# INVERSE ACTIVE LOAD INPUTS ESTIMATION OF THE 3D SPATIAL TRUSS STRUCTURE SYSTEM

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## ABSTRACT

This study presents an innovative fuzzy inverse method with the finite-element scheme for estimating the unknown time-varying load inputs on a three-dimensional (3D) spatial truss structural system. The finite-element scheme is employed to discretize the problem in space, allowing multidimensional problems of various geometries to be treated. This method is based on the fuzzy Kalman Filter (FKF) technology and the fuzzy weighting recursive least square method (FWRLSM). The fuzzy Kalman filter measures the system responses at two distinct nodes in the 3D spatial truss structure. The fuzzy weighting recursive least square method is derived using the residual innovation sequence to compute the input loads. The proposed method's superiority is demonstrated using several typical simulation cases that vary with different estimator and the distinct levels of the initial process noise covariance and the measurement noise covariance. The results show that this method has great stability and accuracy.

**Keywords:** Finite-element scheme, Fuzzy kalman filter.

## 1. INTRODUCTION

In the structural design and reliability assessment, the most important procedure is to obtain the values of active input forces to the system. Therefore, the dynamic input loads on the structure must be determined using the inverse or measurement technique. In practical engineering problems, there are always difficulties in installing the force transducers used to measure the active input forces to the structural system [1]. To resolve this situation, the active input forces to the structural system can be estimated on-line using the inverse estimation technique.

Force estimation is a process of determining the applied loads from the system output responses. The estimation of dynamic forces acting on a structure is an old problem that has been treated with only partial success. Methods for such estimation fall into two categories; direct methods and indirect methods. In general, the indirect approach to force estimation is more difficult and is called an "inverse problem" [2]. As such, it is ill-posed and ill-conditioned [3]. To treat these ill-conditional problems, the input estimation method is used to determine the unknown inputs. This method is based on the Kalman Filter and the recursive least square algorithm weighted by a fading factor, to determine the unknown inputs acting on the system. The input estimation is widely adopted to cope with a system with inputs that cannot be easily measured directly. Tuan *et al*. [4,5] presented an input estimation approach which can recursively solve the IHCPs in real time. Ma *et al*. [6] presented an inverse method to estimate the excitation forces by analyzing the dynamic responses of the structural system. Lee *et al*. [7] utilized the adaptive weighted input estimation method to inversely solve the burst load on a truss structural system. Chen *et al*. [8] investigated the adaptive input estimation method applied to the inverse estimation of load input in a multi-layer shearing stress structure. The unknown inputs usually have a slow or quick time-varying state which cannot be predicted in the estimation process. Therefore, it is difficult to choose an adaptive and efficient weighting function for any input variation. The overall input estimation performance is acceptable by analyzing the dynamic response of the structural system. However, the estimates converge slowly in the initial state when the adaptive weighting function is used in the RLSE. The increase in process noise variance will influence the estimation precision. With a larger process noise variance assumed, better capability of tracking the time-varying force inputs can be obtained, but the overall measurement noise reduction effectiveness will be degraded.

This paper presents an intelligent fuzzy estimator with higher target tracking performance and better noise reduction effectiveness. This efficient and robust estimator is accelerated and weighted by adopting

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the fuzzy accelerating and weighting factor based on the fuzzy logic inference system. The practicability of the proposed method can be verified using numerical simulations of the input loads estimation for a simple 3D spatial truss bridge. The results will be compared with the results produced using other inverse estimation methods. The results reveal that this method has the properties of better feasibility and effectiveness in estimating the input loads.

## 2. PROBLEM FORMULATION

To illustrate the practicability and precision of the presented approach in estimating the unknown input loads, numerical simulations of the 3D spatial truss structure are investigated here. The 3D spatial truss structural system is shown in Fig. 1. Input estimation is based on the state-space analysis method. In this work, we used the FEM to construct a state-space model of a truss structural system. The finite element model of a truss structure is considered to be an "$n$"-degree-of-freedom system. Therefore, the differential equations of motion for the system in terms of mass, stiffness and damping matrices are shown below:

$$M\ddot{Y}(t)+C\dot{Y}(t)+KY(t)=F_b(t) \tag{1}$$

where $M$ is the $n \times n$ mass matrix. $C$ is the $n \times n$ damping coefficient matrix. $K$ is the $n \times n$ stiffness matrix. $\ddot{Y}(t)$, $\dot{Y}(t)$, and $Y(t)$ are the $n \times 1$ acceleration, speed and displacement vectors, respectively. $F_b(t)$ is the $n \times 1$ input load vector. The matrices, $M$ and $K$, can be obtained using FEM. The matrix $C$ was obtained by assembling the matrices $M$ and $K$ as a proportional damping model.

The input estimation algorithm is a calculation method using the state space. Therefore, the state equation and the measurement equation must be constructed before implementing this method. After converting to the state-space model, the state variables of the second order dynamic system with $n$ degrees of freedom are represented by a $2n \times 1$ state vector, *i.e.*, $X=\left[Y(t) \quad \dot{Y}(t)\right]^T$. From Eq. (1), the continuous-time state equation and measurement equation of the structure system can be formulated as follows [4]:

$$\dot{X}(t)=AX(t)+BF_b(t) \tag{2}$$

$$Z(t)=HX(t) \tag{3}$$

where

$$A=\begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$$

$$B=\begin{bmatrix} 0_{n\times n} \\ M^{-1} \end{bmatrix}$$

$$X(t)=\left[X_1(t) \quad X_2(t) \quad \cdots \quad X_{2n-1}(t) \quad X_{2n}(t)\right]^T$$

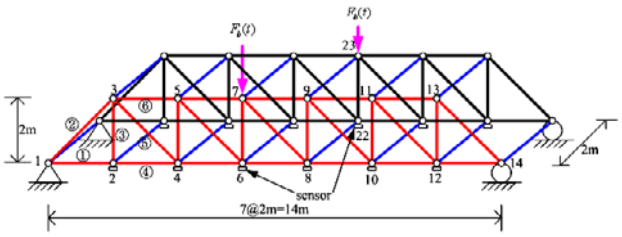



Fig. 1 Finite element model of the 3D spatial truss structural system. (64 elements with 28 nodes)

$A$ and $B$ are constant matrices composed of mass, damping and stiffness of the truss structural system. $X(t)$ is the state vector. $Z(t)$ is the observation vector and $H$ is the measurement matrix in Eq. (3). The vertical displacement of nodes 6 and 22 were been measured in this study.

Noise turbulence always exists in the practical environment. This is the reason that any physical system contains two portions: The first is the deterministic portion; the second is the random portion, which is distributed around the deterministic portion. Eqs. (2), (3) do not take the noise turbulence into account. In order to construct a statistic model of the system state characteristics, a noise disturbance term, which can reflect these state characteristics, will need to be added into these two equations. Up to now, one of the random noise disturbances that can be completely resolved is the Gaussian white noise, which has been statistically illustrated in full using the probability distribution function and the probability density function. Practically, any function corresponding to the functions mentioned above has the same effect. The characteristic function of the random variable is one example. Two most important characteristic values are the mean and the variance, which represent the statistic properties of the random process [9]. Taking the above consideration into account, the continuous-time state equation is to be sampled with the sampling interval, $\Delta t$, to obtain the discrete-time statistic model of the state equation shown below [10]:

$$X(k+1)=\Phi X(k)+\Gamma\left[F_b(k)+w(k)\right] \tag{4}$$

where

$$X(k)=\left[X_1(k) \quad X_2(k) \quad \cdots \quad X_{2n-1}(k) \quad X_{2n}(k)\right]^T$$

$$\Phi=\exp(A\Delta t)$$

$$\Gamma=\int_{k\Delta t}^{(k+1)\Delta t}\exp\left\{A\left[(k+1)\Delta t-\tau\right]\right\}Bd\tau$$

$$F_b(k)=\left[F_{b1}(k) \quad F_{b2}(k) \quad \cdots \quad F_{bn-1}(k) \quad F_{bn}(k)\right]^T$$

$$w(k)=\left[w_1(k) \quad w_2(k) \quad \cdots \quad w_{n-1}(k) \quad w_n(k)\right]^T$$

$X(k)$ is the state vector. $\Phi$ is the state transition matrix. $\Gamma$ is the input matrix. $\Delta t$ is the sampling interval. $w(k)$ is the processing error vector, which is assumed as the Gaussian white noise. Note that $E\{w(k)$



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$w^T(j)\} = Q\delta_{kj}$, and $Q = Q_w \times I_{2n\times 2n}$. $Q$ is the discrete-time processing noise covariance matrix. $\delta_{kj}$ is the Kronecker delta function. When describing the active characteristics of the structural system, an additional term, $w(k)$, can be used to present the uncertainty in a numerical manner. The uncertainty could be the random disturbance, uncertain parameters, or the error due to the over-simplified numerical model.

Generally speaking, the system state can be determined by measuring the system output. This measurement usually has a certain relationship with the system output. However, there is also the noise issue with the measurement. As a result, the discrete-time statistical model of the measurement vector can be presented below:

$$Z(k) = HX(k) + v(k) \tag{5}$$

where

$$Z(k) = \begin{bmatrix} Z_1(k) & Z_2(k) & \dots & Z_{2n}(k) \end{bmatrix}^T$$

$$v(k) = \begin{bmatrix} v_1(k) & v_2(k) & \dots & v_{2n}(k) \end{bmatrix}^T$$

$Z(k)$ is the observation vector. $v(k)$ represents the measurement noise vector and is assumed to be the Gaussian white noise with zero mean and the variance, $E\{v(k)\ v^T(j)\} = R\delta_{kj}$, where $R = R_v \times I_{2n\times 2n}$. $R$ is the discrete-time measurement noise covariance matrix. $H$ is the measurement matrix.

## 3. THE FUZZY INPUT ESTIMATION METHOD

In this section we will construct a fuzzy accelerated factor and weighted factor which can be operated at each step based on the innovation from the Kalman filter. The presented method can inversely estimate the unknown inputs by applying the active reaction of the structural system. This method is composed of the fuzzy Kalman filter without the input term and the fuzzy weighted recursive least square estimator. The fuzzy estimator are accelerated and weighted by the fuzzy accelerating factor for the processing noise covariance matrix and weighting factor of the input estimation method proposed based on the fuzzy logic inference system. The fuzzy Kalman filter can produce the residual innovation sequence, which contains the bias or systematic error caused by the unknown time-varying inputs, and the variance or random error caused by the measurement error. Therefore, the estimator utilizes the innovation sequence to estimate the inputs over time by adopting the fuzzy weighted recursive least square method. The Kalman filter without the input term is shown as follows [4]:

The state prediction is

$$\overline{X}(k/k-1) = \Phi\overline{X}(k-1/k-1) \tag{6}$$

The prediction error covariance matrix is

$$P(k/k-1) = \Phi P(k-1/k-1)\Phi^T + \Gamma Q\Gamma^T \tag{7}$$

The bias innovation produced by the measurement noise and input disturbance is expressed by

$$\overline{Z}(k) = Z(k) - H\overline{X}(k/k-1) \tag{8}$$

The covariance of residual is $S(k)$

$$S(k) = HP(k/k-1)H^T + R \tag{9}$$

The Kalman gain is

$$K_a(k) = P(k/k-1)H^T S^{-1}(k) \tag{10}$$

The state filter value is expressed as

$$\overline{X}(k/k) = \overline{X}(k/k-1) + K_a(k)\,\overline{Z}(k) \tag{11}$$

The filter error covariance matrix is expressed by

$$P(k/k) = \left[I - K_a(k)H\right]P(k/k-1) \tag{12}$$

Above Eqs. (6) to (12), a superscript '–' indicates filter estimation. The related equations of the recursive least square estimator are as follows [4]:

The sensitivity matrices are $B(k)$ and $M(k)$

$$B_s(k) = H\left[\Phi M_s(k-1) + I\right]\Gamma \tag{13}$$

$$M_s(k) = \left[I - K_a(k)H\right]\left[\Phi M_s(k-1) + I\right] \tag{14}$$

The correction gain is expressed as

$$K_b(k) = \gamma^{-1}P_b(k-1)\,B_s^T(k)\left[B_s(k)\,\gamma^{-1}P_b(k-1)\,B_s^T(k) + S(k)\right]^{-1} \tag{15}$$

where $\gamma$ is the weighting factor. The error covariance of the input estimation process is

$$P_b(k) = \left[I - K_b(k)B_s(k)\right]\gamma^{-1}P_b(k-1) \tag{16}$$

The estimated load input is

$$\hat{F}_b(k) = \hat{F}_b(k-1) + K_b(k)\left[\overline{Z}(k) - B_s(k)\,\hat{F}_b(k-1)\right] \tag{17}$$

Some filter parameters must be obtained before the filtering process. Such as, the state transition matrix of the structural system, $\Phi$, the measurement matrix, $H$, the discrete-time processing noise covariance matrix, $Q$ and the discrete-time measurement noise covariance matrix, $R$. The on-line state estimation, $\overline{X}(k/k-1)$ and state estimation error covariance, $P(k/k-1)$ of the filter will be acquired when the observation vector is unceasingly input immediately after the initial conditions $X_0$ and $P_0$ are drawn into the estimator. According to the error covariance matrix $p(k/k)$ in Eq. (12), in the meantime when the Kalman filter produces the state estimates, the error analysis is on going to determine if the error is a symmetric matrix greater or equal to zero, so that the precision of the estimation can be obtained. $K_a(k)$ gets smaller as the processing noise covariance,



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$Q_w$ and the state filter error covariance get smaller according to Eqs. (7) and (10), indicate that the new measurement is mitigating to the state predicted correction. $K_a(k)$ gets smaller as the measurement noise covariance, $R_v$ get larger according to Eqs. (9) and (10), that is to say, the measurement error is mitigating to the state estimation of the estimator. This means that the new measurements produce lower influence of correction on state estimation. The further use of the least square theory can detect the system bias due to the unknown time-varying inputs. The estimated inputs can be converged rapidly to the exact value as the correction gain $K_b(k)$ gets smaller with the increased simulation time. In other words, the Kalman gain $K_a(k)$ depends on $R_v$ and $Q_w$. The above-mentioned is an important principle and a key problem that the appropriate $R_v$ and $Q_w$ can be chosen in accordance with the system property and the magnitude of noise interference in the estimation process. $R_v$ can be chosen in accordance with the measurement instrument precision. $Q_w$ can be chosen in accordance with the system modular error. The Kalman gain can be slightly corrected with the higher measurement instrument precision, that is to say, the modular error of the system changes from big to small. For this reason, the processing noise covariance can be defined as follows:

$$Q_w(k+1) = Q_w(k) \times 10^{\alpha(k)} \tag{18}$$

where $\alpha(k)$ is the fuzzy accelerating factor, which is chosen in the interval, [−1, 1]. The estimation precision gets better as $\alpha(k)$ gets smaller. Conversely, the estimation precision gets worse as $\alpha(k)$ gets larger. Since the processing noise covariance, $Q_w$ is normally unknown, processing noise covariance of the estimator can be initialized as large number. This has the effect of treating the initial errors as large numbers, so that the estimator usually ignores the first few estimates.

The weighting factor $\gamma(k)$ is another important parameter which affecting the estimation precision in the estimation process. It also plays the role as an adjustable parameter to control the estimator bandwidth or the gain magnitude of the recursive least square estimator. It can operate at each step based on the innovation produced by the Kalman filter. Furthermore, in the equations of the recursive least square estimator, $\hat{F}_b(k)$ is the estimate of the unknown input. $P_b(k)$ is the error covariance of the input estimation process. $K_b(k)$ is the correction gain. $\gamma(k)$ is the weighting factor in the range between 0 and 1. The two functions of $\gamma(k)$ are smoothing and forgetting. The forgetting effectiveness depends on the value of $\gamma(k)$. $K_b(k)$ gets larger as $\gamma(k)$ gets smaller according to Eq. (15). The forgetting effect becomes more conspicuous according to Eq. (16). Note that the faster the forgetting effect is, the lower the smoothing effect will be, that is, it introduces oscillation. The weighting factor $\gamma(k)$ is employed to compromise between the upgrade of tracking capability and the loss of estimation precision.

The processing noise covariance and the weighting factor can be adjusted using the each step innovation produced by the Kalman filter. In other word, the magnitude of fuzzy accelerating and weighting factor is determined according to the modulus of bias innovation, $\left|\Delta\bar{Z}(k)/\bar{Z}(k)\right|$. The unknown input prompt variation will cause the large modulus of bias innovation. In the meantime, the smaller accelerating and weighting factor is obtained when the modulus of bias innovation is larger. Therefore, the estimator accelerates the tracking speed and produces larger vibration in the estimation process. On the contrary, the smaller variation of unknown input causes the smaller modulus of bias innovation. In the meantime, the larger accelerating and weighting factor is obtained according to the small modulus of bias innovation. The estimator is unable to estimate the unknown input effectively. For this reason, the fuzzy input estimation method which efficiently and robustly estimates the time-varying unknown input will be constructed in this study.

The fuzzy input estimator in this work is proposed based on the fuzzy logic inference system. The fuzzy logic system includes four basic components, which are the fuzzy rule base, fuzzy inference engine, fuzzifier, and defuzzifier. The value of fuzzy logic system input, $\theta(k)$, may be chosen within the interval, [0,1]. The Pythagorean theorem with the transverse axle (time, $t$) and the vertical axle (residual of predictor, $\bar{Z}$) can be used to solve the length of the hypotenuse. In other words, the length of the hypotenuse is the variation rate of the residual in the sampling interval. The dimensionless input variable is defined as follows:

$$\theta(k) = \frac{\left|\dfrac{\Delta\bar{Z}(k)}{\bar{Z}(k)}\right|}{\sqrt{\left(\dfrac{\Delta\bar{Z}(k)}{\bar{Z}(k)}\right)^2 + \left(\dfrac{\Delta t}{t_f}\right)^2}} \tag{19}$$

where $\Delta\bar{Z}(k) = \bar{Z}(k) - \bar{Z}(k-1)$, $\Delta t$ is the sampling interval, let $t_f = 1$. The fuzzy sets for $\theta(k)$, $\alpha(k)$ and $\gamma(k)$ are labeled in the linguistic terms of EL (extremely large), VL (very large), LV (large value), MV (medium value), SV (small value), VS (very small), and ES (extremely small). A fuzzy rule base is a collection of fuzzy IF-THEN rules which are shown in Table 1.

where $\theta(k)$ is input variable, $\alpha(k)$ and $\gamma(k)$ are the output variables of the fuzzy logic system, respectively. The fuzzier maps a crisp point $\theta(k)$ into a fuzzy set $A$. Therefore, the non-singleton fuzzier can be expressed as follows [11].

$$\mu_A\left(\theta(k)\right) = \exp\left(-\frac{\left(\theta(k) - \bar{x}_i^l\right)^2}{2(\sigma_i^l)^2}\right) \tag{20}$$



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Table 1 The fuzzy rule base

| | | input variable $\theta(k)$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | ES | VS | SV | MV | LV | VL | EL |
| Output variables | $\alpha(k)$ | EL | VL | LV | MV | SV | VS | ES |
| | $\gamma(k)$ | EL | VL | LV | MV | SV | VS | ES |

$\mu_A(\theta(k))$ decreases from 1 as $\theta(k)$ moves away from $\bar{x}_i^l$. $(\sigma_i^l)^2$ is a parameter characterizing the shape of $\mu_A(\theta(k))$.

The Mamdani maximum-minimum inference engine is used in this work. The max-min-operation rule of fuzzy implication of the output variable, $\alpha(k)$ is shown as follows [11].

$$\mu_B(\alpha(k)) = \max_{j=1}^{c}\left\{\min_{i=1}^{d}\left[\mu_{A_i^j}(\theta(k)), \mu_{A_i^j\to B^j}(\theta(k), \alpha(k))\right]\right\} \quad (21)$$

The output variable, $\gamma(k)$ can be similarly shown as follows [11]:

$$\mu_B(\gamma(k)) = \max_{j=1}^{c}\left\{\min_{i=1}^{d}\left[\mu_{A_i^j}(\theta(k)), \mu_{A_i^j\to B^j}(\theta(k), \gamma(k))\right]\right\} \quad (22)$$

where $c$ is the fuzzy rule, and $d$ is the dimension of input variables.

The defuzzifier maps a fuzzy set $B$ to a crisp point $\alpha \in V$. The fuzzy logic system with the center of gravity is defined in the following [11].

$$\alpha(k) = \frac{\sum_{l=1}^{n} \bar{y}^l \mu_B(\alpha^l(k))}{\sum_{l=1}^{n} \mu_B(\alpha^l(k))} \quad (23)$$

The defuzzifier of the output variable, $\gamma(k)$ can be similarly shown as following [11]:

$$\gamma^*(k) = \frac{\sum_{l=1}^{n} \bar{y}^l \mu_B(\gamma^l(k))}{\sum_{l=1}^{n} \mu_B(\gamma^l(k))} \quad (24)$$

$n$ is the number of outputs. $\bar{y}^l$ is the value of the $l$th output. $\mu_B(\alpha^l(k))$ and $\mu_B(\gamma^l(k))$ represent the membership of $\alpha^l(k)$ and $\gamma^l(k)$ in the fuzzy set $B$, respectively. Substituting $\alpha(k)$ in Eq. (23) into Eq. (18) and $\gamma^*(k)$ in Eq. (24) into Eqs. (15), (16) allows us to configure the fuzzy estimator. A design flow chart of the fuzzy estimator is given in Fig. 2.

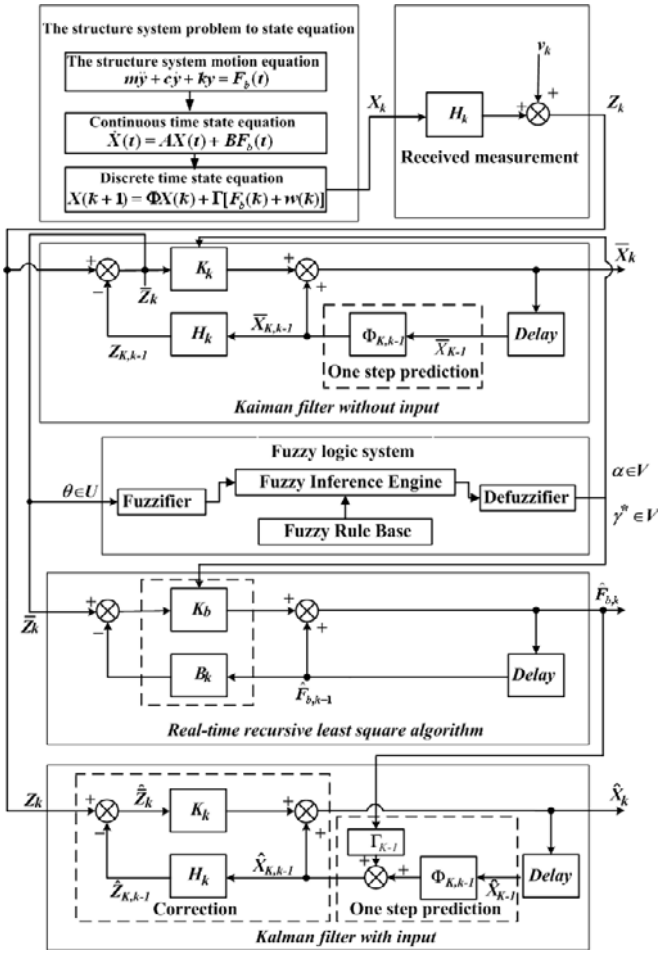



Fig. 2 Flowchart of the finite element method combined with the fuzzy estimation algorithm

## 4. RESULTS AND DISCUSSION

To illustrate the accuracy and practicability of the proposed approach in predicting the unknown input load, a 3D spatial example is used to check the feasibility of the inverse fuzzy input estimation method combined with the finite-element scheme. The consistent mass matrix element $M^e$ and the element stiffness matrix $K^e$ of the space truss are shown as follows [12]:

$$M^e = \frac{\rho A^e l}{6}\begin{bmatrix} 2\xi_1^2 & 2\xi_1\xi_2 & 2\xi_1\xi_3 & \xi_1^2 & \xi_1\xi_2 & \xi_1\xi_3 \\ & 2\xi_2^2 & 2\xi_2\xi_3 & \xi_1\xi_2 & \xi_2^2 & \xi_2\xi_3 \\ & & 2\xi_3^2 & \xi_1\xi_3 & \xi_2\xi_3 & \xi_3^2 \\ & & & 2\xi_1^2 & 2\xi_1\xi_2 & 2\xi_1\xi_3 \\ & & & & 2\xi_2^2 & 2\xi_2\xi_3 \\ SYM & & & & & 2\xi_3^2 \end{bmatrix}$$

and



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$$K^e = \frac{A^e E}{l}\begin{bmatrix} \xi_1^2 & 2\xi_1\xi_2 & 2\xi_1\xi_3 & -\xi_1^2 & -\xi_1\xi_2 & -\xi_1\xi_3 \\ & \xi_2^2 & 2\xi_2\xi_3 & -\xi_1\xi_2 & -\xi_2^2 & -\xi_2\xi_3 \\ & & \xi_3^2 & -\xi_1\xi_3 & -\xi_2\xi_3 & -\xi_3^2 \\ & & & \xi_1^2 & 2\xi_1\xi_2 & 2\xi_1\xi_3 \\ & & & & \xi_2^2 & 2\xi_2\xi_3 \\ SYM & & & & & \xi_3^2 \end{bmatrix}$$

where $\{\xi_1\ \xi_2\ \xi_3\}$ is the direction cosines of the $\bar{x}$ -, $\bar{y}$ - and $\bar{z}$ - with respect to the $x$-axis of the $xyz$-coordinate system, respectively. The density, $\rho = 7860\text{kg/m}^3$. The cross-sectional area, $A^e = 2.5 \times 10^{-3}\text{m}^2$. $l$ is the length. The elastic modulus of all members, $E = 200\text{GPa}$. The proportional damping coefficient, $C_n = \alpha M_n + \beta K_n$, where $\alpha = 0.002$ and $\beta = 0.00005$. The initial conditions of the error covariance are given as $p(0/0) = \text{diag}[10^8]$ for the fuzzy Kalman filter and $p_b(0) = 10^8$ for the fuzzy weighted recursive least square estimator. The simulation parameters are set as follows. The sampling interval, $\Delta t = 0.01$ss. The sensitivity matrix $M(0)$ is null.

### 4.1 Example: Decaying Exponential Input Loads

The input load produces a rapid release of energy when the explosive detonates. In the meantime, a tremendous input load is produced and spreads out along with the vibrational wave. This kind of input load has the properties of decaying and transient existence. This is the reason why the input load is often approximated in the form of a decaying exponent. This simulation adopts the decaying exponent input loads with different values of amplitude on Nodes 7 and 23 of the truss structural system. The numerical model of the load inputs are shown as follows:

$$F_b 7(t) = \begin{cases} 2000000 \times \exp(-3 \times t) & 2 \le t \le 7 \\ 0 & 0 \le t < 2 \end{cases} \quad (N) \qquad (25)$$

$$F_b 23(t) = \begin{cases} 1000000 \times \exp(-2 \times t) & 2 \le t \le 7 \\ 0 & 0 \le t < 2 \end{cases} \quad (N) \qquad (26)$$

The initial processing noise variance, $Q_w(0) = 10^{10}$. The measurement noise variance, $R_v = \sigma^2 = 10^{-10}$. The vertical displacement of nodes 6 and 22 were been measured by simulation sensors. By applying the active dynamic reaction which contains noise to the presented algorithm, the estimation result of the decaying exponent input load inputs, $F_b 7(t)$ and $F_b 23(t)$, can be obtained and shown in Figs. 3(a) and 4(a). The results of coarse estimation in the initial response on account of the larger initial processing noise variance are shown in Figs. 3(a) and 4(a). Figures 3(b) and 4(b) show the estimated dynamic responses of truss structural system to the input loads. The simulation results demonstrate that the proposed estimator has the property of faster convergence following the regulated processing noise variance in the estimation process. All in all, the result reveals good estimation ability. The estimation values converge to actual values rapidly.

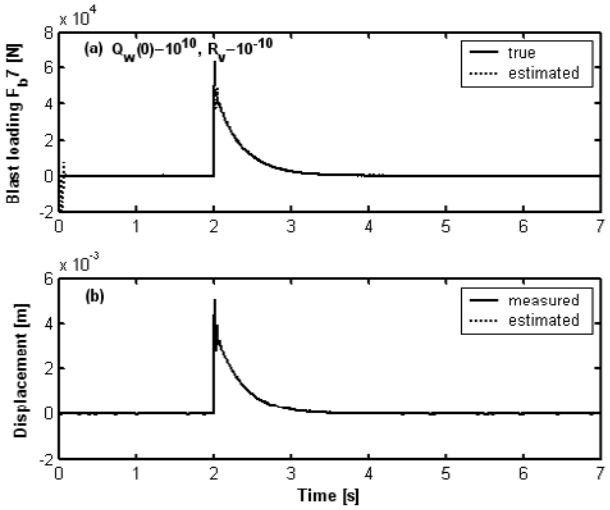



Fig. 3 (a) Inverse estimation of the load input, $F_b 7(t)$. (b) The measured and estimated displacement of the node 6

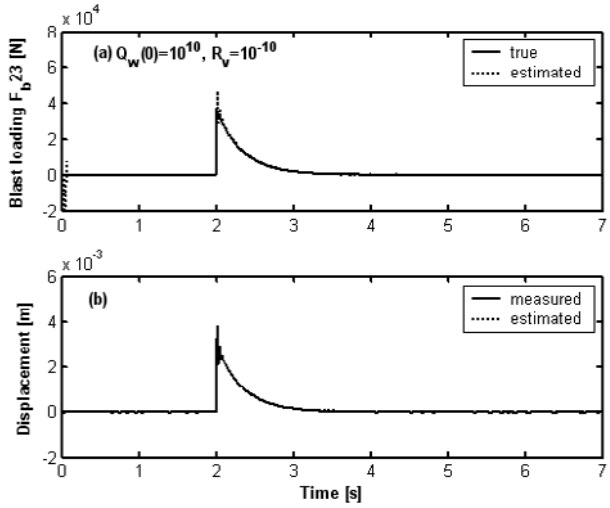



Fig. 4 (a) Inverse estimation of the load input, $F_b 23(t)$. (b) The measured and estimated displacement of the node 22

Figure 5 shows that the estimator has great tracking performance when the larger output variable, $\alpha(k)$, can be chosen to generate a larger processing noise variance, $Q_w(k)$, according to Eq. (18). The estimator has great success in reducing the noise effect when a smaller output variable, $\alpha(k)$, can be chosen to generate a smaller processing noise variance, $Q_w(k)$, when the unknown is a steady input system. Figure 6 shows that a smaller weighting factor can be chosen in the fuzzy recursive least square method when a larger unknown is input into the system. Note that the faster the forgetting effect is, the lower the smoothing effect will be, that is, it introduces oscillation. The fuzzy weighting factor $\gamma(k)$ is employed to compromise between the tracking capability upgrade and the loss of estimation precision. Figure 7 shows that the presented method compares estimation results with the fuzzy weighting function, the adaptive weighting function and the constant weighting factor input estimation method. Simulation results demonstrate that the presented input



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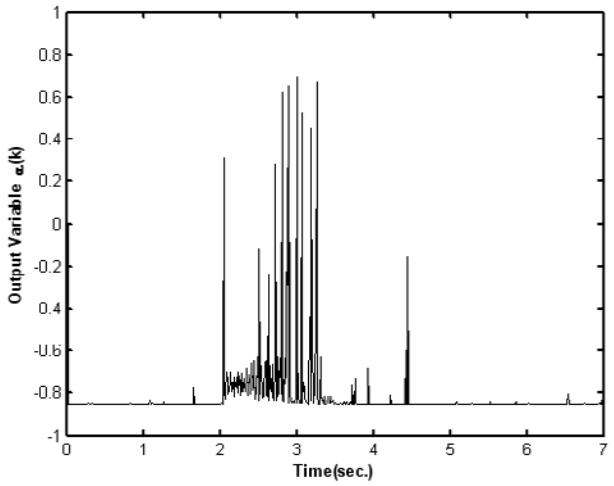



Fig. 5 The varied value of output variable, $\alpha(k)$

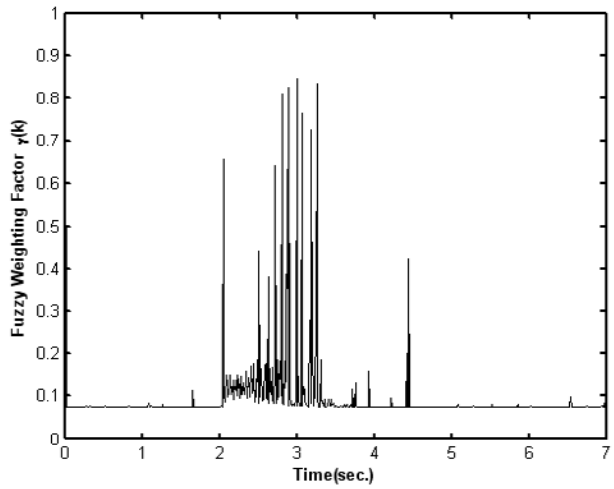



Fig. 6 The varied value of fuzzy weighting factor, $\gamma(k)$

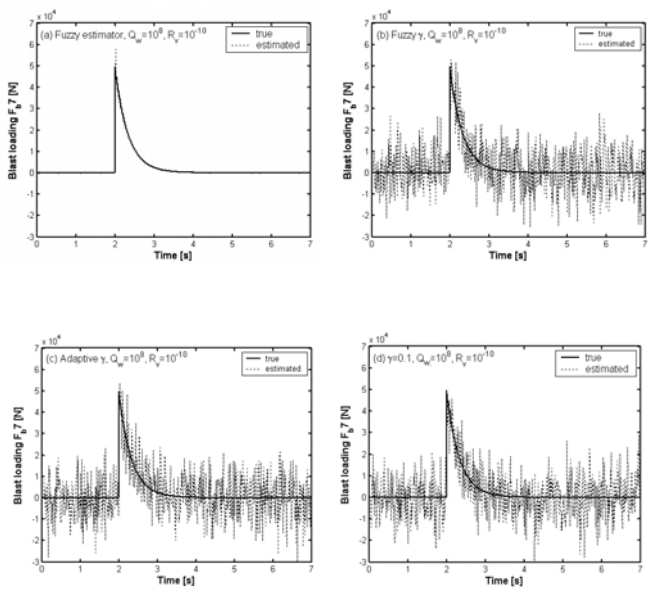



Fig. 7 Inverse estimation of the load input, $F_b7(t)$, with different estimator

estimation inverse methodology has good performance in tracking the decaying exponent load input in the 3D spatial truss structural system. This case was compared using different initial process noise variance $Q_w(0) = 10^8$, $10^{10}$, $10^{14}$ and $10^{18}$ as in Fig. 8. Figure 8 shows that if the initial process noise variance $Q_w(0)$ increases, it will influence the estimation resolution. A larger initial process noise variance will affect the capability of tracking the decaying exponent load input in the initial state. Figure 9 shows the estimation results with the initial process noise variance fixed ($Q_w(0) = 10^8$), and different measurement error variances ($R_v = 10^{-10}$, $10^{-8}$, $10^{-6}$ and $10^{-4}$). The result shows that when $R_v$ is large, the filter transient performance will be slower with noise filtered out. Conversely, the fluctuation will become more severe when $R_v$ decreases. The filter transient performance will be faster with more influence induced by the noise. The estimation results for the proposed method agree highly with the actual values. The results show that a larger measurement error can cause estimation lag and estimate precision degradation when estimating the decaying exponent load input. In practice, the measurement error should depend on sensors, measurement locations and users. This study assumes the sensor location is under the truss structural system nodes and the users' error is neglected. The measurement error can be restricted by an extremely accurate sensor. The signal-to-noise ratio (*SNR*) is a measure used in science and engineering to quantify how much a signal has been corrupted by noise. It is can be defined as the ratio of mean square amplitude of the signal to that of the noise. Here we define that ratio, $SNR\ (dB) = 20 \times \log_{10}(R_s / R_n)$ [13]. $R_s$ is the mean square amplitude of the measurement original signal, $R_n$ is that of the measurement noise. Figure 9 shows that the simulation results of the estimator with different level measurement noise (*SNR*). Observe that, the proposed estimator is therefore quite sensitive to *SNR*. Because the initial process noise covariance is bigger, we are really saying that the coarse estimation result in the initial response. We can see that after a brief transient period the estimate and the actual signal are virtually identical. Therefore, we can say that the proposed estimator gives excellent estimates of the signal when the $SNR \geq 16.26dB$. Figure 10 shows the responses (displacement of the node 6) without and with measured noise ($R_v = 10^{-8}$).

The above simulation results demonstrate that the proposed method has good performance in tracking the time-varying unknown loads in the 3D truss structural system.

## 5. CONCLUSIONS

This study presented an on-line methodology based on the fuzzy inverse input estimation method combined with the finite-element scheme. The proposed method was developed for estimating the unknown time-varying input loads in a 3D spatial truss structural system. The FEM is adopted to construct the state equation of the truss structure. The fuzzy Kalman Filter is



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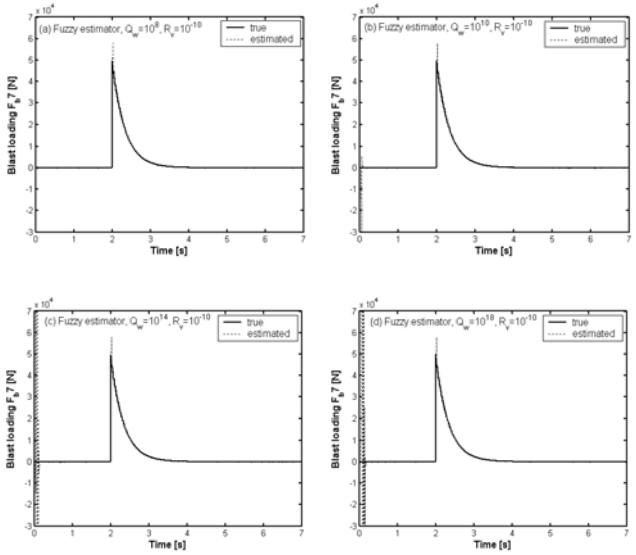



Fig. 8 Inverse estimation of the load input, $F_b7(t)$, with $Q_w(0) = 10^8$, $10^{10}$, $10^{14}$ and $10^{18}$, respectively

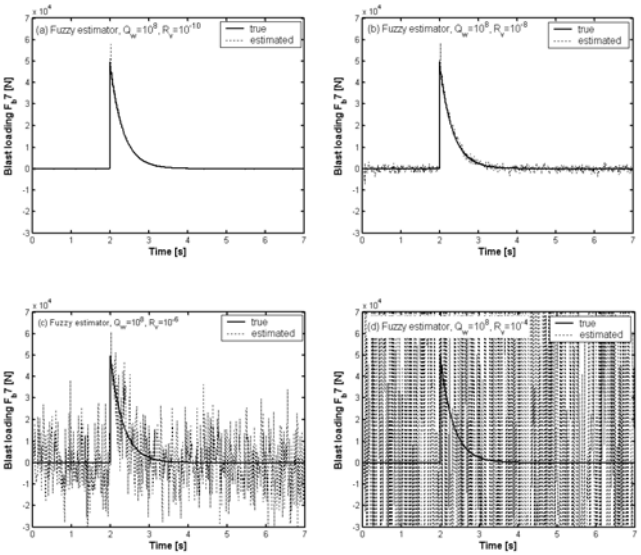



Fig. 9 Inverse estimation of the load input, $F_b7(t)$, with (a) $R_v = 10^{-10}$, $SNR$ = 35.67dB, (b) $10^{-8}$, $SNR$ = 16.26dB, (c) $10^{-6}$, $SNR$ = 1.611dB and (d) $10^{-4}$, $SNR$ = 0.002dB respectively

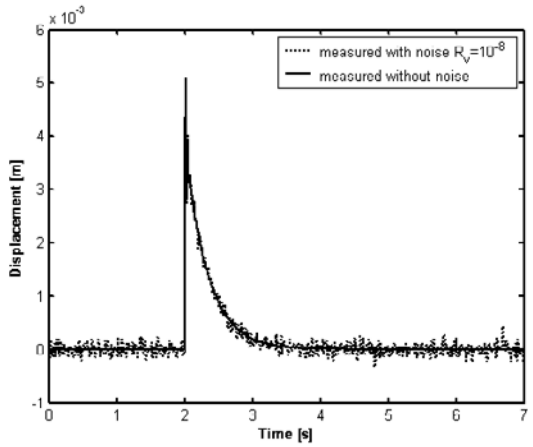



Fig. 10 The responses (displacement of the node 6) with and without measured noise

further combined with the fuzzy weighted recursive least square algorithm to estimate the input loads. The assumption of inaccurate initial process noise variance and measurement noise were considered, that is to say, the larger random variable will be added into the statistical system state model characteristics in the estimation process. The simulation results show that the fuzzy estimator has the properties of fast tracking and effective noise reduction, because it is accelerated and weighted by the fuzzy accelerating factor and a weighting factor based on the fuzzy logic inference system. The simulation results were compared by alternating between the fuzzy weighting, adaptive and constant factors. The results demonstrate that this method has faster convergence in the initial response, better target tracking capability and more effective noise reduction.

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