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Friction factors for smooth pipe flow

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Friction factor data from two recent pipe flow experiments are combined to provide a comprehensive picture of the friction factor variation for Reynolds numbers from 10 to 36,000,000.

For pipe flow, as long as entrance effects, roughness, and temperature variations are small, dimensional analysis indicates that the friction factor λ is only a function of the Reynolds number Re. That is,

$$\lambda = f\,(Re)$$

where

$$\lambda = \frac{-(\mathrm{d}P/\mathrm{d}x)D}{\frac{1}{2}\rho\overline{U}^2} \qquad \text{and} \qquad Re = \frac{\overline{U}D}{\nu}.$$

Here f denotes a functional relationship, $\mathrm{d}P/\mathrm{d}x$ is the pressure drop per unit length, D is the diameter of the pipe, ρ is the fluid density, ν is the kinematic viscosity, and $\overline{U}$ is the flow velocity averaged over the cross-sectional area of the pipe.

The Princeton (Zagarola & Smits 1998; McKeon *et al.* 2004*a*; McKeon, Zagarola & Smits 2004*b*) and Oregon (Swanson *et al.* 2002) research groups have recently examined fully developed pipe flow using very different apparatus. Compressed air is used in the Princeton 'Superpipe', whereas the Oregon device uses several room-temperature gases: helium, oxygen, nitrogen, carbon dioxide and sulphur hexafluoride are used for relatively small Reynolds numbers, and normal liquid helium (helium I) is used for the highest Reynolds numbers. The difference in scale of the two devices is dramatic: for example, the Superpipe weighs about 25 tons, whereas the Oregon tube weighs about 1 ounce.

The combined data sets are tabulated in table 1 and plotted in figures 1 and 2. We see that the range of experimentally known friction factors has been increased from $Re = 10^3$ to $Re = 3.2 \times 10^6$ (see, for example, figure 20.1 in Schlichting 1979), and $Re = 10$ to $Re = 35.7 \times 10^6$. Where they overlap, the data lie within an average of 1.7%, compared with experimental errors of 2% to 4% for the Oregon data and

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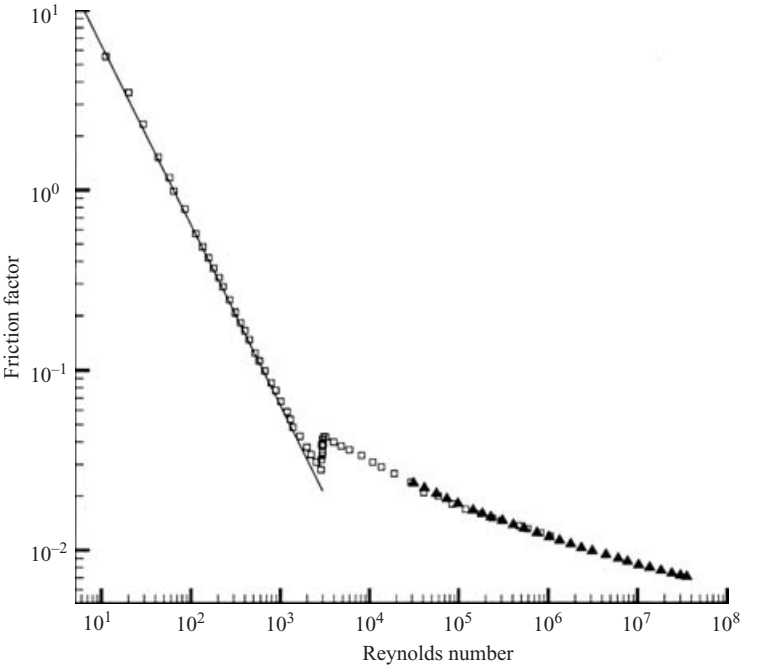


FIGURE 1. Combined data sets from table 1: ▲, Princeton; □, Oregon; —, theoretical friction factor for laminar flow, $\lambda = 64/Re$. Since the error bars are smaller than the size of the symbols, they are not shown.

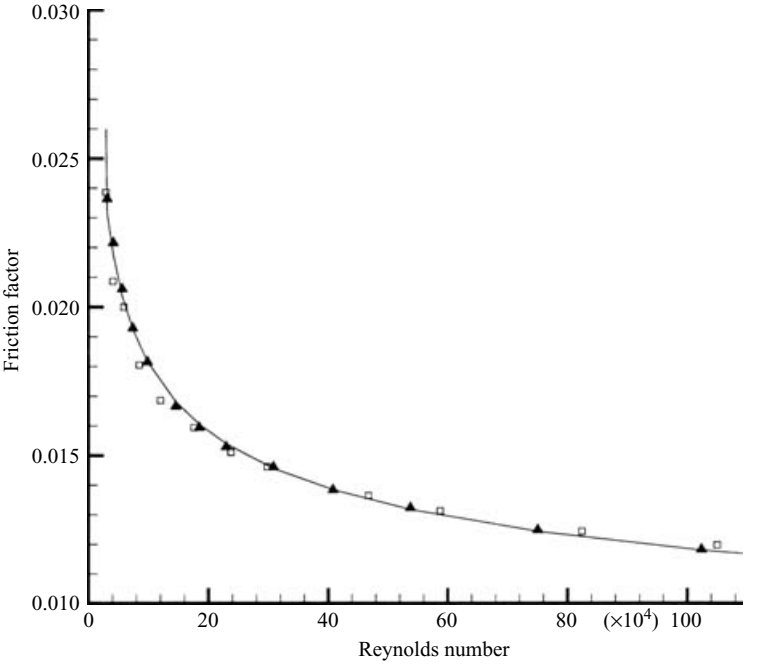


FIGURE 2. Linear plot of the points where the two data sets overlap. ▲, Princeton; □, Oregon. The line corresponds to equation (1).

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Oregon data				Princeton data	
Re	λ	*Re*	λ	*Re*	λ
1.121E+01	5.537E+00	2.227E+03	3.405E-02	3.131E+04	2.364E-02
2.022E+01	3.492E+00	2.554E+03	3.091E-02	4.144E+04	2.216E-02
2.928E+01	2.329E+00	2.868E+03	2.804E-02	5.636E+04	2.061E-02
4.319E+01	1.523E+00	2.903E+03	3.182E-02	7.397E+04	1.929E-02
5.773E+01	1.173E+00	2.926E+03	3.846E-02	9.846E+04	1.815E-02
6.458E+01	9.863E-01	2.955E+03	3.363E-02	1.456E+05	1.666E-02
8.605E+01	7.826E-01	2.991E+03	4.124E-02	1.848E+05	1.594E-02
1.133E+02	5.709E-01	2.997E+03	3.500E-02	2.296E+05	1.529E-02
1.353E+02	4.815E-01	3.047E+03	3.875E-02	3.085E+05	1.461E-02
1.575E+02	4.182E-01	3.080E+03	4.285E-02	4.081E+05	1.384E-02
1.794E+02	3.655E-01	3.264E+03	4.260E-02	5.378E+05	1.324E-02
2.064E+02	3.237E-01	3.980E+03	3.995E-02	7.507E+05	1.249E-02
2.280E+02	2.884E-01	4.835E+03	3.797E-02	1.024E+06	1.183E-02
2.709E+02	2.433E-01	5.959E+03	3.610E-02	1.342E+06	1.131E-02
3.152E+02	2.077E-01	8.162E+03	3.364E-02	1.791E+06	1.079E-02
3.589E+02	1.834E-01	1.090E+04	3.088E-02	2.352E+06	1.028E-02
4.029E+02	1.656E-01	1.365E+04	2.903E-02	3.109E+06	9.890E-03
4.502E+02	1.475E-01	1.899E+04	2.670E-02	4.438E+06	9.410E-03
5.225E+02	1.245E-01	2.943E+04	2.386E-02	6.103E+06	8.970E-03
5.831E+02	1.126E-01	4.085E+04	2.086E-02	7.757E+06	8.620E-03
6.718E+02	9.917E-02	5.922E+04	2.000E-02	1.031E+07	8.250E-03
7.898E+02	8.501E-02	8.476E+04	1.805E-02	1.368E+07	7.980E-03
8.910E+02	7.722E-02	1.200E+05	1.686E-02	1.830E+07	7.670E-03
1.013E+03	6.707E-02	1.760E+05	1.594E-02	2.413E+07	7.400E-03
1.197E+03	5.880E-02	2.377E+05	1.511E-02	3.015E+07	7.200E-03
1.300E+03	5.328E-02	2.982E+05	1.462E-02	3.554E+07	7.080E-03
1.390E+03	4.815E-02	4.678E+05	1.365E-02		
1.669E+03	4.304E-02	5.875E+05	1.313E-02		
1.994E+03	3.739E-02	8.242E+05	1.244E-02		
		1.050E+06	1.198E-02		

TABLE 1. Friction factor data from the two experiments.

1.1% for the Princeton data. The line shown in figure 2 is the curve fit proposed by McKeon *et al.* (2004*b*) for $Re_D > 300 \times 10^3$:

$$\frac{1}{\lambda^{1/2}} = 1.930 \log\left(Re\lambda^{1/2}\right) - 0.537. \tag{1}$$

The maximum difference between equation (1) and the Princeton data for $31 \times 10^3 \leqslant Re \leqslant 35 \times 10^6$ is always less than 1.25%. For $300 \times 10^3 \leqslant Re \leqslant 13.6 \times 10^6$, the deviation from the proposed formula is less than $\pm 0.5\%$. For Reynolds numbers above 13.6×10^6 the friction factors start to move above this line, indicating the possible effects of roughness, but the trend is within the experimental error on λ for all but the highest Reynolds number.

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