THE CHARACTERISTIC OF CUMULATIVE DAMAGE STUDY ABOUT ELECTRICAL INSULATION BASED ON ACCELERATED LIFE TESTS

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ABSTRACT

In this study the Maximum Likelihood Estimator is utilized to identify the characteristics of failure of class-H insulation by considering accelerated life test data under censored situations from Nelson. The hazard rate function is considered in terms of the reliability, $h(R)$, so-called AE model. The AE model is used to model the failures which are expressed as the serial connection between three modes, namely the turn, phase, and ground. This is the so-called competing failure. The main concern in the present investigation relates to the characteristic of changes in cumulative damage with temperature. The characteristic of the damage process basically change, with less capability of cumulation. The failure tends to be unpredictable in a constant hazard rate situation in much higher temperature environments. The parameters of the model are related to the temperature and follow the Arrhenius law. The numerical results indicate that the AE model is well fitted to the data and gives more information to identify the failure modes with fewer parameters. This is better than the using Weibull distribution with both parameters varied with temperature. According to the predicted lifetime, the turn needs to be rearranged primarily, followed by the phase. The ground mode only has influence on the failure at much higher temperatures.

Keywords: Hazard rate-reliability model, Accelerated life test, Competing failure mode, Cumulative damage.

1. INTRODUCTION

The electrical insulation system is an important part of motor. Statistics show that the most common cause of motor failure is due to electrical insulation failure. Thus, the insulation systems of electrical devices have to satisfy the requirement for high reliability, possibly having a designed mean-time-to-failure (MTTF) of over 100,000 hours of use under normal conditions. Accelerated life test (ALT) is usually carried out by stressing the specimens at overstress conditions to explore defects and estimate the MTTF under normal use. The basic assumption in conducting ALT is that the tests should be designed to produce the same failure mechanism that would lead to product failure under normal conditions. The failure mechanism is related to the degradation of material due to stress from certain physical phenomena. Temperature is usually chosen as an environmental condition for ALT when activating failure mechanisms in electric devices by varying the rates of chemical reactions. The chemical reactions which occur in insulating materials under elevated temperatures are multi-fold compared to the normal use temperature. The main failure mechanism is deterioration due to oxidation which occurs with the introduction of carboxylic acid groups into the insulation and release H^+ to increase the conductivity and power factor of the insulation. Dakin [1] proposed using the Arrhenius relationship to describe the variation with temperature throughout the lifetime of various types of electrical insulation and dielectrics, a method which is now widely used in ALT analysis.

For a specimen operating under specific conditions, including environmental condition, the stress is decided

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by the geometry, which will vary with time as degradation becomes worse and stress is redistributed. The local geometrical shape changed eventually the failure occurred place. Due to the complexity of the product geometry, failure could occur at different locations, each being characterized by its own particular features. These are the so-called failure modes, *i.e.*, there is a specific probability for product failure at different positions. The lifetime estimation can become overly optimistic if only the most frequently occurring failure modes are considered. Competing failure modes analysis (CFMA) takes all potential failure modes into account at the same time. The mission of CFMA is to predict the proportion of each mode in the apparent failure which is observed under the same mechanism without the location details. Such information helps in product development. In the early study, Nelson [2] applied the concept of competing risk in the analysis of ALT. To shorten the total time for ALT, the process is usually terminated with a certain amount of survivals, the so-called censored situation [3]. In this case the estimation becomes much more sophisticated. The Maximum Likelihood Estimation (MLE) [4] is a typical approach adopted for the prediction in such case. In the present study, motor failure due to deterioration of the insulation under temperature-ALT is investigated with censored data as multiple failure modes are involved.

Another type of deterioration of the insulating material gradually becomes more serious in higher temperature environments, the thermal decomposition of the insulation [5,6]. The polymer chains break into shorter units occurs in all polymers including cellulose at elevated temperatures even in the absence of oxygen. The variance of the dielectric strength distribution of the insulation increases with rising temperatures. The experimental results from Bowler *et al.* [7] confirm this behavior. According to the Stress Strength Influence (SSI) theory, the characteristic of the cumulative process of reliability is affected by the variance of the stress/strength distribution. Large variance of distribution cause cumulative damage phenomenon unobvious. When the insulation is at higher temperatures, the variance of dielectric strength distribution becomes larger. Thus, the characteristics of the damage process will change with less capability of cumulation. Insulation failure tends to be unpredictable in a constant hazard rate situation (*i.e.*, exponential distribution).

To model the characteristic of the damage process varying with temperature, the Weibull distribution is proper to be utilized. Both shape and scale parameters are varied with temperature. Such a point of view has already been mentioned by Nelson [8]. Seo and Kim [9] designed an accelerated life test sampling plan with consideration of the Weibull distribution where both the shape and scale parameters are following in the power-law forms of temperature. Yang and Wang proposed a Gompertz-relation of shape parameter [10]. However, the likelihood function becomes much more sophisticated when both parameters are functions of the temperature, especially when there are multiple failure modes involved. In addition, the MTTF evaluation of the Weibull distribution is a gamma function with both parameters. Thus, the life estimation does not coincide with the Arrhenius relation if the shape parameters are not the same at every stress level when following the Arrhenius relation.

Therefore, a hazard rate-reliability dependent model which has the merit of demonstrating the characteristic of cumulative damage, is presented in this study for ALT analysis. In the past two decades, the relations between hazard rate and reliability have been studied intensively [11-15]. This is based on the consideration that there exists a one-to-one correspondence between R and the time, t, meanwhile the reliability always monotonically decays with t. More details of the relation, h vs. R, is discussed in Sec.2.

2. HAZARD RATE IN TERMS OF THE RELIABILITY MODEL

Usually the reliability decreases monotonically with time, thus there is a one-to- one correspondence between R and t, *i.e.* the hazard rate function can also be expressed as:

$$h(t) = -\frac{1}{R(t)}\frac{dR}{dt} = f(R). \tag{1}$$

the change of the expression $h(t)$ to $h(R)$ has certain advantages [12]. The hazard rate is investigated in finite domain [1,0] as comparing with that in infinite domain of time sequence. Sometimes with the fitting of $h(t)$ has to be concerned with whether or not the collected data in the time domain are enough to depict the actual circumstances, but this is not a concern in the fitting of $h(R)$. In the past years, we proposed the relationship of $h(R)$ based on the available data, the following hazard function were studied

$$h(R) = e_0 + c_n(1-R) + \text{higher order terms}. \tag{2}$$

In the research of these years, we proved Eq. (2) into a simple form named the (e_0 c_n n) model, and the hazard function can be re written as

$$h(R) = e_0 + c_n(1-R)^n, \tag{3}$$

where e_0 concerns a constant with unpredictable failure occurrence from the sudden increase of applied load due to environmental variations or its intrinsic weakness embedded within a new system. Moreover, take e_0 as symbolization the initial state of the system. Parameter c_n equals to substantial decrease in reliability as the cumulative damage becomes significant in the failure possibility. Hence, this can be thought of as the coefficient of material degradation. The parameter exponent n denotes the beginning of obvious change in reliability, referring to the memory characteristics of damage. This type of hazard rate emphasizes the damage characteristics of material degradation. In order to simplify the reliability and probability density functions to obtain a closed form, let exponent n be 1,

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and the effect of n is replaced by e_0. Equation (3) can now be simplified as follows:

$$h(R) = e_0 + c_n(1-R)\,,\ 1 > e_0 > 0\,,\ 1 > c_n > 0\,. \tag{4}$$

Equation (4) is also known as the Algebraic-Equation model (AE model). Following the definition in Eq. (1), the Reliability function and Probability density function are

$$R(t) = \frac{e_0 + c_n}{c_n + e_0 \exp\left[(e_0 + c_n)t\right]}\,, \tag{5}$$

and

$$f(t) = \frac{e_0(e_0 + c_n)^2 \exp\left[(e_0 + c_n)\,t\right]}{\left\{c_n + e_0 \exp\left[(e_0 + c_n)\,t\right]\right\}^2}\,. \tag{6}$$

In reliability theory, the mean-time-to-failure (MTTF) can be calculated by integrating reliability $R(t)$ with time t as

$$\mathrm{MTTF} = \int_0^\infty R(t)\,dt = \frac{1}{c_n}\ln\left(\frac{c_n}{e_0} + 1\right), \tag{7}$$

Equation (7) is much simpler than the MTTF calculation of the other distribution, *i.e.*, the Weibull distribution which is related to the gamma function of both parameters.

The phenomenon of cumulative failure can be understood through the Strength-Stress Interference (SSI) theory; see Fig. 1. The probability of failure comes from the variance of strength (σ_S) and stress (σ_L) of components. When both σ_S and σ_L are extremely small, all components fail at the same specific time, resulting in complete cumulative failure behavior; see the solid line in Fig. 2. Otherwise, components fail randomly in a constant failure rate situation if σ_S or σ_L are extremely large, see the thin dashed line in Fig. 2. To determine the phenomenon of cumulative damage, two properties in probability theory provide related information. One is the variance (σ) of probability density about failure which distinguishes the concentration of the appearance of failure and shows the intensity of cumulative damage in reliability degradation. The other depends on the location of inflection point in reliability degradation curve [15]. The third moment of distribution, the skewness, confirms the symmetry of distribution and gives information about the inflection point of reliability degradation. Following this definition, in the AE model, variance (σ) and skewness (γ) are derived from Eq. (6) and shown in the Appendix. The functions of the variance and skewness are complicated but it is easy to determine the ratio of the parameters, k ($k = c_n / e_0$) , which is the main dependence of these two functions. The variations of variance and skewness are shown in Figs. 3(a) and 3(b), and the variations of the variance and skewness of the Weibull distribution are also shown. According to Fig. 3(a), the variance of the AE model distribution becomes gradually smaller with k increased. It is similar to the shape parameter of the Weibull distribution. In Fig. 3(b), the skewness

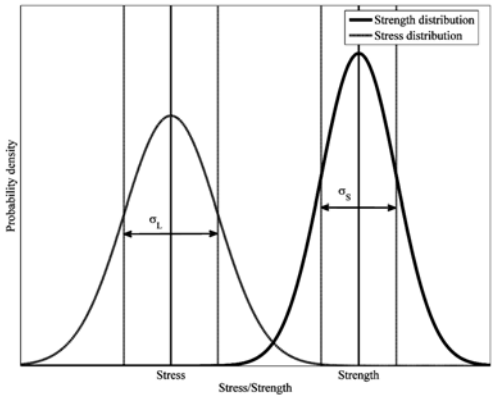


Fig. 1 Schematic diagram for the Stress Strength Interference (SSI) theory

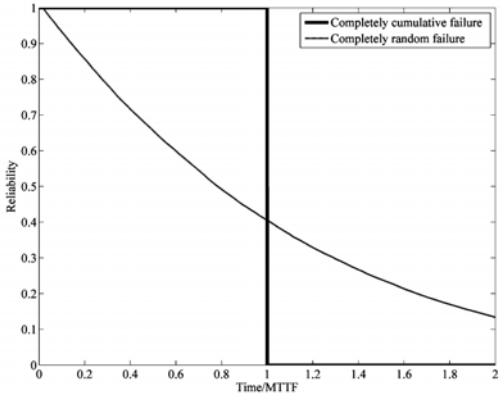
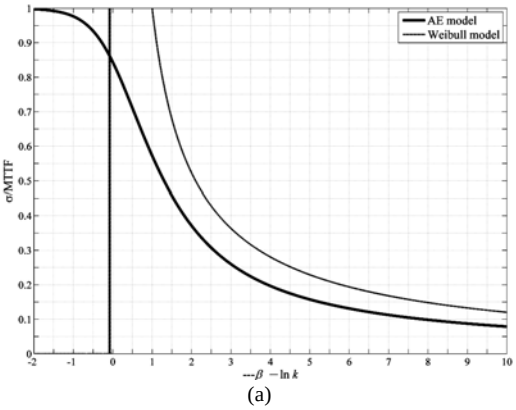
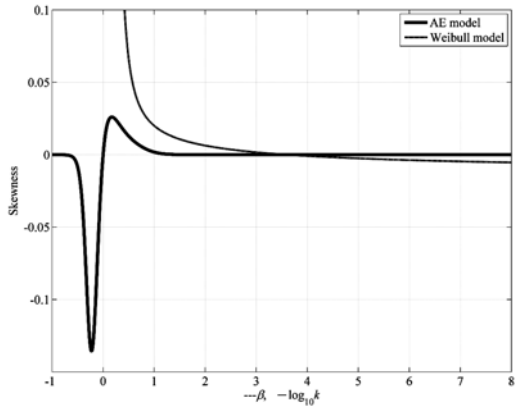


Fig. 2 Schematic diagram for the completely cumulative failure and completely random failure



(a)



(b)

Fig. 3 Variation of (a) variance and (b) skewness of the parameters using the AE model and the Weibull model

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of the AE model distribution is closed to zero when k is large. It means that the distribution is extremely symmetrical when $e_0 << c_n$. See Fig. 4, the probability density of the AE model with different k. Also, the descriptive ability of the AE model is as good as the Weibull distribution, see Fig. 5. Furthermore, as shown in Figs. 3(a) and 3(b), the margin of change about variance and skewness with k in the AE model are limited. These properties make the regressions more stable when the model is applied to failure data, especially for incomplete data. Two examples of different kinds of failure are practiced below. Both the AE and Weibull model are adopted to fit the data with a least squares regression.

A set of data are collected on the lives of 100 brakes for a particular model of automobile [16]. The fitting results obtained with both the AE model and Weibull model are shown in Fig. 6. The break of automobile was weaved with uniform composite. Failure occurs rarely until the brake is seriously abraded, after which point, failures suddenly appear due to the decreasing of the friction coefficient. The ratio k ($k = 2.48E3$) is large, which means the phenomenon of cumulative damage is significant. Compared with the results from the Weibull model, the value of the shape parameter is quite large, with $\beta = 5.24$.

A designated test fleet of vehicles is in operation in a high corrosion producing area in the northeastern United States [17]. 100 vehicles were monitored for evidence of corrosion failure of certain underbody components. The fitting results are shown in Fig. 7. Corrosion is the gradual destruction of material caused by chemical reactions with the surrounding environment. The characteristic of cumulative damage is not that obvious in the brakes of automobiles. The value of k ($k = 1.28E1$) is much smaller than brake failure and its corresponding shape parameter for the Weibull model is 2.01. Thus, the ratio k can be taken as a shape parameter to the AE model. These two examples demonstrate the merit of AE model in describing the phenomenon of cumulative damage. Moreover, another consideration is about the failure data are incomplete (censored). Assume that data of brakes failure of automobile from Ref. [16] are truncated at specific distance, D_c, such as in a censored data set. Least squares regression is adopted to fit the data into two models. The mean life of brakes about complete data is 91181.23 kilometers. The results are shown in Table 1 and life estimation of different D_c are compared to complete data. The life estimation is acceptable if difference less than 5%. In Table 1, the estimation of the AE model is acceptable when D_c > 65,000 kilometers, but the Weibull model only get acceptable estimation when D_c > 85,000 kilometers. This result shows the advantage of the AE model for dealing with incomplete data.

3. TEMPERATURE DEPENDENCE OF THE AE MODEL PARAMETERS AND MLES

In temperature accelerated life test analysis, the concepts of the Arrhenius relation are widely used for

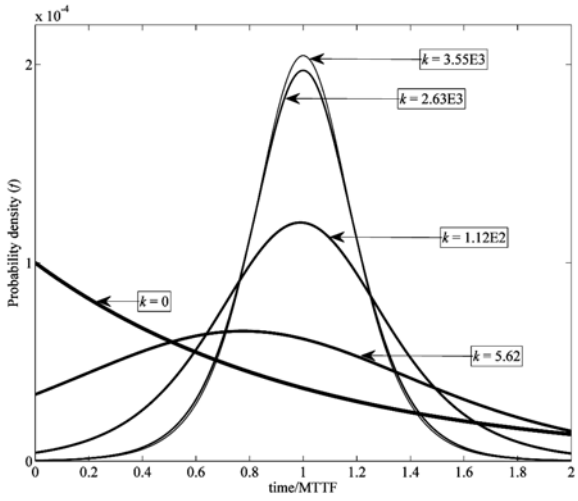


Fig. 4 Probability density of the AE model with different k

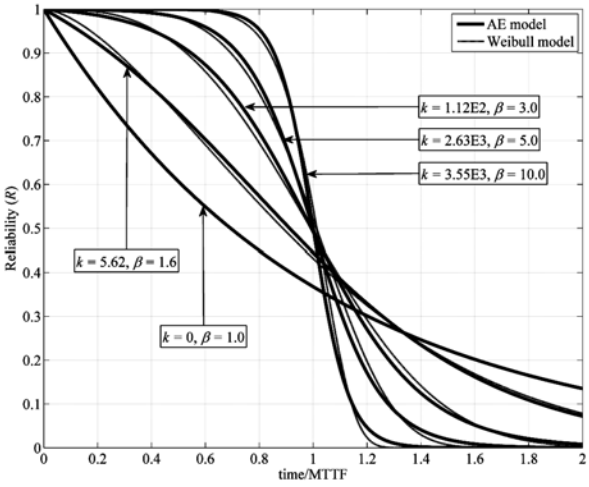


Fig. 5 Comparison of reliability of the AE model with the Weibull representation

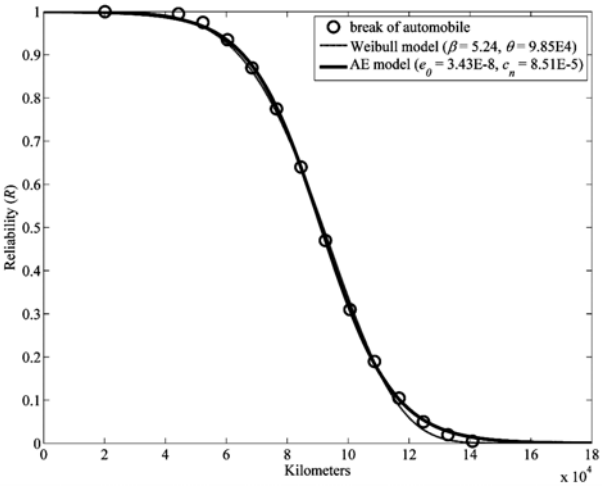


Fig. 6 Fitted reliability for the break system of an automobile. (data from ref. [16])

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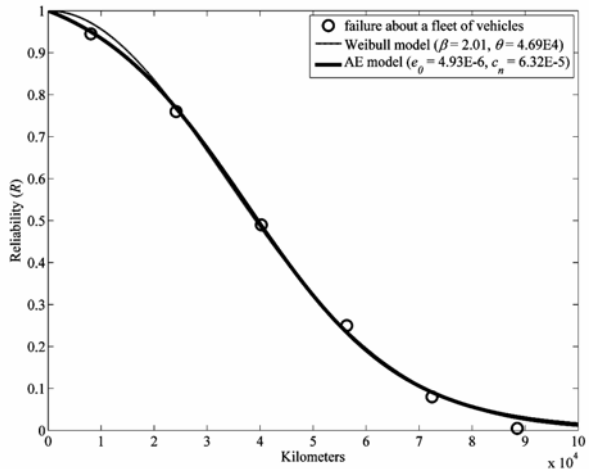


Fig. 7 Fitted reliability for the corrosion failure of a fleet of vehicles (data from ref. [17])

Table 1 Comparison of life estimation with truncated data from ref [16]

AE model				
Censored data (truncated distance)	e_0	c_n	Life estimation	Difference of life (%)
140,000	3.12E-8	8.43E-5	91298	0.13
110,000	3.10E-8	8.41E-5	91241	0.07
80,000	2.29E-8	9.12E-5	89414	1.94
65,000	1.19E-8	9.98E-5	87203	4.36
Weibull model				
Censored data (truncated distance)	Scale parameter θ	Shape parameter β	Life estimation	Difference of life (%)
140,000	9.85E4	5.24	90634	0.60
110,000	9.80E4	5.43	90429	0.82
80,000	9.67E4	5.78	89503	1.84
65,000	8.75E4	7.26	82041	10.02

life estimation. In this section, two major features are presented. First, the relation of parameters to the lifetime at different temperatures is calculated through the Arrhenius relation. Secondly, the corresponding maximum likelihood estimate function is presented.

3.1 Arrhenius Relation for Ae Model Parameters

The Arrhenius relation presents the chemical reaction rate among the material components under a specific thermal environment. It is expressed as

$$r = A^* \times \exp\left[-\frac{E_a}{KT}\right],$$

where r is the rate constant of reaction, A^* the characteristic parameter about the chemical process, K is the Boltzmann constant, E_a is the activation energy of the reaction and T is the temperature on the Kelvin scale. Since a higher reaction rate yields a shorter lifetime of the insulation, a straight-forward assumption can be made for the $r \cdot L_T \sim$ constant, while L_T is the estimation of the lifetime. Therefore we have

$$L_T = \exp\left[A + \frac{B}{T}\right]. \tag{8}$$

Since the relationship between MTTF and temperature should obey the Arrhenius relation, then, based on Eq. (7), the parameters of the AE model are related to temperature as

$$\frac{1}{c_n}\ln\left(\frac{c_n}{e_0}+1\right) = \exp\left[A + \frac{B}{T}\right]. \tag{9}$$

The variation of ratio k is expressed with temperature; Eq. (9) can now be rewritten as follows:

$$\frac{1}{k+1} = \exp\left[-c_n \times \exp\left[A + \frac{B}{T}\right]\right]. \tag{10}$$

Equation (10) is similar to the Gompertz function [18]. Thus, the value of Eq. (10), $1 / k + 1$, should slip from its saturation value to 1, in the constant failure rate situation under testing at higher temperature. Furthermore, as the temperature is lowered to normal use conditions, the characteristic of cumulative damage becomes significant. This phenomenon corresponds to the thermal degradation of the insulating material. Based on Eq. (10), the reliability function can also expressed with temperature as

$$R(t) = \frac{k+1}{k + \exp\left[c_n\left(\frac{k+1}{k}\right)t\right]}, \tag{11}$$

where k is also a function of temperature. Equation (11) makes the calculation of the likelihood function more concise and the information about the variation of the characteristic of cumulative damage with temperature is obtained.

3.2 Maximum Likelihood Estimation for Competing Failure Mode

Consider in the presence of m failure causes, the observed lifetime, τ, of a set of tests is the minimum among $X_1, X_2, \ldots, X_m$; X_i the random failure lifetime of mode i, *i.e.*

$$\tau = \min(X_1, X_2, ..., X_m). \tag{12}$$

Let the failure modes be independent to each other, thus a series connection of these modes represents the apparent failure in the tests. Now we have the reliability, $R_{cf}(t)$, as

$$R_{cf}(t) = P(\tau > t) = P(X_1 > t, X_2 > t, ..., X_m > t)$$
$$= \prod_{i=1}^{m} P(X_i > t) = \prod_{i=1}^{m} R_i(t), \tag{13}$$

with a probability density function, $f_{cf}(t)$, as follows:

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$$f_{cf}(t) = -\frac{dR(t)}{dt} = -\sum_{i=1}^{m}\left[\frac{dR_i(t)}{dt}\left(\prod_{\substack{j=1\\ j\neq i}}^{m} R_j(t)\right)\right]. \qquad (14)$$

Let the r-th among m causes lead to n specimen failures, with each failure coming only from a single cause. The corresponding likelihood function, L, is defined as

$$L = \prod_{j=1}^{r}\prod_{i=1}^{m}\left(f_i^{\delta_{ij}}(t_j)\times R_i^{1-\delta_{ij}}(t_j)\right)\times\prod_{j=r+1}^{n} R_{cf}(t_j). \qquad (15)$$

where δ_{ij} is the decision factor, *i.e.*, $\delta_{ij} = 1$ as the specimen fails at test time point t_j with the i-th failure mode; otherwise $\delta_{ij} = 0$. For censored testing data the effect of the j-th survival on the likelihood function is accounted for in the term $\prod_{j=r+1}^{n} R_{cf}(t_j)$. For a p-constant temperature levels ALT, the likelihood estimator (LE) function becomes

$$L = \prod_{k=1}^{p}\prod_{j=1}^{r}\prod_{i=1}^{m}\left(f_{ik}^{\delta_{ijk}}(t_{jk})\times R_{ik}^{1-\delta_{ijk}}(t_{jk})\right)\times\prod_{k=1}^{p}\prod_{j=r+1}^{n} R_{cf}(t_{jk}). \qquad (16)$$

Therefore, based on Eqs. (10) and (11), only three parameters (c_n, A, B) have to be solved in the likelihood estimation for each failure mode.

4. CASE STUDY

Consider a set of data by Nelson [2], see Table 2, which collected from a thermal-accelerated life tests of Class-H motor insulation. The material of Class-H insulation consists of silicone elastomeric and the combination of others such as mica, glass fiber, asbestos *etc*. A median life over 20,000 hours is expected with temperature below 453K. In the tests, Class-H insulation failed according to three failure modes, namely turn, phase, and ground, each occurring at different parts of the insulation. In a standard test of motor insulation [19] the turn insulation is located in the winding coils, the phase is a sheet of insulation laid between the phases and the ground insulation is located the slot cell. In thermal-accelerated life test, specimens go into an oven which is then heated to the test temperature. There is no voltage applied to the insulation in the oven. After a specified time at that temperature, the specimens are removed from the oven and cooled to room temperature after which a specified voltage is applied to the insulation, at which break down (failure) occurs or the specimen survives. In the table it presents the recorded failure times for each mode (turn, phase and ground, respectively) under different thermal environment, as the temperature held at 463K, 493K, 513K and 533K, respectively. During the testing the motors were kept going on run till the rest failure modes appeared after the first shown. The failure is assumed as the voltage drops below the threshold defined for the particular mode. The recorded time in Table 2 is the midway between periodical inspections for the failure. It is quite different in real application; the first appearance of the defined failure may not cause the end of motor life at once but lead to less efficiency. For details, the motors appeared as in the sequence 1, 2 and 3 in the table at temperature 463K, of which the insulation decays did not reach the thresholds of the defined failure modes till at 7228 hours all of these three motors were disqualified on the turn mode and then the motor 1 failed at 10511 hours, motor 2 and 3 at 11855 hours all on the mode phase afterward quitted the tests with mode ground still qualified by marked plus to denote the censoring time. The other tests in Table 2 were also carried out in the same manner.

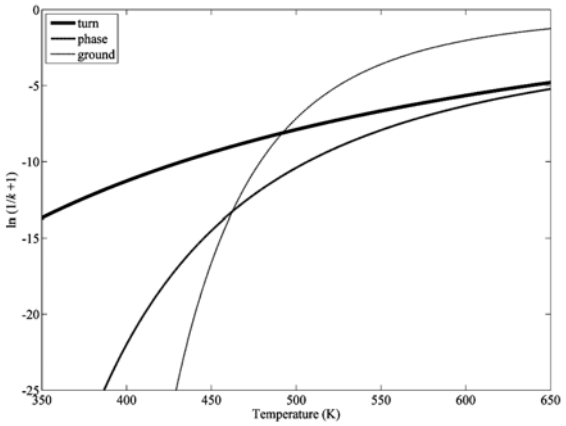
Table 2 Class-H motor insulation data under competing failure modes

463K				513K			
S \ M	Turn	Phase	Ground	S \ M	Turn	Phase	Ground
1	7228	10511	10511+	1	1175	1175+	1175
2	7228	11855	11855+	2	1881+	1881+	1175
3	7228	11855	11855+	3	1521	1881+	1881+
4	8448	11855	11855+	4	1569	1761	1761+
5	9167	12191+	12191+	5	1617	1881+	1881+
6	9167	12191+	12191+	6	1665	1881+	1881+
7	9167	12191+	12191+	7	1665	1881+	1881+
8	9167	12191+	12191+	8	1713	1881+	1881+
9	10511	12191+	12191+	9	1761	1881+	1881+
10	10511	12191+	12191+	10	1953	1953+	1953+
493K				533K			
S \ M	Turn	Phase	Ground	S \ M	Turn	Phase	Ground
1	1764	2436	2436	1	1632+	1632+	600
2	2436	2436	2490	2	1632+	1632+	744
3	2436	2436	2436	3	1632+	1632+	744
4	2436	2772+	2772	4	1632+	1632+	744
5	2436	2436+	2436	5	1632+	1632+	912
6	2436	4116+	4116+	6	1128	1128+	1128
7	3108	4116+	4116+	7	1512	1512+	1320
8	3108	4116+	4116+	8	1464	1632+	1632+
9	3108	3108	3108+	9	1608	1608+	1608
10	3108	4116+	4116+	10	1896	1896	1896

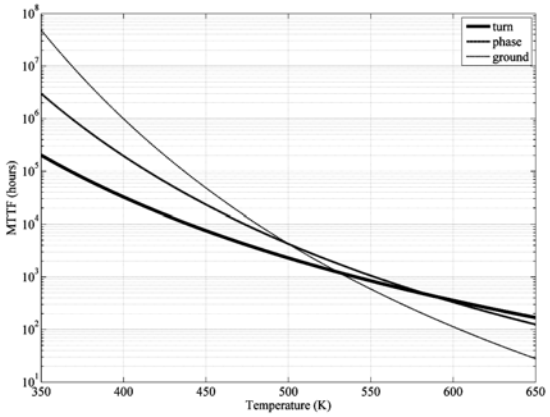
M: mode, S: sequence

The LE in Eq. (16) is maximized with the parameters shown in the relation of k's and c_n's, see Eq. (11), taken from the data in Table 2. The identified parameters and MTTF for each failure mode are given in Table 3. The result in Table 3 for k indicates that there is a decrease in the characteristic of cumulative damage of the three modes with increased temperature. Among these three modes, the characteristic of cumulative damage for the turn and phase modes show less influence from temperature, but the ground mode shows less tendency to accumulate due to the small value of k at higher temperature, see Fig. 8(a). The variations of MTTF with the temperature for these modes based on Eq. (9) are given in Fig. 8(b). The plots show that all have a similar trend. From the CFMA, the mode with the lowest MTTF dominates the apparent failure, *i.e.*, the

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(a)



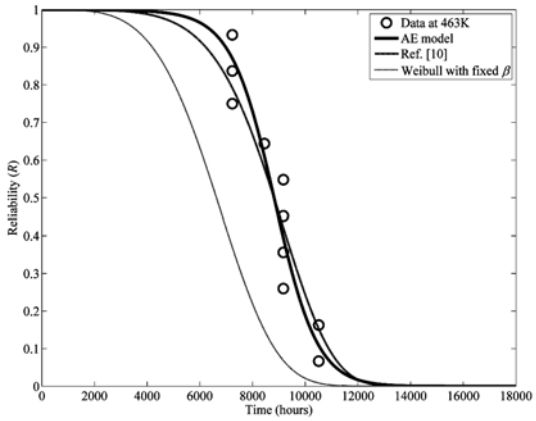
(b)

Fig. 8 Variations of (a) k and (b) MTTF with the temperature for each failure mode

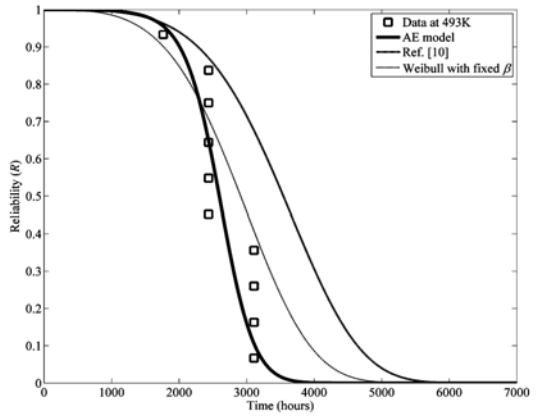
Table 3 Fitted parameters of the AE model with Arrhenius relation for the ALT data in Table 2

Mode	Temperature	c_n	k	L_T	(A, B)
Turn	463K	1.41E-3	2.44E5	8792	(–5.65, 6768)
	493K	3.90E-3	3.06E4	2647	
	513K	4.85E-3	3.05E3	1654	
	533K	4.98E-3	8.04E2	1344	
Phase	463K	7.80E-4	6.59E4	14229	(–6.94, 6768)
	493K	2.60E-3	3.59E4	4049	
	513K	3.60E-3	1.02E4	3472	
	533K	4.32E-3	7.32E3	2060	
Ground	463K	4.03E-4	1.41E5	29419	(–13.43, 11894)
	493K	2.56E-3	1.47E4	3613	
	513K	2.71E-3	3.26E3	2983	
	533K	3.04E-3	3.62E1	1190	

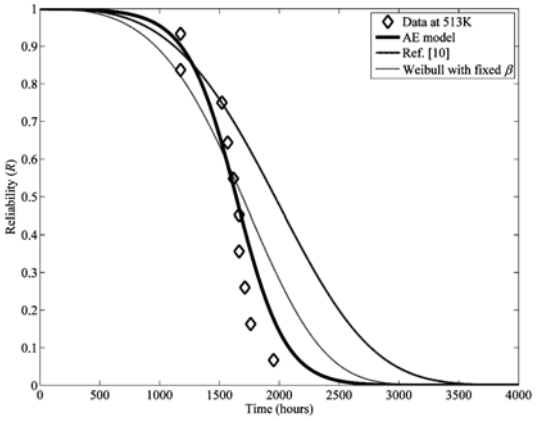
actual lifetime is somewhat below that of the dominant mode. At lower temperature, say below 450K, the failure due to the phase and ground modes can be ignored, because their MTTF is much larger than the turn mode; however when temperature is higher than 530K, the effect of ground mode failure becomes significant. For comparison of the models see Fig. 9 which shows the estimated reliability regressions from the data for four temperatures. Two other regressions [10] are also shown in Fig. 9. One is based on the Gompertz-type variation of β with Weibull distribution. The other



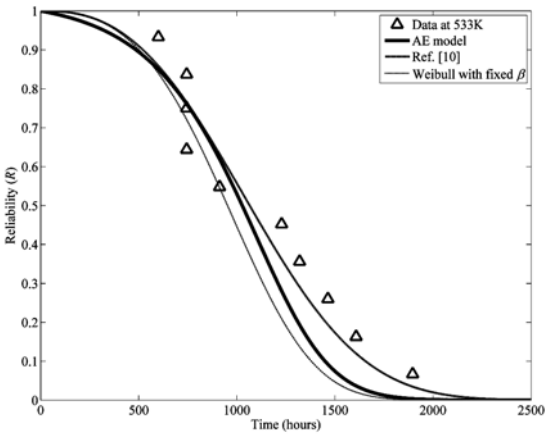
(a) 463K



(b) 493K



(c) 513K



(d) 533K

Fig. 9 Reliability fittings based on the MLE

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follows Weibull distribution with fixed β. It can be seen in the present study that the AE model fits the data better than others at four temperatures, even when the parameters are functions of the temperature. The MTTFs estimation at the maximum allowed temperature (453K) and use condition (365K) for these failure modes and the corresponding apparent failures are shown in Table 4. The predictions based on the AE model are quite similar to the Gompertz-type variation of β with the Weibull distribution. The MTTF of the apparent failure which is primarily dominated by the turn mode, followed by the phase mode, while the ground mode has little influence.

Table 4 Comparison of the MTTF in the hours for each failure mode at the maximum allowed temperature (453K) and use condition (363K)

Mode	AE model		β with Gompertz-type-relation [10]		β fixed	
	453K	363K	453K	363K	453K	363K
Turn	1.08E4*	4.40E5	1.25E4*	4.79E5	9.82E3*	2.54E5
Phase	2.08E4	1.37E6	1.71E4*	5.55E5	1.70E4*	5.58E5
Ground	4.10E4	1.59E7	5.92E4	9.70E6	3.95E4	9.99E6
Mode in series connection	1.08E4	4.40E5	1.19E4*	4.43E5	9.52E3*	2.51E5

*does not meet the required lifetime for class-H insulation (20,000 hours at 453K)

5. CONCLUSIONS

In the present study, the hazard rate-reliability dependent model is utilized to investigate the characteristics of insulation failure in class-H motors during accelerated life tests data in a censored situation. Three failure modes, namely the turn, phase and ground are considered in the estimation. This is the so-called competing failure analysis. Several remarks can be summarized as follows:

a. About the Model

1. The AE model representation of the reliability for each failure mode is suitable since the cumulative damage decreased with the temperature. This is necessary due to its influence on the estimation of k which is closely related to the MTTF prediction;
2. The parameters can be related to Arrhenius relations with function of MTTF. And the parameters solved in MLE are less than in the Weibull distribution when both parameters varied with temperature;
3. The AE model fits the data well even when the parameters are functions of the temperature.
4. The comparison shows that the AE model provides better life estimations for incomplete data.

b. About the Motor in the Tests

1. The apparent failure mode changes with variations in the temperature. Thus, the CFMA is necessary if there is more than one failure mode causing the system to shut down;
2. As the temperature rises to 440K, which is still within the allowable range of application, it is most important to rearrange the turn structure, then the phase. However, the characteristic of the cumulative process for the ground mode gradually decreases at higher temperatures, thus the ground should also be rearranged if the device must operate at specific thermal conditions;

The highest temperature of the test should be ignored in ALT analysis if the failure behavior is the same at all temperatures, because the characteristic of cumulative damage is quite different with other temperatures.

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APPENDIX A
FIRST THREE MOMENT OF THE AE MODEL

1. Mean (μ), the first moment

$$\mu = E(T) = \int_{-\infty}^{\infty} t \times f(t)\, dt = -\frac{\ln e_0 - \ln c_n}{c_n}. \tag{A1}$$

2. Variance (σ), the second moment

$$\sigma^2 = V(T) = \int_{-\infty}^{\infty} \left[t - E(T)\right]^2 dt = -\frac{(c_n^2 - e_0^2)(\ln e_0 - \ln c_n)^2 + 2c_n\left(Li_2\left(-\frac{c_n}{e_0}\right) + Li_2\left(-\frac{e_0}{c_n}\right)\right)}{c_n^3(e_0 + c_n)}. \tag{A2}$$

where Li is polylogarithm function.

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3. Skewness (γ), the third moment

$$\gamma = \frac{\int_{-\infty}^{\infty} \left[t - E(T)\right]^3 \times f(t)\, dt}{\sigma^3}$$

$$= \left(\begin{array}{l} \left(e_0 (e_0 + c_n)(4 e_0 c_n^2 - e_0 c_n^3) - \left(3 c_n (c_n^2 + e_0^2) + (e_0^2 - 2 e_0 c_n)(e_0 + c_n) \left(\ln \frac{e_0}{c_n} \right)^2 \right) \left(\ln \frac{e_0}{c_n} \right) \right) \\ + e_0 c_n^2 \left(3(e_0 + c_n) \left(\ln \frac{e_0}{c_n} \right) \left(Li_2 \left(-\frac{c_n}{e_0} \right) + Li_2 \left(-\frac{e_0}{c_n} \right) + 2 c_n \left(-Li_3 \left(-\frac{c_n}{e_0} \right) + Li_3 \left(-\frac{e_0}{c_n} \right) \right) \right) \right) \end{array} \right) \quad \text{(A3)}$$

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