Feedback

On 'On the frequency of Friday the thirteenth': Henry Ricardo writes: In the May 1933 issue of the *American Mathematical Monthly*, B. H. Brown posed the problem (E 36) of showing that the thirteenth of the month is more likely to be Friday than any other day of the week. In his eleven-line featured solution (December, 1933), Raphael Robinson showed that in a 400 year period, the thirteenth of the month occurs 4800 times, distributed as follows:

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
687	685	685	687	684	688	684

The problem later became one of the "Four Hundred 'Best' Problems (1918-1950)" that constituted *The Otto Dunkel Memorial Problem Book*. This was Part II of the August-September 1957 issue of the *Monthly*.

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On 105.16: Nick Lord writes: Des MacHale's neat elementary proofs that, if p is prime, then $2^n + p$ is composite for infinitely many values of n (Theorem 2) with a similar result for $2^n - p$ (Theorem 1), set me thinking about the underlying general results.

Suppose that a and b are coprime positive integers with $a > 1, b > 0$.

Generalisation of Theorem 2: $a^n + b$ is composite for infinitely many values of n.

Proof: Let p be any prime divisor of $a + b$. Then a and b are coprime so, by Fermat's little theorem, $a^{p-1} \equiv 1 \pmod{p}$. Let $n = 1 + r(p-1)$ for $r = 1, 2, \ldots$. Then $a^n = a\left(a^{p-1}\right)^r \equiv a \pmod{p}$, so that

$$a^n + b = \left(a^n - a\right) + (a + b) \equiv 0 \pmod{p}$$

is composite.

Generalisation of Theorem 1: $a^n - b$ is composite for infinitely many values of n.

Proof: The same proof works with prime p dividing $a - b$, unless $a - b = \pm 1$. In the latter case $a^3 - b \geqslant 5$ and we choose any prime divisor, p, of $a^3 - b$. Then a and b are coprime so, by Fermat's little theorem, $a^{p-1} \equiv 1 \pmod{p}$. Let $n = 3 + r(p-1)$ for $r = 1, 2, \ldots$. Then $a^n \equiv a^3 \pmod{p}$, so that $a^n - b = \left(a^n - a^3\right) + \left(a^3 - b\right) \equiv 0 \pmod{p}$ is composite.

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On 105.17 and 105.18: Chris Starr writes: The Notes in [1] and [2] discuss ideal multigrades of order 5, that is, sets of 6 integers such that the sums of the zeroth to the fifth powers are equal for each set.

The parametric formula asserted by Tyke in [1, p. 286] can be derived from Theorem 4 in [2, p. 297] by using the following transformation:

https://doi.org/10.1017/mag.2021.134 Published online by Cambridge University Press