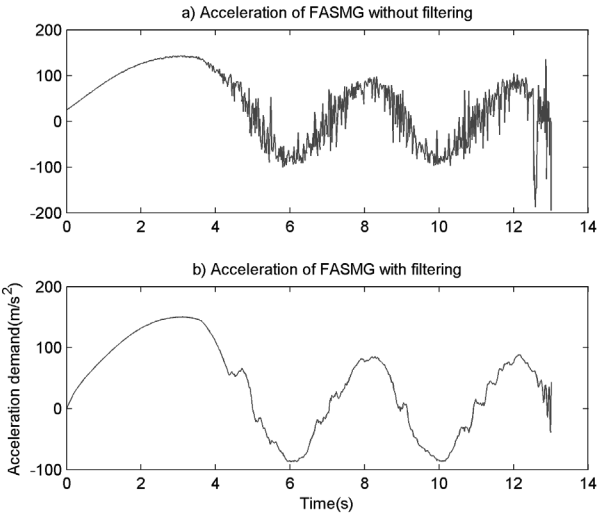


Figure 10. The effects of the filtering on the acceleration demand in the tail-chase.

Compared with the ASMG, the FASMG owns a flat trajectory at the beginning of guidance. Figure 7(b) shows that the LOS angular rate of both guidance laws converges to zero in the end, while the convergence of the FASMG is delayed for about two seconds. As a result, the acceleration demand of the FASMG at the beginning of the guidance is reduced significantly compared with the ASMG as shown in Fig. 7(c). At the beginning of the engagement, the largest acceleration demand of the ASMG is $221{\cdot}6\text{ms}^{-1}$, while the largest acceleration demand of the FASMG is $142{\cdot}8\text{ms}^{-1}$. After about 4 seconds, the two guidance laws perform almost equivalently in both the LOS angular rate and the acceleration demand. In the simulations, because the distance is closing to zero, the LOS angular rate and the acceleration demand at the end of guidance wave drastically as shown in Figs 7(b) and 7(c).

Due to the acceleration of the target and the noise of $\dot{q}$, $\dot{q}$ of both the ASMG and the FASMG does not strictly converge to zero. Meanwhile, the acceleration demand is mainly related to the acceleration of the target. If the acceleration of the target is increased, the acceleration demands of both guidance laws are increased respectively as shown in Fig.8.

Figure (9) presents how the variance and the frequency of $\Delta\dot{q}$ affect the acceleration demand. If the variance is increased, the chattering of the acceleration demand is enhanced. Due to the filtering system, the chattering of the acceleration demand is not increased obviously when the frequency of $\Delta\dot{q}$ is increased. Moreover, by introducing the filtering system, the chattering of the acceleration demand is significantly reduced as shown in Fig. 10.

To be mentioned, although $\dot{q}/(|\dot{q}|+\delta)$ is used to replace $\text{sgn}(\dot{q})$ to alleviate the chattering of the ASMG and the FASMG, the chattering still exists in the LOS angular rate and the acceleration performances, especially at the end of the guidance[5,10].

4.5.2 Head-on interception engagement

Considering the hypersonic vehicles in the interception environment, the speed of the interceptor and the target is set to be 4Ma and 5Ma respectively, and the initial distance is 15,000m. The initial σ is 165°, the initial q is 170°, and the initial σ_T is 0°.

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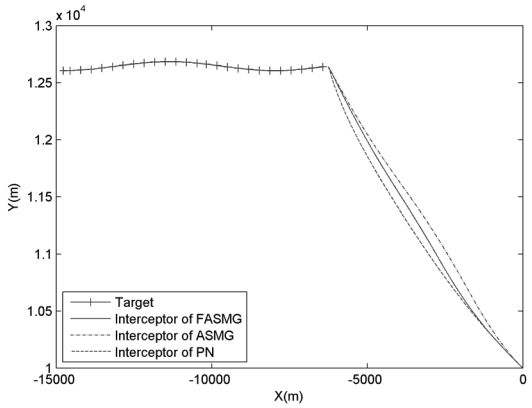


Figure 11. The trajectory of the head-on interception.

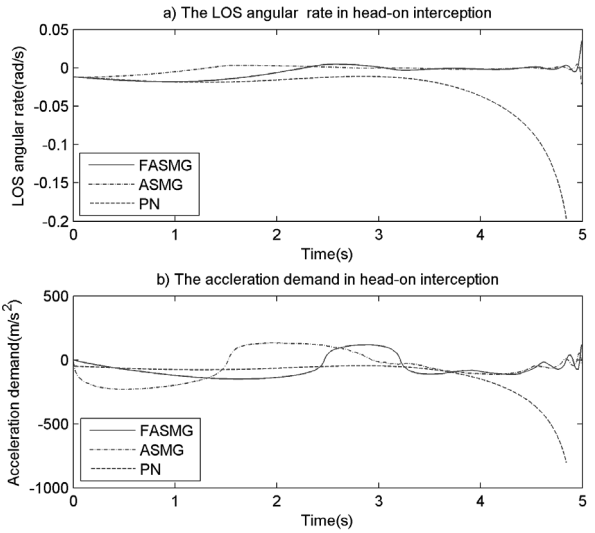


Figure 12. The LOS angular rate and acceleration demand of the head-on interception.

In head-on interception engagement, the closing speed is extreme large, leading to a short guidance time ($t_f - t_s$). As a result, a small value of $\delta t_{1,2}$ is generated by the fuzzy controller. As shown in Fig. 11, the FASMG performs the similar flat trajectory to the PN law at the beginning and medium bending after a short period of time. Although the value of $\delta t_{1,2}$ is small, the convergence of the FASMG is delayed for about 0·8 seconds compared with the ASMG as shown in Fig. 12(a). As expected, a smaller acceleration demand of the ASMG at the beginning of the guidance is incurred compared with the ASMG as shown in Fig. 12(b). Compared with the PN law, $\dot{q}$ of both the ASMG and the FASMG converges to zero rapidly, leading to smaller acceleration demands than that of the PN law in the end.

4.5.3 Head pursuit engagement

The speed of the interceptor and the target is set to be 4Ma and 5Ma respectively. The initial distance is 5,000m , the initial σ is 10°, the initial q is 160°, and the initial σ_T is 0°.

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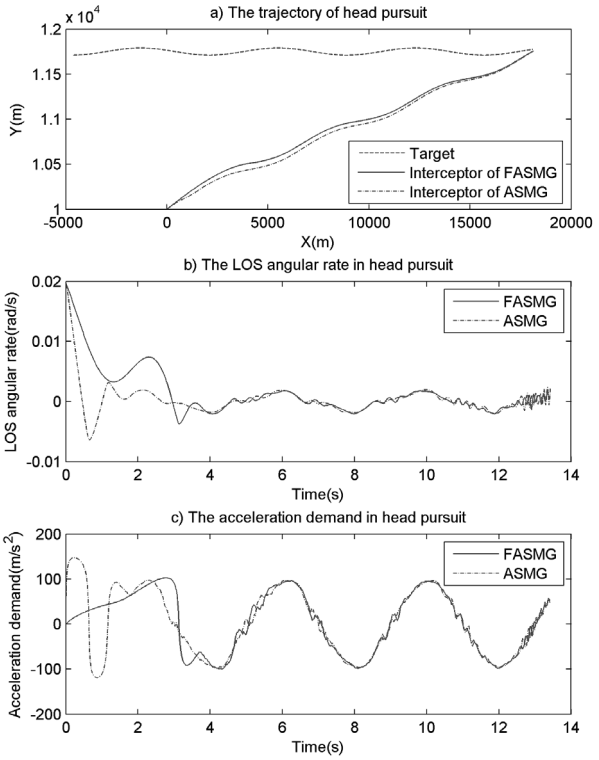


Figure 13. The trajectory, LOS angular rate, and acceleration demand of the head pursuit.

Similarly, the head pursuit engagement using the ASMG and the FASMG with the target acceleration and the noise of $\dot{q}$ considered are simulated. The convergence to the sliding surface of the FASMG is delayed for about 2·2 seconds compared with the ASMG as shown in Fig. 13(b). Moreover, the acceleration demand obtained by the FASMG is significantly reduced as shown in Fig.13(c).

As presented and analysed above, the fuzzy coefficients strategy is able to delay the convergence to the sliding surface by a short duration in the three types of engagement. Thus, the acceleration demand at the beginning of the guidance can be significantly reduced. Both the ASMG and the FASMG are influenced by the disturbance and noise, and the fuzzy controller does not provide extra burden on the robustness to the disturbances.

5.0 CONCLUSIONS

Based on the ASMG with the LOS angular rate converging to zero, a variable coefficients strategy is proposed to decrease the acceleration demand at the beginning of the terminal guidance. The fuzzy control is introduced in this article to generate a short period of time when the coefficients perform linear variation. The simulation results of the tail-chase, the head-on interception, and the head pursuit engagements prove that the approach is efficient. Compared with the method of selecting fixed coefficient, the fuzzy variable coefficients strategy significantly contributes to the decrease of the acceleration demand of the interceptor at the beginning of the terminal guidance.

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The simulations show that the convergence of the LOS angular rate is delayed instead of damaged due to the fuzzy controller. The extra requirement of the improved guidance is the estimation of the terminal guidance time, which is easy to achieve.

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