NOTE ON THE PRECEDING PAPER

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PART I. It is worth remarking that if e, f are two primitive idempotents in a semigroup S which commute, then:

(i) if S has a zero 0, either $e = f$ or $ef = 0$;

(ii) if S has no zero, $e = f$.

Clearly the product ef is under each of the primitive idempotents e and f.

Thus the following results can be quickly deduced:

1. If S is an inverse semigroup such that every idempotent is primitive, then

(i) if S has a zero 0, the product of any two distinct idempotents is 0;

(ii) if S has no zero, S is a group.

Part (i) follows immediately from Preston's definition of inverse semigroups (4);* in (ii), S has a single idempotent e, which must be both the left and right unit of every element of S, i.e. (in the notation of (4)) S coincides with its subgroup G_e.

It is clear from the first example, due to R. McFadden, in the above paper (where the only idempotents a and b are primitive), that S is not necessarily simple here (and hence not necessarily completely simple).

2. If S is a completely simple semigroup without zero, whose idempotents commute, S is a group. This follows immediately from Preston's remark in (5) that a completely simple semigroup (without zero) with only one idempotent, is a group.

PART II. Arising immediately out of the preceding paper

THEOREM. *If S is a simple semigroup with zero then under any of conditions* (i)–(vii) *of the Theorem of the preceding paper*:

Either (i′) *S is the union of* one *group G with zero together with zero semigroups.*

Or (ii′) *for any $x \in S$, either $x^2 = 0$ or there exists a, b in S with $a^2 = b^2 = 0$ such that $x = ab$.*

Proof. Since S is completely simple, then to each pair L, R of minimal left and right ideals, respectively, such that $LR = S$, there corresponds a group G with zero, where $G = RL$, and S is the union of all such sets G, together with zero semigroups, by (8). Now by condition (vi) in the theorem of the preceding paper, $R = G \cup Z$, $L = G \cup Z'$, where $ZR = LZ' = (0)$. Hence

$$S = (G \cup Z')(G \cup Z) = G \cup GZ \cup Z'G \cup Z'Z = G \cup A \cup B \cup Z'Z, \tag{1}$$

where $A^2 = B^2 = Z^2 = (Z')^2 = (0)$.

* References are as in the preceding paper.

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If (i′) does not hold, there exists at least one other such pair L_i, R_i of minimal left and right ideals, and a corresponding decomposition

$$S = G_i \cup A_i \cup B_i \cup Z_i' Z_i. \tag{2}$$

But clearly $G_i \cap G = A_i \cap G = B_i \cap G = (0)$, since $A_i^2 = B_i^2 = (0)$. Therefore $G \subset Z_i' Z_i$, that is, from (1)

$$S = A \cup B \cup Z'Z \cup Z_i' Z_i,$$

from which result (ii′) immediately follows.

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