J. Plasma Physics (2011), *vol.* 77, *part* 2, *pp.* 271–276. © Cambridge University Press 2010
doi:10.1017/S0022377810000139

On the drag force on non-spherical dust grain

S. I. KRASHENINNIKOV and D. A. MENDIS

University of California, San Diego, La Jolla, CA 92093, USA
(skrash@mechanics.ucsd.edu)

(Received 20 January 2010; revised 28 January 2010 and accepted 8 February 2010,
first published online 10 March 2010)

Abstract. It is shown that the direction of the drag force imposed on non-spherical grain by plasma flow is not necessarily aligned with the flow velocity. The implications of this fact for a grain dynamics are discussed.

Model

Dust plays an important role in different environments ranging from laboratory experiments (e.g. low temperature and fusion plasmas) to space [1–11]. However, practically, all theoretical studies of dust in plasmas deal or assume spherical shape of the dust grains. Although there are some works considering the impact of irregular shape of dust on grain charging and ion scattering, so far there is no analysis on the impact of the shape of dust grain on the drag force, which plays a vital role in many applications. In an early work Jeans [12] considered the distribution of surface charges on perfectly conducting grains assuming them to be prolate or oblate spheroids. In [13] it was shown that the non-uniform electrical stress on non-spherical conducting bodies could lead to their electrostatic erosion. More recently, in [14] the impact of irregular shape of the grain on grain charging and ion scattering by the grain were considered numerically for a small Knudsen number by using diffusive approximation of both ion and electron motion. In [15] the role of irregular grain shape on the torque imposed by plasma flow was discussed, and in [16] the effect of non-spherical grain shape on grain charging in collisionless plasma was treated numerically.

In this brief communication we consider the simplest possible version of the impact of grain shape on the direction of the drag force and show that it is not necessarily aligned with plasma flow velocity. We also discuss some implications of this fact.

Let us consider a non-spherical stationary grain idealized as a prolate spheroid having major and minor diameters a and b correspondingly, which is situated in the plasma flowing with the velocity $\vec{u}$. The grain orientation can be characterized by the vector $\vec{\mathbf{d}}$ such that $|\vec{\mathbf{d}}| = (a-b)/(a+b)$ (see Fig. 1). Then there are only two vectors, which can be related to the direction of the drag force imposed by the plasma flow, they are $\vec{\mathbf{u}}$ and $\vec{\mathbf{d}}$, although the change of the sign of $\vec{\mathbf{d}}$ should not affect the direction of the drag force due to symmetry of spheroidal grain. Taking into account that at small $|\vec{\mathbf{u}}|$, the magnitude of the drag force, $\vec{\mathbf{F}}_{\mathrm{drag}}$, should be linearly proportional to $|\vec{\mathbf{u}}|$, we arrive at the following expression

$$\vec{\mathbf{F}}_{\mathrm{drag}} = C_1\vec{\mathbf{u}} + C_2\vec{\mathbf{d}}(\vec{\mathbf{d}} \times \vec{\mathbf{u}}), \tag{1}$$

https://doi.org/10.1017/S0022377810000139 Published online by Cambridge University Press

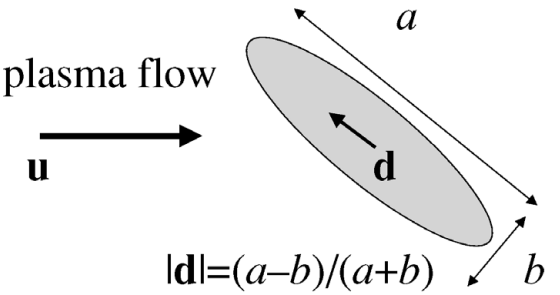


Figure 1. The orientation of spheroidal grain with respect to plasma flow velocity.

where C_1 and C_2 depend on plasma parameters, grain size and Much number. As we see, the second term in (1) is not necessarily aligned with plasma flow velocity $\vec{\mathbf{u}}$. The physical reason for this force is somewhat similar to the "sail" force imposed on the sail airflow. We notice that changing the sign of $\vec{\mathbf{d}}$ in (1) does not change $\vec{\mathbf{F}}_{\text{drag}}$.

In order to access the value of coefficient C_2 with respect to C_1, we consider the model potential of interactions of plasma electron and ions with the grain, $U(\vec{\mathbf{r}})$:

$$U(\vec{\mathbf{r}}) = \frac{U_0}{\mathrm{r}} \left\{ 1 + \left(\frac{\vec{\mathbf{r}} \cdot \vec{\mathbf{d}}}{r} \right)^2 \right\}, \tag{2}$$

where $\vec{\mathbf{r}}$ is the radius vector of electron/ion with respect to the grain and U_0 is the normalization constant.

To calculate the drag force imposed by plasma flow on stationary grain, we need to solve the equation of motion of plasma ions (impact of electrons can be neglected) in potential (2)

$$M\frac{d\vec{\mathbf{V}}}{dt} \equiv M\frac{d^2\vec{\mathbf{r}}}{dt^2} = -\nabla U(\vec{\mathbf{r}}) \equiv U_0 \left\{ \frac{\vec{\mathbf{r}}}{r^3} + \left(3\frac{\vec{\mathbf{r}}(\vec{\mathbf{r}} \cdot \vec{\mathbf{d}})^2}{r^5} - 2\frac{\vec{\mathbf{d}}(\vec{\mathbf{r}} \cdot \vec{\mathbf{d}})}{r^3} \right) \right\}, \tag{3}$$

(where M and $\vec{\mathbf{V}}$ are the ion mass and velocity) and find the deflection of the ion velocity, $\delta\vec{\mathbf{V}}$, caused by interactions with the grain

$$\delta\vec{\mathbf{V}} = -\int_{-\infty}^{\infty} \nabla U\left(\vec{\mathbf{r}}(t)\right) dt/M. \tag{4}$$

However, solution of ion motion equation with non-spherically symmetric potential (2) for a general case requires significant effort and goes beyond the purpose of this paper to demonstrate just the effect of irregular shape of the grain on the direction of the drag force. Therefore, we will consider (3) assuming that $|\vec{\mathbf{d}}| \ll 1$. Then the non-spherical part of the force in (3) is small and can be considered as a perturbation to a well-known problem of a charged particle scattering in a Coulomb potential

$$M\frac{d^2\vec{\mathbf{r}}}{dt^2} = U_0\frac{\vec{\mathbf{r}}}{r^3}. \tag{5}$$

https://doi.org/10.1017/S0022377810000139 Published online by Cambridge University Press

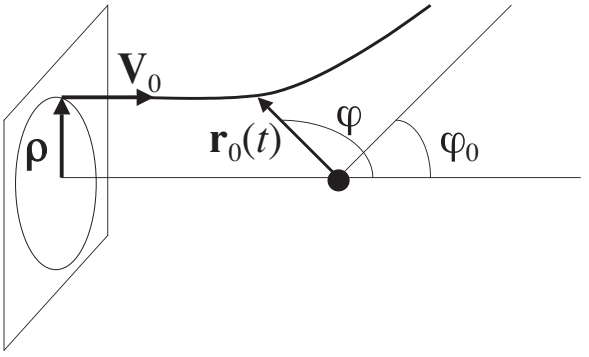


Figure 2. Schematic view of particle scattering by spherical potential.

Specifying initial velocity, $\vec{\mathbf{V}}_0$, and an impact parameter, $\vec{\boldsymbol{\rho}}$ ($\vec{\boldsymbol{\rho}} \cdot \vec{\mathbf{V}}_0 = 0$) of the ion, we can solve (5) and find ion trajectory in a zero-order approximation, $\vec{\mathbf{r}}_0(t)$ (see Fig. 2). However, the scattering by the spherically symmetric potential gives the resulting drag force in the direction of plasma flow velocity. To find the impact of non-spherical part of the potential on the deflection of the ion velocity, $\delta\vec{\mathbf{V}}_{\vec{\mathbf{d}}}$, we can use trajectory $\vec{\mathbf{r}}_0(t)$, which gives

$$\delta\vec{\mathbf{V}}_{\vec{\mathbf{d}}} = \frac{U_0}{M}\int\limits_{-\infty}^{\infty}\left(3\frac{\vec{\mathbf{r}}_0(\vec{\mathbf{r}}_0\cdot\vec{\mathbf{d}})^2}{r_0^5} - 2\frac{\vec{\mathbf{d}}(\vec{\mathbf{r}}_0\cdot\vec{\mathbf{d}})}{r_0^3}\right)dt. \tag{6}$$

As we are interested in the force imposed by scattered ions on the grain, we need to average $\delta\vec{\mathbf{V}}_{\vec{\mathbf{d}}}$ over the direction of the impact vector $\vec{\boldsymbol{\rho}}$ in the plane normal to the velocity $\vec{\mathbf{V}}_0$. Expanding $\vec{\mathbf{r}}_0(t)$ along vectors $\vec{\mathbf{V}}_0$ and $\vec{\boldsymbol{\rho}}$, $\vec{\mathbf{r}}_0(t) = \eta_v(t)\vec{\mathbf{V}}_0 + \eta_\rho(t)\vec{\boldsymbol{\rho}}$, after some algebra we find the terms which are not aligned with $\vec{\mathbf{V}}_0$

$$\left\langle\delta\vec{\mathbf{V}}_{\vec{\mathbf{d}}}\right\rangle_{\vec{\boldsymbol{\rho}}} = \vec{\mathbf{d}}(\vec{\mathbf{d}}\cdot\vec{\mathbf{V}}_0)\frac{U_0}{M}\int\limits_{-\infty}^{\infty}\left(3\frac{\rho^2\eta_\rho^2\eta_v}{r_0^5} - 2\frac{\eta_v}{r_0^3}\right)dt. \tag{7}$$

As we see from (7), the averaged deflection of ion velocity depends on the orientation of the grain. This is consistent with the results of [14], even though in [14] the diffusive motion of plasma ions and electrons was assumed.

To proceed with the evaluation of the integrals in (7) we note (see Fig. 2) that

$$\eta_\rho\,\rho = r_0\sin\varphi, \quad \eta_v V_0 = r_0\cos\varphi, \quad \text{and} \quad \frac{d\varphi}{dt} = -\frac{\rho V_0}{r_0^2}. \tag{8}$$

As a result, from (7) and (8) we find

$$\left\langle\delta\vec{\mathbf{V}}_{\vec{\mathbf{d}}}\right\rangle_{\vec{\boldsymbol{\rho}}} = \vec{\mathbf{d}}(\vec{\mathbf{d}}\cdot\vec{\mathbf{V}}_0)\frac{U_0}{M}\left(\frac{2\sin\varphi_0 - \sin^3\varphi_0}{\rho V_0^2}\right), \tag{9}$$

where φ_0 is the scattering angle in spherically symmetric part of the potential, which is determined by the well-known relations (e.g. see [17]).

In (9) the first term in the brackets describes contribution of small angle scattering, $\varphi_0 = 2U_0/\rho M V_0^2 \ll 1$, which dominates when the size of the grain, R_d, is much smaller than the Debye length, λ_D, which we will assume to be the case. Then for

https://doi.org/10.1017/S0022377810000139 Published online by Cambridge University Press

$|U_0|/\rho \ll MV_0^2$ from (9) we find

$$\left\langle \delta \vec{\mathbf{V}}_{\vec{\mathbf{d}}} \right\rangle_{\vec{\rho}} = \vec{\mathbf{d}}(\vec{\mathbf{d}} \cdot \vec{\mathbf{V}}_0) \left(\frac{2U_0}{\rho M V_0^2} \right)^2. \tag{10}$$

Taking into account different impact parameters, the contribution of non-spherically symmetric potential to the drag force on the grain, $\vec{\mathbf{F}}_{\vec{\mathbf{d}}}$, which is not aligned with ion velocity, $\vec{\mathbf{V}}_0$, can be written as follows:

$$\vec{\mathbf{F}}_{\vec{\mathbf{d}}} = -M n_{\vec{\mathbf{V}}_0} V_0 2\pi \int_{\rho_{\min}}^{\rho_{\max}} \left\langle \delta \vec{\mathbf{V}}_{\vec{\mathbf{d}}} \right\rangle_{\vec{\rho}} \rho d\rho = -\vec{\mathbf{d}}(\vec{\mathbf{d}} \cdot \vec{\mathbf{V}}_0) \frac{8\pi \Lambda U_0^2}{M V_0^3} n_{\vec{\mathbf{V}}_0}, \tag{11}$$

where Λ is the Coulomb logarithm and $n_{\vec{\mathbf{V}}_0}$ is the density of ions with velocity $\vec{\mathbf{V}}_0$. Comparing (11) with the corresponding force along $\vec{\mathbf{V}}_0$ (e.g. see [18])

$$\vec{\mathbf{F}}_{\vec{\mathbf{V}}_0} = \vec{\mathbf{V}}_0 \frac{4\pi \Lambda U_0^2}{M V_0^3} n_{\vec{\mathbf{V}}_0}, \tag{12}$$

we find that in case ion–grain interaction potential is described by (2), the ratio $C_2/C_1 = -2$. Therefore, in this case (1) the drag force, which can be found by the averaging (11) and (12) over ion distribution function, can be written as

$$\vec{\mathbf{F}}_{\text{drag}} = C \left(\vec{\mathbf{u}} - 2\vec{\mathbf{d}}(\vec{\mathbf{d}} \cdot \vec{\mathbf{u}}) \right), \tag{13}$$

where C is the function of plasma parameters, Much number, and grain size.

Thus, using a model potential (2) we confirm that the structure of the expression for the drag force imposed by plasma flow on elliptical grain agrees with general expression (1). Moreover, relative contribution of spherical and non-spherical part of (2) into drag force for $R_d \ll \lambda_D$ and $|\vec{\mathbf{d}}| \sim 1$ is comparable. However, in practice at large distances from the grain with $R_d \ll \lambda_D$ the non-spherical part of actual Coulomb potential will decrease faster than the spherical one. It means that the contribution of non-spherical part to the drag force will be limited by large angle scattering and absorption of ion momentum caused by ion–grain collisions. As a result, for $R_d \ll \lambda_D$ we have $|C_2/C_1| = 1/\Lambda < 1$, although, for $R_d \gtrsim \lambda_D$ and $|\vec{\mathbf{d}}| \sim 1$ we expect $|C_2/C_1| \sim 1$.

So far we consider plasma–grain drag force assuming complete absorption of ion momentum by the grain during ion–grain collisions. However, in practice it is not the case and this effect can also be important for our consideration. For example, for the hydrogenic ion energies typical for fusion plasmas (~few tenths of eV), a significant fraction (~50% [19]) of the ions are reflected from the surface as neutrals with the energies comparable (~50% [19]) to the energy of impinging ions. The rest of ions can be thermally desorbed later. As a result, energetically reflected neutrals will carry significant momentum (somewhat comparable to the momentum of impinging ions), which should be accounted for in the calculation of the drag force. For a non-spherical grain it can result in the component of friction force, the direction of which is not aligned with the plasma velocity and also causing "lifting" force effect.

In addition to the "lifting" force, the synergy of plasma flow and irregular shape of the grain can result in a torque, $\vec{\tau}_d$, acting on the grain [15] (although there are other mechanisms which can result in a torque on even spherical grain,

https://doi.org/10.1017/S0022377810000139 Published online by Cambridge University Press



Figure 3. Schematic view of "dithering" of trajectory of irregular grain.

e.g. see [20]). Therefore, grain will spin and the direction of the "lifting" force will change in time. It will cause the "dithering" of grain trajectory (see Fig. 3), which in laboratory experiments can be monitored with fast cameras. However, the amplitude of "dithering" will decrease with the increase of angular velocity, $\vec{\Omega}_{\rm d}$, of the grain spinning and for $\Omega_d \to \infty$ there is an effective "symmetrization" of the grain shape. In case the effect of torque orients grain in a way that $\vec{\tau}_{\rm d} = 0$ (if such orientation is available and stable, e.g. in case of "propeller"-like grain $\vec{\tau}_{\rm d} \neq 0$), the "lifting" force, which still exists, will simply push grain in the direction perpendicular to plasma flow. The details of the dynamics of irregular rotating grain immersed in plasma flow will be discussed elsewhere.

In conclusion, we show that the direction of the drag force imposed on non-spherical grain by plasma flow is not necessarily aligned with flow velocity. This fact, which can significantly alter the dynamics of non-spherical grain in plasma, should be taken into account.

Acknowledgements

This work is performed under auspices of USDOE by the grant DE-FG02-04ER54852 at UCSD.

References

[1] Merlino, R. L. and Goree, J. A. 1994 *Phys. Today* **July**, 32.
[2] Selwynd, G. E., Heidenrich, J. E. and Haller, K. L. 1990 *Appl. Phys. Lett*. **46**, 1887.
[3] Mendis, A. and Rosenberg, M. 1994 *Ann. Rev. Astron. Astrophys.* **32**, 419.
[4] Tsytovich, V. N. 1997 *Phys. Usp.* **40**, 53.
[5] Grün, E., Gustafson, B. A. S., Dermott, S. and Fechtig, H. (eds) 2001 *Interplanetary Dust*. Berlin, Germany: Springer.
[6] Shukla, P. K. and Mamun, A. A. 2002 *Introduction to Dusty Plasma Physics*. Bristol, UK: Institute of Physics.
[7] Winter, J. 2004 *Plasma Phys. Control. Fusion* **46**, B583.
[8] Fortov, V. E. et al. 2005 *Physics Reports* **421**, 1.
[9] Krasheninnikov, S. I. et al. 2008 *Plasma Phys. Control. Fusion* **50**, 124054.
[10] Shukla, P. K. and Eliasson, B. 2009 *Rev. Mod. Phys.* **81**, 25.
[11] Morfill, G. E. and Ivlev, A. V. 2009 *Rev. Mod. Phys.* **81**, 1353.
[12] Jeans, J. H. 1927 *The Mathematical Theory of Electricity and Magnetizm*, Cambridge, UK: Cambridge Univ. Press, p. 246.
[13] Hill, J. R. and Mendis, D. A. 1981 *Can. J. Phys.* **59**, 897.
[14] Keiter, R. and Kushner, M. J. 1998 *J. Appl. Phys.* **83**, 5670.
[15] Tsytovich, V. N., Sato, N. and Morfill, G. E. 2003 *New J. Phys.* **5**, 1.
[16] Auer, S., Kempf, S. and Grun, E. 2007 *Computed Electric Charges of Grains with Highly Irregular Shapes*, Vol. 643. Workshop on dust in planetary systems. Noordwijk, The Netherlands: ESA Publications Division c/o ESTEC, p. 177.

https://doi.org/10.1017/S0022377810000139 Published online by Cambridge University Press

[17] Landau, L. D. and Lifshits, E. M. 2003 *Course of Theoretical Physics, Vol. 1 Mechanics.* Amsterdam, The Netherlands: Elsevier.

[18] Trubnikov, B. A. 1965 Particle Collisions in Fully Ionized Plasmas In: *Review of Plasma Physics*, Vol. 1 (ed. M. A. Leontovich). New York: Consultants Bureau.

[19] Eckstain, W. 1991 *Suppl. Nucl. Fusion* **1**, 17; Tahada, T. et al. 1985 *Nucl. Instum. Meth.* B **9**, 113.

[20] Sato, N. 2005 Spinning Motion of fine Particles in Plasmas In: *AIP Conference Proceeding 799*, New York: AIP, p. 97; Hutchinson, I. 2004 *New J. Phys.* **6**, 43; Krasheninnikov, S. I. 2006 *Phys. Plasmas* **13**, 114502; Krasheninnikov, S. I., Shevchenko, V. I. and Shukla, P. K. 2007 *Phys. Lett*. A **361**, 133.

https://doi.org/10.1017/S0022377810000139 Published online by Cambridge University Press