102.10 Pythagoras' theorem: a proof by contradiction

If possible, let there be a triangle ABC, right-angled at A, which defies the Pythagorean condition, so that $a^2 \neq b^2 + c^2$.

There are two possibilities:

1. Triangle is 'less-type' (i.e. $a^2 < b^2 + c^2$);
2. Triangle is 'more-type' (i.e. $a^2 > b^2 + c^2$).

We also note that $h^2 = xy$ (by similarity conditions), where h is the altitude on BC.

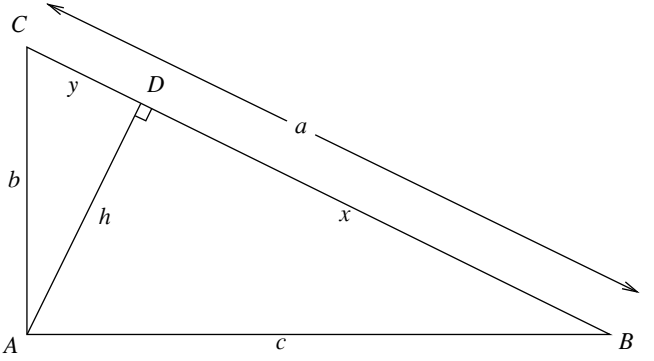


FIGURE 1

When triangle ABC is 'less-type', by virtue of similarity, triangles DBA and DAC are also 'less-type'.

So we have

$$b^2 < h^2 + y^2 \qquad \text{and} \qquad c^2 < x^2 + h^2.$$

As a consequence,

$$b^2 + c^2 < x^2 + y^2 + 2h^2 = (x + y)^2 = a^2.$$

But this leads to a contradiction.

The other case, viz. when triangle ABC is 'more-type', is similar.

Thus, every right-angled triangle must satisfy the Pythagorean condition.

Acknowledgement

The author is grateful to the Editor for his useful suggestions so that this Note could assume a more concise form.

10.1017/mag.2018.19

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https://doi.org/10.1017/mag.2018.19 Published online by Cambridge University Press