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Letter to the Editor

A new electromagnetic wave in a pair plasma

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Abstract. A new nonlinear electromagnetic wave mode in a plasma is reported. Its existence depends on the interaction of an intense circularly polarized electromagnetic wave with a plasma, where quantum electrodynamical photon–photon scattering is taken into account. As an illustration, we consider a pair plasma and show that the new mode can be significant in astrophysical settings and in the next generation of laser–plasma systems.

The concept of photon–photon scattering, which is due to the interaction of photons with virtual electron–positron pairs in quantum electrodynamics (QED), has attracted attention in a number of recent publications (see, e.g., [1–4] and references therein). Although it is a weak effect, it is believed to be of interest for highly magnetized neutron stars [5], giving rise to frequency down-shifting [6] and lensing [7]. Moreover, it has been suggested that the next generation of laser–plasma systems would be able to produce field strengths close to the Schwinger field $\sim 10^{18}$ V m^{-1} [8], at which the vacuum becomes fully nonlinear, and photon–photon scattering then has to be taken into account for a full understanding of the laser–plasma dynamics.

Lasers are currently reaching powers of 10^{21}–10^{22} W cm^{-2} [9,10], and electrons in such fields can therefore attain highly relativistic quiver velocities. A plasma under the influence of such intense lasers will be highly nonlinear and the relativistic dynamics of the laser–plasma system will thus be very complex, giving rise to, for example, soliton formation and pulse collapse [11]. It has been noted that the latter of these effects could be of interest for generating electromagnetic field intensities above the Schwinger limit [8, 12–14]. Furthermore, laser pulse collapse and subsequent electron expulsion due to the immense ponderomotive force is able to create plasma channels in which ultra-high-intensity field strengths are reached [15], such that the nonlinear vacuum effect of elastic photon–photon scattering could become important [2].

In pulsar surroundings, pair production and pair plasmas play an important role in the dynamics of these environments (see, e.g., [16–18]). Charged particles will attain relativistic energies close to the pulsar magnetic poles and radiate γ-ray photons. This, together with the super-strong magnetic fields present around these objects [16], is believed to produce a pair plasma [19]. Thus, nonlinear QED effects are already known to be an important ingredient for pulsar physics. Since the pair

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plasma gives rise to radio wave emissions, and because of the large energy scales involved, pulsar atmospheres are likely to host other QED effects as well, such as vacuum nonlinearities in the form of photon–photon scattering.

An effective theory of photon–photon scattering can be formulated in terms of the Heisenberg–Euler Lagrangian [20, 21]. When a dilute plasma is present, the nonlinearities associated with QED are normally many orders of magnitude smaller than those due to the classical plasma medium. However, in the present paper we will show that the QED processes may, for certain parameters, give a significant contribution even in the presence of a plasma. In particular, we shall investigate the propagation of circularly polarized electromagnetic waves in a multi-component plasma. The QED effects then give rise to a novel low-frequency mode, which can be described by a nonlinear dispersion relation. The significance of our results will be discussed together with various applications.

According to QED, the non-classical phenomenon of photon–photon scattering can take place due to the exchange of virtual electron–positron pairs. This gives rise to vacuum polarization and magnetization currents. In terms of the Heisenberg–Euler Lagrangian density [20, 21] we have

$$\mathscr{L}_{\rm EH} = \epsilon_0 \mathscr{F} + \xi(4\mathscr{F}^2 + 7\mathscr{G}^2), \tag{1}$$

where $\xi \equiv 2\alpha^2\epsilon_0^2\hbar^3/45m_{\rm e}^4c^5$, $\mathscr{F} \equiv \frac{1}{2}(E^2 - c^2B^2)$, $\mathscr{G} \equiv c\mathbf{E}\cdot\mathbf{B}$, $\alpha = e^2/4\pi\epsilon_0\hbar c$ is the fine structure constant, ϵ_0 is the vacuum permittivity, e is the magnitude of the electron charge, c the speed of light, $2\pi\hbar$ the Planck constant and $m_{\rm e}$ the electron mass. The last terms in (1) represent the effects of vacuum polarization and magnetization, and is the result of a perturbative expansion of the full Heisenberg–Euler Lagrangian. The fields are therefore formally constrained by $\omega \ll \omega_{\rm e}$ and $|\mathbf{E}| \ll E_{\rm crit}$, where $\omega_{\rm e} = c/\lambda_{\rm e} = m_{\rm e}c^2/\hbar \simeq 8\times10^{20}\,{\rm s}^{-1}$ and $E_{\rm crit} = m_{\rm e}^2c^3/e\hbar \sim 10^{18}\,{\rm V\,m}^{-1}$, $\lambda_{\rm e} \approx 4\times10^{-13}$ m being the Compton wavelength and $E_{\rm crit}$ the Schwinger field strength. However, it can be shown that the Lagrangian (1) is valid up to field strengths surpassing the Schwinger field, as long as $c|\mathbf{B}| \geqslant |\mathbf{E}|$ in order to avoid pair creation [22–24]. The QED corrected Maxwell's vacuum equations take the classical form using $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$ and $\mathbf{H} = (\mathbf{B}/\mu_0) - \mathbf{M}$, where $\mathbf{P}$ and $\mathbf{M}$ are of third order in the field amplitudes $\mathbf{E}$ and $\mathbf{B}$, as shown below in (3a) and (3b), respectively, and $\mu_0 = 1/c^2\varepsilon_0$.

In a medium with polarization $\mathbf{P}$ and magnetization $\mathbf{M}$ the general wave equations for $\mathbf{E}$ and $\mathbf{B}$ are

$$\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \nabla^2\mathbf{E} = -\mu_0\left[\frac{\partial^2\mathbf{P}}{\partial t^2} + c^2\nabla(\nabla\cdot\mathbf{P}) + \frac{\partial}{\partial t}(\nabla\times\mathbf{M})\right], \tag{2a}$$

and

$$\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} - \nabla^2\mathbf{B} = \mu_0\left[\nabla\times(\nabla\times\mathbf{M}) + \frac{\partial}{\partial t}(\nabla\times\mathbf{P})\right]. \tag{2b}$$

Furthermore, the effective polarization and magnetization in vacuum due to photon–photon scattering induced by the exchange of virtual electron–positron pairs are given by (see, e.g., [2])

$$\mathbf{P} = 2\xi[2(E^2 - c^2B^2)\mathbf{E} + 7c^2(\mathbf{E}\cdot\mathbf{B})\mathbf{B}], \tag{3a}$$

and

$$\mathbf{M} = -2c^2\xi[2(E^2 - c^2B^2)\mathbf{B} - 7(\mathbf{E}\cdot\mathbf{B})\mathbf{E}]. \tag{3b}$$

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For circularly polarized electromagnetic waves propagating in a cold multicomponent plasma rather than in vacuum, the wave operator on the left-hand sides of (2a) and (2b) is replaced by

$$\Box \to \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{\omega_{\rm p}^2}{c^2} \to \frac{-\omega^2 + \omega_{\rm p}^2}{c^2} + k^2, \tag{4}$$

where we have assumed that the EM fields vary as $\exp(ikz - i\omega t)$. Due to the high symmetry of the circularly polarized EM waves, most plasma nonlinearities cancel, and the above substitution therefore holds for arbitrary wave amplitudes, provided that the plasma frequency depends on the electromagnetic amplitude through $\omega_{\rm p} = (\sum_j n_{0j} q_j^2/\epsilon_0 m_j \gamma_j)^{1/2}$, where the sum is over particle species, n_{0j} denotes the particle density in the laboratory frame, and the relativistic factor of each particle species is $\gamma_j = (1 + q_j^2 E_0^2/m_j^2 c^2 \omega^2)^{1/2}$, where E_0 denotes the absolute value of the electric field amplitude [25, 26].

Next, we investigate the regime $\omega^2 \ll k^2 c^2$. From Faraday's law and the above inequality we note that the dominating QED contribution to (2b) comes from the term proportional to $B^2\mathbf{B}$. Combining (2b) and (3b), noting that $B^2 = B_0^2 = k^2 E_0^2/\omega^2$ is constant for circularly polarized EM waves, and using $\omega^2 \ll k^2 c^2$, i.e. $\mathbf{M} \simeq 4c^4 \xi B^2 \mathbf{B}$ and $|\mathbf{M}| \gg \omega |\mathbf{P}|/k$, we thus obtain from (2b) the nonlinear relation

$$\omega^2 = \frac{2\alpha}{45\pi}\left(\frac{E_0}{E_{\rm crit}}\right)^2 \frac{k^4 c^4}{\omega_{\rm p}^2 + k^2 c^2}. \tag{5}$$

This low-frequency mode makes the particle motion ultra-relativistic even for rather modest wave amplitudes. For electrons and positrons in ultra-relativistic motion with equal densities n_0 and elementary charge $\pm e$, we thus use the approximation $\omega_{\rm p}^2 \simeq 2en_0 c\omega/\epsilon_0 E_0 = 2\omega_{\rm p0}^2(\omega/\omega_{\rm e})(E_{\rm crit}/E_0)$, where $\omega_{\rm p0} = (e^2 n_0/\epsilon_0 m_{\rm e})^{1/2}$. The relation (5) then reduces to

$$\omega^3 = \frac{\alpha}{45\pi}\left(\frac{\omega_{\rm e}}{\omega_{\rm p0}}\right)\left(\frac{E_0}{E_{\rm crit}}\right)^3 \frac{k^4 c^4}{\omega_{\rm p0} + (E_0/E_{\rm crit})(kc\omega_{\rm e}/2\omega\omega_{\rm p0})kc}. \tag{6}$$

We note that the ratio $\omega_{\rm e}/\omega_{\rm p0}$ is much larger than unity for virtually all plasmas, i.e. for electron densities up to $\sim 10^{38}$ m^{-3}. In some applications, such as in pulsar astrophysics, it is convenient to re-express the dispersion relation in terms of the relativistic gamma factor using $E_0/E_{\rm crit} \simeq (\omega/\omega_{\rm e})\gamma$. Thus, we obtain

$$\lambda = \frac{2}{3}\gamma\lambda_e\sqrt{\frac{4}{5}\pi\alpha}\left[1 + \sqrt{1 + \frac{16\alpha}{45\pi}\left(\frac{\omega_{\rm p0}}{\omega_{\rm e}}\right)^2\gamma}\right]^{-1/2} \tag{7}$$

from (6) for the wavelength $\lambda = 2\pi/k$.

Astrophysical environments may exhibit extreme fields. Neutron stars have surface magnetic field strengths of the order of 10^{10}–10^{13} G, while magnetars can reach 10^{14}–10^{15} G [5], coming close to energy densities corresponding to the Schwinger limit; here, the quantum vacuum becomes fully nonlinear. Single-particle QED effects, such as photon splitting can play a significant role in the understanding and interpretation of observations from these extreme systems [6, 27]. In fact, the pair plasma creation in pulsar environments itself rests on nonlinear QED vacuum effects. The emission of short wavelength photons due to the acceleration of plasma particles close to the polar caps results in a production of electrons and

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positrons as the photons propagate through the pulsar intense magnetic field [16]. The precise density of the pair plasma created in this fashion is difficult to estimate, and the answer is model dependent. However, given the Goldreich–Julian density $n_{\rm GJ} = 7 \times 10^{15}(0.1\ {\rm s}/P)(B/10^8\ {\rm T})\ {\rm m}^{-3}$, where P is the pulsar period and B the pulsar magnetic field, the pair plasma density is expected to satisfy $n_0 = Mn_{\rm GJ}$, M being the multiplicity [16, 28]. The multiplicity is determined by the model through which the pair plasma is assumed to be created, but a moderate estimate is $M = 10$ [28]. Thus, with these pre-requisites, the density in a hot dense pair plasma is of the order $10^{18}\ {\rm m}^{-3}$, and the pair plasma experiences a relativistic factor $\sim 10^2$–10^3 [18]. Using these estimates in (7), we obtain $\lambda \sim 10^{-12} - 10^{-11}$ m. On the other hand, the primary beam will have $n_0 \sim n_{\rm GJ}$ and $\gamma \sim 10^6$–10^7 [18], at which (7) yields $\lambda \sim 10^{-8}$–10^{-7} m. However, we note that the pulsar stationary magnetic field and relativistic temperature have been neglected in the derivation of (6), and the results could therefore be modified accordingly. In particular, it is possible to take the effects of the temperature T partially into account by substituting for $m_{\rm e}$ an effective mass $m(T)$, thus including the thermal contribution [29].

At the laboratory level, current high laser powers are able to accelerate particles to highly relativistic speeds. Furthermore, pulse self-compression in laser–plasma systems may play an important role in attaining power levels well above current laser limits (see, e.g., [8, 30]). As the intensities approach the Schwinger limit in future laser–plasma setups, effects of pair-creation and photon–photon scattering have to be taken into account [8, 31]. Laser–plasma systems can have electron densities of the order $10^{26}\ {\rm m}^{-3}$, and laser intensities can be close to 10^{23}–$10^{25}\ {\rm W\,cm}^{-2}$ [9, 13]. Moreover, as stated in [8], laser self-focusing in plasmas could reach $E_{\rm crit}$, at which pair creation is likely to follow. In fact, it has been estimated that the National Ignition Facility would be able to produce pairs by direct irradiation of a deuterium pellet [32]. On the other hand, the creation of laboratory electron–positron plasmas is already a feasible task [33, 34], as is the usage of these plasmas for making pair-plasma experiments [35]. Thus, the possibility to study laser–plasma interactions in pair plasmas could be a reality in the nearby future. The currently available positron densities in the laboratories are well below those of regular laser–plasma systems, but according to [32] there is a possibility of reaching densities of order $10^{27}\ {\rm m}^{-3}$. Thus, using $n_0 \sim 10^{26}\ {\rm m}^{-3}$, and the field intensity $10^{16}\ {\rm V\,m}^{-1}$ (due to laser self-compression [8]) at the wavelength 0.3×10^{-6} m, we find from (6) that $\omega \simeq 7.8 \times 10^5\ {\rm rad\,s}^{-1}$, i.e. the frequency is in the low-frequency band.

To summarize, we have presented a new low-frequency circularly polarized electromagnetic wave in an electron–positron plasma, taking into account the combined action of relativistic charged particle mass variation and the QED effect involving photon–photon scattering. The wave frequency strongly depends on the wave amplitude. The present mode is shown to play an important role in astrophysical settings as well as in the next generation of laser–plasma systems.

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