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REGULARITY REFORMULATED

WENG HONG TANG
wenghong@gmail.com

ABSTRACT

This paper focuses on the view that rationality requires that our credences be regular. I go through different formulations of the requirement, and show that they face several problems. I then formulate a version of the requirement that solves most of, if not all, these problems. I conclude by showing that an argument thought to support the requirement as traditionally formulated actually does not; if anything, the argument, slightly modified, supports my version of the requirement.

1 INTRODUCTION

What requirements must your credences satisfy in order for them to count as rational? According to orthodox Bayesianism, they must obey the probability axioms. But various other requirements have been suggested. For example, philosophers have proposed that, on pain of irrationality, credences should also satisfy the Principal Principle, or satisfy the Reflection Principle, or be updated by conditioning, or be regular.[1]

This paper focuses on *regularity*. More precisely, it focuses on the view that rationality requires that credences be regular. Call such a requirement RR – short for the *regularity requirement*. As we shall see, we can understand RR in different ways depending on what we take regularity to be. As we shall also see, different ways of understanding regularity lead to different problems for RR. In fact, one may think that various versions of RR make for rather easy targets. But my aim in this paper is neither to argue for nor to argue against RR *per se*. Rather, it is to ask: how should we formulate RR to make it more plausible – while still keeping to the spirit of the requirement as understood by most of its proponents? This question is worth asking whether we ultimately wish to defend or to reject RR. Obviously, if we wish to defend RR, we shouldn't defend a version of it that makes for an easy target. But even if we wish to reject RR, we should make sure that we are rejecting the most plausible version of it, lest we be accused of attacking a straw person.

To begin, I shall go through below a few different versions of RR. In the next section, I shall look at a host of problems that they face. In §3, I shall formulate an alternative version of the requirement that solves most of, if not all, such problems. In §4, I shall show that an argument traditionally thought to support RR, as interpreted by the likes of Abner Shimony (1955), John G. Kemeny (1955) and Rudolf Carnap (1971), actually does not. If anything, a modified version of the argument supports my version of RR instead.

[1] Lewis (1980) defends the Principal Principle, van Fraassen (1984) defends the Reflection Principle, and Lewis (1999) provides a Dutch Book Argument for why credences ought to be updated by conditioning.

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Timothy Williamson (2007: 173) writes that 'a probability distribution over sets of possibilities is *regular* just in case it assigns probability 0 only to the null set, and therefore probability 1 only to the set of all possibilities'. But just what regularity amounts to, and hence, what RR amounts to, will depend on how we interpret the term 'possibility'. Should we take 'possibility' to mean logical possibility, metaphysical possibility, epistemic possibility, doxastic possibility or some other kind of possibility altogether?[2]

Shimony (1955: 17), together with Kemeny (1955) and Carnap (1971), has *logical* possibility in mind – he holds that rationality requires that if one's conditional credence in p given q is 1, then q logically implies p, for any p and q. Suppose q is a logical truth. Then, according to Shimony, rationality requires that we assign credence 1 to p only if p is a logical truth, that is, only if there is no logical possibility in which not-p. David Lewis, however, seems to have *metaphysical* possibility in mind. According to him, RR is the requirement that we assign credence 0 to p only if p is 'the empty proposition, true at no worlds' (Lewis 1980: 88).[3] Now, Lewis's worlds are metaphysically possible worlds. So, according to him, RR is the requirement that our credence in p be 1 only if there is no metaphysical possibility in which not-p. For Robert C. Stalnaker (1970: 68), the relevant notion of possibility is neither logical possibility nor metaphysical possibility, but *epistemic* possibility. According to him, RR is the requirement that we assign credence 1 to p only if there is no epistemically possible outcome in which not-p. Last but not least, Alan Hájek (forthcoming) holds that the relevant notion of possibility is *doxastic* possibility. According to him, RR is the requirement that we assign credence 1 to p only if there is no doxastic possibility in which not-p. Note that whereas Shimony (1955), Lewis (1980), and Stalnaker (1970) all embrace the respective versions of RR that they formulate, Hájek (forthcoming) does not embrace any version of RR spelt out above, not even the one that he formulates.

Why subscribe to RR? Various reasons have been offered, but I shall focus on three of them. My interest is *not* in endorsing or defending such reasons for subscribing to RR. Rather, going through some of these reasons will tell us something about the spirit in which RR is put forward, and this will help us later in comparing certain versions of RR with others.

First, regularity is thought to encapsulate the supposed norm that our credences ought to reflect the strength of our evidence.[4] The higher our credence, the stronger our evidence ought to be, and credence 1 ought to go with incontrovertibility. But since our evidence for possibly false propositions is never incontrovertible, we ought not to assign credence 1 to them. Or so the reasoning goes.

Second, regularity is thought to embody a kind of open-mindedness. By not assigning credence 1 to a proposition that is possibly false, we signal a willingness to admit that we might be wrong about it; we signal that we have not closed off all lines of enquiry as to its truth.

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[2] As Hájek (MS) notes, we may also understand regularity in different ways depending on how we understand the term 'probability'. For my purposes, I shall take it to refer to subjective probability or credence. But you may also take it to refer to probability or some kind of objective probability.

[3] If your notion of logical possibility is such that something is logically possible if and only if it is metaphysically possible, then you'll maintain that there's no difference between the two versions of RR mentioned thus far. But in order not to prejudge issues concerning how logical possibility and metaphysical possibility are related, I'll keep the two versions separate.

[4] Hájek (forthcoming) cites this as a possible reason for subscribing to RR without endorsing it.

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Correspondingly, a failure of regularity – or *irregularity* – 'is tantamount to a firm resolve to never change your mind no matter what, and that is objectionable' (Lewis 1981: 14). In particular, suppose that credences are to be updated by conditioning, and we assign credence 1 to a proposition that is possibly false. Since the proposition is possibly false, evidence for its falsity *might* arise. But since conditioning can't lower our credence in the proposition, assigning it a credence of 1 signals a dogmatic unwillingness to change our minds about it no matter what evidence comes our way (Skyrms 1980: 74).[5] Such dogmatism seems irrational, and as such, there seems to be something irrational about irregularity. Or so the reasoning goes.

Third, regularity is thought to help us eschew making pragmatically blameworthy decisions. Shimony (1955) holds that rationality requires what Carnap (1971) has called *strict coherence*, where your credence function is *strictly coherent* just in case it does not lead you to a system of bets that will result in a net loss for you in at least one possible outcome, and no net gain in all other possible outcomes, i.e. just in case it does not leave you susceptible to a *Weak Dutch Book*. Given that such susceptibility is pragmatically blameworthy, and given that strict coherence requires regularity – as Shimony (1955) argues – there seems to be something irrational about violating regularity.[6] Or so the reasoning goes.

2 THE REGULARITY REQUIREMENT: DIFFERENT VERSIONS

The three reasons adduced in favour of RR are prima facie plausible, even if you may ultimately find them unconvincing.[7] But we shall see that each version of RR mentioned above faces problems of its own. Note that I shall assume, as proponents of RR typically do, that rational credence functions are probability functions (Shimony 1955; Kemeny 1955; Carnap 1971). For the purposes of my paper, this assumption can be treated as a harmless idealisation, since the problems that arise for the various versions of RR that I discuss do not arise because of it.

2.1 *Regularity, logical possibility and metaphysical possibility*

Consider the version of RR espoused by Shimony, according to which credence 1 is to be assigned to p only if p is logically necessary. Consider also the version of RR put forward by Lewis, according to which credence 1 is to be assigned to p only if p is metaphysically necessary. Call the first *RR-logical* and the second *RR-metaphysical*.

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[5] $Cr(q|p)$ is usually taken to be *defined* by $Cr(p\&q)/Cr(p)$, where $Cr(p) \neq 0$—the so-called *ratio formula*. When $Cr(q) = 1$, $Cr'(q) = Cr(q|p) = Cr(p)/Cr(p) = 1$ for any p, where $Cr(.|.)$ is one's credence function before conditioning, and $Cr'(.|.)$ one's credence function after conditioning. Admittedly, you may hold that conditioning can reduce our credence from 1 to some value lower than 1 if you follow Hájek (2003) in holding that conditional probabilities are primitive, and in denying that conditional probability is to be analysed in terms of the ratio formula. For in such a case, even if $Cr(q) = 1$, $Cr(q|p)$ may equal some value other than 1.

[6] In section 4, we will take a closer look at an argument that is supposed to show that strict coherence requires regularity.

[7] For arguments against taking such reasons to be good reasons for subscribing to RR, see Weintraub (1993) and Hájek (forthcoming).

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What the difference between the two amounts to depends on what the difference between logical possibility and metaphysical possibility amounts to. Consider a narrow notion of logical possibility, according to which it is logically possible that *p* just in case *p* is not ruled out by the laws of classical logic. Then logical possibility is much weaker than metaphysical possibility in the sense that it is much easier for something to be logically possible than for it to be metaphysically possible. For example, it is logically possible that I have different parents from the ones I actually have, but it is at least arguable that such a scenario is metaphysically impossible.

But we may also interpret 'logical' more broadly, so that logical truths include not just the tautologies of classical logic, but a whole range of mathematical and other a priori or conceptual truths, such as 'All bachelors are unmarried'. This broader notion of logical possibility – call it *a priori possibility* – is stronger than the narrower notion mentioned above, but is still arguably weaker than metaphysical possibility.[8] For example, you may think that the proposition that water is H_2O is metaphysically necessary though not a priori true. Given that there is a narrow and a broad notion of logical possibility, there are at least two versions of RR-logical. The distinction between the two will not matter much in this section, though it will be important in §3, when I formulate my version of RR.

Both RR-logical and RR-metaphysical have it that it is irrational to assign credence 1 to any proposition that is not logically or metaphysically necessary. But this makes them too demanding. First, even though it is not logically or metaphysically necessary that I exist, it is arguably rational for me to assign credence 1 to my own existence. In fact, you may think that it is downright irrational for me not to do so (Hájek forthcoming). Second, it is arguably rational for me to assign credence 1 to certain propositions concerning certain phenomenal experiences that I have, while I am having them. For example, it is arguably rational for me to assign credence 1 to the proposition that *this* is an experience of blackness, while I'm having such an experience.[9] But even while I'm having such an experience, it is not logically or metaphysically necessary that I am having it. Third, it seems possible for there to be a rational and omniscient being (such as God) who assigns credence 1 to various propositions that are not logically or metaphysically necessary (Hájek forthcoming; Weintraub 1993: 257).[10] But the versions of RR under consideration are incompatible with such a possibility, for according to them, anyone who violates regularity is irrational. Fourth, supposing that there are contingent a priori truths such as 'Julius invented the zip', it seems rational for us to assign credence 1 to them.[11] But RR-metaphysical (and

[8] Anthony Appiah (1985) subscribes to a version of RR-logical according to which 'logical' is understood in the broad sense. He writes: 'On my view, every representation with 0 probability is a priori false – as I shall say "impossible"' (219).

[9] A phenomenal belief may be necessarily constituted by the experience that the belief is about—in which case it is impossible for one to have the belief without having the experience. For more on issues related to the infallibility of certain phenomenal beliefs, see Chalmers 2010: 277–304.

[10] Of course, if we understand an omniscient being to be one who knows all truths, but we also hold that knowing that *p* does not require assigning *p* a credence of 1, then it may well be that even the credence function of an omniscient being ought to be regular. But we may imagine beings (such as God) who are omniscient in the sense that they know for *certain* all truths.

[11] The above example of a contingent a priori proposition comes from Evans (1979). 'Julius invented the zip' is a priori because 'Julius' is stipulated to be a proper name of the person who invented the zip—whoever he is. But it's possible that the person we pick out by the name 'Julius' did not invent the zip, and that someone else did. Hence it's contingent that Julius invented the zip.

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the narrow version of) RR-logical would deem us irrational in doing so, since such truths are neither metaphysically necessary nor logically necessary (in the narrow sense). Fifth – and this problem afflicts all the versions of RR considered in this paper, including the one I favour – it seems rational for us to assign credence 1 to the proposition that an infinitely fine dart thrown randomly at a dartboard will fail to land on some arbitrary point X (Hájek 2003: 289). For supposing that probabilities are real-valued, then since there are uncountably many points on the dartboard, and the dart is as likely to land on one point as another, the probability of it landing on any particular point has got to be 0. But it is logically and metaphysically possible for the dart to land on point X.[12]

RR-logical faces a problem that RR-metaphysical avoids if logical possibility is construed narrowly. Consider a metaphysically necessary proposition that is a priori true, but is not a logical truth in the narrow sense, for example, the proposition that all bachelors are unmarried. There is nothing wrong with assigning credence 1 to such a proposition, but RR-logical will indict us for doing so on account of the proposition not being a truth of (classical) logic.

Also, RR-metaphysical faces a problem that RR-logical avoids. All the problems raised so far have to do with RR being too demanding. But this problem has to do with it being too *liberal*. Suppose there are metaphysically necessary propositions that are a posteriori true, for example, the proposition that water is H_2O. RR-metaphysical, unlike, RR-logical, will not indict us for assigning credence 1 to the proposition. But one would expect typical proponents of RR to think that it is irrational to assign credence 1 to a proposition whose truth we have to discover empirically. After all, an empirical proposition seems to be the kind of proposition for we lack incontrovertible evidence, and about which our opinions should be open to revision. It also seems to be the kind of proposition in which a credence of 1 will lead us to a bet in which we stand to gain nothing but might well lose something. (Recall the reasons adduced in favour of RR.)

I am not saying that all the problems raised against RR-logical and RR-metaphysical cannot be resolved. For example, you may hold that it is irrational for us to assign credence 1 to our own existence, or deny that we have phenomenal beliefs that are infallible, or deny that there are contingent a priori or necessary a posteriori propositions. Still, ideally, a proponent of RR should not be forced to take a stand on such (relatively controversial) issues. For instance, in deciding whether to accept some version of RR, it would be ideal if you don't first have to make up your mind about whether there are infallible phenomenal beliefs or about whether there are necessary a posteriori propositions.

2.2 *Regularity, epistemic possibility and doxastic possibility*

Traditionally, most proponents of RR endorse RR-logical or RR-metaphysical, or something like them. But to avoid the above problems, you may decide to embrace Stalnaker's or Hájek's version of the requirement – call them *RR-epistemic* and *RR-doxastic*

[12] You may argue that in dartboard cases, it is not rational to assign credence 1 to the proposition that the dart will fail to land on X. You may argue that since it's possible that the dart will land on X, we should invoke hyperreals, and maintain that the probability that the dart will fail to land on X is not 1 but some value infinitely close to 1 (Skyrms 1980: 74–8; Lewis 1980: 267–8). But the issue of whether such a response works remains controversial. See Williamson (2007), who argues that problems remain even if we invoke hyperreals.

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respectively. Unfortunately, while the latter do avoid some of the above problems, they face other problems of their own.

How do they avoid some of the above problems? In epistemic and doxastic logic, knowledge and belief are often cashed out in the following ways:

S knows that *p* just in case *p* is true in all epistemically possible worlds relative to S, i.e., worlds compatible with what S knows.
S believes (in the all-or-nothing sense) that *p* just in case *p* is true in all doxastically possible worlds relative to S, i.e., worlds compatible with what S believes.

Thus RR-epistemic can be understood as saying that we ought to assign credence 1 to *p* only if we know that *p*. And one way of understanding RR-doxastic is to take it to say that we ought to assign credence 1 to *p* only if we believe that *p*. (We'll look at a different way of understanding RR-doxastic below.) Now suppose that I know or believe that I exist. Suppose also there are some propositions concerning my phenomenal experiences that I know or believe to be true. Further, suppose that given any true proposition, God would know or believe that it is true, whether or not it is logically or metaphysically necessary. In such cases, assigning credence 1 to the relevant propositions does not violate the versions of RR under consideration. For suppose you know or believe that *p* is true. Then, in assigning credence 1 to *p*, you are not violating the requirement to assign credence 1 only to the set of all epistemic or doxastic possibilities, since *p* would be true in all such possibilities.

RR-epistemic and RR-doxastic also avoid problems having to do with contingent a priori truths, with necessary a posteriori truths and with necessary propositions that are a priori true but not logically true (in the narrow sense of the word 'logical'). Suppose I know or believe that Julius invented the zip, or that water is H_2O, or that all bachelors are unmarried. Then I won't be indicted for assigning credence 1 to such propositions, for they would be true in all epistemically or doxastically possible worlds. Furthermore, suppose we maintain (rather controversially) that the probability of an infinitely fine dart failing to land on point *X* on a dartboard is high enough for us to know or believe that it will fail to land on point *X*. In such a case, RR-epistemic and RR-doxastic will not indict us for assigning credence 1 to the proposition that the dart will fail to land on point *X*.

So far so good. But RR-epistemic and RR-doxastic face problems of their own. First, it is controversial whether we really know or believe in the dartboard case that the dart will fail to land on point *X*. You may think that the case is akin to the case of a lottery with uncountably many tickets, of which exactly one is the winner. If you think that in the latter case we do not know or believe of any ticket that it is a losing ticket, you'll probably be inclined to hold that in the dartboard case we do not know or believe of any point on the dartboard that the dart will fail to land on it.[13] But if you're right, then RR-epistemic and RR-doxastic are too demanding – they will indict us for assigning credence 1 to the proposition that the dart will fail to land on *X*, even though it may well be rational for us to do so.

Second, the problems that RR-logical and RR-metaphysical face suggest that their proponents may well have to do with a slightly weaker version of RR that prohibits most

[13] If we know or believe of each ticket that it's a losing ticket, an uncountable version of the lottery paradox arises—see Maher 1990: 388 and van Fraassen 1995: 351.

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agents (those who, unlike God, are non-omniscient) from assigning credence 1 to a vast swath of contingent or a posteriori propositions, without prohibiting them from assigning credence 1 to all such propositions. But RR-epistemic and RR-doxastic try to handle the problems faced by RR-logical and RR-metaphysical by being very liberal – to the point that they risk being uninteresting to typical proponents of RR. Suppose we know or believe an ordinary contingent proposition such as the following:

(Asteroid) No asteroid collided with Earth in 400 BCE.

In such a case, no asteroid collided with Earth in 400 BCE in all epistemically or doxastically possible worlds. Then assigning credence 1 to (Asteroid) does not violate RR-epistemic or RR-doxastic. This is so even if the objective probability of (Asteroid) being true is less than 1, insofar as we may know or believe propositions whose objective probabilities of being true are less than 1. But this violates the spirit – if not the letter – of RR.[14] As we've seen, RR is thought to encapsulate the norm that our credence ought to reflect the strength of our evidence. Since our evidence for an ordinary contingent proposition such as (Asteroid) is presumably not incontrovertible, it may be thought that, accordingly, our credence in the proposition should be less than 1. Furthermore, regularity is thought to embody a kind of open-mindedness, and an ordinary contingent proposition such as (Asteroid) seems to be the kind of proposition that we should be open-minded about, if we are to be open-minded about any proposition at all. We may well know or believe that such a proposition is true, but unless knowledge or belief entails certainty, it may be thought unnecessary – in fact, it may well be hubris – to assign the proposition credence 1. Finally, regularity is thought to help us eschew making pragmatically blameworthy decisions. And for typical proponents of RR, (Asteroid) is presumably the kind of proposition in which a credence of 1 will lead us to a bet in which we stand to gain nothing but might well lose something.

Third, with their invocation of knowledge and of all-or-nothing belief, there is something rather *un-Bayesian* about RR-epistemic and RR-doxastic. Perhaps the Bayesian is ultimately unwise to eschew such important notions in traditional epistemology. Nonetheless, it would be defeatist for the Bayesian to give up so quickly: it is worth her while to try looking for some formulation of regularity that does not appeal to knowledge or to all-or-nothing belief.

Fourth, RR-doxastic (but not RR-epistemic) risks being *vacuous*. Suppose that, setting aside dartboard cases, assigning credence 1 to a proposition is sufficient for you to count as believing it. Then you can't violate RR-doxastic (setting aside dartboard cases). For once you've assigned credence 1 to a proposition, it will become doxastically necessary. Suppose, furthermore, that you subscribe to the Threshold View of belief, according to which to believe that p is to have a sufficiently high credence in p. Hájek (manuscript), who is no fan of RR, notes that someone may appeal to the view to support RR-doxastic. But any such support comes at the cost of vacuity. For if the Threshold View is true, the requirement cannot be violated – in such a case, it says nothing

[14] There's another way in which RR-doxastic may be too liberal. RR-doxastic does not indict me for assigning credence 1 to (Asteroid) even if I believe (Asteroid) without good reason. But this problem might be avoided by amending RR-doxastic slightly, so that it becomes the requirement that we assign credence 1 only to propositions that we *rationally* believe.

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more than that you ought to assign credence 1 to p only if your credence in p is sufficiently high.

Let's consider a response to the second problem, and then a response to the third. To avoid the second problem, you may suggest that we amend RR-epistemic and RR-doxastic as follows:

RR-epistemic*: Assign credence 1 to p only if p is knowable a priori, where a proposition is knowable a priori just in case it can be known independently of experience.
RR-doxastic*: Assign credence 1 to p only if p can be rationally believed a priori, where a proposition can be rationally believed a priori just in case it can be rationally believed independently of experience.

Since (Asteroid) is not the kind of proposition that is knowable a priori or that can be rationally believed a priori, both RR-epistemic* and RR-doxastic* will indict you for assigning credence 1 to it. Unfortunately, they will also indict an omniscient God for assigning credence 1 to a posteriori propositions. Furthermore, RR-epistemic* and RR-doxastic* will indict me for assigning credence 1 to the proposition that *this* is an experience of blackness even while I'm having the experience, since the relevant item of knowledge or belief is not acquired independently of experience.

To avoid the third problem, you may suggest that we stick with RR-doxastic, but instead of taking a doxastic possibility to be one compatible with all that we believe, we should take it to be one compatible with all that we assign credence 1 to. This allows us to avoid talk of knowledge and of all-or-nothing belief. Unfortunately, this suggestion runs into a problem similar to one that I've raised against the original version of RR-doxastic. Suppose you're wondering if you may assign credence 1 to a proposition. According to the version of RR-doxastic under consideration, you may do so only if the proposition is true at all worlds compatible with what you assign credence 1 to. But then, you *can't* violate the requirement – merely assigning a proposition a credence of 1 allows you to fulfil the requirement vacuously. Say, for example, that you're wondering whether you may assign credence 1 to (Asteroid). Well, don't worry – just assign credence 1 to it. For once you've done so, the proposition will become doxastically necessary: it will become true at all worlds compatible with what you assign credence 1 to.[15]

3 REGULARITY REFORMULATED

The various formulations of RR that I have discussed face a number of problems. In this section, I shall formulate a new version of RR that avoids most of those problems.

Recall that RR is the view that a credence of 1 is to be assigned to p only if p is true in all possibilities. As we've seen, we get different versions of RR depending on how 'possibility' is understood. To come up with a more plausible version of RR than those we've considered so far, I propose that we take 'possibility' to mean *a priori possibility* (i.e. logical possibility in the broad sense mentioned above), and hold that one's credence

[15] Suggesting that a doxastic possibility is one that is compatible with what you rationally assign credence 1 to won't help. This is because we then have to ask: what may we rationally assign credence 1 to? But the answer to this question depends in the first place on whether we should accept RR.

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function $Cr(.)$ is regular just in case it assigns credence 1 to p only if it is a priori impossible that $Cr(p) = 1$ and p is false. The version of RR corresponding to such a version of regularity will then say: assign credence 1 to p only if it is a priori impossible that you assign credence 1 to p and p is false. More formally:

RR-new: For any proposition p and any credence function $Cr(.)$, it ought to be that $Cr(p) = 1$ only if p is true in all a priori possibilities in which $Cr(p) = 1$.

Note that one agent may satisfy RR-new whereas another doesn't, even if both assign credence 1 to the same proposition. For example, RR-new prohibits me from assigning credence 1 to (Asteroid), since it is not true in all a priori possibilities in which I assign credence 1 to it. But, according to RR-new, an omniscient God is not prohibited from doing so, since (Asteroid) is true in all a priori possibilities in which God assigns credence 1 to it.

RR-new avoids most of the problems that plague other versions of RR. First, it does not deem me irrational in assigning credence 1 to my own existence if it is a priori impossible that I assign credence 1 to my existence but not exist. Second, suppose that I assign credence 1 to the proposition '*This* is an experience of blackness'. If it is a priori impossible for me to do so without having the experience in question, RR-new does not deem me irrational. Now you may worry that it isn't a priori true that my introspection is infallible in such a case – it may well be a priori possible that I assign credence 1 to the proposition in question without having the relevant experience.[16] But this poses no problem for RR-new, for in such a case, it simply says that I shouldn't assign credence 1 to the proposition. RR-new is compatible with it being a priori true, as well as with it not being a priori true, that our introspection is infallible in the kind of case under consideration. Third, suppose that for any p, it is a priori impossible for an omniscient God to assign credence 1 to p and for p to be false. Then RR-new does not deem God irrational for assigning credence 1 to various contingent or a posteriori propositions. Fourth, suppose there are contingent a priori propositions. RR does not deem us irrational in assigning credence 1 to such propositions, for example, to the proposition 'Julius invented the zip'. Since such propositions are a priori true, it would be a priori impossible for us to assign credence 1 to them and for them to be false. Fifth, RR-new does not deem us irrational for assigning credence 1 to metaphysical necessities that are not logical truths in the narrow sense, but are nonetheless a priori true. For example, it does not deem us irrational for assigning credence 1 to the proposition that all bachelors are unmarried. For though the proposition is not a logical truth in the narrow sense, it is metaphysically necessary and a priori true. As such, it is a priori impossible for us to assign credence 1 to it and for it to be false. Sixth, RR-new is not too liberal in the sense that if there are propositions that are metaphysically necessary but true a posteriori, it indicts us for assigning credence 1 to them. For example, even if the proposition that water is H_2O is metaphysically necessary, it is not a priori true. RR-new will therefore indict us for assigning credence 1 to it, since it is a priori possible that it is false and we assign credence 1 to it. (Note that if the notion of possibility invoked by RR-new were not a priori possibility but metaphysical possibility, then RR-new would not indict us for assigning credence 1 to necessary a

[16] Thanks to an anonymous reviewer for raising this point, and for suggesting a reply to it.

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posteriori propositions.) Seventh, RR-new is not too liberal in the way that RR-epistemic and RR-doxastic are – it indicts us for assigning credence 1 to an ordinary contingent proposition such as (Asteroid), since it is a priori possible that the proposition is false and we assign credence 1 to it. Eighth, unlike the version of RR-doxastic according to which a doxastic possibility is that which is compatible with what one assigns credence 1 to, RR-new isn't vacuous. You violate RR-new if you assign credence 1 to (Asteroid), since it's a priori possible that (Asteroid) is false and you assign credence 1 to it. Last but not least, RR-new is Bayesian through and through – unlike RR-epistemic and RR-doxastic, it eschews talk of knowledge and binary belief.[17]

RR-new also maintains the spirit of RR: the three reasons that have been offered in support of RR do not clash with RR-new. For example, suppose it is a priori impossible for me to assign credence 1 to my own existence and not exist. Then RR-new does not indict me for assigning credence 1 to my own existence. This does not go against the supposed norm that our credence ought to reflect the strength of our evidence – in such a case, I have incontrovertible evidence for my own existence. My assigning credence 1 to my own existence also needn't be dogmatic – or if dogmatic, not unreasonably so. Even proponents of RR won't want to hold that all assignments of credence 1 are unreasonably dogmatic – after all, they hold that rational credence functions are probability functions, and as such, hold that we ought to assign credence 1 to logical truths. The difference between assigning credence 1 to a logical truth or to my own existence on the one hand, and assigning credence 1 to a proposition such as (Asteroid) on the other, is as follows: in the latter case, but not in the former, it is a priori possible for us to be mistaken. This, I suggest, is why in the latter case, but not in the former, assigning credence 1 to the relevant proposition seems unreasonably dogmatic. Finally, my assigning credence 1 to my own existence will not lead me into making pragmatically blameworthy decisions, since it is a priori impossible for such a credence to lead me into losing a bet on my own existence.

Alas, dartboard cases remain a problem for RR-new. RR-new will indict me for assigning credence 1 to the proposition that an infinitely fine dart thrown at a dartboard will fail to land on an arbitrary point X, since it is a priori possible for me to do so and for the dart to land on X. But if such an assignment of credence is reasonable, then RR-new seems unreasonably demanding.

However, the problem is one that plagues both RR-logical and RR-metaphysical too. Insofar as RR-new solves the other problems that plague other versions of RR, it is superior to them. How about RR-epistemic and RR-doxastic? *If* we can be said to know or believe in dartboard cases that the dart will fail to land on X, then RR-epistemic and RR-doxastic have at least one advantage over RR-new. But that's a big 'if'. Supposing that we can't be said to know or believe such a thing, then RR-new

[17] I'm not claiming that it is indeed rational for me to assign credence 1 to my own existence, or that there are necessary a posteriori propositions, or that there are contingent a priori propositions, or that there are phenomenal beliefs that are infallible. As I mentioned in section 2.1, it would be ideal if a proponent of RR isn't forced to take a stand on such issues. And RR-new does not automatically force one to take a stand. For instance, whether RR-new indicts me for assigning credence 1 to my own existence depends on the answer to the question 'Is it a priori possible that I assign credence 1 to my existence but not exist'? If the answer is 'yes', then I'm in violation of RR-new; but if the answer is 'no', then I'm not. And someone who subscribes to RR-new doesn't have to deny that there are necessary a posteriori propositions or contingent a priori propositions or infallible phenomenal beliefs, since their existence won't cause trouble for RR-new.

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is clearly superior to RR-epistemic and RR-doxastic: it faces a problem that the latter also face, but avoids problems that the latter do not avoid.

We have also seen that RR-epistemic and RR-doxastic are too liberal in the sense that they allow assignments of credence 1 that typical proponents of RR would frown upon. In particular, they do not prohibit us from assigning credence 1 to *any* contingent or a posteriori proposition so long as we know or believe the proposition in question. Assuming that we may know or believe that p even if the objective probability that p is true is less than 1, this means that RR-doxastic and RR-epistemic do not prohibit us from assigning credence 1 to propositions whose objective probabilities are less than 1. It is small wonder then that the dartboard case is not a problem for RR-epistemic and RR-doxastic if we know or believe that the dart in question will fail to land on X. This is especially since the probability that the dart will fail to land on X is higher than the probabilities of most other contingent or a posteriori propositions – such as (Asteroid) – that we may know or believe to be true. If RR-epistemic and RR-doxastic do not prohibit us from assigning credence 1 to the latter propositions, it is not surprising that they do not prohibit us from assigning credence 1 to the former. But, as mentioned, RR-epistemic and RR-doxastic go against the spirit of RR, and it is unlikely that a typical defender of RR has RR-epistemic or RR-doxastic in mind. In fact, a typical opponent of RR may even be sympathetic to RR-epistemic or RR-doxastic.[18]

Now, is there a way for proponents of RR-new to respond to dartboard cases? Suppose that an infinitely fine dart is thrown at a dartboard, and it is rational to assign credence 1 to the dart failing to land on point X. Then dartboard cases can be taken to show that there is a difference between assigning credence 1 to a proposition and being *certain* that it is true. Even if it is rational to assign credence 1 to the dart failing to land on X, it is intuitively not rational to be certain that it will fail to do so. For, from our point of view, it *might*. Hence, it seems that there is something lacking in the Bayesian machinery – Bayesians need to improve their framework to make room for the representation of certainty, and to allow us to distinguish between certainty and credence 1. Now suppose that they manage to find a way to represent certainty using new and improved Bayesian machinery. There's a very natural way to modify RR-new to reflect this:

RR-new*: For any proposition p and any agent S, it ought to be that S is certain that p only if p is true in all a priori possibilities in which S is certain that p.

Of course, it remains open whether Bayesians will succeed in designing such new machinery. But suppose that they do so. Then, given that there is a distinction between credence 1 and certainty, RR-new* seems to do justice to the spirit of RR. For it is not too much of a stretch to think that proponents of RR really have – or should have – certainty and not

[18] Of course, you may be perfectly happy not keeping to the spirit of RR. You may reject all the three reasons that I went through in section 1 for subscribing to RR. And unlike typical proponents of RR, you may be happy with a weak version of RR that does not prohibit us from assigning credence 1 to a vast swath of contingent or a posteriori propositions. In such a case, I won't say that your version of RR is false. After all, the weaker a claim, the more likely that it is true. But insofar as we want to explore a bolder and more interesting version of RR that prohibits us from assigning credence 1 to a vast swath of contingent and a posteriori propositions, but that nonetheless avoids the problems that RR-logical and RR-metaphysical face, RR-new is the way to go.

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merely credence 1 in mind. (Perhaps they were just mistaken in conflating the two.) For, again, recall the three reasons adduced in favour of RR in §1. If there is a difference between credence 1 and certainty, then it seems that certainty – and not a mere credence of 1 – should go with incontrovertible evidence. Also, it is certainty in a proposition like (Asteroid) – and not a mere credence of 1 – that seems to indicate a lack of open-mindedness. Finally, it is certainty that p – and not a mere credence of 1 in p – that should make you indifferent between accepting and rejecting a bet on which you gain nothing if p but lose something if not-p.

Let me set aside RR-new* for the rest of this paper, and focus on RR-new. If one day, we find new Bayesian machinery that gives us a way to represent certainty, and hence a way to distinguish between certainty and credence 1, we can replace RR-new with RR-new* to reflect such machinery.

4 RR-NEW AND STRICT COHERENCE

I've argued that RR-new is superior to the other versions of RR discussed in this paper. In this section, I shall show that an argument traditionally thought to support RR-logical actually does not; if anything, a modified version of the argument supports RR-new.

Shimony (1955) argues that your credence function is strictly coherent only if it is regular, while Kemeny (1955) argues that it is strictly coherent if it is regular. Carnap (1971) takes both Shimony and Kemeny to have *shown* that your credence function is strictly coherent if and only if it is regular. If Carnap is right, and if, like Carnap, you think that rationality demands strict coherence, then you should hold that rationality demands regularity. But strict coherence does not require regularity, if 'regularity' is understood in the way that proponents of RR-logical understand it. Or so I shall argue.

Recall that a credence function is strictly coherent just in case it does not make you susceptible to a Weak Dutch Book, i.e. a system of bets that will lead you to a net loss in at least one possible outcome, and no net gain in all other possible outcomes.[19] And according to Shimony, Kemeny and Carnap, regularity says that for any p and any $Cr(.)$ representing an agent's credence function, $Cr(p) = 1$ only if not-p is impossible.[20] Now let's consider a typical argument for the claim that strict coherence requires regularity. Following Shimony (1955: 16), suppose that for any p and $Cr(.)$ such that it is possible that not-p but $Cr(p) = 1$, an agent with the credence function $Cr(.)$ will be willing to accept the following bet on p, for any y: she gains nothing if she wins, but will lose y dollars if she loses.[21] Now if p is true, the agent will not gain anything. But if p is false, she will lose

[19] You may question the link between credences and betting dispositions that Shimony, Kememy, and Carnap seem to take for granted. But my aim is to show that, even if there is such a link, strict coherence does not demand regularity in the sense understood by proponents of RR-logical.

[20] Strictly speaking, according to them, regularity is the thesis that $Cr(p|q) = 1$ only if q logically implies p. But we may focus on the case in which q is a logical truth.

[21] Arguments from Kemeny (1955) and Carnap (1971) run along similar lines. Note that Shimony's argument applies originally to conditional credences. But I've adapted it so that it applies to unconditional credences. The point that I make about the adapted argument should apply to his original argument as well. Note also that Shimony (1955) uses the term 'coherent' to describe a credence function that is a probability function and that does not leave one susceptible to a Weak Dutch Book. The term 'strict coherence' is Carnap's.

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something. So, according to Shimony, if an agent assigns credence 1 to p when it is possible that not-p, she falls short of strict coherence. For she is susceptible to accepting a set of bets in which she is bound not to win anything but might lose something.[22]

Shimony's argument is unsound. According to him:

> (i) For any p such that it is possible that not-p and for any agent S such that S's credence in p is 1, if not-p and S's credence of 1 in p leads her to bet on p in the way described, she will lose the bet.

But it's too quick to move from (i) to the claim that for any agent S and any p such that it is possible that not-p, S's having a credence of 1 in p might lead her to a losing bet. For there might be some p and some agent S such that it's *impossible* for the antecedent of the conditional in (i) to be true. In order for his argument to go through, Shimony needs the following claim to be true as well:

> (ii) For any p such that it is possible that not-p and for any agent S such that S's credence in p is 1, it's possible both that not-p and that S's credence of 1 in p leads her to bet on p in the way described.

Given (ii), and given that the conditional in (i) is understood to be a strict conditional, we may indeed infer that for any agent S and any p such that it is possible that not-p, S's having a credence of 1 in p might lead her to a losing bet. But is (ii) true?

No. For even if it is possible that not-p, it is not possible for an omniscient God to bet on p and for p to be false. And if p is the proposition that you exist, then it is not possible for you to bet on p and for p to be false, even if it is possible that you don't exist. You may protest that (ii) is true if we understand 'possibility' to mean logical possibility in the narrow sense mentioned earlier. For example, it is not a violation of the laws of classical logic for (Asteroid) to be false, but for God to assign credence 1 to (Asteroid) being true. And it is not a violation of the laws of classical logic for you to assign credence 1 to your own existence but for you not to exist. In this sense, it is possible for God to lose a bet on (Asteroid) and for you to lose a bet on your own existence. But it is implausible in either case that there is any violation of rationality: the link between rationality and the possibility of a loss in the weak sense under consideration is tenuous. To restore the link between rationality and the possibility of a loss, it seems that we would do better if we understand 'possibility' to mean something stronger like a priori possibility. But if we understand 'possibility' in this way, then (ii) is false.

In sum, strict coherence does not require regularity, if regularity is understood as the claim that if $Cr(p) = 1$, then p is (a priori) necessary. However, suppose we understand regularity as the claim that if $Cr(p) = 1$, then it is impossible that $Cr(p) = 1$ and p is false. A failure of regularity in this sense *does* lead to a failure of strict coherence – it does leave one susceptible to a Weak Dutch Book. For suppose there is some possibility in which $Cr(p) = 1$ but not-p. Then $Cr(p)$ will lead one to a system of bets on p that will result in a net loss in least one possible outcome – namely, a case in which $Cr(p) = 1$ but not-p – and no net gain in all other possible outcomes.

It might be worth noting that Shimony (1955) also argues that the converse of RR is true, but there is no parallel argument that shows that the converse of RR-new is true.

[22] Let us assume that agents do not desire to lose the bets they make. This assumption is often made implicitly in Dutch Book Arguments.

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According to Shimony, rationality requires that if it is impossible that not-*p*, then *Cr(p)* = 1. The thought is that a positive credence in not-*p* when not-*p* is impossible would lead one to accept a bet on not-*p* that guarantees a loss. But rationality does not require that if it is impossible both that *Cr(p)* = 1 and that not-*p*, then *Cr(p)* = 1. For there will be some *p* and *Cr(.)* such that it is impossible both that *Cr(p)* = 1 and that not-*p*, but possible both that *Cr(p)* < 1 and that not-*p*. Suppose, for example, that *Cr(.)* stands for God's credence function, or that *p* stands for 'I have credence 1 in this proposition'. In such cases, if a positive credence in not-*p* leads one to bet on not-p, the bet will not guarantee a loss.[23]

5 CONCLUSION

Regularity is thought to encapsulate the norm that our credences ought to reflect the strength of our evidence, to embody a kind of open-mindedness and to help us eschew making pragmatically blameworthy decisions. That is all well and good; unfortunately, various versions of RR are problematic. In this paper, I have formulated yet another version of RR, namely, RR-new, that I claim avoids most – if not all – of the problems that various other versions of RR face. I have also argued that an argument traditionally thought to support RR, as interpreted by the likes of Shimony (1955), Kemeny (1955) and Carnap (1971), actually does not; if anything, a modified version of the argument supports my interpretation of it instead. Unfortunately, dartboard cases remain a problem for RR-new. But if we take them to show that there is a difference between credence 1 and certainty, and if new Bayesian machinery can be devised that will allow us to represent certainty, we can easily tweak RR-new to reflect such machinery.

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[23] I'm indebted to an anonymous reviewer for the point discussed in this paragraph, and for the example of a proposition *p* which is such that it's impossible both that *Cr(p)* = 1 and that not-*p*, but possible both that *Cr(p)* < 1 and that not-*p*.

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WENG HONG TANG is assistant professor of philosophy at the National University of Singapore. His areas of research include epistemology and the philosophy of probability.

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