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Design and analysis of a 3-DOF spherical parallel manipulator – CORRECTED VERSION

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SUMMARY
In this paper, we propose a unique, 3 degree-of-freedom (DOF) parallel wrist. Specifically, an over-constrained spherical 3-DOF parallel mechanism is presented and the modified structure, which avoids the redundant constraints, is also introduced.

KEYWORDS: Parallel manipulators; Design; Automation; Control of robotic systems; Mechatronic systems.

1. Introduction

In order to fulfill a certain task, a robot requires both position and orientation adjustment. The wrist that connects the arm to the end-effector is the most complex component in a robot. Among the existing 1-DOF (degree of freedom), 2-DOF, and 3-DOF wrists, the 3-DOF is the most promising candidate, as it can achieve pitch, yaw, and roll motions and can also perform a variety of tasks.

The "Motoman KS" industrial robots of YASKAWA Electric Corporation (Kurosakishiroishi, YahatanishikuKitakyushu, Japan) adopted epicyclical gear trains to achieve three rotational motions. However, the wear on the gear surface can significantly affect the life of the gear systems. Furthermore, it is difficult for the gear train to reach high precision and compact size.

With the increasing demands for a high dexterity wrist, a new type of robot, the spherical parallel mechanism (SPM), has been developed. Gosselin and Angeles[1] first proposed an SPM, which can generate spherical motion, in 1989. This mechanism can satisfy most of the desired requirements such as compact size, low inertia, high accuracy, and mechanical stiffness. Many researchers, such as Wu and Deng,[2] Li and Payandeh,[3] Di Gregorio,[4] and Chablat and Angeles,[5] have developed different types of 3-DOF spherical parallel wrists. Gosselin and Gagné[10] analyzed the forward kinematic problem of a spherical 3-DOF parallel mechanism.

In this paper, a spherical mechanism, which contains rotational motions around the x, y, and z axes, is presented. Consequently, an over-constrained 3-DOF SPM is obtained and a revised structure that avoids the redundant constraints is subsequently introduced. Overall, this mechanism has a simple structure and is easy to control.

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2. Synthesis for Parallel Mechanism

The Jacobian matrix is a powerful tool to analyze the kinematic performance of the parallel mechanism. However, in this paper, it will be proved to be very useful to synthesize the parallel mechanism. First, we should get started from the screw form of Jacobian matrix.

The velocity of the moving platform would be denoted as:

$$\mathbf{T} = [\omega_x \quad \omega_y \quad \omega_z \quad v_x \quad v_y \quad v_z]^{\mathbf{T}} = [\boldsymbol{\omega}^{\mathbf{T}} \ \mathbf{v}^{\mathbf{T}}]^{\mathbf{T}}$$

In screw theory, the instantaneous motion of the moving platform of $F-DOF$ parallel mechanism would be expressed as[6]:

$$\$_{\mathbf{p}} = \sum_{j=1}^{C_i} \$_{\mathbf{ji}} \dot{q}_{\mathbf{ji}} \quad i = 1, \ldots, F \tag{1}$$

where $\dot{q}_{\mathbf{ji}}$ denotes the intensity, $\$_{ji}$ denotes the unit screw associated with the jth joint of ith kinematic chain, and C_i denotes the number of $1-DOF$ kinematic joints, as the spherical joint and the universal joint could be replaced by three and two intersecting nonlinear revolute joints, respectively.

When the actuated joint in each kinematic chain is locked, the DOF of the ith kinematic chain will decrease to $C_i - 1$, that is, the reciprocal screw of kinematic chain will include the $6 - C_i$ constraints and the $6 - C_i + 1$ reciprocal screw. The $6 - C_i + 1$ reciprocal screw is considered as the wrench of actuation, as the work done by wrench to twist can be represented by the orthogonal product of twist and wrench.[7] Let the 1st joint of the ith kinematic chain actuated, and the actuation is represented as $\$_{a,i}$, and to take the orthogonal product of both sides of Eq. (1), yields:

$$\$^{\mathbf{T}}_{\mathbf{a,i}} \$_{\mathbf{p}} = \$^{\mathbf{T}}_{\mathbf{a,i}} \$_{\mathbf{1i}} \dot{q}_{\mathbf{1i}} \quad i = 1, \ldots, F$$

where $k_i = 1/\$^T_{ai}\$_{1i}$, the velocity of actuated joint of the ith kinematic chain, could be obtained:

$$\dot{\mathbf{q}}_{1i} = k_{\mathbf{i}} \$_{\mathbf{p}} \quad i = 1, \ldots, F \tag{2}$$

Rewriting Eq. (2) for each kinematic chain, we have:

$$\dot{q}_i = J_a \$_p \tag{3}$$

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where

$$\mathbf{J_a} = \begin{bmatrix} k_1\$_{a1}^T \\ k_2\$_{a2}^T \\ \cdots \\ k_F\$_{aF}^T \end{bmatrix}, \quad \dot{\mathbf{q}}_\mathbf{i} = \begin{bmatrix} \dot{q}_{11} & \dot{q}_{12} & \cdots & \dot{q}_{1F} \end{bmatrix}^\mathbf{T}$$

The matrix J_a is actuation Jacobian matrix of parallel mechanism.[8] The row vector of actuation Jacobian matrix is the multiplication of actuation force $\$_{ai}$ by constant k_i. For the mechanism, each DOF of the moving platform is actuated by a motor. The kinematic equation of the mechanism can be represented as:

$$\begin{cases} t_1 = f_1(q_1) \\ t_2 = f_2(q_2) \\ \cdots \\ t_F = f_F(q_F) \end{cases} \quad i = 1, 2, 3$$

where t_i denotes the output of a parallel manipulator and q_i denotes the intensity of actuated joint of the ith kinematic chain. The standard form of Jacobian matrix of the mechanism can be obtained:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial t_1}{\partial q_1} & 0 & \ldots & 0 \\ 0 & \frac{\partial t_2}{\partial q_2} & \ldots & 0 \\ . & . & . & . \\ 0 & 0 & \ldots & \frac{\partial t_F}{\partial q_F} \end{bmatrix}_{F\times n} \tag{4}$$

Comparing Eq. (3) with Eq. (4), we can find that:

$$\begin{cases} \$_{\mathbf{a},\mathbf{1}}^\mathbf{T} = \left[k_1' \frac{\partial t_1}{\partial q_1}, 0, 0, \ldots, 0\right]^T \\ \$_{\mathbf{a},\mathbf{2}}^\mathbf{T} = \left[0, k_2' \frac{\partial t_2}{\partial q_2}, 0, \ldots, 0\right]^T \\ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \$_{\mathbf{a},\mathbf{F}}^\mathbf{T} = \left[0, 0, 0, \ldots, k_F' \frac{\partial t_F}{\partial q_F}\right]^T \end{cases}$$

The actuations of the parallel mechanism are perpendicular to each other. As the actuated joints are mostly revolute joints or prismatic joints, the actuation of them are the couple along the axis of revolute joint and the force along the axis of prismatic joint. In these cases, the actuations are accordant with the axis of actuated joints. Thus, the topology condition of parallel mechanism finally can be concluded: the actuated joints of all kinematic chains are perpendicular to each other.

3. Type Synthesis of Spherical Parallel Mechanism

The motion of moving platform is the intersection of motion of all kinematic chains of parallel mechanism.[9] Assuming that the SPM is composed of three kinematic chains, the conditions of the SPM should be: (1) Each kinematic chain of SPM should have at least three rotational DOFs; (2) The actuated joints of three kinematic chains are perpendicular to each other.

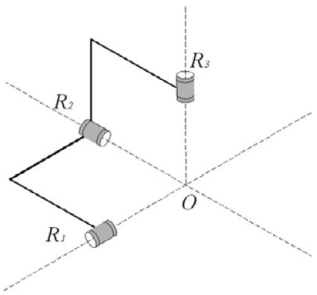


Fig. 1. The *RRR* chain with constraints f_1, f_2, f_3.

3.1. Type synthesis of the kinematic chain

On the basis of the reciprocal screw theory, the constraint system of an SPM should be:

$$\mathbf{f_1} = [l_1, m_1, n_1; 0, 0, 0]$$
$$\mathbf{f_2} = [l_2, m_2, n_2; 0, 0, 0]$$
$$\mathbf{f_3} = [l_3, m_3, n_3; 0, 0, 0]$$

where $\mathbf{f_1}, \mathbf{f_2}, \mathbf{f_3}$ represent the constraint force passing across the origin (the reference point of the moving platform). According to the number of constraints, the feasible kinematic chains of SPM could be divided into four types: the zero-constraint kinematic chain, the single-constraint kinematic chain, the two-constraint kinematic chain, and the three-constraint kinematic chain.

3.1.1. Type synthesis of three-constraint kinematic chain. As the constraint system includes three constraint forces f_1, f_2, f_3, the twists of kinematic chain can be obtained:

$$\mathbf{R_1} = [a_1, b_1, c_1; 0, 0, 0]$$
$$\mathbf{R_2} = [a_2, b_2, c_2; 0, 0, 0]$$
$$\mathbf{R_3} = [a_3, b_3, c_3; 0, 0, 0]$$

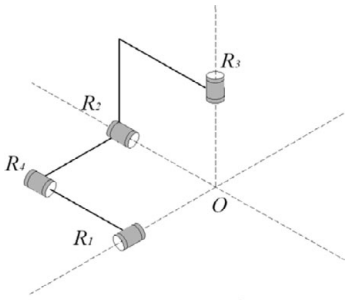
where $\mathbf{R_1}, \mathbf{R_2}, \mathbf{R_3}$ denote the revolute joints whose axis passes across the origin. The kinematic chain with constraints $\mathbf{f_1}, \mathbf{f_2}, \mathbf{f_3}$ is shown in Fig. 1.

3.1.2. Type synthesis of two-constraint kinematic chain. Suppose the constraint system includes two constraint forces $\mathbf{f_1}, \mathbf{f_2}$, then the twists of kinematic chain will be:

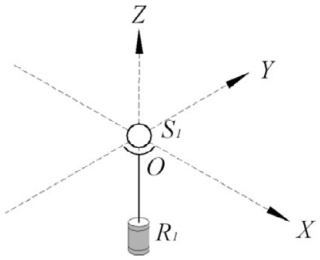
$$\mathbf{R_1} = [a_1, b_1, c_1; 0, 0, 0]$$
$$\mathbf{R_2} = [a_2, b_2, c_2; 0, 0, 0]$$
$$\mathbf{R_3} = [a_3, b_3, c_3; 0, 0, 0]$$
$$\mathbf{P_1} = [0, 0, 0; d_4, e_4, f_4]$$

where R_1, R_2, R_3 denote the revolute joints whose axis passes across the origin. P_1 denotes a prismatic joint, and this translational DOF could be used by a pair of parallel revolute joints, $R_4//R_2$. The new revolute joint R_4 is obtained by the

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(a) The *PRRR* Chain



(b) The *PS* Chain

Fig. 2. The chain that affords constraints f_1, f_2.

linear combination of joint R_2 and joint P_1:

$$\mathbf{R_4} = \mathbf{R_2} + \mathbf{P_1} = [a_2, b_2, c_2; d_4, e_4, f_4]$$

So, the twist system of kinematic chain will become:

$$\begin{aligned} \mathbf{R_1} &= [a_1, b_1, c_1; 0, 0, 0] \\ \mathbf{R_2} &= [a_2, b_2, c_2; 0, 0, 0] \\ \mathbf{R_3} &= [a_3, b_3, c_3; 0, 0, 0] \\ \mathbf{R_4} &= [a_2, b_2, c_2; d_4, e_4, f_4] \end{aligned} \tag{5a}$$

The *PRRR* kinematic chain can be obtained with the constraint force $\mathbf{f_1}$, $\mathbf{f_2}$, as shown in Fig. 2(a).

The revolute joints R_5, R_6 can be obtained by the linear combination of revolute joint and prismatic joint:

$$\mathbf{R_5} = \mathbf{R_2} + \mathbf{P_1} = [a_2, b_2, c_2; d_4, e_4, f_4]$$
$$\mathbf{R_6} = \mathbf{R_3} + \mathbf{P_1} = [a_3, b_3, c_3; d_4, e_4, f_4]$$

When the revolute joints R_2, R_3 are replaced by R_5, R_6, then the twist system of kinematic chain will become:

$$\begin{aligned} \mathbf{R_1} &= [a_1, b_1, c_1; 0, 0, 0] \\ \mathbf{R_4} &= [a_1, b_1, c_1; d_4, e_4, f_4] \\ \mathbf{R_5} &= [a_2, b_2, c_2; d_4, e_4, f_4] \\ \mathbf{R_6} &= [a_3, b_3, c_3; d_4, e_4, f_4] \end{aligned} \tag{5b}$$

So, the PS kinematic chain can be obtained to afford the constraint force $\mathbf{f_1}$, $\mathbf{f_2}$, as shown in Fig. 2(b).

3.1.3. Type synthesis of one-constraint kinematic chain. Assuming that the constraint system includes one constraint force f_1, the twists of kinematic chain can be obtained:

$$\begin{aligned} \mathbf{R_1} &= [a_1, b_1, c_1; 0, 0, 0] \\ \mathbf{R_2} &= [a_2, b_2, c_2; 0, 0, 0] \\ \mathbf{R_3} &= [a_3, b_3, c_3; 0, 0, 0] \\ \mathbf{P_1} &= [0, 0, 0; d_4, e_4, f_4] \\ \mathbf{P_2} &= [0, 0, 0; d_5, e_5, f_5] \end{aligned}$$

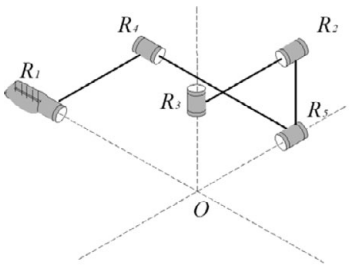
The new revolute joint can be obtained by the linear combination of revolute joint and prismatic joint:

$$\mathbf{R_4} = \mathbf{R_1} + \mathbf{P_1} = [a_1, b_1, c_1; d_4, e_4, f_4]$$
$$\mathbf{R_5} = \mathbf{R_2} + \mathbf{P_2} = [a_2, b_2, c_2; d_5, e_5, f_5]$$

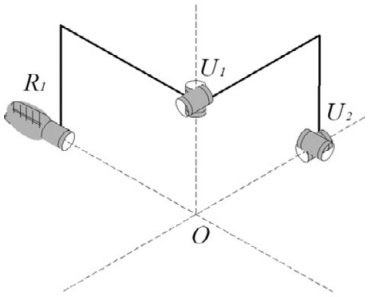
So, the twist system of kinematic chain will become:

$$\begin{aligned} \mathbf{R_1} &= [a_1, b_1, c_1; 0, 0, 0] \\ \mathbf{R_2} &= [a_2, b_2, c_2; 0, 0, 0] \\ \mathbf{R_3} &= [a_3, b_3, c_3; 0, 0, 0] \\ \mathbf{R_4} &= [a_1, b_1, c_1; d_4, e_4, f_4] \\ \mathbf{R_5} &= [a_2, b_2, c_2; d_5, e_5, f_5] \end{aligned} \tag{6a}$$

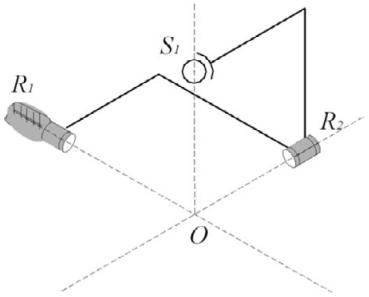
Therefore, the **RRRRR** kinematic chain can be obtained to afford the constraint force f_1, as shown in Fig. 3(a).



(a) The *RRRRR* chain



(b) The *RUU* chain



(c) The *RRS* chain

Fig. 3. The *PPRRR* chain that affords constraint f_1.

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The new revolute joints $\mathbf{R_6}, \mathbf{R_7}$ can be obtained by the linear combination of revolute joint and prismatic joint:

$$\mathbf{R_6} = \mathbf{R_2} + \mathbf{P_1} = [a_2, b_2, c_2; d_4, e_4, f_4]$$
$$\mathbf{R_7} = \mathbf{R_1} + \mathbf{P_2} = [a_1, b_1, c_1; d_5, e_5, f_5]$$

The two universal joints $\mathbf{U_1}, \mathbf{U_2}$ will be obtained when $\mathbf{P_1}, \mathbf{P_2}$ are replaced by $\mathbf{R_6}, \mathbf{R_7}$, and the twist system of kinematic chain will become:

$$\begin{aligned} \mathbf{R}_1 &= [a_1, b_1, c_1; 0, 0, 0] \\ \mathbf{R_2} &= [a_2, b_2, c_2; 0, 0, 0] \\ \mathbf{R_6} &= [a_2, b_2, c_2; d_4, e_4, f_4] \\ \mathbf{R_3} &= [a_3, b_3, c_3; 0, 0, 0] \\ \mathbf{R_7} &= [a_1, b_1, c_1; d_5, e_5, f_5] \end{aligned} \tag{6b}$$

So, the **RUU** kinematic chain can be obtained to afford the constraint force $\mathbf{f_1}$, as shown in Fig. 3(b).

The spherical joint $\mathbf{S}_1$ will be obtained when $\mathbf{P_1}, \mathbf{P_2}$ are replaced by $\mathbf{R_6}, \mathbf{R_7}$, and the twist system of kinematic chain will become:

$$\mathbf{R_8} = \mathbf{R_1} + \mathbf{P_1} = [a_1, b_1, c_1; d_4, e_4, f_4]$$
$$\mathbf{R_9} = \mathbf{R_2} + \mathbf{P_1} = [a_2, b_2, c_2; d_4, e_4, f_4]$$
$$\mathbf{R_{10}} = \mathbf{R_3} + \mathbf{P_1} = [a_3, b_3, c_3; d_4, e_4, f_4]$$

The spherical joint S_1 will be obtained by the combination of $\mathbf{R_8}, \mathbf{R_9}$, and $\mathbf{R_{10}}$, and the twist system of kinematic chain will become:

$$\begin{aligned} \mathbf{R_1} &= [a_1, b_1, c_1; 0, 0, 0] \\ \mathbf{R_2} &= [a_2, b_2, c_2; 0, 0, 0] \\ \mathbf{R_8} &= [a_1, b_1, c_1; d_4, e_4, f_4] \\ \mathbf{R_9} &= [a_2, b_2, c_2; d_4, e_4, f_4] \\ \mathbf{R_{10}} &= [a_3, b_3, c_3; d_4, e_4, f_4] \end{aligned} \tag{6c}$$

Hence, the **RRS** kinematic chain could be obtained to afford the constraint force $\mathbf{f_1}$, as shown in Fig. 3(c).

3.1.4. Type synthesis of zero-constraint kinematic chain. This type of kinematic chain is simple and is shown in Fig. 4.

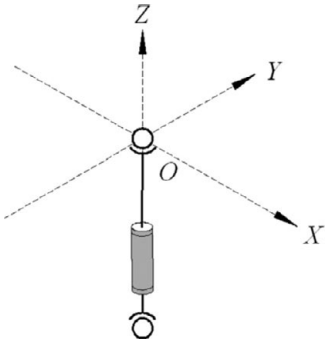


Fig. 4. One kinematic chain that affords zero-constraint.

3.2. Type synthesis of the spherical parallel mechanism
The SPM can be obtained by assembling these kinematic chains while the actuators are perpendicular to each other. Figures 5(a) and 5(b) are symmetrical SPMs. As the *RRRRR* kinematic chain affords three constraints, the $3-RRRRR$ parallel mechanism is non-overconstrained. Figure 5(d) is another example of non-overconstrained.

It should be noted that the synthesis method of parallel mechanisms is applicable only instantaneously for some specific configurations. Kong and Gosselin[11] investigated more detailed case studies about type synthesis.

4. Analysis and Verification

4.1. Motion verification
As shown in Fig. 6, the three motors of the manipulator are fixed to the base. The unit vectors directed along the axes of the actuators are referred to as vectors $\boldsymbol{u}_i$ for $i = 1, 2, 3$, while the unit vectors directed along the axes of the revolute joints of the end-effector are represented as vectors $\boldsymbol{v}_i$, $i = 1, 2, 3$. Moreover, the orientation of the end-effector with respect to the base is given by a rotation matrix,Q. The actuator angles are represented as θ_i, $i = 1, 2, 3$. Finally, the unit vectors directed along the axis of the intermediate revolute joint for each of the legs are noted as $\boldsymbol{w}_i$ for $i = 1, 2, 3$.

The rotation matrix of end-effector is presented as:

$$\begin{aligned} {}^0\boldsymbol{T}_2(\theta_x, \theta_y, \theta_z) &= Rot(z, \theta_z)Rot(y, \theta_y)Rot(x, \theta_x) \\ &= \begin{bmatrix} \cos\theta_z\cos\theta_y & \cos\theta_z\sin\theta_y\sin\theta_x - \sin\theta_z\cos\theta_x & \cos\theta_z\sin\theta_y\cos\theta_x + \sin\theta_z\sin\theta_x \\ \sin\theta_z\cos\theta_y & \sin\theta_z\sin\theta_y\sin\theta_x + \cos\theta_z\cos\theta_x & \sin\theta_z\sin\theta_y\cos\theta_x - \cos\theta_z\sin\theta_x \\ -\sin\theta_y & \cos\theta_y\sin\theta_x & \cos\theta_y\cos\theta_x \end{bmatrix}, \end{aligned}$$

where θ_x, θ_y, and θ_z are the rotation angles with respect to each axis. The motion under generic configurations is shown in Fig. 7.

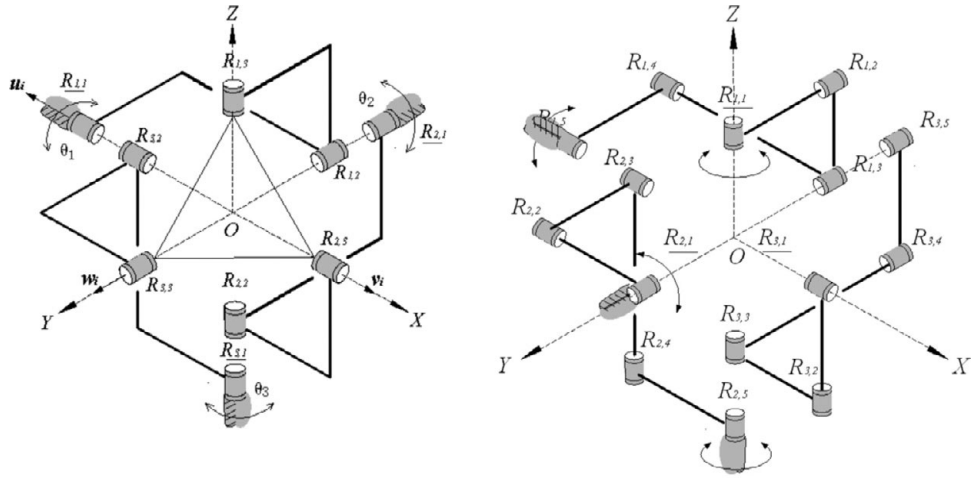
For the ith kinematic chain, the unit vectors of the 2nd revolute joints with respect to frame $O - XYZ$ can be written as

$$[\boldsymbol{w}_1]^0_{R0} = \{0, -1, 0\}, [\boldsymbol{w}_2]^0_{R0} = \{0, 0, -1\}, [\boldsymbol{w}_3]^0_{R0} = \{-1, 0, 0\}.$$

When the actuator works with angle θ_i, the unit vector of the intermediate revolute joint can be obtained:

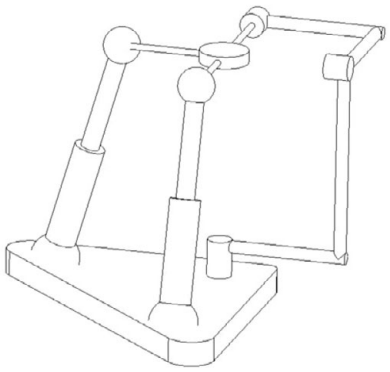
$$[\boldsymbol{w}_1]_{R0} = Rot(x, \theta_1)[\boldsymbol{w}_1]^0_{R0}, \ [\boldsymbol{w}_2]_{R0} = Rot(y, \theta_2)[\boldsymbol{w}_2]^0_{R0},$$
$$[\boldsymbol{w}_3]_{R0} = Rot(z, \theta_3)[\boldsymbol{w}_3]^0_{R0}.$$

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(a) The 3 – *RRR* Decoupled SPM

(b) The 3 – *RRRRR* Decoupled Spherical PM



(c) The 2*SPS* – *RRR* SPM

Fig. 5. The kinematic models of spherical parallel mechanism (SPM).

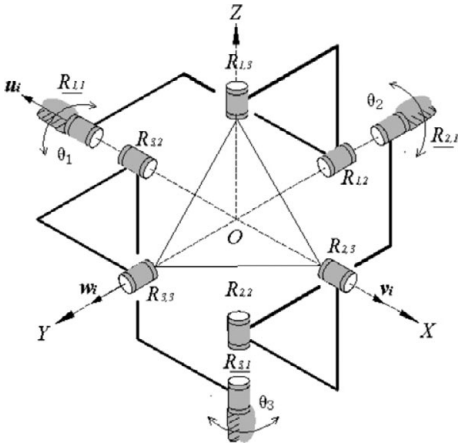


Fig. 6. The kinematic model of 3-*RRR* parallel mechanism.

Thus,

$$[\boldsymbol{w}_1]_{R0} = \begin{bmatrix} 0 \\ -\cos\theta_1 \\ -\sin\theta_1 \end{bmatrix}, \quad [\boldsymbol{w}_2]_{R0} = \begin{bmatrix} -\sin\theta_2 \\ 0 \\ -\cos\theta_2 \end{bmatrix},$$

$$[\boldsymbol{w}_3]_{R0} = \begin{bmatrix} -\cos\theta_3 \\ -\sin\theta_3 \\ 0 \end{bmatrix}. \tag{7}$$

The unit vector of the third revolute joint can be obtained by inverse kinematics for the parallel mechanism as

$$[\boldsymbol{v}_1]_{R2} = \{0, 0, 1\}, [\boldsymbol{v}_2]_{R2} = \{1, 0, 0\}, [\boldsymbol{v}_3]_{R2} = \{0, 1, 0\}.$$

The vectors of v_i under the frame $O - XYZ$ are obtained:

$$[\boldsymbol{v}_i]_{R0} = {}^0\boldsymbol{T}_2[\boldsymbol{v}_i]_{R2}.$$

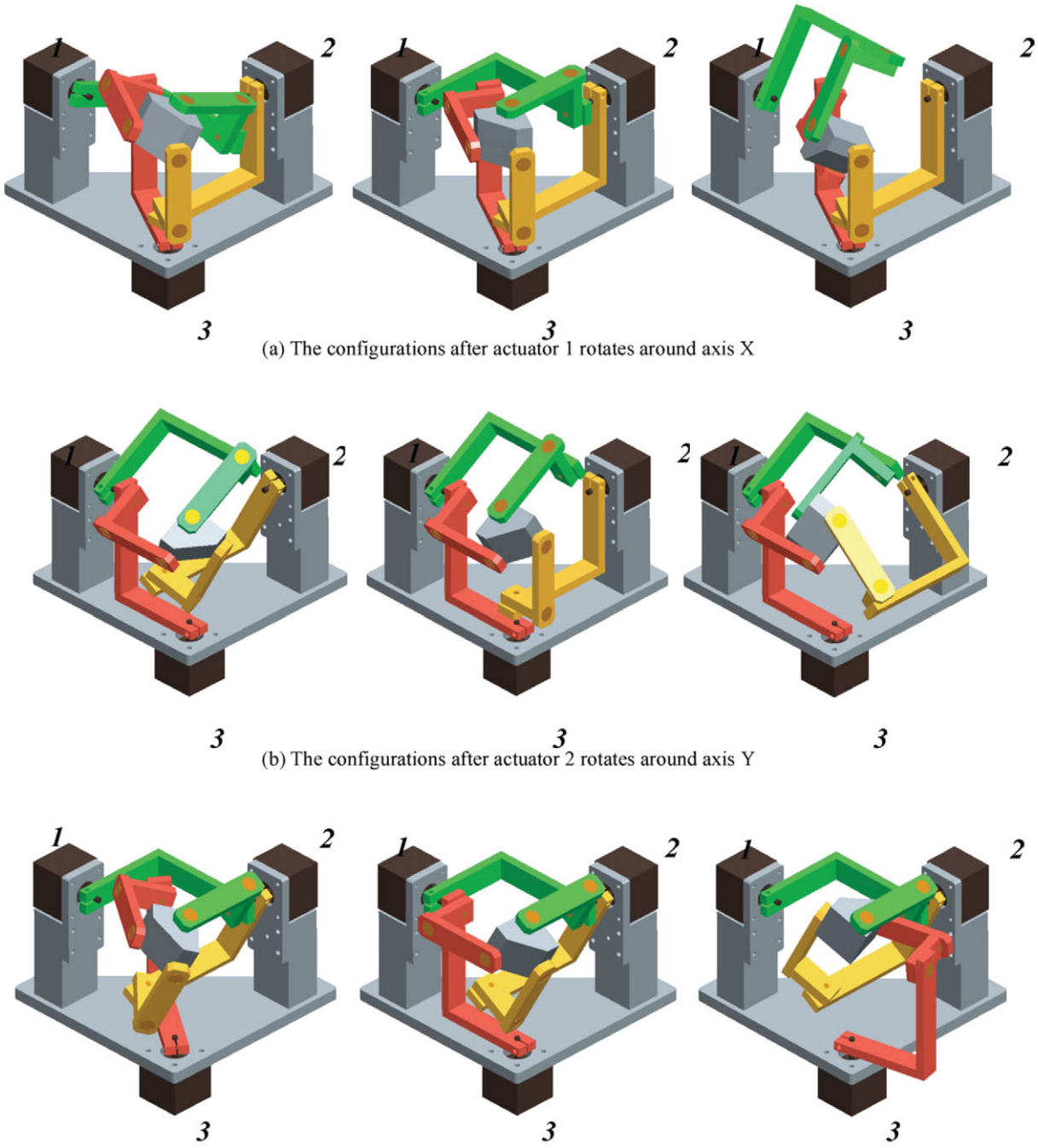
Thus,

$$[\boldsymbol{v}_1]_{R0} = \begin{bmatrix} \cos\theta_z \sin\theta_y \cos\theta_x + \sin\theta_z \sin\theta_x \\ \sin\theta_z \sin\theta_y \cos\theta_x - \cos\theta_z \sin\theta_x \\ \cos\theta_y \cos\theta_x \end{bmatrix},$$

$$[\boldsymbol{v}_2]_{R0} = \begin{bmatrix} \cos\theta_z \cos\theta_y \\ \sin\theta_z \cos\theta_y \\ -\sin\theta_y \end{bmatrix},$$

$$[\boldsymbol{v}_3]_{R0} = \begin{bmatrix} \cos\theta_z \sin\theta_y \sin\theta_x - \sin\theta_z \cos\theta_x \\ \sin\theta_z \sin\theta_y \sin\theta_x + \cos\theta_z \cos\theta_x \\ \cos\theta_y \sin\theta_x \end{bmatrix}. \tag{8}$$

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(a) The configurations after actuator 1 rotates around axis X

(b) The configurations after actuator 2 rotates around axis Y

(c) The configurations after actuator-3 rotates around axis Z

Fig. 7. (Colour online) The simulation of motion under generic configurations.

From the geometry of the kinematic chain, we can find the equation for vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ as

$$\boldsymbol{w} \cdot \boldsymbol{v} = \cos a_i, \tag{9}$$

where $a_i = \frac{\pi}{2}$.

Substituting Eq. (7) and Eq. (8) into Eq. (9) yields

$$\begin{aligned}
&-\cos\theta_1(\sin\theta_z \sin\theta_y \cos\theta_x - \cos\theta_z \sin\theta_x) \\
&-\cos\theta_y \cos\theta_x \sin\theta_1 = 0, \\
&-\cos\theta_z \cos\theta_y \sin\theta_2 + \sin\theta_y \cos\theta_2 = 0, \\
&-\cos\theta_3(\cos\theta_z \sin\theta_y \sin\theta_x - \sin\theta_z \cos\theta_x) \\
&-\sin\theta_3(\sin\theta_z \sin\theta_y \sin\theta_x + \cos\theta_z \cos\theta_x) = 0.
\end{aligned}$$

Furthermore, we can obtain

$$\theta_1 = \arctan \frac{-(\sin\theta_z \sin\theta_y \cos\theta_x - \cos\theta_z \sin\theta_x)}{\cos\theta_y \cos\theta_x},$$

$$\theta_2 = \arctan \frac{\sin\theta_y}{\cos\theta_z \cos\theta_y},$$

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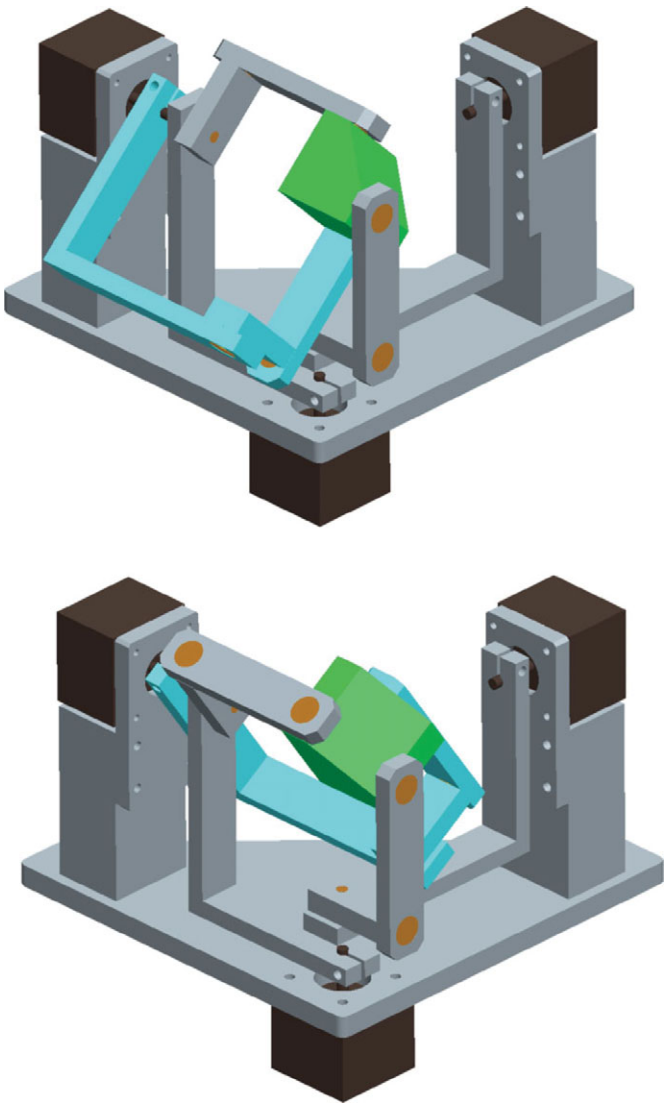


Fig. 8. (Colour online) The extreme roll angle loci of the spherical mechanism.

$$\theta_3 = -\arctan\frac{\cos\theta_z \sin\theta_y \sin\theta_x - \sin\theta_z \cos\theta_x}{\sin\theta_z \sin\theta_y \sin\theta_x + \cos\theta_z \cos\theta_x}.$$

4.2. The extreme roll angle loci of the spherical mechanism

The extreme roll angle of rotation around a single axis can be measured with the tool supplied by Pro/E software. For example, in the rotation around the z-axis, except for the locus with which the link is intervened, the roll range can achieve over 270°, as shown in Fig. 8.

5. Condition for Isotropic

The relationship between the accuracy of the Cartesian velocity of the end-effector and the joint velocity can be represented as:

$$\frac{\|\delta\mathbf{q}\|}{\|\mathbf{q}\|} \leq \|\mathbf{J}^{-1}\|\,\|\mathbf{J}\|\,\frac{\|\delta\dot{\phi}\|}{\|\dot{\phi}\|}$$

where $\|\cdot\|$ represents the spectral norm of matrix. If the matrix J is nonsingular, the matrix $\mathbf{J}^{\mathbf{T}}\mathbf{J}$ is a positive matrix, and the eigenvalue of the matrix is positive, then the spectral norm of Jacobian matrix $\mathbf{J}$, $\boldsymbol{\sigma}_{\max}$, is the largest singular value of the matrix J:

$$\|\mathbf{J}\| = \boldsymbol{\sigma}_{\max}(\mathbf{J}) = [\max \boldsymbol{\lambda}(\mathbf{J}^{\mathbf{T}}\mathbf{J})]^{\frac{1}{2}}$$

where $\boldsymbol{\lambda}(\cdot)$ denotes the eigenvalue of the matrix. Furthermore, we have:

$$\|\mathbf{J}^{-1}\| = \mathbf{1}/\sigma_{\mathbf{Min}}(\mathbf{J})$$

Thus, the condition number of the Jacobian matrix can be represented as:

$$\mathbf{C}(\mathbf{G}) = \|\mathbf{G}\|\,\|\mathbf{G}^{-1}\| = \frac{\sigma_{\mathbf{Max}}(\mathbf{J})}{\sigma_{\mathbf{Min}}(\mathbf{J})}$$

where $\sigma_{\mathbf{Max}}(\mathbf{J})$ and $\sigma_{\mathbf{Min}}(\mathbf{J})$ are the largest and the smallest singular values of the Jacobian matrix J, respectively.

If the singular values of the Jacobian matrix are equivalent, such as in the case where

$$\sigma_{\mathbf{Max}}(\mathbf{J}) = \sigma_{\mathbf{Min}}(\mathbf{J}),$$

then the condition number of the Jacobian matrix will be 1. This is the condition for a spatial mechanism to be a fully isotropic mechanism.

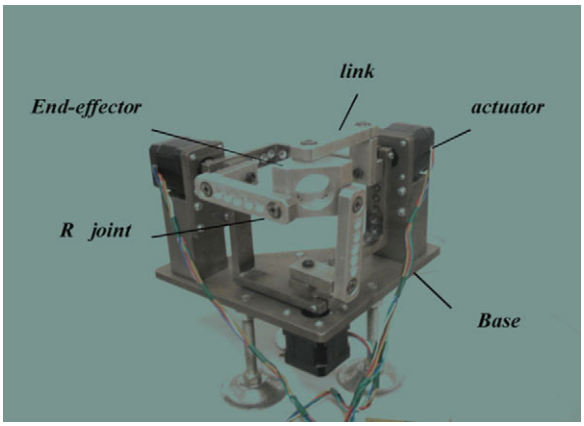


Fig. 9. (Colour online) The prototype of a 3-*RRR* spherical parallel wrist.

6. Conclusion

In this paper, a new 3-DOF spherical parallel manipulator is presented and the prototype of the 3-*RRR* spherical parallel wrist has been built in the lab (Fig. 9). Specifically, a modified structure is introduced to avoid any redundant rotational constraints. Hence, the manipulator has potential for extensive industrial applications.

Thus far, we have discovered that when the active forces are perpendicular and the output DOF can be controlled by one actuator.

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