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Ionizing drift-wave instability in a low-temperature plasma

S. V. VLADIMIROV[1]† and LIN I[2]

[1] Special Research Centre for Theoretical Astrophysics, School of Physics, The University of Sydney, NSW 2006, Australia
[2] Department of Physics, National Central University, Chungli 32054, Taiwan ROC

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The dissipative drift-wave instability is considered in a non-uniform weakly ionized magnetoplasma when additional ionization is caused by electrons moving in the field of the wave. It is demonstrated that the instability can be considerably enhanced due to the wave-induced perturbations of the ionization/recombination balance and may therefore strongly affect transport properties of the plasma.

1. Introduction

It is well known (Kadomtsev 1965) that a dissipative instability of low-frequency electrostatic drift waves can develop in a non-uniform collisional plasma. The driving mechanism for the instability is the induced non-adiabaticity of plasma electrons due to their collisions when the electron–ion drag produces a channel for the free energy of plasma density gradients to be conveyed to drift waves in a dissipative plasma. The standard theory of the dissipative drift-wave instability (Kadomtsev 1965) assumes the plasma to be fully ionized or ignores possible changes in electron and ion number densities due to ionization/recombination processes.

However, at the edges of weakly ionized plasmas of low-temperature radiofrequency discharges, ionization (and, generally speaking, recombination) processes are very important (Chen 1984). Because of the relatively large amplitudes of waves observed in these plasmas (Lin and Wu 1987; Lin *et al.* 1995), they can affect the ionization process, which in turn influences their propagation characteristics (Vladimirov and Yu 1983). Therefore it is of significant interest to examine the influence of ionization/recombination processes on drift-wave dispersion and instability in non-uniform collisional magnetoplasmas.

Here we study the linear dispersion characteristics of drift waves, taking into account their perturbation of the plasma ionization process, and consider the dissipative drift-wave instability in this case. We demonstrate that the deviation from the electron Boltzmann distribution caused by collisions of plasma electrons with neutrals and perturbation of the ionization/recombination equilibrium can lead to dissipative instability of low-frequency (frequency lower than the ion cyclotron frequency) ion drift waves. At the same

† E-mail: S.Vladimirov@physics.usyd.edu.au; http://www.physics.usyd.edu.au/∼vladimi.

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time, a change in the ion number density due to the wave-induced perturbations of the ionization/recombination process leads to wave damping, and under certain conditions can effectively slow down the instability.

2. Analysis

Consider a weakly ionized plasma, non-uniform in the x direction, embedded in a z-directed homogeneous magnetic field $\mathbf{B}$. The electrostatic field $\mathbf{E} = -\boldsymbol{\nabla}\phi$, where ϕ is the wave potential, of frequency low compared with the ion cyclotron frequency $\Omega_i = eB/m_i c$, where $B = |\mathbf{B}|$, e and m_i are the ion charge and mass, and c is the speed of light, causes a drift of plasma particles (Chen 1984). Thus the components of the electron and ion velocities perpendicular to the magnetic field are given by

$$\mathbf{v}_{e\perp} \approx \frac{c}{B}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\phi - \frac{cT_e}{eBn_0}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\delta n_e, \tag{1}$$

$$\mathbf{v}_{i\perp} \approx \frac{c}{B}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\phi - \frac{c}{B\Omega_i}(\partial_t + \nu_{i\perp})\,\boldsymbol{\nabla}_\perp\phi. \tag{2}$$

Here $\hat{\mathbf{z}}$ is the unit vector along the z axis, $\delta n_e \ll n_0$ is the electron density perturbation of the background plasma density (we have $n_{e0} = n_{i0} = n_0$), T_e is the electron temperature, and $\nu_{i\perp}$ is the ion–neutral collision rate in the direction perpendicular to the external magnetic field (we assume $\nu_{i\perp} \ll \Omega_i$).

The parallel component of the linear ion fluid velocity is given by

$$(\partial_t + \nu_{iz})\,v_{iz} = -\frac{e}{m_i}\partial_z\phi, \tag{3}$$

while, under the assumption that the characteristic wave frequency is much smaller than the electron collision frequency, we obtain for the parallel component of the linear electron fluid velocity

$$v_{ez} = \frac{v_{Te}^2}{\nu_{ez}}\partial_z\left(\frac{e\phi}{T_e} - \frac{\delta n_e}{n_0}\right), \tag{4}$$

where $v_{Te} = (T_e/m_e)^{1/2}$ is the electron thermal velocity and $\nu_{e(i)z}$ is the electron (ion)–neutral collision frequency in the direction parallel to the magnetic field.

Substituting (1) and (4) into the electron continuity equation and taking into account a linear perturbation of the ionization/recombination process, we find

$$\partial_t\,\delta\tilde{n}_e + \mathbf{v}_D\cdot\boldsymbol{\nabla}\tilde{\phi} + \frac{v_{Te}^2}{\nu_{ez}}\partial_z^2(\tilde{\phi} - \delta\tilde{n}_e) = \frac{1}{n_0}\delta[n_e(N - n_e)\langle\sigma\mathbf{v}_e\rangle - \beta n_e^2 + \nu_{\mathrm{ext}}\,n_e]. \tag{5}$$

where

$$\delta\tilde{n}_e = \frac{\delta n_e}{n_0}, \qquad \tilde{\phi} = \frac{e\phi}{T_e},$$

$$\mathbf{v}_D = -\frac{cT_e}{eB}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\ln n_0$$

is the equilibrium drift velocity, and the right-hand side includes the density N of neutral particles, the electron–neutral collision cross-section σ, and the three-body recombination and external ionization rates β and ν_{ext} respectively; the

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angular brackets denote averaging over the directions of the electron velocity. For simplicity, we assume that the coefficients σ, β and $\nu_{\rm ext}$ are constants. In the linear approximation, using (1) and (4), we assume

$$\left|\frac{v_{e\perp}}{v_{ez}}\right| \sim \frac{\lambda_z}{\lambda_\perp}\frac{\nu_{ez}}{\Omega_e} \ll 1, \tag{6}$$

where Ω_e is the electron cyclotron frequency. Thus perturbations of the electron parallel velocity dominate, and for the right-hand side of (5) we obtain

$$\frac{1}{n_0}\delta[n_e(N-n_e)\langle\sigma\mathbf{v}_e\rangle-\beta n_e^2+\nu_{\rm ext}\,n_e] \approx v_{Te}\,\partial_z(\tilde{\phi}-\delta\tilde{n}_e)-\beta n_0\,\delta\tilde{n}_e. \tag{7}$$

Note that in the considered collisional regime when the electron–neutral collision rate is much higher than the characteristic frequency of the wave, but is restricted by the condition (6), the electron–neutral collisional frequency cancels out on the right-hand side of (7). However, it can be present for other regimes, for example when the inequality opposite to (6) holds.

Substituting for the ion velocity from (2) into the ion continuity equation, and taking into account that the electron ionization/recombination process equally affects ions (at the same time, we ignore the ion contribution to the plasma ionization), we find

$$\partial_t\,\delta\tilde{n}_i+\mathbf{v}_D\cdot\boldsymbol{\nabla}\tilde{\phi}-\rho_s^2(\partial_t+\nu_{i\perp})\,\nabla_\perp^2\,\tilde{\phi}+\partial_z\,v_{iz} = v_{Te}\,\partial_z(\tilde{\phi}-\delta\tilde{n}_e)-\beta n_0\,\delta\tilde{n}_e, \tag{8}$$

where $\delta\tilde{n}_i = \delta n_i/n_0 \ll 1$ is the normalized ion number density perturbation, and $\rho_s = (T_e/m_i)^{1/2}/\Omega_i$ is the ion Larmor radius at the electron temperature.

Equations (5) and (8) are closed using the quasineutrality condition $\delta\tilde{n}_e \approx \delta\tilde{n}_i \equiv \delta\tilde{n}$. Furthermore, we consider all perturbations in the form of propagating waves, i.e. proportional to $\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$, and obtain

$$\delta\tilde{n} = \left(1-ik_\perp^2\rho_s^2\frac{\omega+i\nu_{i\perp}}{\omega_d}+i\frac{k_z^2\,v_s^2}{\omega^2+\nu_{iz}^2}\frac{\omega-i\nu_{iz}}{\omega_d}\right)\tilde{\phi}, \tag{9}$$

where $\mathbf{k}_\perp$ is the component of the wave vector perpendicular to $\hat{\mathbf{z}}$, $v_s = (T_e/m_i)^{1/2}$ is the ion sound speed and $\omega_d = k_z^2\,v_{Te}^2/\nu_{ez}$. Hence we find the following dispersion equation:

$$\omega\left[1+\rho_s^2\left(\kappa_\perp^2-\frac{k_z^2\,\Omega_i^2}{\omega^2+\nu_{iz}^2}\right)\left(1+\frac{\tilde{\beta}}{\omega_d}\right)-i\frac{\omega-k_z\,v_{Te}}{\omega_d}\rho_s^2\left(k_\perp^2-\frac{k_z^2\,\Omega_i^2}{\omega^2+\nu_{iz}^2}\right)\right]$$
$$= \omega_D-i\tilde{\beta}-i\rho_s^2\left(\kappa_\perp^2\nu_{i\perp}+\frac{k_z^2\,\Omega_i^2\,\nu_{iz}}{\omega^2+\nu_{iz}^2}\right)\left(1+\frac{\tilde{\beta}}{\omega_d}-i\frac{\omega-k_z\,v_{Te}}{\omega_d}\right), \tag{10}$$

where $\tilde{\beta} = \beta n_0$ and $\omega_D = \mathbf{k}\cdot\mathbf{v}_D$. Letting $\omega = \omega_r + i\gamma$ and assuming $\gamma \ll \omega_r$, we obtain from (10) the real part of the drift wave frequency ($\omega_* \gg \nu_{iz}$)

$$\omega_r = \omega_*-\rho_s^2\left(k_\perp^2\,\nu_{i\perp}+k_z^2\,\nu_{iz}\frac{\Omega_i^2}{\omega_*^2}\right)\frac{\omega_*-k_z\,v_{Te}}{\omega_d}, \tag{11}$$

where

$$\omega_* \approx \omega_D\left[1+\rho_s^2\left(k_\perp^2-k_z^2\frac{\Omega_i^2}{\omega_D^2}\right)\left(1+\frac{\tilde{\beta}}{\omega_d}\right)\right]^{-1}. \tag{12}$$

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The imaginary part of the wave frequency is defined by

$$\gamma\frac{\omega_D}{\omega_*}=\omega_*\frac{\omega_*-k_z v_{Te}}{\omega_d}\rho_s^2\left(k_\perp^2-k_z^2\frac{\Omega_i^2}{\omega_*^2}\right)-\rho_s^2\left(k_\perp^2\nu_{i\perp}+k_z^2\nu_{iz}\frac{\Omega_i^2}{\omega_*^2}\right)\left(1+\frac{\tilde{\beta}}{\omega_d}\right)-\tilde{\beta}. \quad (13)$$

We see that recombination as well as ion dissipative processes effectively slow down the instability. In the absence of electron (ion)–neutral collisions (and, as a consequence, in the absence of perturbations of the ionization/recombination process), (13) gives the growth rate, which is similar to that of the classical dissipative drift-wave instability (Kadomtsev 1965) in a fully ionized plasma:

$$\gamma_e=\frac{\omega_{**}^2 k_\perp^2\rho_s^2}{(1+k_\perp^2\rho_s^2)}\frac{\nu_{ez}}{k_z^2 v_{Te}^2}, \quad (14)$$

where $\omega_{**}=\omega_D/(1+k_\perp^2\rho_s^2)$. Note, however, that, unlike the well-known case of a fully ionized plasma (Kadomtsev 1965), here the main dissipation mechanism is due to electron–neutral collisions, and therefore the electron–neutral collision rate ν_{ez} appears in (14).

The instability described by (13) is associated with the presence of finite ion polarization drift, which is essential for the instability to be triggered; we note that in principle it can develop even for lower wave frequencies when $\omega_r<\nu_{i\perp}$ if $|\omega_r k_z v_{Te}|>\nu_{i\perp}\omega_d$, which gives $|k_y v_D \nu_{ez}|>|k_z v_{Te}\nu_{i\perp}|$. The growth rate of the ionization-driven instability can be faster than the rate (14), provided that $k_z\nu_{Te}\gg\omega_*\gg k_z\Omega_i/k_\perp$ and effects of recombination and of ion motion in the z direction, as well as $\nu_{i\perp}$, can be neglected:

$$\gamma\approx\nu_e\frac{k_z^2 v_s^2}{\omega_*}\approx\nu_{ez}\frac{k_\perp^2\rho_s^2}{1+k_\perp^2\rho_s^2}\frac{\lambda_z}{\lambda_y}\frac{\rho_e}{L}, \quad (15)$$

where $L\approx|d_x\ln n_0|^{-1}$ is the characteristic inhomogeneity scale in the x direction and $\rho_e=v_{Te}/\Omega_e$ is the electron Larmor radius. Note that only perturbations propagating in certain directions with respect to the external magnetic field and plasma inhomogeneity, namely those with $k_y k_z<0$, are unstable.

It is useful to compare the obtained instability rates with kinetic dissipative effects. There are two types of the latter. First, we note that the effects of finite Larmor radius (of electrons and/or ions) on wave dissipation can be neglected when the wavelength along the magnetic field is much greater than the Larmor radius, i.e. $k_z v_{Te(i)}\ll\Omega_{e(i)}$, which is often the case for low-temperature gas-discharge plasmas (Lin and Wu 1987; Lin *et al.* 1995). Secondly, the parallel (to the external magnetic field) Čerenkov resonance makes an exponentially small contribution for both electrons and ions when $\omega_*\gg k_z v_{Te}$, which is the case of the instability (14). Finally, under the conditions $k_z v_{Ti}\ll k_z v_{Te}$, corresponding to (15), ion Landau damping is exponentially small, and electron Landau damping can be estimated as

$$\gamma_L\approx\frac{\omega_*^2}{|k_z|v_{Te}}. \quad (16)$$

Comparison of (15) and (16) gives the condition when the rate of the collisional dissipative instability exceeds the kinetic effects

$$\nu_e k_z^2 v_s^2\gg\omega_*^3. \quad (17)$$

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The above dissipative drift-wave instability can affect non-thermal particle transport. Indeed, the ion flux is given by

$$\Gamma_{ix} = -D_i \frac{dn_0}{dx}, \tag{18}$$

where the ion diffusion coefficient is defined by the wave potential fluctuations

$$D_i = 2\left(\frac{T_e c}{eB}\right)^2 \sum_k k_y^2 \frac{\gamma}{\omega_r^2} |\tilde{\phi}_k|^2, \tag{19}$$

where we have $\omega_r \approx \omega_{**} = k_y \rho_s^2 \Omega_i / L$. The Fourier component of the saturated drift-wave potential can be estimated as (Kadomtsev 1965)

$$\tilde{\phi}_k \approx \frac{\pi}{k_\perp L}. \tag{20}$$

Thus for the ion diffusion coefficient we find

$$D_i \approx 2\pi^2 \frac{\lambda_z}{\lambda_y} \frac{\rho_e}{L} \nu_{ez} \rho_s^2. \tag{21}$$

Note that the standard result using the rate (14) contains the squared factor $\lambda_z \rho_e / \lambda_y L$. Finally, we note that when the inequality opposite to (6) holds, we have, instead of (7),

$$\frac{1}{n_0} \delta [n_e (N - n_e) \langle \sigma \mathbf{v}_{e\perp} \rangle - \beta n_e^2 + \nu_{\text{ext}} n_e] \approx -\frac{v_{Te} \nu_{e\perp}}{\Omega_e} \partial_y (\tilde{\phi} - \delta \tilde{n}_e) - \beta n_0 \delta \tilde{n}_e. \tag{22}$$

Thus we have analogous results with $\omega_* + k_y \rho_e \nu_{e\perp}$ instead of $\omega_* - k_z v_{Te}$ in (11) and (13), and the following estimate of the maximum instability rate ($k_\perp^2 \rho_s^2 \ll 1$):

$$\gamma \approx \nu_{ez} k_\perp^2 \rho_s^2 \left(\frac{\lambda_z}{\lambda_y} \frac{\rho_c}{L}\right)^2 \left(1 + \frac{L}{l_x}\right) \approx \gamma_{ei} \left(1 + \frac{L}{l_x}\right), \tag{23}$$

where l_x is the electron mean free path in the direction of plasma inhomogeneity. A similar factor $1 + L/l_x$ also appears for the ion diffusion coefficient in this case.

3. Conclusions

To summarize, we have investigated the dissipative drift-wave instability in the presence of wave-induced perturbations of the ionization/recombination process in a non-uniform collisional weakly ionized magnetoplasma. We have found that in this case the dissipative drift-wave instability, which is caused by the effects of electron–neutral collisions, as well as cross-field ion diffusion, can be enhanced (compared with the classic case in a fully ionized collisional plasma) in collisional weakly ionized plasmas, such as those found in radiofrequency discharges as well as in the Earth's lower ionosphere. This investigation should be useful in understanding the properties of low-frequency drift waves, their turbulence, and associated particle transport in various weakly ionized plasmas, such as those of typical glow discharges when the ionization degree is less than 1% and the electron–neutral collision frequency can be comparable to the electron gyrofrequency.

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