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Adiabatic dust-acoustic waves with dust-charge fluctuations

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We study the effect of charge fluctuations on the propagation of adiabatic linear and nonlinear dust-acoustic waves by considering the electrons and ions to be in Boltzmann equilibria, and the dust grains to satisfy the fluid equations with full adiabatic equation of state. Linear dust-acoustic waves are damped owing to the dust-charge fluctuations, and the damping rate decreases with increasing adiabatic dust pressure. Nonlinear dust-acoustic waves are governed by the set of coupled Boussinesq-like and dust-charge perturbation equations. It is shown that for unidirectional propagation, the Boussinesq-like equation reduces to usual Korteweg–de Vries (KdV) equation. At early times, the localized solutions of the KdV equation are damped owing to the dust-charge perturbations. The soliton amplitude decreases with increasing adiabatic dust plasma pressure and increases with Mach number. Soliton solutions are found only in the supersonic regime.

1. Introduction

In recent years, there has been much interest in dusty plasmas because of their importance in space environments such as planetary rings, cometary tails, the lower part of the Earth's ionosphere and laboratory plasmas (Whipple *et al.* 1985; de Angelis *et al.* 1988; Goertz 1989; Sheehan *et al.* 1990; Carlile *et al.* 1991). Observations of dust-acoustic waves and formation of dusty plasma crystals in the laboratory provided much impetus to dusty plasma research (Bouchoule *et al.* 1991; Boufendi *et al.* 1992; Thomas *et al.* 1994; Barkan *et al.* 1995; Morfill and Thomas 1996; Prabhakara and Tanna 1996; Thompson *et al.* 1997). A number of authors have studied linear and nonlinear waves in dusty plasmas, and most of these studies were confined to the constant-charge case (Rao *et al.* 1990; Shukla *et al.* 1991; Shukla 1992; Bharuthram and Shukla 1992; Rao 1993; Rosenberg and Krall 1996; Shchekinov 1997; Vidhya *et al.* 1997; Singh and Rao 1997). However, in reality the dust grains have variable charge. The dust grains are usually of micrometre or submicrometre size, and are negatively charged. Charging of the dust grains is caused by the plasma currents, photo-emission and secondary emission, etc. In thermal plasmas, the charging of the dust grains is dominated by the ion and electron currents over the photo-emission and secondary emission. The charge on the dust-grain surface is thus affected by the nearby plasma conditions. Therefore

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one must consider dust-grain charge to be varying with time and include its fluctuations. The charge fluctuations may lead to new kinds of phenomena in dusty plasmas.

Varma *et al.* (1993) investigated the effect of dust-charge perturbations on electrostatic oscillations and instabilities in a uniform unmagnetized dusty plasma. Melandsø *et al.* (1993) studied the effect of charge fluctuations on dust-acoustic waves (DAWs) in dusty plasmas. They showed that charge fluctuations lead to the damping of the DAW, which is not present in a constant-charge case. D'Angelo (1993) studied the Rayleigh–Taylor (RT) instability by including the dust-charge variation. It is found that the effect of charge variation is important for wave frequencies close to the dust charging frequency. Nonlinear dust-acoustic waves in an unmagnetized dusty plasma were studied by Rao and Shukla (1994) by a considering the effect of charge variation. These are shown to be governed by Boussinesq-like equation coupled to the charge fluctuations. For unidirectional propagation, it reduces to the Korteweg–de Vries (KdV) equation. The localized solutions, at early times, are found to be damped owing to the dust-charge fluctuations. Bhatt (1997) studied Langmuir waves in a dusty plasma with variable grain charge. Ma and Liu (1997) have shown that on a very slow time scale of the dust motion and with grain-charge variations, dust-acoustic solitons exist for a range of velocities, and the amplitude increases linearly with Mach number. More recently, Nejoh (1997) studied the effect of dust-charge fluctuations on electrostatic ion waves in a dusty plasma with trapped electrons.

Recently, Singh and Rao (1997) developed a the theory of adiabatic dust-acoustic solitons in a dusty plasma, and compared the results with the usual γ model. It is suggested that the adiabatic model should be preferred over the γ model, since the predictions of the latter are limited to low temperatures. In this paper, we examine the effect of dust-charge fluctuations on adiabatic dust-acoustic waves. In our model, we consider electrons and ions to be in Boltzmann equilibria and the dust grains to follow the fluid equations with full adiabatic equation of state. We derive a Boussinesq-like equation with a source term for the dust-acoustic waves. For unidirectional propagation, the Boussinesq equation reduces to the KdV equation with a source term. The plan of the paper is as follows. In Sec. 2, we derive two coupled equations. Solutions of these coupled equations for limiting cases are discussed in Sec. 3. A summary of the results is given in Sec. 4.

2. Analysis

We consider a three-component dusty plasma with electrons, ions and dust grains. For very low-frequency waves such as DAWs, the electron and ion inertia can be neglected, and their number densities are governed by

$$n_e = n_{e0} \exp\left(\frac{e}{T_e}\phi\right), \tag{1}$$

$$n_i = n_{i0} \exp\left(-\frac{q_i}{T_i}\phi\right), \tag{2}$$

where $n_{e0}(n_{i0})$, $T_e(T_i)$ and $-e\,(q_i = Z_i e)$ are the number density, temperature and charge of the electron (ion) respectively, and Z_i is the ion charge number. The

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dynamics of the dust grains is governed by the fluid equations

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0, \tag{3}$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = -\frac{q_d}{m_d}\frac{\partial \phi}{\partial x} - \frac{1}{n_d m_d}\frac{\partial P_d}{\partial x}, \tag{4}$$

$$\frac{\partial P_d}{\partial t} + v_d \frac{\partial P_d}{\partial x} + \gamma_d P_d \frac{\partial v_d}{\partial x} = 0, \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi(q_i n_i + q_d n_d - e n_e), \tag{6}$$

$$\frac{\partial q_d}{\partial t} + v_d \frac{\partial q_d}{\partial x} = I_e + I_i, \tag{7}$$

where m_d, n_d, P_d and q_d are the mass, number density, pressure and charge of the dust grains respectively, I_e and I_i are the electron and ion currents (Barnes *et al.* 1992), given by

$$I_e = -\pi e R^2 \left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_e \exp\left(\frac{e}{T_e}\psi\right), \tag{8}$$

$$I_i = \pi q_i R^2 \left(\frac{8T_i}{\pi m_i}\right)^{1/2} n_i \left(1 - \frac{q_i}{T_i}\psi\right), \tag{9}$$

where $m_e\,(m_i)$ is the mass of the electron (ion), R is the dust-grain radius and $\psi = q_d/R$ is the dust grain surface potential relative to the plasma potential ϕ. If the ion streaming velocity v_0 is much larger than the ion thermal speed then an approximate expression for the ion current is $I_i = \pi R^2 q_i n_i v_0 (1 - 2e\psi/m_i v_0^2)$. At equilibrium, we obtain the floating potential ψ_0 and equilibrium dust charge $q_{d0} = C\psi_0$ (where C is the dust grain capacitance) by equating $I_e + I_i = 0$.

In order to derive the nonlinear equations for small- but finite-amplitude DAWs, we expand $\Psi = \Psi_0 + \Psi_1$ ($\Psi \equiv n_d, q_d, P_d$ and ψ respectively), $\phi = \phi_1$ and $v_d = v_{d1}$, where subscripts 0 and 1 denote the equilibrium and perturbed quantities respectively. Applying the above expansions in (3)–(7) and assuming $e\phi/T_j$ ($j = i, e$ for electrons and ions respectively), and retaining the lowest-order nonlinear terms, we obtain

$$\frac{\partial n_{d1}}{\partial t} + n_{d0}\frac{\partial v_{d1}}{\partial x} + \frac{\partial}{\partial x}(n_{d1} v_{d1}) = 0, \tag{10}$$

$$\frac{\partial v_{d1}}{\partial t} + v_{d1}\frac{\partial v_{d1}}{\partial x} + \frac{q_{d0}}{m_d}\frac{\partial \phi_1}{\partial x} + \frac{1}{n_{d0} m_d}\frac{\partial P_{d1}}{\partial x} - \frac{n_{d1}}{n_{d0}^2 m_d}\frac{\partial P_{d1}}{\partial x} = 0, \tag{11}$$

$$\frac{\partial P_{d1}}{\partial t} + v_{d1}\frac{\partial P_{d1}}{\partial x} + \gamma_d P_{d0}\frac{\partial v_{d1}}{\partial x} + \gamma_d P_{d1}\frac{\partial v_{d1}}{\partial x} = 0, \tag{12}$$

$$\frac{1}{4\pi}\frac{\partial^2 \phi_1}{\partial x^2} = a_1\phi_1 - a_2\phi_1^2 - n_{d0}q_{d1} - q_{d0}n_{d1}, \tag{13}$$

$$\frac{\partial q_{d1}}{\partial t} + v_{d1}\frac{\partial q_{d1}}{\partial x} = -\omega_1 q_{d1} - R\omega_2\phi_1 - \frac{e}{2RT_e}\omega_3 q_{d1}^2 - \frac{eR}{2T_e}\omega_4\phi_1^2, \tag{14}$$

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where

$$a_1 = \frac{q_i^2 n_{i0}}{T_i} + \frac{e^2 n_{e0}}{T_e}, \tag{15}$$

$$a_2 = \frac{q_i^3 n_{i0}}{2T_i^2} - \frac{e^3 n_{e0}}{2T_e^2}, \tag{16}$$

$$\omega_1 = \lambda + \left(\frac{8\pi}{m_e T_e}\right)^{1/2} e^2 R n_{e0} \exp\left(\frac{e}{T_e}\psi_0\right), \tag{17}$$

$$\omega_2 = \omega_1 - \lambda \frac{q_i}{T_i}\psi_0, \tag{18}$$

$$\omega_3 = \omega_1 - \lambda, \tag{19}$$

$$\omega_4 = \omega_2 - \lambda\left(1 - \frac{q_i}{T_i}\psi_0\right)\left(1 + Z_i\frac{T_e}{T_i}\right) \tag{20}$$

and

$$\lambda = \frac{1}{(2\pi)^{1/2}}\frac{R}{\lambda_{di}}\omega_{pi}, \tag{21}$$

where

$$\omega_{pi} = \left(\frac{4\pi n_{i0} q_i^2}{m_i}\right)^{1/2} \quad \text{and} \quad \lambda_{di} = \left(\frac{T_i}{4\pi n_{i0} q_i^2}\right)^{1/2}$$

are the plasma frequency and Debye length of the ions respectively. The various frequencies $\omega_k, k = 1, 2, 3, 4$, are related by (Rao and Shukla 1994)

$$\omega_2 - \omega_4 = (\omega_1 - \omega_3)\left(1 - \frac{q_i}{T_i}\psi_0\right)\left(1 + Z_i\frac{T_e}{T_i}\right). \tag{22}$$

For typical dusty-plasma parameters, the frequencies $\omega_1 \approx \omega_2 \approx \omega_3 \approx \omega_4$. The equilibrium potential ψ_0 can be calculated as a function of the dust number-density parameter (Melandsø *et al.* 1993). For $T_i = T_e = T$, $e\psi_0/T$ is typically in the range from 0 to -3.

The zeroth-order terms yield the charge-neutrality condition, given by

$$e n_{e0} = q_i n_{i0} + q_{d0} n_{d0}. \tag{23}$$

In (11) and (13) we have neglected the nonlinear terms involving the product of the dust-charge perturbation q_{d1} and the wave potential ϕ_1, as well as the dust number-density perturbation n_{d1}. For acoustic-like perturbations with phase velocity near the dust-acoustic speed, i.e. $\partial/\partial t \approx -C_{DA}\partial/\partial x$, the linear terms in (10)–(12) yield

$$\frac{n_{d1}}{n_{d0}} = \frac{v_{d1}}{C_{DA}} = \frac{P_{d1}}{\gamma_d P_{d0}} = \frac{q_{d0}\phi_1}{m_d c_d^2}, \tag{24}$$

where

$$c_d = \left[\frac{T_e T_i n_{d0} q_{d0}^2}{e^2 m_d (Z_i^2 n_{i0} T_e + n_{e0} T_i)}\right]^{1/2}, \quad C_{DA} = \left(c_d^2 + \frac{\gamma_d P_{d0}}{n_{d0} m_d}\right)^{1/2}.$$

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Combining (10)–(13) and using (24), we obtain

$$\frac{1}{4\pi}\frac{\partial^4\phi_1}{\partial x^3\partial t} = -\frac{n_{d0}q_{d0}^3}{m_d c_d^4}C_{DA}\frac{\partial^2\phi_1}{\partial t^2} + \frac{n_{d0}q_{d0}^2 C_{DA}^3}{m_d c_d^4}\frac{\partial^2\phi_1}{\partial x^2} + \frac{n_{d0}q_{d0}^3 C_{DA}}{m_d^2 c_d^4}\left(1+\frac{\gamma_d^2 P_{d0}}{2n_{d0}m_d c_d^2}\right)\frac{\partial^2\phi_1^2}{\partial x^2} - \frac{1}{2}\left(\beta + \frac{n_{d0}q_{d0}^3}{m_d^2 c_d^4}\frac{\gamma_d P_{d0}}{n_{d0}m_d c_d^2}\right)\frac{\partial^2\phi_1^2}{\partial x\partial t} - n_{d0}\frac{\partial}{\partial x}\left(\frac{\partial q_{d1}}{\partial t}\right), \tag{25}$$

where

$$\beta = \frac{q_i}{q_{d0}}\frac{e^3 n_{i0}n_{e0}}{n_{d0}}\left(\frac{T_i + Z_i T_e}{T_e T_i}\right)^2. \tag{26}$$

In the cold dusty-plasma limit, (25) reduces to equation (17) of Rao and Shukla (1994). Using $\partial/\partial t = -C_{DA}\partial/\partial x$ in the nonlinear and dispersive terms in (25), we obtain a Boussinesq-like equation for the DAWs:

$$\frac{\partial^2\phi_1}{\partial t^2} - C_{DA}^2\frac{\partial^2\phi_1}{\partial x^2} - \frac{c_d^4}{\omega_{pd}^2}\frac{\partial^4\phi_1}{\partial x^4} - \frac{1}{2}\left\{\beta\frac{m_d c_d^4}{n_{d0}q_{d0}^2} + \frac{2q_{d0}}{m_d}\left[1+\frac{\gamma_d(\gamma_d+1)P_{d0}}{2n_{d0}m_d c_d^2}\right]\right\}\frac{\partial^2\phi_1^2}{\partial x^2} = -\frac{m_d c_d^4}{q_{d0}^2 C_{DA}}\frac{\partial}{\partial x}\left(\frac{\partial q_{d1}}{\partial t}\right), \tag{27}$$

where $\omega_{pd} = (4\pi n_{d0}q_{d0}^2/m_d)^{1/2}$ is the plasma frequency of the dust particles. Using (24) in (14), we obtain

$$\frac{\partial q_{d1}}{\partial t} + \frac{q_{d0}C_{DA}}{m_d c_d^2}\phi_1\frac{\partial q_{d1}}{\partial x} = -\omega_1 q_{d1} - R\omega_2\phi_1 - \frac{e}{2RT_e}\omega_3 q_{d1}^2 - \frac{eR}{2T_e}\omega_4\phi_1^2 \tag{28}$$

Thus the small- but finite-amplitude DAWs are governed by the coupled Boussinesq-like equation (27) and the charge-fluctuation equation (28). In the absence of dust-charge fluctuations and adiabatic dust pressure, and up to the lowest-order nonlinearity, (27) reduces to the Boussinesq equation (9) of Rao *et al.* (1990).

3. Discussion

From (27) and (28), we obtain the following linear dispersion relation for the DAWs in the presence of charge fluctuations and adiabatic pressure:

$$\frac{\omega^2}{k^2} = c_d^2 - c_d^2 k^2\lambda_d^2 - c_d^4\frac{m_d R}{q_{d0}^2}\frac{\omega_2}{\omega_1 - i\omega} + \frac{\gamma_d P_{d0}}{n_{d0}m_d}, \tag{29}$$

where $\lambda_d = c_d/\omega_{pd}$ is the dust-grain Debye length. Substituting for $\omega = \omega_r + i\gamma$ in (29) with $\gamma \ll \omega_r$, we obtain the real frequency as

$$\omega_r \approx kc_d\left(1 - k^2\lambda_d^2 + \frac{\gamma_d P_{d0}}{n_{d0}m_d c_d^2}\right)^{1/2} \tag{30}$$

and the damping rate as

$$\gamma \approx -2\pi n_{d0}\lambda_d^2 R\frac{\omega_r^2}{\omega_1^2 + \omega_r^2}\frac{\omega_2}{1 - k^2\lambda_d^2 + \dfrac{\gamma_d P_{d0}}{n_{d0}m_d c_d^2}}. \tag{31}$$

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In the long-wavelength limit ($k\lambda_d \ll 1$), (30) corresponds to a non-dispersive mode with phase speed C_{DA}, and is identical to the dispersion relation obtained by Rao (1993). Also, for a cold plasma and in the long-wavelength limit, the real frequency and damping rate are identical to the corresponding expressions obtained by Melandsø *et al.* (1993). The damping of the dust-acoustic wave arises because of the phase difference between the perturbed electron and ion number densities and the dust-charge fluctuation q_{d1}.

For unidirectional propagation near the dust-acoustic phase speed C_{DA}, (27) can be reduced to a driven KdV equation:

$$\frac{\partial \phi_1}{\partial t} + C_{DA}\frac{\partial \phi_1}{\partial x} + \frac{c_d^4}{2C_{DA}\omega_{pd}^2}\frac{\partial^3 \phi_1}{\partial x^3} + \frac{q_{d0}}{m_d C_{DA}}\left[1 + \mu + \frac{\gamma_d(\gamma_d+1)P_{d0}}{2n_{d0}m_d c_d^2}\right]\phi_1\frac{\partial \phi_1}{\partial x} = \frac{1}{2}\frac{m_d c_d^4}{q_{d0}^2 C_{DA}^2}\frac{\partial q_{d1}}{\partial t}, \tag{32}$$

where

$$\mu = \tfrac{1}{2} n_{i0} n_{e0} Z_i \left(\frac{T_i + Z_i T_e}{n_{e0}T_i + Z_i^2 n_{i0} T_e}\right)^2. \tag{33}$$

Equation (32) and the linearized dust charge perturbation equation

$$\frac{\partial q_{d1}}{\partial t} = -\omega_1 q_{d1} - R\omega_2 \phi_1 \tag{34}$$

are coupled equations and constitute a complete set of governing equations for the nonlinear DAWs. We now discuss two limiting cases.

For time scales much shorter than the charging time, i.e. $\partial/\partial t \gg \omega_1$, the coupled equations (32) and (34) can be reduced to

$$\frac{\partial \phi_1}{\partial t} + C_{DA}\frac{\partial \phi_1}{\partial x} + \frac{c_d^4}{2C_{DA}\omega_{pd}^2}\frac{\partial^3 \phi_1}{\partial x^3} + \frac{q_{d0}}{m_d C_{DA}}\left[1 + \mu + \frac{\gamma_d(\gamma_d+1)P_{d0}}{2n_{d0}m_d c_d^2}\right]\phi_1\frac{\partial \phi_1}{\partial x} = -\hat{\gamma}\phi_1, \tag{35}$$

where $\hat{\gamma} = 2\pi n_{d0}\lambda_d^2 R\omega_2 c_d^2/C_{DA}^2$. Now, defining $\phi_1 = \Phi \exp(-\hat{\gamma}t)$ and using it in the above equation, we obtain

$$\frac{\partial \Phi}{\partial t} + C_{DA}\frac{\partial \Phi}{\partial x} + \frac{c_d^4}{2C_{DA}\omega_{pd}^2}\frac{\partial^3 \Phi}{\partial x^3} + \frac{q_{d0}}{m_d C_{DA}}\left[1 + \mu + \frac{\gamma_d(\gamma_d+1)P_{d0}}{2n_{d0}m_d c_d^2}\right]\Phi\frac{\partial \Phi}{\partial x}\exp(-\hat{\gamma}t) = 0. \tag{36}$$

The localized solution of (36) can be found at early times ($t \to 0$). Defining $\xi = x - Mt$ and using it in (36), we obtain

$$\phi_1 = 3(M-1)\frac{b_1}{b_2}\exp(-\hat{\gamma}t)\,\mathrm{sech}^2(b_3\xi), \tag{37}$$

where

$$b_1 = 1 + \frac{\gamma_d P_{d0}}{n_{d0}m_d c_d^2},$$

$$b_2 = 1 + \mu + \frac{\gamma_d(\gamma_d+1)P_{d0}}{2n_{d0}m_d c_d^2},$$

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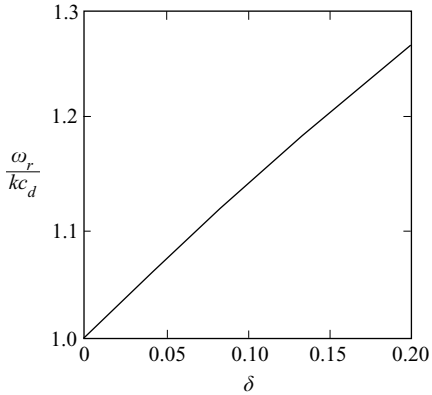


Figure 1. Variation of normalized real frequency $\bar{\omega_r}$ versus δ $(= P_{d0}/n_{d0}m_dc_d^2)$ for $\gamma_d = 3$.

$$b_3 = [\tfrac{1}{2}(M-1)\,b_1]^{1/2}.$$

It is clear from (37) that at early times the localized solutions of (35) are damped owing to the dust-charge fluctuations. For $\omega_r \gg \omega_1$ and in the long-wavelength limit, the damping rate $\hat{\gamma}$ is the same as that predicted by the linear theory. Also, from (37) it is observed that the solitons are possible only for Mach number $M > 1$. The amplitude and width of the solitons decrease with increasing adiabatic dust plasma pressure. The soliton amplitude increases with increasing Mach number.

On the other hand, for time scales much larger than the dust-charging time, i.e. $\partial/\partial t \ll \omega_1$, we obtain the usual KdV equation

$$\eta\frac{\partial\phi_1}{\partial t} + C_{DA}\frac{\partial\phi_1}{\partial x} + \frac{c_d^4}{2C_{DA}\omega_{pd}^2}\frac{\partial^3\phi_1}{\partial x^3} + \frac{q_{d0}}{m_dC_{DA}}\left[1+\mu+\frac{\gamma_d(\gamma_d+1)P_{d0}}{2n_{d0}m_dc_d^2}\right]\phi_1\frac{\partial\phi_1}{\partial x} = 0, \tag{38}$$

where

$$\eta = 1 + 2\pi n_{d0}\lambda_d^2 R\frac{\omega_2}{\omega_1}\frac{c_d^2}{c_{DA}^2}.$$

The above equation will have undamped soliton solutions. This is consistent with the prediction of the linear theory as $\gamma \to 0$ for real frequencies much smaller than the charging frequency, i.e. $\omega_r \ll \omega_1$.

Next, we discuss the numerical results. We plot the normalized real frequency and damping rate versus a parameter δ $(= P_0/n_{d0}m_dc_d^2)$. In all the calculations performed in this paper, we have assumed that $k\lambda_d \ll 1$ and all the frequencies are normalized with respect to kc_d. Figure 1 shows the variation of the normalized real frequency $\bar{\omega_r}$ versus δ for $\gamma_d = 3$. The real frequency increases almost linearly with adiabatic dust plasma pressure.

Figure 2 shows the variation of the effective damping rate $\gamma_{\text{eff}} = \bar{\gamma}/a$ versus δ for $\gamma_d = 3$ and for various values of $\alpha = \omega_1/\omega_r$, the ratio of the dust-charging frequency to the real frequency of DAWs; $\bar{\gamma}$ is the normalized damping rate and $a = 2\pi n_{d0}\lambda_d^2 R$ is related to a parameter P given by Havnes *et al.* (1990). This parameter $P \geqslant 1$ $(\ll 1)$ for denser (tenuous) plasmas. It can be seen from the figure that for $\omega_1 \gg \omega$ $(\alpha = 10)$, the effective damping rate $\gamma_{\text{eff}} \approx -1/\bar{\omega_1}$, and the damping is weak. This

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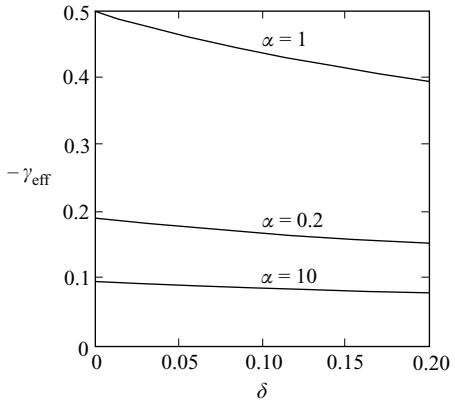


Figure 2. Variation of effective damping rate ($-\gamma_{\text{eff}}$) versus δ for $\gamma_d = 3$, $\alpha = 0.2$, 1.0 and 10.

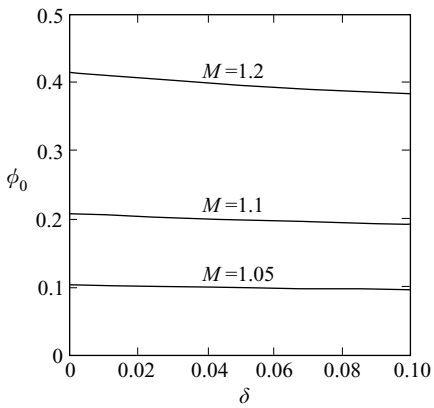


Figure 3. Variation of soliton amplitude ϕ_0 versus δ for $Z_i = 1$, $T_e/T_i = 1$, $n_{i0}/n_{e0} = 0.5$, $\gamma_d = 3$ and Mach number $M = 1.05$, 1.1 and 1.2.

can be understood from the fact that the phase difference between the dust-charge perturbation $q_{d1} \approx -R\omega_2\phi_1/\omega_1$ and the wave potential ϕ_1 is zero, and the damping is unaffected by the amount of dust-charge perturbation. In this case, we get a non-dispersive and non-dissipative dust-acoustic mode. For $\omega_1 \ll \omega$ ($\alpha = 0.2$), the effective damping rate $\gamma_{\text{eff}} \approx -\bar{\omega_2}/\bar{\omega_r^2}$, and the phase difference is $\frac{1}{2}\pi$. This would give damping if the charge variation $q_{d1} \approx -iR\omega_2\phi_1/\omega$ were appreciable, but the charge will hardly react, since the dust-charge perturbation $q_{d1} \to 0$ in this case, and again damping is weak. Now, for $\omega_1 = \omega$ ($\alpha = 1$), the phase difference will be $\frac{1}{4}\pi$ and the charge perturbation $q_{d1} = -R\omega_2\phi_1/\omega\,(1-i)$ and effective damping rate $\gamma_{\text{eff}} \approx -\bar{\omega_2}/2\bar{\omega_r}^2$. Damping in this case will be maximum, as illustrated in Fig. 2. Furthermore, the effect of dust plasma pressure is to reduce the damping.

Figure 3 shows the variation of the soliton amplitude ϕ_0 with dust plasma pressure for $Z_i = 1$, $T_e/T_i = 1$, $n_{i0}/n_{e0} = 0.5$, $\gamma_d = 3$, and various values of Mach number $M = 1.05$, 1.1 and 1.2. It can be seen that the amplitude decreases with increasing dust plasma pressure, and increases with the Mach number.

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4. Summary

To summarize, we have studied the effect of dust-charge fluctuations and adiabatic dust pressure on linear and nonlinear DAWs by considering the electrons and ions to be in Boltzmann equilibria. The dust grains are considered to follow the fluid equations with full adiabatic equation of state. Linear dust-acoustic waves are damped owing to the dust-charge fluctuations. Damping is caused by the phase difference between the dust-charge fluctuations and the wave potential. It is found that the damping is high when the wave frequency is close to the dust-charging frequency and the phase difference is $\frac{1}{4}\pi$. The damping rate decreases with the adiabatic dust pressure. Nonlinear DAWs are governed by a Boussinesq-like and dust-charge perturbation equation. For unidirectional propagation, the Boussinesq-like equation reduces to the usual Korteweg–de Vries (KdV) equation. At early times, for wave frequencies much greater than the dust charging frequency, the localized solutions of the KdV equation are damped owing to the dust-charge perturbations. The soliton amplitude decreases with increasing adiabatic dust plasma pressure. It is found that soliton solutions exist only for supersonic speeds ($M > 1$). The soliton amplitude increases with Mach number M. However, for wave frequencies much smaller than the charging frequency, the usual soliton solutions are possible for the nonlinear DAWs. Our results may be applied to planetary rings, where the plasma is tenuous ($P \ll 1$).

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