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Surface waves in a plasma flow

E. BENOVA,[1]★ S. T. IVANOV[2]
and A. A. RUKHADZE[3]

[1] Department of Physics, Institute for Foreign Students, BG-1111 Sofia, Bulgaria
[2] Faculty of Physics, Sofia University, BG-1164 Sofia, Bulgaria
[3] Faculty of Physics, Moscow State University, 119899 Moscow, Russia

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Abstract. Dispersion characteristics of surface waves in a semibounded plasma flow with relativistic velocity $\mathbf{u}_0$ parallel to the plasma–vacuum interface are presented. The plasma is considered to be cold and collisionless, which allows us to take into account only the electron motion. It is shown that – in contrast to the bulk waves, which are invariant with respect to Lorentz transformations and whose spectrum is independent of the flow velocity – the surface waves are not invariant, which leads to a dependence of their spectrum on the flow velocity and, correspondingly, to non-reciprocity. The latter peculiarity is due to the fact that the boundary conditions are not invariant.

1. Introduction and plasma permittivity tensor

In the last few years powerful plasmatrons at plasma flow velocity $|\mathbf{u}_0|$ higher than 10^8 cm s^{-1}, as well as high-density relativistic electron beams with velocities close to the speed of light have become available to the plasma physics community. This makes the problem of propagation of surface waves in moving plasmas of great interest. In particular, the problem of ionization of a high-speed gas flow and sustaining a plasma is important for plasmatrons. The most promising gas ionizer to ensure a high-density plasma flow is the surfatron operating on surface waves. In the case of relativistic beams, the surface modes play an important role in relativistic plasma electronics as a source of powerful coherent radiation.

Here, we consider the simplest case of a semi-space isotropic collisionless plasma flow. The permittivity of the flow in its proper reference frame can be presented in the form

$$\varepsilon_{ij} = \delta_{ij}\,\varepsilon(\omega), \tag{1.1a}$$

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2(x)}{\omega^2}, \tag{1.1b}$$

where $\omega_p(x) = \sqrt{4\pi e^2 n(x)/m}$ is the electron plasma frequency; the remaining notation has its usual meaning. It is obvious that ions can be considered as immobile. The plasma density $n(x)$ is a function of the transverse coordinate x; the longitudinal/axial coordinate is z. We study only surface waves travelling along the z axis, i.e. in the direction of the flow velocity $\mathbf{u}_0 \parallel \hat{\mathbf{z}}$.

It is well known (Bolotovskii and Rukhadze 1959) that bulk waves in a

★ E-mail: benova@phys.uni-sofia.bg

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plasma with permittivity (1.1) are invariant in infinite space, and their dispersion relation (independently of the coordinate frame) reads

$$k_x^2 + k_z^2 - \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right) = 0. \tag{1.2}$$

For surface waves on the plasma–vacuum interface in the same geometry, this statement is no longer valid. We have to find the dispersion equation in the laboratory frame, where the flow velocity $\mathbf{u}_0 \parallel \hat{\mathbf{z}}$. Let us first recall that for an immobile plasma, the dispersion relation is (Alexandrov et al. 1984)

$$\sqrt{k_z^2 - \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)} + \sqrt{k_z^2 - \frac{\omega^2}{c^2}}\left(1 - \frac{\omega_p^2}{\omega^2}\right) = 0. \tag{1.3}$$

Two important conclusions follow from this equation:

(i) for surface-wave frequencies $\omega < \omega_p/\sqrt{2}$ and in the low-frequency region $\omega \ll \omega_p$, the waves are strongly non-potential (transverse), with a phase velocity close to the speed of light c, i.e. $\omega \approx |k_z|\, c \ll \omega_p$. However, in the high-frequency region, the waves are potential ones with frequency $\omega = \omega_p/\sqrt{2} \ll |k_z|\, c$.

(ii) These waves are always reciprocal, i.e.

$$\omega(-k_z) = \omega(k_z). \tag{1.4}$$

Let us examine the change of wave characteristics as the plasma flows along the interface. We consider, in the same geometry, waves travelling on the plasma flux with $k_z \neq 0$ and $k_y = 0$ (because $\mathbf{u}_0 \parallel \hat{\mathbf{z}}$). In this case, the plasma permittivity tensor is (Alexandrov et al. 1984)

$$\hat{\varepsilon}_{ij} = \begin{pmatrix} \hat{\varepsilon}_{xx} & 0 & \hat{\varepsilon}_{xz} \\ 0 & \hat{\varepsilon}_{yy} & 0 \\ \hat{\varepsilon}_{zx} & 0 & \hat{\varepsilon}_{zz} \end{pmatrix}, \tag{1.5}$$

where the tensor elements $\hat{\varepsilon}_{ij}$ are operators because of the presence of a plasma–vacuum interface at $x = 0$, and are given by

$$\hat{\varepsilon}_{yy} = \hat{\varepsilon}_{xx} \equiv \varepsilon_{xx} = 1 - \frac{\omega_p^2(x)\,\gamma^{-1}}{\omega'^2}, \tag{1.6a, b}$$

$$\hat{\varepsilon}_{xz} = \frac{i\,u_0\,\omega_p^2(x)}{\omega^2\omega'}\frac{\partial}{\partial x}, \tag{1.6c}$$

$$\hat{\varepsilon}_{zx} = \frac{\partial}{\partial x}\frac{i\,u_0\,\omega_p^2(x)}{\omega^2\omega'}, \tag{1.6d}$$

$$\hat{\varepsilon}_{zz} = 1 - \frac{\omega_p^2(x)\,\gamma^{-1}}{\omega'^2} + \frac{\partial}{\partial x}\frac{u_0^2\,\omega_p^2(x)\,\gamma}{\omega^2\omega'^2}\frac{\partial}{\partial x}, \tag{1.6e}$$

with $\omega' = (\omega - k_z u_0)\,\gamma$, and $\gamma = (1 - u_0^2/c^2)^{-1/2}$. Here $\omega_p^2\,\gamma^{-1}$ is an invariant; the factor γ appears because ω_p is expressed via the plasma density and the electron rest mass m. The derivative $\partial/\partial x$ acts on all the quantities following it when some $\hat{\varepsilon}_{ij}$ enters the wave equations.

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2. Wave equations and boundary conditions

It is easy to show that the surface wave can only be an E mode with non-zero components E_x, B_y, E_z (Alexandrov et al. 1984), and the wave equations are

$$k_z E_x - \frac{\omega}{c} B_y = -i \frac{\partial E_z}{\partial x}, \tag{2.1a}$$

$$\frac{\omega}{c} \varepsilon_{xx} E_x - k_z B_y = -\frac{\omega}{c} \hat{\varepsilon}_{xz} E_z, \tag{2.1b}$$

$$\frac{\partial B_y}{\partial x} + i \frac{\omega}{c} (\hat{\varepsilon}_{zx} E_x + \hat{\varepsilon}_{zz} E_z) = 0. \tag{2.1c}$$

From (2.1), one can obtain the following equation for the component E_z:

$$\frac{\partial^2 E_z}{\partial x^2} - \left(k_z^2 - \frac{\omega^2}{c^2} \varepsilon_{xx} \right) E_z = 0. \tag{2.2}$$

This equation is valid for both the plasma, where $n = n_0 = \text{const}$, and for vacuum, where $n = 0$ and $\varepsilon_{xx} = 1$. We note that in obtaining (2.2) from (2.1), the term

$$\varepsilon^1 \equiv \varepsilon_{xx} = 1 - \frac{\omega_p^2 \gamma^{-1}}{\omega'^2} \tag{2.3}$$

has been omitted. The equation $\varepsilon^1 = 0$ is for pure longitudinal waves in the plasma bulk. That is why, further on, $\varepsilon^1 \neq 0$ is considered.

The boundary conditions are obtained by integrating (2.1):

$$\{E_z\}_{x=0} = 0, \tag{2.4a}$$

$$\left\{ \frac{1}{k_z^2 - (\omega^2/c^2)\, \varepsilon_{xx}} \left[k_z \hat{\varepsilon}_{zx} E_x - i \varepsilon_{xx} \frac{\partial E_z}{\partial x} \right.\right.$$

$$\left.\left. - \frac{u_0 \omega_p^2}{\omega^2 \omega'} \left(\frac{\omega^2}{c^2} \hat{\varepsilon}_{xz} E_z - i k_z \frac{\partial E_z}{\partial x} \right) \right] + i \frac{u_0^2 \omega_p^2 \gamma}{\omega^2 \omega'^2} \frac{\partial E_z}{\partial x} \right\}_{x=0} = 0. \tag{2.4b}$$

Here the symbol $\{A(x)\}_{x=0}$ denotes the difference between the values of a function $A(x)$ on each side of the interface $x = 0$; i.e. $\{A(x)\}_{x=0} = 0$ means that $A(x)$ is continuous at the interface $x = 0$.

The solutions to (2.2) for the regions $x > 0$ (vacuum) and $x < 0$ (plasma) can be presented in the form

$$E_z(x) = \begin{cases} C_1 \exp\left(-\sqrt{k_z^2 - \dfrac{\omega^2}{c^2}}\, x \right) & (x > 0), \\ C_2 \exp\left(\sqrt{k_z^2 - \dfrac{\varepsilon_{xx} \omega^2}{c^2}}\, x \right) & (x < 0). \end{cases} \tag{2.5}$$

Inserting these solutions into the boundary conditions (2.4), we obtain the dispersion relation for surface waves in a plasma flow (compare with (1.3)):

$$\sqrt{k_z^2 - \frac{\omega^2}{c^2} \left(1 - \frac{\omega_p^2 \gamma^{-1}}{\omega^2} \right)} + \sqrt{k_z^2 - \frac{\omega^2}{c^2}} \left(1 - \frac{\omega_p^2 \gamma^{-1}}{\omega'^2} \right) = 0. \tag{2.6}$$

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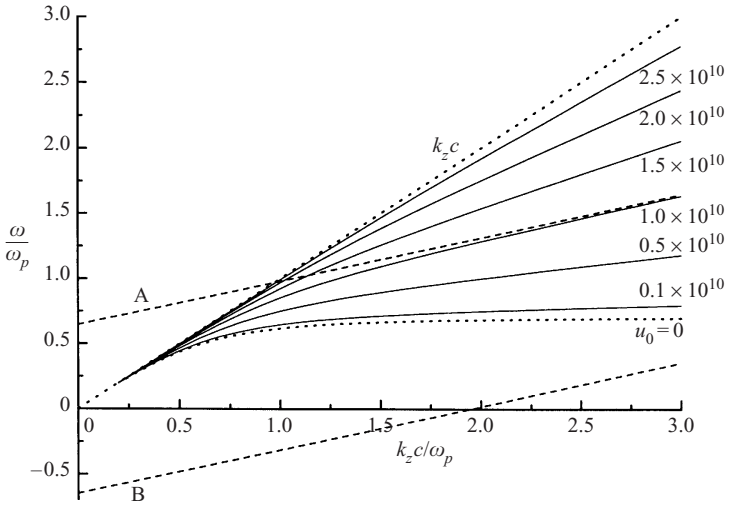


Figure 1. Dispersion curves of surface waves in a semi-infinite plasma flow at various flow velocities (in cm s^{-1}). The dashed lines A and B correspond to $\omega = k_z u_0 \pm \gamma^{-3/2}\omega_p/\sqrt{2}$ respectively, at $u_0 = 1 \times 10^{10}$ cm s^{-1}.

The same result can be obtained from (1.3), by transforming the $(\mathbf{k}, \omega)$ four-vector to the frame in which the plasma is moving, bearing in mind that both $k_z^2 - \omega^2/c^2$ and $\omega_p^2 \gamma^{-1}$ are invariants.

3. Discussion of wave dispersion

Comparing the obtained dispersion equation for surface waves and its spectrum depicted in Fig. 1 with those of an immobile plasma, (1.3), we deduce the following:

(i) In a plasma flow, as in an immobile plasma the relation

$$\omega < k_z c \tag{3.1}$$

is valid.

(ii) While in an immobile plasma the surface modes are only in the frequency range $|\omega| < \omega_p/\sqrt{2}$, for a plasma flow the corresponding inequality reads

$$\omega' = (\omega - k_z u_0)\gamma < \omega_p \gamma^{-1/2}. \tag{3.2}$$

This means that the frequency region of wave existence is moved owing to the Doppler effect.

(iii) The most important difference between (2.6) and its solutions and (1.3) is the breaking of the symmetry, which means that the spectrum of surface waves in a plasma flow is non-reciprocal:

$$\omega(-k_z) \neq \omega(k_z). \tag{3.3}$$

This is due to the Doppler effect too.

(iv) In the short-wavelength limit ($k_z \to \infty$ or $k_z^2 c^2 \gg \omega_p^2$), the asymptotic solution of (2.6) is

$$\omega' = \frac{\omega_p \gamma^{-1/2}}{\sqrt{2}}.$$

This corresponds to potential longitudinal waves, as in the case of (1.3) in the same limit.

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(v) In a high-density plasma, which corresponds to the long-wavelength limit when $|k_z c| \ll \omega_p \gamma^{-1/2}$, the asymptotic solutions to (2.6) and (1.3) coincide, and are

$$\omega \approx k_z c.$$

In this region, the surface waves become reciprocal ones.

(vi) Finally, we note that the surface waves are divided into *fast* and *slow* waves, which is also due to the Doppler effect and becomes noticeable in the region of short wavelength. This can be seen in Fig. 1.

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