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Analysis and Experimentation of Transpose Jacobian-based Cartesian Regulators*

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SUMMARY

Transpose Jacobian-based controllers represent an attractive approach to robot control in Cartesian space. These controllers attempt to drive the robot end-effector posture to a specified desired position and orientation without solving either the inverse kinematics nor computing the robot inverse Jacobian. A wide class of transpose Jacobian-based regulators obtained from the energy shaping plus damping injection technique is analyzed in this paper. Our main theoretical contribution is the introduction of a novel analysis which does not invoke any assumption on Jacobian singularities to ensure local asymptotic stability for a family of nonredundant robots. The performance of four transpose Jacobian-based regulators is illustrated via experimental tests conducted on a direct-drive vertical arm.

KEYWORDS: Robot control; Transpose Jacobian control; Regulation; Stability; Energy-shaping; Task space.

1. INTRODUCTION

Tasks for robot manipulators are usually specified as trajectories of the robot end-effector posture in the Cartesian space. This goal is usually achieved by first computing the time histories of the corresponding desired joint position (velocity) trajectory through the inverse kinematics (inverse Jacobian). Once these data are available, then they are used to compute the joint position (velocity) error which drives joint-based control schemes.[1]

An alternative approach is given by the so-called Cartesian-based control schemes. Here, the actual Cartesian description which may be directly measured or computed from joint position by means of the kinematic equation, is compared to the desired Cartesian position in order to form errors in Cartesian space. Cartesian-based control schemes are those driven by Cartesian position errors instead of joint position or velocity errors. The main practical advantage conferred to Cartesian-based controllers is that the solution of the inverse kinematics is removed because the joint position error is not required anymore.

The main paradigms of Cartesian-based regulators are the *inverse Jacobian control* and *transpose Jacobian control*.[1] A wide study including stability and robustness of the inverse Jacobian control has been reported in Miyazaki and Arimoto.[2] The present paper is devoted to the study of transpose Jacobian-based control schemes.

Since the first transpose Jacobian regulators proposed by Takegaki and Arimoto,[3] a number of analysis and extensions have been reported in the literature. Two transpose Jacobian regulators were originally proposed in Takegaki and Arimoto.[3] One of them possesses a joint velocity damping term while the other has a Cartesian velocity one. These control schemes together with the extension reported in Miyazaki and Arimoto[2] are the main objects of analysis in this paper. For the sake of presentation, the literature review is deferred to Section 3.

The contribution of this paper is twofold. First, we introduce a novel analysis of transpose Jacobian-based controllers whose main feature is that assumption on Jacobian singularities is not required. Second, we present an experimental evaluation of four transpose Jacobian-based regulators on a two degrees of freedom direct-drive arm. Practical features and limitations of these regulators are emphasized.

The paper is organized in the following way. Section 2 presents the robot dynamics and the regulation goal. The related previous literature on transpose Jacobian-based controllers is presented in Section 3. Our main theoretical studies concerning transpose Jacobian controllers with joint space damping are described in Section 4. In Section 5, we introduce the analysis of transpose Jacobian controllers with Cartesian space damping. Section 6 is devoted to the experimental evaluation of several transpose Jacobian controllers on a two degrees of freedom direct-drive arm. Finally, Section 7 offers a brief summary.

2. ROBOT MODEL AND REGULATION GOAL

The dynamics of rigid serial n-link robot manipulators can be described by[4]

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \tau \tag{1}$$

where $\boldsymbol{q}$ is the $n\times 1$ vector of joint displacements, τ is the $n\times 1$ vector of applied joint torques, $M(\boldsymbol{q})$ is the $n\times n$ symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ is the $n\times 1$ vector of centripetal and Coriolis torques, and $\boldsymbol{g}(\boldsymbol{q})$ is the $n\times 1$ vector of gravitational torques due to gravity.

The posture of the end-effector with respect to the robot reference frame is denoted by

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$$\boldsymbol{y}=\boldsymbol{k}(\boldsymbol{q}) \tag{2}$$

where $\boldsymbol{k}$: $\mathbb{R}^n \rightarrow \mathbb{R}^3 \times SO(3)$ is the forward kinematic transformation taking joint positions into Cartesian position and orientation. The read-out map $\boldsymbol{k}(\boldsymbol{q})$ is also called the task function.[5]

For the purpose of this paper, a local description of the end-effector posture is considered by using a minimal representation of the end-effector orientation. In this way, it is possible to describe the end-effector posture by means of an $m \times 1$ *vector, with* $m \leq 6$; hence, hereafter we consider $\boldsymbol{k}: \mathbb{R}^n \rightarrow \mathbb{R}^m$. The differential kinematics is obtained as the time derivative of the forward kinematics (2), i.e., $\dot{\boldsymbol{y}}=J_A(\boldsymbol{q})\dot{\boldsymbol{q}}$ where the analytical Jacobian is defined by

$$J_A(\boldsymbol{q})=\frac{\partial \boldsymbol{k}(\boldsymbol{q})}{\partial \boldsymbol{q}} \in \mathbb{R}^{m \times n}. \tag{3}$$

For the purposes of this paper it is convenient to recall the concept of self-motion.[6,7]

Definition 1. A robot manipulator possesses self-motion if there exist joint motions that do not change the end-effector posture.

□ □ □

Nonredundant robots ($m=n$) either possess no self-motion or, if it does, the self-motion only happens at singular configuration.[7]

From definition 1 we extract the following consequences:

Fact 1. Consider a manipulator for which no self-motion exists. Let $\boldsymbol{y}^* \in \mathbb{R}^m$ be an admissible end-effector posture, then any solution $\boldsymbol{q}=\boldsymbol{q}^*$ of the inverse kinematics $\boldsymbol{y}^*=\boldsymbol{k}(\boldsymbol{q})$ is isolated.

□ □ □

Fact 2. Consider a manipulator for which no self-motion exists. Then, no change on the end-effector posture ($\dot{\boldsymbol{y}}(t) \equiv \boldsymbol{0}$) implies that no joint motion is possible, i.e.

$$J_A(\boldsymbol{q}(t))\dot{\boldsymbol{q}}(t) \equiv \boldsymbol{0} \Rightarrow \dot{\boldsymbol{q}}(t) \equiv \boldsymbol{0}.$$

□ □ □

The regulation aim in Cartesian space is to ensure

$$\lim_{t \rightarrow \infty} \boldsymbol{y}(t)=\boldsymbol{y}_d \tag{4}$$

where $\boldsymbol{y}_d \in \mathbb{R}^m$ is the desired Cartesian posture which is assumed to be constant. This paper will narrow the scope of the presentation considering the control goal in a local sense, i.e. to ensure (4) provided that $\|\boldsymbol{y}_d-\boldsymbol{y}(0)\|$ and $\|\dot{\boldsymbol{q}}(0)\|$ are small enough, where $\|\cdot\|$ stands for the Euclidean norm. It is assumed that the joint position $\boldsymbol{q}$ and joint velocity $\dot{\boldsymbol{q}}$ are available for measurement. Also the kinematics $\boldsymbol{k}(\boldsymbol{q})$ is assumed to be known. Posture $\boldsymbol{y}$ may be measured directly or computed through the forward kinematics (2).

3. LITERATURE REVIEW

Early solutions to the stated regulation problem were proposed by Takegaki and Arimoto[3], where the following two transpose Jacobian control schemes were introduced

$$\boldsymbol{\tau}=J_A(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}-K_v \dot{\boldsymbol{q}}+\boldsymbol{g}(\boldsymbol{q}) \tag{5}$$

and

$$\boldsymbol{\tau}=J_A(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}-J_A(\boldsymbol{q})^T K_v \dot{\boldsymbol{y}}+\boldsymbol{g}(\boldsymbol{q}) \tag{6}$$

where $\tilde{\boldsymbol{y}}=\boldsymbol{y}_d-\boldsymbol{y}$ is the Cartesian posture error, and K_v, K_p are symmetric positive definite matrices having suitable dimensions.

The only difference between these control schemes is the second right hand term. The first regulator (5) involves the term $-K_v\dot{\boldsymbol{q}}$ while the second (6) has the term $-J_A(\boldsymbol{q})^T K_v \dot{\boldsymbol{y}}$. In turn, using the differential kinematics, the latter term is equivalent to $-J_A(\boldsymbol{q})^T K_v J_A(\boldsymbol{q})\dot{\boldsymbol{q}}$ which can be implemented without needing measurement of the posture rate $\dot{\boldsymbol{y}}$. It is worth noticing that both regulators compensate for the gravitational torque vector $\boldsymbol{g}(\boldsymbol{q})$.

The controllers have attracted the attention of various researchers and their motivations and analysis have been addressed in several textbooks.[1,8-10] Usually, the studies have been carried out assuming the existence of an unknown *desired* joint configuration $\boldsymbol{q}_d \in \mathbb{R}^n$ such that $\boldsymbol{y}_d=\boldsymbol{k}(\boldsymbol{q}_d)$. This is called the regulation solvability assumption.

The original analysis conducted by Takegaki and Arimoto[3] showed that for kinematic nonredundant robots ($m=n$) and under the assumption that the robot Jacobian $J_A(\boldsymbol{q})$ evaluated at $\boldsymbol{q}=\boldsymbol{q}_d$ is nonsingular, then the local regulation aim is achieved. These assumptions and claims are also reported in References 8 and 11.

The textbooks[9,10] present the analysis of both controllers (5) and (6). They consider redundant as well as nonredundant robots, and under the assumption that the Jacobian $J_A(\boldsymbol{q})$ is full-rank, they get the conclusion of global regulation. However, the fact that the Jacobian must be full-rank for all joint configuration $\boldsymbol{q}$ limits the application spectrum to a small class of manipulators (e.g. Cartesian manipulators) for which asymptotic stability is guaranteed in a global sense.

Excluding the boundary of the workspace, Koditschek[12] characterized the closed-loop global attractor and pointed out the problem of "stationary stall" states. A further complete study for the case $m=n=2$ was presented in Reference 13.

The analysis of the transpose Jacobian with joint space damping regulator (5) against imperfect Jacobian has been addressed in Reference 14. Focusing to the study of a planar revolute joint robot ($n=m=2$), they presented conditions for local asymptotic stability. In the same direction, References 15 and 16 treat the stability analysis with uncertainties in the Jacobian matrix under the transpose Jacobian with Cartesian space damping regulator. The application of these studies to visual servoing is described in Reference 15 while grasping for multi-fingered robot hands is considered in Reference 16. Both works propose adaptive schemes to deal with parametric uncertainty in the gravitational torque vector $\boldsymbol{g}(\boldsymbol{q})$.

Transpose Jacobian-based regulators have also been proposed in References 13, 17, 18 and 19 for the case when the end-effector position and orientation is defined in $\mathbb{R}^3 \times SO(3)$. In this situation the control structure involves the geometric or basic Jacobian.

In a more general framework, Miyazaki and Arimoto[2] motivated by the energy shaping plus damping regulators

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design technique, introduced explicitly a generic artificial potential function yielding the transpose Jacobian-based family of controllers

$$\tau = J_A(\boldsymbol{q})^T \nabla_{\tilde{\boldsymbol{y}}} \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}) - K_v \dot{\boldsymbol{q}} + \boldsymbol{g}(\mathbf{q}) \tag{7}$$

and

$$\tau = J_A(\boldsymbol{q})^T \nabla_{\tilde{\boldsymbol{y}}} \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}) - J_A(\boldsymbol{q})^T K_v \dot{\boldsymbol{y}} + \boldsymbol{g}(\boldsymbol{q}) \tag{8}$$

where $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}) : \mathbb{R}^m \times \mathbb{R}^m \rightarrow \mathbb{R}$ is a continuously differentiable positive definite function called *artificial potential energy*, and $\nabla_{\tilde{\boldsymbol{y}}} \bar{\mathfrak{U}}_a(\boldsymbol{y}_d, \tilde{\boldsymbol{y}})$ stands for the gradient with respect to $\tilde{\boldsymbol{y}}$ of $\bar{\mathfrak{U}}_a(\boldsymbol{y}_d, \tilde{\boldsymbol{y}})$. The quadratic form

$$\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}) = \frac{1}{2} \tilde{\boldsymbol{y}}^T K_p \tilde{\boldsymbol{y}} \tag{9}$$

yields the controllers (5) and (6). Assuming nonredundant robots ($m=n$) and nonsingularities on the robot Jacobian $J_A(\boldsymbol{q})$, it was shown that the local regulation is achieved. Possible artificial potential functions are also mentioned in Reference 15.

With respect to the quoted previous literature, the contribution of this paper is twofold. First, we provide a novel analysis of transpose Jacobian-based Cartesian controllers (5) and (6) for nonredundant robots possessing no self-motion. We establish conditions for asymptotic stability without invoking any assumption on Jacobian singularities. Our second contribution is of a practical matter; we present an experimental evaluation of four transpose Jacobian-based controllers carried out on a two degrees of freedom direct-drive arm. We point out the practical advantages and limitations of these controllers. These issues will be addressed in the next sections.

4. TRANSPOSE JACOBIAN WITH JOINT SPACE DAMPING

This section deals with the analysis of the transpose Jacobian with joint space damping control scheme given by (5):

$$\tau = J_A(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}} - K_v \dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \tag{10}$$

where $\tilde{\boldsymbol{y}}(\boldsymbol{q}) = \boldsymbol{y}_d - \boldsymbol{y}(\boldsymbol{q}) = \boldsymbol{y}_d - \boldsymbol{k}(\boldsymbol{q})$, and $K_v \in \mathbb{R}^{n \times n}$ is symmetric and positive definite.

The following assumptions are made:

A1. The robot is nonredundant ($m=n$) and no self-motion exists.

A2. There exists $\boldsymbol{q}_d \in \mathbb{R}^n$ such that $\boldsymbol{y}_d = \boldsymbol{k}(\boldsymbol{q}_d)$.

Sufficient conditions for a robot to have self-motion are reported in Reference 7. Assumption A2 is a necessary condition for the problem solvability. According to Fact 1, the no self-motion existence established in assumption A1 means that $\boldsymbol{q} = \boldsymbol{q}_d$ is an isolated solution of $\boldsymbol{y}_d = \boldsymbol{k}(\boldsymbol{q})$.

The closed-loop system obtained by substituting the control law (10) into the robot dynamics (1) can be written as

$$\frac{d}{dt} \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1} [J_A(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q}) - K_v \dot{\boldsymbol{q}} - C(\boldsymbol{q}, \dot{\boldsymbol{q}}) \dot{\boldsymbol{q}}] \end{bmatrix}. \tag{11}$$

The main result is stated in the following

Proposition 1. Consider the transpose Jacobian with joint space damping control scheme (10), and assume that A1-A2 are satisfied. Then, $[\boldsymbol{q}^T \ \dot{\boldsymbol{q}}^T]^T = [\boldsymbol{q}_d^T \ \mathbf{0}^T]^T \in \mathbb{R}^{2n}$ is an asymptotically stable equilibrium of the closed-loop system (11), and the regulation goal $\lim_{t \rightarrow \infty} \boldsymbol{y}(t) = \boldsymbol{y}_d$ is locally achieved.

▽ ▽ ▽

Proof. Assumptions A1 and A2 imply that $\boldsymbol{q} = \boldsymbol{q}_d$ is an isolated solution of $\boldsymbol{y}_d = \boldsymbol{k}(\boldsymbol{q})$. From this fact, $\boldsymbol{q} = \boldsymbol{q}_d$, is an isolated minimum point of $\frac{1}{2} \tilde{\boldsymbol{y}}(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q})$, and therefore, its gradient with respect to $\boldsymbol{q}$, i.e., $-J_A(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q})$ must vanish only at $\boldsymbol{q} = \boldsymbol{q}_d$ in a neighborhood of $\boldsymbol{q}_d$. This allows the conclusions that $[\boldsymbol{q}_d^T \ \mathbf{0}^T]^T$ is an isolated equilibrium of the closed-loop equation (11).

To carry out the stability analysis, we borrow the Lyapunov function candidate proposed in Reference 3:

$$V(\boldsymbol{q}_d - \boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2} \dot{\boldsymbol{q}}^T M(\boldsymbol{q}) \dot{\boldsymbol{q}} + \frac{1}{2} \tilde{\boldsymbol{y}}(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q}). \tag{12}$$

The first right hand side term is a globally positive definite function with respect to $\dot{\boldsymbol{q}}$ because of the positive definiteness of $M(\boldsymbol{q})$. The remaining term defines a locally positive definite function with respect to $\boldsymbol{q}_d - \boldsymbol{q}$ because $\boldsymbol{q}_d$ is an isolated minimum point of $\frac{1}{2} \tilde{\boldsymbol{y}}(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q})$.

The time derivative of the Lyapunov function candidate along the trajectories of the closed-loop equation (11) yields.

$$\dot{V}(\boldsymbol{q}_d - \boldsymbol{q}, \dot{\boldsymbol{q}}) = -\dot{\boldsymbol{q}}^T K_v \dot{\boldsymbol{q}} \tag{13}$$

where we have used the property $\dot{\boldsymbol{q}}^T [\frac{1}{2} \dot{M}(\boldsymbol{q}) - C(\boldsymbol{q}, \dot{\boldsymbol{q}})] \dot{\boldsymbol{q}} = 0$ for all $\boldsymbol{q}, \dot{\boldsymbol{q}} \in \mathbb{R}^n$ (see e.g. Reference 20).

The time derivative (13) is a globally negative semi-definite in virtue of positive definiteness of K_v; therefore, the Lyapunov's direct method allows the conclusion of stability. In order to find the largest invariant set in a neighbourhood of $[\boldsymbol{q}^T \ \dot{\boldsymbol{q}}^T]^T = [\boldsymbol{q}_d^T \ \mathbf{0}^T]^T$ we need the solution of

$$J_A(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q}) = 0.$$

But, we have already established that $\boldsymbol{q} = \boldsymbol{q}_d$ is an isolated minimum point of $\frac{1}{2} \tilde{\boldsymbol{y}}(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q})$, therefore, we have the conclusion that $\boldsymbol{q} = \boldsymbol{q}_d$ is the unique solution of $J_A(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q}) = 0$ in a neighborhood of $\boldsymbol{q}_d$. These arguments lead to claim that $[\boldsymbol{q}^T \ \dot{\boldsymbol{q}}^T]^T = [\boldsymbol{q}_d^T \ \mathbf{0}^T]^T$ is the largest invariant set in a neighborhood of this equilibrium point. Finally, asymptotic stability follows from the Krasovskii-LaSalle's theorem (see e.g. Reference 21) and therefore $\lim_{t \rightarrow \infty} \boldsymbol{q}(t) = \boldsymbol{q}_d$ provided that $\|\boldsymbol{q}_d - \boldsymbol{q}(0)\|$ and $\|\dot{\boldsymbol{q}}(0)\|$ are sufficiently small.

□ □ □

Remark 1. None assumption on the Jacobian singularities has been invoked to establish the claims in proposition 1. The unknown desired joint position $\boldsymbol{q}_d$ is allowed to be a singular configuration where the Jacobian $J_A(\boldsymbol{q}_d)$ is singular. In other words, it is still possible to achieve $\lim_{t \rightarrow \infty} \boldsymbol{y}(t) = \boldsymbol{y}_d$ even though $\boldsymbol{y}_d$ corresponds to a singular robot configuration (for instance at the boundary of the workspace).

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4.1 Generalized transpose Jacobian with joint space damping

The remainder of this section is devoted to studying the following extension of the control scheme (10) introduced in Takegaki and Arimoto[3]

$$\tau = J_A(\boldsymbol{q})^T \nabla_{\tilde{\boldsymbol{y}}} \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}) - K_v \dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \tag{14}$$

where $\tilde{\boldsymbol{y}}(\boldsymbol{q}) = \boldsymbol{y}_d - \boldsymbol{y}(\boldsymbol{q}) = \boldsymbol{y}_d - \boldsymbol{k}(\boldsymbol{q})$, $K_v \in \mathbb{R}^{n \times n}$ is a symmetric positive matrix.

This control structure holds the closed-loop system:

$$\frac{d}{dt}\begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ -M(\boldsymbol{q})^{-1}[\nabla_{\boldsymbol{q}} \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q})) + K_v \dot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}] \end{bmatrix} \tag{15}$$

where the gradient of $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$ with respect to $\boldsymbol{q}$ is given by

$$\nabla_{\boldsymbol{q}} \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q})) = -J_A(\boldsymbol{q})^T \nabla_{\tilde{\boldsymbol{y}}} \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}).$$

The following assumptions are made:

B1. The robot is nonredundant ($m = n$) and no self-motion exists.
B2. The artificial potential energy $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$ as a function of $\boldsymbol{q}$ possesses isolated minimum points.

Define the nonempty set $\mathfrak{Q}^{as} \subset \mathbb{R}^n$ as the collection of the isolated minimum points of the artificial potential energy $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$.

The following proposition describes stability characteristics of the closed-loop system.

Proposition 2. The state space points $[\boldsymbol{q}^T \ \dot{\boldsymbol{q}}^T]^T = [\bar{\boldsymbol{q}}^T \ \boldsymbol{0}^T]^T \in \mathbb{R}^{2n}$ with $\bar{\boldsymbol{q}} \in \mathfrak{Q}^{as}$, are asymptotically stable equilibria of the closed-loop system (15). Furthermore,

$$\lim_{t \to \infty} \boldsymbol{y}(t) = \boldsymbol{k}(\bar{\boldsymbol{q}})$$

provided that $\|\bar{\boldsymbol{q}} - \boldsymbol{q}(0)\|$ and $\|\dot{\boldsymbol{q}}(0)\|$ are small enough.

▽ ▽ ▽

Proof. First notice that because $\bar{\boldsymbol{q}}$ is by definition a minimum point of $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$, then its gradient with respect to $\boldsymbol{q}$ must vanish at $\boldsymbol{q} = \bar{\boldsymbol{q}}$. Hence, $[\bar{\boldsymbol{q}}^T \ \boldsymbol{0}^T]^T$ are equilibrium points of the closed-loop system (11) for each $\bar{\boldsymbol{q}} \in \mathfrak{Q}^{as}$.

The stability of these equilibria can be studied by the following Lyapunov function candidate

$$V(\bar{\boldsymbol{q}} - \boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q})) - \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\bar{\boldsymbol{q}})).$$

This is a locally positive definite function because $\frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}$ is a globally positive definite function with respect to $\dot{\boldsymbol{q}}$ and $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q})) - \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\bar{\boldsymbol{q}}))$ defines a locally positive definite function with respect to $\bar{\boldsymbol{q}} - \boldsymbol{q}$ because $\bar{\boldsymbol{q}}$ is an isolated minimum point of $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$.

In virtue of the property $\dot{\boldsymbol{q}}^T[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q}, \dot{\boldsymbol{q}})]\dot{\boldsymbol{q}} = 0$ for all $\boldsymbol{q}, \dot{\boldsymbol{q}} \in \mathbb{R}^n$ (see e.g. Reference 20), the time derivative of the Lyapunov function candidate yields $\dot{V}(\boldsymbol{q}_d - \boldsymbol{q}, \dot{\boldsymbol{q}}) = -\dot{\boldsymbol{q}}^T K_v \dot{\boldsymbol{q}}$. Since by definition K_v is positive definite, then the Lyapunov's direct method allows the conclusion of stablility for the equilibria $[\boldsymbol{q}^T \ \dot{\boldsymbol{q}}^T]^T = [\bar{\boldsymbol{q}}^T \ \boldsymbol{0}^T]^T \in \mathbb{R}^{2n}$ with $\bar{\boldsymbol{q}} \in \mathfrak{Q}^{as}$. The asymptotic stability of these equilibria follows from the KrasovskiiLaSalle's theorem by invoking similar arguments to those in proof of proposition 1.

□ □ □

It is worth mentioning the following remarks.

Remark 2. The result stated in proposition 2, like in proposition 1, does not invoke any assumption on the Jacobian singularities.

Remark 3. Proposition 2 is valid even though the control problem is not solvable, i.e., in the situation where there does not exist a constant vector $\boldsymbol{q}_d \in \mathbb{R}^n$ such that $\boldsymbol{y}_d = \boldsymbol{k}(\boldsymbol{q}_d)$.

Remark 4. An interesting situation appears for two degrees of freedom ($n = m = 2$) planar revolute arm when $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}) = \frac{1}{2}\tilde{\boldsymbol{y}}^T K_p \tilde{\boldsymbol{y}}$ and the desired Cartesian position $\boldsymbol{y}_d \in \mathbb{R}^2$ is beyond the workspace. Obviously there is no way to solve the control problem; however, if $K_p = k_p I_{2\times 2}$ with $k_p > 0$, then the arm will reach a singular configuration $\bar{\boldsymbol{q}}$ at the boundary of the workspace minimizing locally $\|\boldsymbol{y}_d - \boldsymbol{y}(\bar{\boldsymbol{q}})\|$.

As a consequence of propositions 1 and 2, the following can be proven straightforward.

Corollary 1. Consider the generalized transpose Jacobian with joint space damping control scheme (14), and suppose that in addition to assumptions A1 and A2, the artificial potential energy $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}})$ possesses a minimum point at $\tilde{\boldsymbol{y}} = \boldsymbol{0}$. Then, $[\boldsymbol{q}^T \ \dot{\boldsymbol{q}}^T]^T = [\boldsymbol{q}_d^T \ \boldsymbol{0}^T]^T \in \mathbb{R}^{2n}$ is an asymptotically stable equilibrium of the closed-loop system (11), and the regulation goal $\lim_{t \to \infty} \boldsymbol{y}(t) = \boldsymbol{y}_d$ is locally achieved.

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This result provides sufficient conditions to ensure local regulation for a wide class of transpose Jacobian-based regulators with joint space damping. Two examples of regulators belonging to this family will be presented in the experimental evaluation section.

5. TRANSPOSE JACOBIAN WITH CARTESIAN SPACE DAMPING

The study of the transpose Jacobian with Cartesian space damping follows similar steps to those evoked in the previous section. However, for the sake of completeness, this section presents the entire analysis.

The transpose Jacobian with Cartesian space damping control scheme given by (6):

$$\tau = J_A(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}} - J_A(\boldsymbol{q})^T K_v J_A(\boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \tag{16}$$

where $\tilde{\boldsymbol{y}}(\boldsymbol{q}) = \boldsymbol{y}_d - \boldsymbol{y}(\boldsymbol{q}) = \boldsymbol{y}_d - \boldsymbol{k}(\boldsymbol{q})$, and $K_v \in \mathbb{R}^{m \times m}$ is symmetric and positive definite.

The analysis will be carried out under the same assumptions as those evoked for the transpose Jacobian with joint space damping, namely:

A1. The robot is nonredundant ($m = n$) and no self-motion exists.
A2. There exists $\boldsymbol{q}_d \in \mathbb{R}^n$ such that $\boldsymbol{y}_d = \boldsymbol{k}(\boldsymbol{q}_d)$.

The closed-loop system obtained by substituting the control law (16) into the robot dynamics (1) can be written as

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$$\frac{d}{dt}\begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1}[J_A(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q}) - J_A(\boldsymbol{q})^T K_v J_A(\boldsymbol{q})\dot{\boldsymbol{q}} - C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}] \end{bmatrix}. \tag{17}$$

The stability result is stated in the following

Proposition 3. Consider the transpose Jacobian with Cartesian space damping control scheme (16), and assume that A1-A2 are satisfied. Then, $[\boldsymbol{q}^T\ \dot{\boldsymbol{q}}^T]^T = [\boldsymbol{q}_d^T\ \boldsymbol{0}^T]^T \in \mathbb{R}^{2n}$ is an asymptotically stable equilibrium of the closed-loop system (17), and the regulation goal $\lim_{t\to\infty} \boldsymbol{y}(t) = \boldsymbol{y}_d$ is locally achieved.

▽ ▽ ▽

Proof. Obviously $[\boldsymbol{q}^T\ \dot{\boldsymbol{q}}^T]^T = [\boldsymbol{q}_d^T\ \boldsymbol{0}^T]^T$ is an equilibrium of the closed-loop equation (17) because $\tilde{\boldsymbol{y}}(\boldsymbol{q}_d) = \boldsymbol{0}$. To carry out the stability analysis, let us consider the Lyapunov function candidate proposed by Takegaki and Arimoto[3]

$$V(\boldsymbol{q}_d - \boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{y}}(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q}).$$

In virtue that $\boldsymbol{q} = \boldsymbol{q}_d$ is an isolated minimum of $\frac{1}{2}\tilde{\boldsymbol{y}}(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q})$, then the Lyapunov function candidate is at least locally positive definite. The subset of the state space where $V(\boldsymbol{q}_d - \boldsymbol{q}, \dot{\boldsymbol{q}})$ is a positive definite function is denoted hereafter by $\mathcal{D} \subset \mathbb{R}^{2n}$.

The time derivative of the Lyapunov function candidate along the trajectories of the closed-loop equation (17) yields

$$\dot{V}(\boldsymbol{q}_d - \boldsymbol{q}, \dot{\boldsymbol{q}}) = -\dot{\boldsymbol{q}}^T J_A(\boldsymbol{q})^T K_v J_A(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{18}$$

where we have used the property $\dot{\boldsymbol{q}}^T[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q}, \dot{\boldsymbol{q}})]\dot{\boldsymbol{q}} = 0$ for all $\boldsymbol{q}, \dot{\boldsymbol{q}} \in \mathbb{R}^n$.

The time derivative (18) is a globally negative semi-definite in virtue of positive definiteness of K_v; therefore, the Lyapunov's direct method allows the conclusion of stability. Due to the autonomous nature of the closed-loop system, the Krasovskii-LaSalle's theorem (see e.g. References 21 and 22) can be invoked to show asymptotic stability. To this end, define the set $\Omega \subset \mathcal{D} \subset \mathbb{R}^{2n}$ as

$$\Omega = \left\{ \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix} \in \mathcal{D} : \dot{V}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = 0 \right\}$$

$$= \left\{ \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix} \in \mathcal{D} : J_A(\boldsymbol{q})\dot{\boldsymbol{q}} = 0 \right\}$$

Any solution of the closed-loop system (17) within Ω must satisfy $J_A(\boldsymbol{q}(t))\dot{\boldsymbol{q}}(t) = \boldsymbol{0}$ for all $t \geq 0$. Since by Assumption A1 the manipulator has no self-motion, then from Fact 2 we get the conclusion that $\dot{\boldsymbol{q}}(t) \equiv \boldsymbol{0}$, and therefore $\ddot{\boldsymbol{q}}(t) \equiv \boldsymbol{0}$. Hence, considering the closed-loop equation, the joint $\boldsymbol{q}$ must satisfy

$$J_A(\boldsymbol{q}(t))^T K_p \underbrace{[\boldsymbol{k}(\boldsymbol{q}_d) - \boldsymbol{k}(\boldsymbol{q}(t))]}_{\tilde{\boldsymbol{y}}(\boldsymbol{q})} = \boldsymbol{0} \quad \forall t \geq 0.$$

But, in a neighborhood of $\boldsymbol{q} = \boldsymbol{q}_d$, the unique solution of $J_A(\boldsymbol{q})^T K_p \tilde{\boldsymbol{y}}(\boldsymbol{q}) = \boldsymbol{0}$ is just $\boldsymbol{q} = \boldsymbol{q}_d$ because $\frac{1}{2}\tilde{\boldsymbol{y}}^T K_p \tilde{\boldsymbol{y}}$ has its unique minimum point (therefore isolated) at $\tilde{\boldsymbol{y}} = \boldsymbol{0}$, and no self-motion assumption A1 which in turn implies form Fact 1 that $\tilde{\boldsymbol{y}}(\boldsymbol{q}) = \boldsymbol{0}$ has the isolated solution $\boldsymbol{q} = \boldsymbol{q}_d$. The consequence of these arguments is that $[\boldsymbol{q}(t)^T\ \dot{\boldsymbol{q}}(t)^T]^T = [\boldsymbol{q}_d^T\ \boldsymbol{0}^T]^T$ is the unique solution starting in Ω and staying there forever. Finally, according to the Krasovskii-LaSalle's theorem, the equilibrium $[\boldsymbol{q}^T\ \dot{\boldsymbol{q}}^T]^T = [\boldsymbol{q}_d^T\ \boldsymbol{0}^T]^T$ is asymptotically stable.

□ □ □

Remark 5. In contrast with the previous stability analysis reported in the literature, Proposition 3 relaxes the non-singularity assumption on the Jacobian at the unknown desired joint configuration $\boldsymbol{q}_d$.

5.1 Generalized transpose Jacobian with Cartesian space damping

A family of transpose Jacobian controllers with Cartesian space damping can be derived according to the ideas presented in Miyazaki and Arimoto.[2] The generalized transpose Jacobian with Cartesian space damping is given by (8):

$$\tau = J_A(\boldsymbol{q})^T \nabla_{\tilde{\boldsymbol{y}}} \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}) - J_A(\boldsymbol{q})^T K_v \dot{\boldsymbol{y}} + \boldsymbol{g}(\boldsymbol{q}) \tag{19}$$

where $\tilde{\boldsymbol{y}}(\boldsymbol{q}) = \boldsymbol{y}_d - \boldsymbol{y}(\boldsymbol{q}) = \boldsymbol{y}_d - \boldsymbol{k}(\boldsymbol{q})$, and $K_v \in \mathbb{R}^{m\times m}$ is a symmetric positive definite matrix.

The closed-loop equation is obtained by substituting the control law (19) into the robot dynamics (1):

$$\frac{d}{dt}\begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\boldsymbol{q}} \\ -M(\boldsymbol{q})^{-1}[\nabla_{\boldsymbol{q}} \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q})) + J_A(\boldsymbol{q})^T K_v J_A(\boldsymbol{q})\dot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}] \end{bmatrix} \tag{20}$$

where the gradient of $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$ with respect to $\boldsymbol{q}$ is given by

$$\nabla_{\boldsymbol{q}} \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q})) = -J_A(\boldsymbol{q})^T \nabla_{\tilde{\boldsymbol{y}}} \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}).$$

The behavior of the closed-loop system will be studied under the same assumptions as the generalized transpose Jacobian with joint space damping, namely:

B1. The robot is nonredundant ($m = n$) and no self-motion exists.
B2. The artificial potential energy $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$ as a function of $\boldsymbol{q}$ possesses isolated minimum points.

Also, remember that $\mathfrak{Q}^{as} \subset \mathbb{R}^n$ denotes the collection of the isolated minimum points of the artifical potential energy $\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$.

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The following proposition states the stability features of the closed-loop system.

Proposition 4. The state space points $[\boldsymbol{q}^T \ \dot{\boldsymbol{q}}^T]^T = [\bar{\boldsymbol{q}}^T \ \boldsymbol{0}^T]^T \in \mathbb{R}^{2n}$ with $\bar{\boldsymbol{q}} \in \mathfrak{Q}^{as}$, are asymptotically stable equilibria of the closed-loop system (20). Furthermore,

$$\lim_{t\to\infty} \boldsymbol{y}(t) = \boldsymbol{k}(\bar{\boldsymbol{q}})$$

provided that $\|\bar{\boldsymbol{q}} - \boldsymbol{q}(0)\|$ and $\|\dot{\boldsymbol{q}}(0)\|$ are small enough.

▽▽▽

Proof. First notice that in virtue of assumption B2, the isolated minimum points $\bar{\boldsymbol{q}} \in \mathfrak{Q}^{as}$ *of* $\mathfrak{U}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$ are isolated critical points of its gradient, so they are also isolated zeroes of $\nabla_{\boldsymbol{q}}\mathfrak{U}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$. Therefore, the state space points $[\boldsymbol{q}^T \ \dot{\boldsymbol{q}}^T]^T = [\bar{\boldsymbol{q}}^T \ \boldsymbol{0}^T]^T$ are isolated equilibrium points of the closed-loop equation (20).

The stability of these equilibria can be studied by the following Lyapunov function candidate

$$V(\bar{\boldsymbol{q}} - \boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q})) - \bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\bar{\boldsymbol{q}})).$$

This is a locally positive definite function because $\frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}$ is a globally positive definite function with respect to $\dot{\boldsymbol{q}}$ and $\mathfrak{U}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q})) - \mathfrak{U}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\bar{\boldsymbol{q}}))$ defines a locally positive definite function with respect to $\bar{\boldsymbol{q}} - \boldsymbol{q}$ because $\bar{\boldsymbol{q}}$ is an isolated minimum point of $\mathfrak{U}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}(\boldsymbol{q}))$.

The robot dynamics property $\dot{\boldsymbol{q}}^T[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q}, \dot{\boldsymbol{q}})]\dot{\boldsymbol{q}} = 0$ for all $\boldsymbol{q}, \dot{\boldsymbol{q}} \in \mathbb{R}^n$ yields the following time derivative of the Lyapunov function candidate: $\dot{V}(\boldsymbol{q}_d - \boldsymbol{q}, \dot{\boldsymbol{q}}) = -\dot{\boldsymbol{q}}^T K_v \dot{\boldsymbol{q}}$. Since by definition K_v is positive definite, then the Lyapunov's direct method allows the conclusion of stability for the equilibria $[\boldsymbol{q}^T \ \dot{\boldsymbol{q}}^T]^T = [\bar{\boldsymbol{q}}^T \ \boldsymbol{0}^T]^T \in \mathbb{R}^{2n}$ with $\bar{\boldsymbol{q}} \in \mathfrak{Q}^{as}$. The asymptotic stability of these equilibria follows from the Krasovskii-LaSalle's theorem by invoking similar arguments to those in proof of proposition 3.

□□□

The following corollary can be extracted easily from propositions 3 and 4.

Corollary 2. Consider the generalized transpose Jacobian with Cartesian space damping control scheme (19), and suppose that in addition to assumptions A1 and A2, the artificial potential energy $\mathfrak{U}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}})$ possesses a minimum point at $\tilde{\boldsymbol{y}} = \boldsymbol{0}$. Then, $[\boldsymbol{q}^T \ \dot{\boldsymbol{q}}^T]^T = [\boldsymbol{q}_d^T \ \boldsymbol{0}^T]^T \in \mathbb{R}^{2n}$ is an asymptotically stable equilibrium of the closed-loop system (20), and the regulation goal $\lim_{t\to\infty} \boldsymbol{y}(t) = \boldsymbol{y}_d$ is locally achieved.

▽ ▽ ▽

The freedom to choose the artifical potential energy $\mathfrak{U}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}})$ in agreement to assumption B2, enables to design a number of transpose Jacobian control laws achieving local regulation. Besides the PD transpose Jacobian with Cartesian damping, one controller derived from a suitable artificial potential energy has been evaluated experimentally on a direct-drive arm. This evaluation will be presented in the next section.

6. EXPERIMENTAL EVALUATION

This section is devoted to the experimental evaluation of four transpose Jacobian-based regulators on a two degrees of freedom arm.

We have designed and built a direct-drive arm with two vertical rigid links (see Figure 1). High torque, brushless direct-drive servos are used to drive the joints without gear reduction. The motors used in the experimental arm are the Compumotor Models DM1200-A and DM1015-B operating

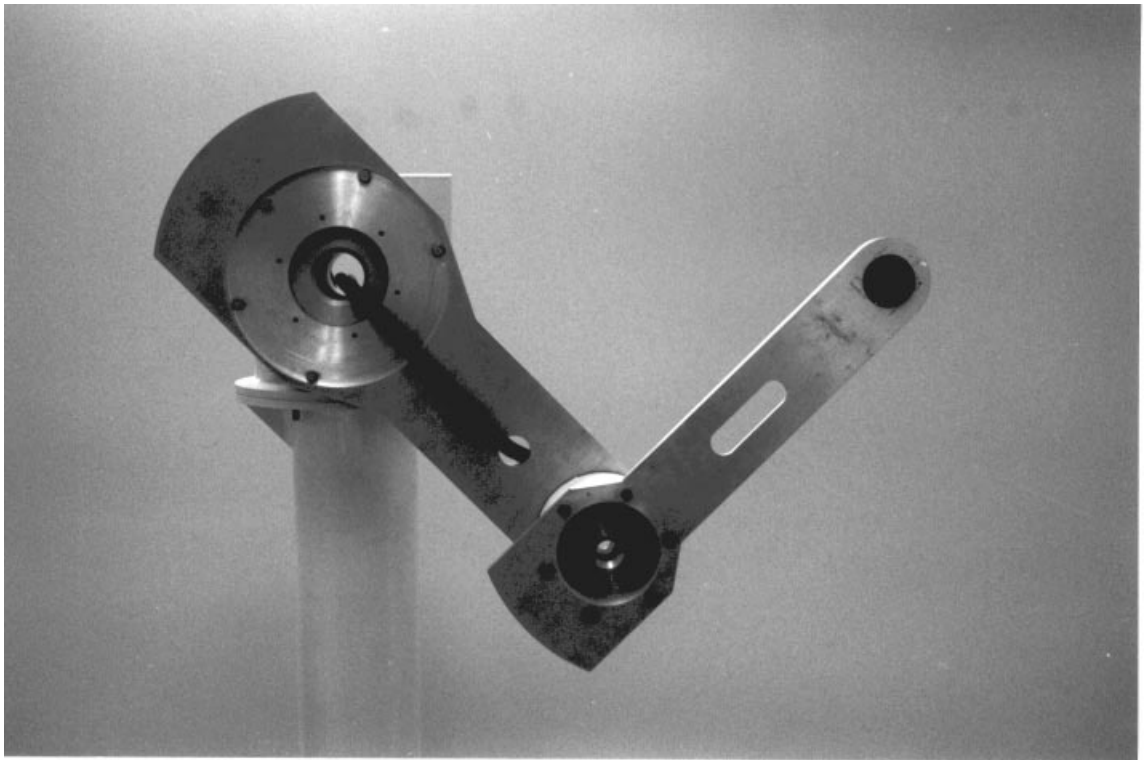


Fig. 1. Experimental arm.

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Table I: Parameters of the manipulator

	notation	value	unit
Length link 1	l_1	0.45	m
Length link 2	l_2	0.45	m
Link (1) center of gravity	l_{c1}	0.091	m
Link (2) center of gravity	l_{c2}	0.048	m
Mass link 1	m_1	23.90	kg
Mass link 2	m_2	3.88	kg
Inertia link 1	I_1	1.27	kg m^2
Inertia link 2	I_2	0.09	kg m^2
Gravity acceleration	g	9.8	m/sec^2

in torque mode for the shoulder and elbow joints respectively.

A motion control board based on a TMS320C31 32-bit floating point microprocessor from Texas Instruments is used to execute the control algorithm. This board is mounted in a PC 486, 66 MHz host computer which provides environment for program generation, compilation, loading data for plotting purposes and downloading programs for real time execution. The control program is written in C programming language for the TMS320 compiler and executed in the control board at a 2.5 msec. sampling rate.

Experiments showed that static, Coulomb, and viscous friction at the motor joint are present and they depend in a complex manner on the joint position and velocity. Instead of modeling these friction phenomena for compensation purposes, we have decided to consider them as unmodeled dynamics.

With reference to the symbols listed in Table I, we present below the entries of the robot dynamics.[23]

The elements $M_{ij}(\boldsymbol{q})$ $(i, j=1, 2)$ of the inertia matrix $M(\boldsymbol{q})$ are

$$M_{11}(\boldsymbol{q})=m_1 l_{c1}^2+m_2(l_1^2+l_{c2}^2+2l_1 l_{c2}\cos(q_2))+I_1+I_2$$
$$M_{12}(\boldsymbol{q})=m_2(l_{C2}^2+l_1 l_{c2}\cos(q_2))+I_2$$
$$M_{21}(\boldsymbol{q})=m_2(l_{c2}^2+l_1 l_{c2}\cos(q_2))+I_2$$
$$M_{22}(\boldsymbol{q})=m_2 l_{c2}^2+I_2.$$

The elements $C_{ij}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ $(i, j=1, 2)$ from the centrifugal and Coriolis matrix $C(\boldsymbol{q}, \dot{\boldsymbol{q}})$ are

$$C_{11}(\boldsymbol{q}, \dot{\boldsymbol{q}})=-m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2$$
$$C_{12}(\boldsymbol{q}, \dot{\boldsymbol{q}})=-m_2 l_1 l_{c2}\sin(q_2)(\dot{q}_1+\dot{q}_2)$$
$$C_{21}(\boldsymbol{q}, \dot{\boldsymbol{q}})=m_2 l_1 l_{c2}\sin(q_2)\dot{q}_1$$
$$C_{22}(\boldsymbol{q}, \dot{\boldsymbol{q}})=0.$$

The entries of the gravitational torque vector $\boldsymbol{g}(\boldsymbol{q})$ are given by

$$g_1(\boldsymbol{q})=(m_1 l_{c1}+m_2 l_1)g\sin(q_1)+m_2 l_{c2} g\sin(q_1+q_2)$$
$$g_2(\boldsymbol{q})=m_2 l_{c2} g\sin(q_1+q_2).$$

Among the entries of the robot dynamics, only the gravitational torque vector is needed for implementation of transpose Jacobian-based regulators.

In our experimental tests we are concerned with the Cartesian position of the arm tip. The origin of the Cartesian frame is attached at the axis of rotation of the first joint while Y_1 and Y_2 denote the horizontal and vertical axes respectively. With this convention, the forward kinematics is given by

$$\boldsymbol{k}(\boldsymbol{q})=\begin{bmatrix} l_1\sin(q_1)+l_2\sin(q_1+q_2) \\ -l_2\cos(q_1)-l_2\cos(q_1+q_2) \end{bmatrix}$$

which leads to the following analytical Jacobian

$$J_A(\boldsymbol{q})=\begin{bmatrix} l_1\cos(q_1)+l_2\cos(q_1+q_2) & l_2\cos(q_1+q_2) \\ l_1\sin(q_1)+l_2\sin(q_1+q_2) & l_2\sin(q_1+q_2) \end{bmatrix}.$$

In all experimental tests we have kept the same initial and desired Cartesian configuration. Specifically, the initial and desired final Cartesian position are sketched in Figure 2. The initial joint position $\boldsymbol{q}(0)=\boldsymbol{0} \in \mathbb{R}^2$ which corresponds to a singular configuration yields the Cartesian initial position $\boldsymbol{y}(0)=[0\ -0.90]^T$ [m]. The desired Cartesian position is set within the workspace at $\boldsymbol{y}_d=[0.5\ 0.25]^T$ [m].

It is worth mentioning that the robot and desired Cartesian position satisfy assumptions A1 and A2, i.e. the experimental arm is nonredundant and it does not possess self-motion, and the inverse kinematics has isolated solutions.

6.1 PD Transpose Jacobian with joint space damping

The transpose Jacobian regulator with joint space damping (10) is perhaps the simplest Cartesian position controller based in transpose Jacobian. The control law is given by

$$\tau=J_A(\boldsymbol{q})^T K_p\tilde{\boldsymbol{y}}-K_v\dot{\boldsymbol{q}}+\boldsymbol{g}(\boldsymbol{q})$$

which can be interpreted as a PD controller with gravity compensation where the proportional term is driven by the Cartesian position error $\tilde{\boldsymbol{y}}$ and the corresponding gain is a nonlinear function of the joint position $\boldsymbol{q}$. The main properties of this control scheme have been summarized in Proposition 1. Since assumptions A1 and A2 are already satisfied, then Proposition 1 establishes that $\lim_{t\to\infty}\boldsymbol{y}(t)=\boldsymbol{y}_d$ locally.

A number of experiments were conducted for a variety of controller gains. The policy followed to choose them was to obtain as lower steady state Cartesian position error as

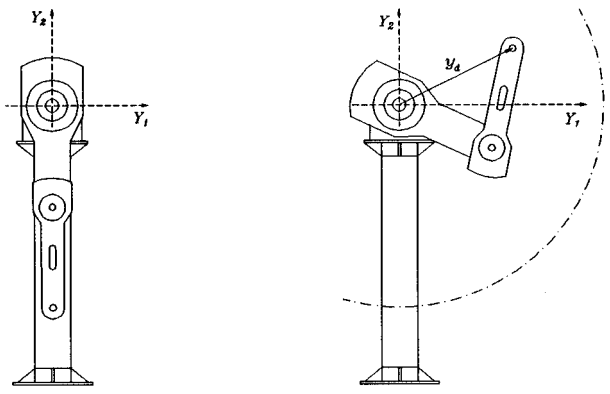


Fig. 2. Inital and desired configurations.

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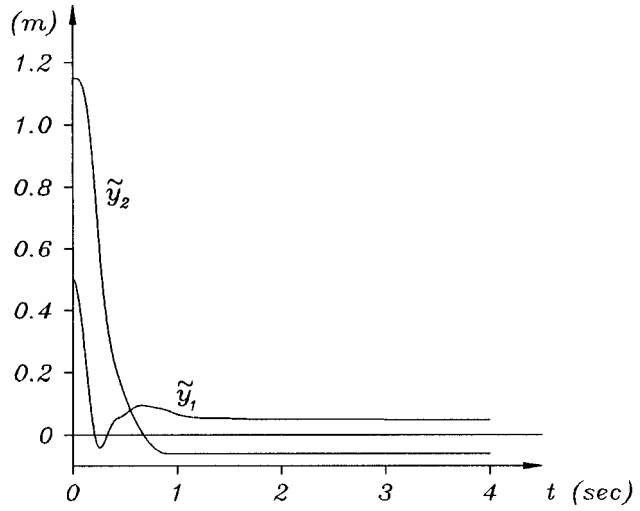


Fig. 3. Cartesian error: PD transpose Jacobian with joint space damping.

possible by keeping the control action within the range of allowable motor torque limits. This was conducted by tedious experimental trial and error procedure. It was observed that re-tuning of the gains is required depending on the desired Cartesian position; this is a severe drawback of this controller which is unacceptable in real applications. For the specified desired output $\boldsymbol{y}_d=[0.5\ 0.25]^T$ $[m]$, the best results were obtained by using

$$K_p=\text{diag}\{66.5,\ 60.0\}\ [\text{N/m}];$$

$$K_v=\text{diag}\{2.415,\ 2.400\}\ [\text{Nm sec/rad}].$$

Figure 3 depicts the time evolution of the components of the Cartesian position error. From this figure we get the conclusion that despite the theoretical claim in the sense that the Cartesian position error must vanish, in practice it remains stuck far from zero. The explanation to this behavior is the presence of static friction in the arm joints.

6.2 Tanh-D transpose Jacobian with joint space damping

Corollary 1 allows to design a wider class of transpose Jacobian-based regulators. According to results stated in this Corollary, the control structure (14) yields effective regulators provided that $\mathfrak{U}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}})$ possess a minimum point at $\tilde{\boldsymbol{y}}=\boldsymbol{0}$. With application to our experimental set-up, the following choice satisfies the requirement

$$\bar{\mathfrak{U}}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}})=\sum_{i=1}^{2}\frac{k_{pi}}{\lambda_i}\ln[\cosh(\lambda_i\tilde{y}_i)]$$

where k_{pi} and λ_i are positive constants. This selection produces the control law

$$\tau=J_A(\boldsymbol{q})^T K_p\begin{bmatrix}\tanh(\lambda_1\tilde{y}_1)\\ \tanh(\lambda_2\tilde{y}_2)\end{bmatrix}-K_v\dot{\boldsymbol{q}}+\boldsymbol{g}(\boldsymbol{q})$$

where $K_p=\text{diag}\{k_{p1},\ k_{p2}\}$. Because of the use of the hyperbolic tangent function, we name it the Tanh-D transpose Jacobian controller.

The tuning procedure of this controller needed experimental trial and error tests. The aim was to achieve small steady state Cartesian position error by maintaining the control actions within the actuator capabilities. The experimental tuning procedure for this controller was certainly more clear than that conducted for the PD transpose Jacobian controller.

The final experiments were carried out with the following controller gains

$$K_p=\text{diag}\{22,\ 55\}\ [\text{Nm}];\ K_v=\text{diag}\{20,\ 5\}\ [\text{Nm sec/rad}];$$

$$\lambda_1=50\ [\text{rad/m}];\ \lambda_2=50\ [\text{rad/m}].$$

The actual behavior of the Cartesian position error is shown in Figure 4. In contrast with the results obtained with the PD transpose Jacobian controller depicted in Figure 3, the Tanh-D transpose Jacobian regulator has better performance as is seen from Figure 4 where both components have a clear trend to zero. The effect of the static friction at the arm joints in the steady state Cartesian position error has been greatly reduced owing to the freedom for increasing the factors λ_i without violating the stability conditions. High values for λ_i produce an equivalent high gain proportional term driven by Cartesian position error which is "turn-on" for small and middle range errors.

6.3 PD Transpose Jacobian with Cartesian space damping

The PD transpose Jacobian with Cartesian space damping was studied in Section 5. Let us rewrite again the control law:

$$\tau=J_A(\boldsymbol{q})^T K_p\tilde{\boldsymbol{y}}-J_A(\boldsymbol{q})^T K_v J_A(\boldsymbol{q})\dot{\boldsymbol{q}}+\boldsymbol{g}(\boldsymbol{q})$$

where $K_v\in\mathbb{R}^{m\times m}$ is symmetric positive definite. The damping is introduced via the term $J_A(\boldsymbol{q})^T K_v\dot{\boldsymbol{y}}$ which feedbacks the Cartesian velocity $\dot{\boldsymbol{y}}$. For implentation purposes, this is equivalent to replacing the Cartesian velocity by $J_A(\boldsymbol{q})\dot{\boldsymbol{q}}$. Proposition 3 states that the regulation aim $\lim_{t\to\infty}\boldsymbol{y}(t)=\boldsymbol{y}_d$ is locally achieved provided that K_p is symmetric and positive definite.

This controller was also tested in our robotic arm. A number of gains K_p and K_v were chosen after trial and error

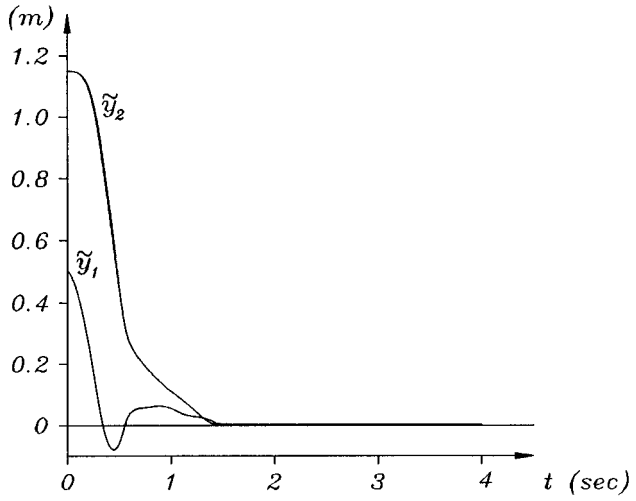


Fig. 4. Cartesian error: Tanh-D transpose Jacobian with joint space damping.

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steps. It was attempted to obtain small steady state Cartesian position errors and at the same time to maintain the applied torques within the allowable limits. This was a tedious experimental task because appropriate gains for a given desired Cartesian position produces bad performance for other references; this impractical drawback of this controller was really disappointing. The final tests were conducted with the following gains

$$K_p = \text{diag}\{66.5, 66.0\}\ [\text{N/m}];$$
$$K_v = \text{diag}\{4.0, 10.0\}\ [\text{kg/sec}].$$

The experimental results are depicted in Figure 5. Despite the theoretical result claims local regulation, from Figure 5 it can be seen that the regulation goal was not achieved in practice because the steady state Cartesian position error was much too large. The explanation to this unexpected phenomenon was attributed to the static friction present in the arm joints. There was no way to reduce them by other choice of the controller gains without saturating the robot actuators.

6.4 Normalized transpose Jacobian with Cartesian space damping

Besides the artificial potential energy $\bar{u}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}) = \frac{1}{2}\tilde{\boldsymbol{y}}^T K_p \tilde{\boldsymbol{y}}$, other expressions can yield controllers with useful features. For example, Miyazaki and Arimoto[2] have suggested:

$$\bar{u}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}}) = \begin{cases} k_p \alpha \|\tilde{\boldsymbol{y}}(\boldsymbol{q})\| - \frac{\alpha}{2} & \text{if } \|\tilde{\boldsymbol{y}}(\boldsymbol{q})\| > \frac{\sigma}{\alpha} \\ k_p \frac{\alpha^2}{2\sigma} \|\tilde{\boldsymbol{y}}(\boldsymbol{q})\|^2 & \text{if } \|\tilde{\boldsymbol{y}}(\boldsymbol{q})\| \le \frac{\sigma}{\alpha} \end{cases}$$

where k_p, α and σ are positive constants. Notice that $\bar{u}(\boldsymbol{y}_d, \tilde{\boldsymbol{y}})$ has its unique minimum point at $\tilde{\boldsymbol{y}} = \boldsymbol{0}$. This artificial potential energy leads to the smooth control law

$$\tau = J_A(\boldsymbol{q})^T \boldsymbol{n}(\tilde{\boldsymbol{y}}) - J_A(\boldsymbol{q})^T K_v J_A(\boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \qquad (21)$$

where

$$\boldsymbol{n}(\tilde{\boldsymbol{y}}) = \begin{cases} k_p \alpha \dfrac{\tilde{\boldsymbol{y}}(\boldsymbol{q})}{\|\tilde{\boldsymbol{y}}(\boldsymbol{q})\|} & \text{if } \|\tilde{\boldsymbol{y}}(\boldsymbol{q})\| > \frac{\sigma}{\alpha} \\ k_p \frac{\alpha^2}{\sigma} \tilde{\boldsymbol{y}}(\boldsymbol{q}) & \text{if } \|\tilde{\boldsymbol{y}}(\boldsymbol{q})\| \le \frac{\sigma}{\alpha} \end{cases}$$

Therefore, Corollary 2 allows the conclusion of local regulation.

As pointed out by Miyazaki and Arimoto[2], this controller has the advantage of demanding low torques for large errors $\tilde{\boldsymbol{y}}$, therefore avoiding saturation of actuators when the manipulator starts from a distant initial configuration, and at the same time, reduces the steady state value of $\tilde{\boldsymbol{y}}$ when static friction is present in the robot joints.

The selection of the controller gains was relatively easy compared with the previous transpose Jacobian-based controllers. Once the gains were chosen after several tests, they yielded acceptable performance for all desired Cartesian positions that we have specified within the robot workspace. The actual experiments were performed with the following gains: $k_p = 45$ [N], $k_v = 103$ [kg/sec], $\alpha = 1$ [m] and $\sigma = 0.01$ [m].

Figure 6 reports the evolution of the Cartesian position error. After a smooth transient, both components of the position error converge to zero. Certainly, the response was the slowest compared with the remaining transpose Jacobian control schemes, but the final steady state Cartesian position was quite acceptable for any desired Cartesian position that we have tested by keeping the same controller gains.

6.5 Discussion

The experimental tests carried out with transpose Jacobian-base controllers allows to make the following conclusions and remarks:

- Perhaps the most disappointing feature of transpose Jacobian-based controllers is the lack of a procedure and intuition for tuning their gains. In our experiments, the gain tuning was made through tedious trial and error maneuvers. This was particularly true for the PD transpose Jacobian with joint and Cartesian space damping controllers.
- The two most well known transpose Jacobian-based controllers, i.e. PD transpose Jacobian with joint damping (10) and PD transpose Jacobian with Cartesian space damping (16) controllers offered the worst performances. This is twofold true because, first, due to static friction, the final steady state Cartesian position error was unacceptably large, and second, re-tuning was needed for different desired Cartesian positions.

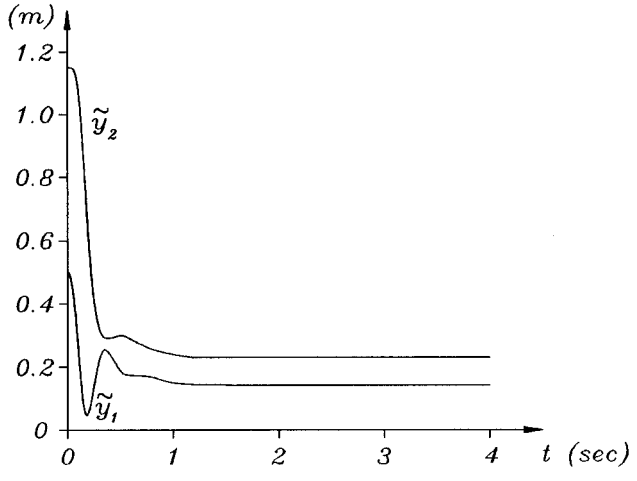


Fig. 5. Cartesian error: PD transpose Jacobian with Cartesian space damping.

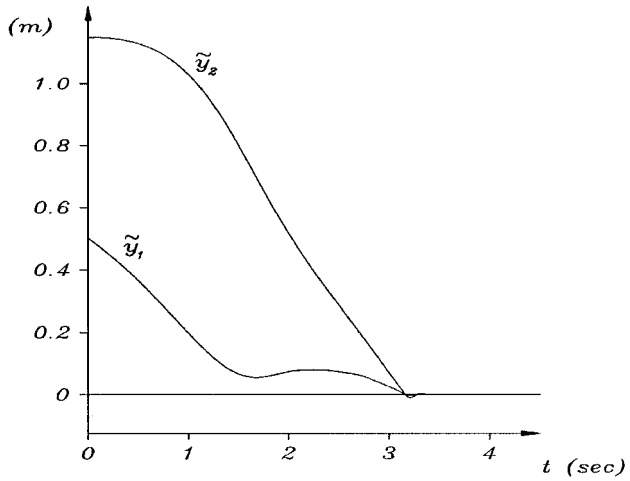


Fig. 6. Cartesian error: Normalized transpose Jacobian with Cartesian space damping.

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- Among the controllers tested, the best performance was obtained using the normalized transpose Jacobian with Cartesian space damping (21). In contrast with the remaining controllers, the gains tuning was relatively easy and re-tuning was not required for desired Cartesian positions within the robot workspace.
- Further experiments were conducted with all transpose Jacobian controllers where a desired Cartesian position located out of the workspace was specified. Obviously, the regulation goal was impossible to achieve; however, in all cases the system behavior showed a safe motion in the sense that the arm reached a configuration and remained there at rest without any oscillation.
- Additional experiments were carried out with the normalized transpose Jacobian controller with Cartesian space damping. More specifically, instead of a fixed desired Cartesian position, we have provided a slow time varying function for the desired Cartesian position. We found that the system behavior was acceptable, and therefore useful for effective Cartesian trajectory tracking.

7. CONCLUSIONS

Transpose Jacobian-based may be recognized as the simplest paradigm of Cartesian-based robot set-point controllers. They are able to solve the robot positioning problem specified in Cartesian space without requiring the solution of the inverse kinematics neither the computation of the inverse robot Jacobian.

In this paper we have presented new results regarding the application of transpose Jacobian-based controllers for nonredundant robots having no self-motion feature. The main theoretical results establish conditions for ensuring local regulation without regard to Jacobian singularities.

Experimental evaluation of several transpose Jacobian controllers has permitted to detect their advantages and drawbacks. From a practical point of view, the main drawback of transpose Jacobian controllers which may limit real applications is the lack of gains tuning policies. This issue still remains open for further research. Despite this criticism, some transpose Jacobian-based controllers appear to be effective in practice even for time varying desired Cartesian positions.

The analysis presented in the paper can also be adapted to study other transpose Jacobian-based controllers where instead of the end-effector Cartesian position be of concern, the output variables of interest is defined with respect to other reference frames, for instance, for applications to visual servoing.

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