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THE THIRD HANKEL DETERMINANT FOR INVERSE COEFFICIENTS OF CONVEX FUNCTIONS

VASUDEVARAO ALLU ✉ and SHOBHIT KUMAR

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Abstract

Let $\mathcal{A}$ denote the class of normalised analytic functions f in the open unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ with $f(0) = 0$ and $f'(0) = 1$. A function $f \in \mathcal{A}$ is said to be convex if $f(\mathbb{D})$ is convex. We establish a sharp upper bound for the third Hankel determinant corresponding to the inverse coefficients of convex univalent (that is, one-to-one) functions in the unit disk $\mathbb{D}$.

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1. Introduction

Let $\mathcal{A}$ denote the class of normalised analytic functions f in the open unit disk defined by $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ and having the Taylor expansion

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathbb{D}.$$

Let $\mathcal{S}$ be the subclass of $\mathcal{A}$ consisting of functions that are univalent in $\mathbb{D}$. Two fundamental subclasses of $\mathcal{S}$ are the classes of starlike functions $\mathcal{S}^*$ and convex functions C, defined respectively by

$$\mathcal{S}^* = \left\{ f \in \mathcal{A} : \operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > 0,\ z \in \mathbb{D} \right\}$$

and

$$C = \left\{ f \in \mathcal{A} : \operatorname{Re}\left(1 + \frac{zf''(z)}{f'(z)}\right) > 0,\ z \in \mathbb{D} \right\}.$$

Functions in C map the unit disk onto convex domains, and enjoy strong geometric and analytic properties. Moreover, the class C is closely connected with the Carathéodory class of functions with positive real part, which plays a central role in deriving sharp coefficient estimates.

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For each $f \in \mathcal{S}$, the inverse function f^{-1} exists in a neighbourhood of the origin and admits the expansion

$$f^{-1}(w) = w + A_2 w^2 + A_3 w^3 + \cdots .$$

Coefficient problems for inverse functions have attracted increasing attention in recent years, as inverse coefficients often exhibit behaviour substantially different from that of the original coefficients, even for classical subclasses such as C.

An important nonlinear functional associated with the Taylor coefficients of f is the qth Hankel determinant, introduced by Pommerenke [12], defined by

$$H(q,n)(f) = \begin{vmatrix} a_n & a_{n+1} & \cdots & a_{n+q-1} \\ a_{n+1} & a_{n+2} & \cdots & a_{n+q} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n+q-1} & a_{n+q} & \cdots & a_{n+2q-2} \end{vmatrix}.$$

Hankel determinants encode nonlinear relations among the coefficients of f and are particularly sensitive to geometric constraints such as starlikeness and convexity. The second Hankel determinant $H(2,2)(f) = a_2 a_4 - a_3^2$ has been extensively studied for many subclasses of univalent functions, whereas sharp estimates for Hankel determinants of order 3 are substantially more difficult to obtain. The third Hankel determinant

$$H(3,1)(f) = 2a_2 a_3 a_4 - a_3^3 - a_4^2 + a_3 a_5 - a_2^2 a_5$$

has been investigated in several recent papers. For the convex class C, Kowalczyk *et al.* [5] proved the sharp bound

$$|H(3,1)(f)| \leq \tfrac{4}{135}.$$

In contrast, comparatively little is known about Hankel determinants associated with inverse coefficients. Even for the classical convex class C, sharp bounds for inverse Hankel determinants remain scarce. This is particularly noteworthy, as inverse coefficient problems typically require techniques distinct from those used for the original function.

The purpose of the present paper is to address this gap. We investigate the third Hankel determinant associated with the inverse function of a convex univalent function and prove that if $f \in C$, then

$$|H(3,1)(f^{-1})| \leq \tfrac{1}{36},$$

and the bound $1/36$ is sharp. Our approach combines explicit coefficient relations for inverse functions with sharp parameter representations for functions in the Carathéodory class, due to Libera and Złotkiewicz [11] and subsequent refinements. By reducing the problem to a constrained optimisation over a compact parameter domain, we identify the extremal configuration and construct an explicit convex function for which equality is attained.

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For completeness, we note that a substantial amount of literature is available on the second Hankel determinant $H(2,2)(f) = a_2a_4 - a_3^2$ for various subclasses of $\mathcal{S}$, including starlike, convex, α-convex and close-to-convex families (see [1–3, 10, 13–16] for representative sharp estimates and related determinant inequalities.

Regarding Hankel determinants of order 3, several sharp bounds for $H(3,1)(f)$ have been obtained for different geometric subclasses (notably within starlike-type families) (see [4, 6, 8, 14] along with the works already cited above). These results further highlight the contrast between the comparatively mature theory for $H(3,1)(f)$ and the still limited understanding of Hankel determinants formed from inverse coefficients, which is precisely the focus of the present paper.

Our result answers negatively a question asked by Thomas [17]. Is the property of invariance true for the determinant $H(3,1)(f)$, and are the estimates for $H(3,1)(f)$ and $H(3,1)(f^{-1})$ the same when f is a convex function?

Let $\mathcal{P}$ denote the class of analytic functions p defined for $z \in \mathbb{D}$ by

$$p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n \tag{1.1}$$

with Re $p(z) > 0$ in the unit disk. Our proof uses the following lemma (see [7, 11]).

LEMMA 1.1. *Let $p \in \mathcal{P}$ be given by (1.1) with $c_1 \geq 0$. Then,*

$$\begin{aligned}
2c_2 &= c_1^2 + \gamma(4 - c_1^2),\\
4c_3 &= c_1^3 + 2(4 - c_1^2)c_1\gamma - (4 - c_1^2)c_1\gamma^2 + 2(4 - c_1^2)(1 - |\gamma|^2)\eta,\\
8c_4 &= c_1^4 + (4 - c_1^2)\gamma(c_1^2(\gamma^2 - 3\gamma + 3) + 4\gamma)\\
&\quad - 4(4 - c_1^2)(1 - |\gamma|^2)(c_1(\gamma - 1)\eta + \overline{\gamma}\eta^2 - (1 - |\eta|^2)\rho)
\end{aligned}$$

for some γ, η, ρ such that $|\gamma| \leq 1$, $|\eta| \leq 1$ and $|\rho| \leq 1$.

2. Main result

THEOREM 2.1. *If $f \in C$, then $|H(3,1)(f^{-1})| \leq 1/36$ and this bound is sharp.*

PROOF. Let $f \in C$. Then, there exists a function $p \in \mathcal{P}$ such that

$$1 + z\frac{f''(z)}{f'(z)} = p(z). \tag{2.1}$$

Let

$$f(z) = z + a_2z^2 + a_3z^3 + a_4z^4 + a_5z^5 + a_6z^6 + \cdots$$

be a normalised convex univalent function in $\mathbb{D}$. Since f is univalent, it possesses an inverse $g = f^{-1}$ in a neighbourhood of $z = 0$, and g has the expansion

$$g(w) = w + b_2w^2 + b_3w^3 + b_4w^4 + b_5w^5 + b_6w^6 + \cdots .$$

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The coefficients of g are determined from the identity $f(g(w)) \equiv w$, and the first few relations between $\{b_n\}$ and $\{a_n\}$ are

$$\begin{aligned} b_2 &= -a_2, \\ b_3 &= 2a_2^2 - a_3, \\ b_4 &= -5a_2^3 + 5a_2a_3 - a_4, \\ b_5 &= 14a_2^4 - 21a_2^2a_3 + 6a_2a_4 + 3a_3^2 - a_5. \end{aligned} \tag{2.2}$$

Comparing coefficients in (2.1) gives

$$\begin{aligned} a_2 &= \frac{c_1}{2}, \\ a_3 &= \frac{1}{6}(c_1^2 + c_2), \\ a_4 &= \frac{1}{24}(c_1^3 + 3c_1c_2 + 2c_3), \\ a_5 &= \frac{1}{120}(c_1^4 + 6c_1^2c_2 + 3c_2^2 + 8c_1c_3 + 6c_4). \end{aligned} \tag{2.3}$$

We now use the representation

$$H(3,1)(f^{-1}) = b_3(b_2b_4 - b_3^2) - b_4(b_4 - b_2b_3) + b_5(b_3 - b_2^2). \tag{2.4}$$

Substituting (2.2) and (2.3) into (2.4) gives

$$\begin{aligned} 8640\, H(3,1)(f^{-1}) = {} & 4c_1^6 - 24c_1^4c_2 - 44c_2^3 + 12c_1^3c_3 + 36c_1c_2c_3 - 60c_3^2 \\ & + c_1^2(39c_2^2 - 36c_4) + 72c_2c_4. \end{aligned} \tag{2.5}$$

Applying Lemma 1.1 to c_2, c_3, c_4 in (2.5),

$$\begin{aligned} 34560\, H(3,1)(f^{-1}) = {} & (-4 + c_1^2)^2(-16\gamma^3 - 60\eta^2 + 72\gamma\rho - 60\eta^2|\gamma|^4 - 72\gamma\rho|\eta|^2 \\ & - 12\gamma\eta\, c_1 - 12\gamma^2\eta\, c_1 + 3\gamma^2c_1^2 + 10\gamma^3c_1^2 + 3\gamma^4c_1^2 - 72\gamma\eta^2\overline{\gamma} \\ & + 12|\gamma|^2(10\eta^2 - 6\gamma\rho + 6\gamma\rho|\eta|^2 + \gamma\eta\, c_1 + \gamma^2\eta\, c_1 + 6\gamma\eta^2\overline{\gamma})). \end{aligned} \tag{2.6}$$

We rewrite the right-hand side of (2.6) as

$$A + B\eta + C\eta^2 + D\rho,$$

where

$$\begin{aligned} A &= \gamma^2(c_1^2 - 4)^2(3c_1^2 + 3\gamma^2c_1^2 + 2\gamma(-8 + 5c_1^2)), \\ B &= 12\gamma(1 + \gamma)(|\gamma|^2 - 1)c_1(c_1^2 - 4)^2, \\ C &= -12(|\gamma|^2 - 1)(c_1^2 - 4)^2(-5 + 5|\gamma|^2 - 6\gamma\overline{\gamma}), \\ D &= 72\gamma(|\gamma|^2 - 1)(|\eta|^2 - 1)(c_1^2 - 4)^2. \end{aligned}$$

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Taking $|\gamma| = x$, $|\eta| = y$, $|\rho| = 1$ and writing $c_1 = p_1$ for convenience,

$$\begin{aligned} H(p_1,x,y) := {} & 3p_1^2(p_1^2-4)^2x^2 + 2(p_1^2-4)^2(8+5p_1^2)x^3 + 3p_1^2(p_1^2-4)^2x^4 \\ & + 12p_1(p_1^2-4)^2x(1+x)(1-x^2)\,y + 12(1-x^2)(p_1^2-4)^2(5-x^2)\,y^2 \\ & + 72x(1-x^2)(1-y^2)(p_1^2-4)^2. \end{aligned}$$

Our aim is to find the maximum of $H(p_1,x,y)$ over

$$D = [0,2]\times[0,1]\times[0,1].$$

Since $H(p_1,x,y)$ is a quadratic polynomial in y, we may write

$$\begin{aligned} H(p_1,x,y) = {} & 3p_1^2(p_1^2-4)^2x^2 + 2(p_1^2-4)^2(8+5p_1^2)x^3 + 3p_1^2(p_1^2-4)^2x^4 \\ & + 72(p_1^2-4)^2x(1-x^2) + 12p_1(p_1^2-4)^2x(1+x)(1-x^2)\,y \\ & + y^2(-72(p_1^2-4)^2x(1-x^2) + 12(p_1^2-4)^2(1-x^2)(5+x^2)). \end{aligned} \tag{2.7}$$

A simple computation gives

$$\frac{\partial^2}{\partial y^2}H(p_1,x,y) = -24(-4+p_1^2)^2(-1+x^2)(5-6x+x^2).$$

This expression is nonnegative for all $(p_1,x)\in[0,2]\times[0,1]$ and, hence, for each fixed (p_1,x), the map $y\mapsto H(p_1,x,y)$ is convex on $[0,1]$. Therefore, the maximum over $y\in[0,1]$ is attained at an endpoint and, hence,

$$\max_D H(p_1,x,y) = \max_{0\le p_1\le 2,\ 0\le x\le 1}\{H(p_1,x,0),\ H(p_1,x,1)\}.$$

We first estimate $H(p_1,x,1)$. Substituting $y=1$ in (2.7) gives

$$\begin{aligned} & H(p_1,x,1) \\ & = (-4+p_1^2)^2(60-48x^2+16x^3-12x^4-12p_1(x-1)x(1+x)^2+p_1^2x^2(3+10x+3x^2)). \end{aligned}$$

It is easy to see that

$$H(p_1,x,1)\le s(p_1,x),\quad (p_1,x)\in[0,2]\times[0,1],$$

where

$$s(p_1,x) := 60(4-p_1^2)^2 + 16p_1(3+p_1)(-4+p_1^2)^2x - 32(4-p_1^2)^2x^2.$$

For each fixed p_1, the function $x\mapsto s(p_1,x)$ is a concave quadratic polynomial on $[0,1]$, so its maximum is attained at its vertex. Writing $s(p_1,x) = Ax^2+Bx+C$,

$$A = -32(4-p_1^2)^2,\quad B = 16p_1(3+p_1)(-4+p_1^2)^2,$$

and, hence, for $p_1\neq 2$,

$$x^* = -\frac{B}{2A} = \frac{p_1(3+p_1)}{4}.$$

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Evaluating $s(p_1, x)$ at x^* gives

$$s(p_1, x^*) = 2(-4 + p_1^2)^2(30 + 9p_1^2 + 6p_1^3 + p_1^4), \quad p_1 \in [0, 2).$$

For $p_1 = 2$, we have $s(2, x) = 0$ for all $x \in [0, 1]$. Set $s_1(p_1) := s(p_1, x^*)$, that is,

$$s_1(p_1) := 2(p_1^2 - 4)^2(p_1^4 + 6p_1^3 + 9p_1^2 + 30). \tag{2.8}$$

Differentiating (2.8) with respect to p_1,

$$s_1'(p_1) = 2(p_1^2 - 4)^2(18p_1 + 18p_1^2 + 4p_1^3) + 8p_1(p_1^2 - 4)(p_1^4 + 6p_1^3 + 9p_1^2 + 30), \quad 0 \le p_1 \le 2,$$

and a direct factorisation gives

$$s_1'(p_1) = 4p_1(p_1 - 2)(p_1 + 2)(4p_1^4 + 21p_1^3 + 19p_1^2 - 36p_1 + 24).$$

The quartic factor is strictly positive on $[0, 2]$; hence, s_1 has no critical point in $(0, 2)$. Therefore, the maximum of s_1 on $[0, 2]$ occurs at an endpoint and a computation gives

$$s_1(0) = 960, \quad s_1(2) = 0.$$

Consequently,

$$H(p_1, x, 1) \le s(p_1, x) \le 960.$$

Since $H(0, 0, 1) = 960$, we conclude that

$$\max_{(p_1,x)\in[0,2]\times[0,1]} H(p_1, x, 1) = 960.$$

We next estimate $H(p_1, x, 0)$. Substituting $y = 0$ in (2.6) gives

$$H(p_1, x, 0) = (p_1^2 - 4)^2 x(72 - 56x^2 + p_1^2 x(3 + 10x + 3x^2)).$$

It is easy to verify that

$$H(p_1, x, 0) \le t(p_1, x), \quad (p_1, x) \in [0, 2] \times [0, 1],$$

where

$$t(p_1, x) = (-4 + p_1^2)^2 x\,(72 + 16p_1^2 - 56x^2) = -56(p_1^2 - 4)^2 x^3 + 8(p_1^2 - 4)^2(9 + 2p_1^2)x. \tag{2.9}$$

Critical points of t in the rectangle $[0, 2] \times [0, 1]$ satisfy

$$\frac{\partial t}{\partial x} = 0, \quad \frac{\partial t}{\partial p_1} = 0.$$

Differentiating (2.9) with respect to x gives

$$\frac{\partial t}{\partial x} = (p_1^2 - 4)^2(8(9 + 2p_1^2) - 168x^2),$$

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so either $p_1 = 2$ or

$$x^2 = \frac{9 + 2p_1^2}{21}.$$

Differentiating (2.9) with respect to p_1 gives

$$\frac{\partial t}{\partial p_1} = 4p_1(p_1^2 - 4)(8(9 + 2p_1^2)x - 56x^3) + 32p_1x(p_1^2 - 4)^2,$$

which simplifies to

$$\frac{\partial t}{\partial p_1} = 32p_1(p_1^2 - 4)x((5 + 3p_1^2) - 7x^2).$$

Thus, $\partial t/\partial p_1 = 0$ if, and only if,

$$p_1 = 0, \quad p_1 = 2, \quad x = 0, \quad \text{or} \quad x^2 = \frac{5 + 3p_1^2}{7}.$$

Solving these conditions in the domain $0 \le p_1 \le 2, 0 \le x \le 1$ shows that the critical set of t consists of the segment $\{(2, x) : 0 \le x \le 1\}$ together with the single point $(0, \sqrt{3/7})$, and there are no further interior critical points.

We now compute the maximum of t on the boundary of the rectangle. On the segment $p_1 = 0$,

$$t(0, x) = 16(72x - 56x^3) = 1152x - 896x^3, \quad 0 \le x \le 1,$$

Therefore,

$$\frac{d}{dx}t(0, x) = 1152 - 2688x^2.$$

The critical point in $[0, 1]$ occurs at $x^2 = 3/7$, that is, $x = \sqrt{3/7}$, and

$$t\left(0, \sqrt{\frac{3}{7}}\right) = 1152\sqrt{\frac{3}{7}} - 896\left(\sqrt{\frac{3}{7}}\right)^3 = 768\sqrt{\frac{3}{7}} = \frac{768\sqrt{21}}{7}.$$

Since $t(0, 0) = 0$ and $t(0, 1) = 256$, it follows that

$$\max_{0 \le x \le 1} t(0, x) = \frac{768\sqrt{21}}{7} = 502.774 \quad \text{attained at } x = \sqrt{\frac{3}{7}}.$$

On the segment $p_1 = 2$, we have $t(2, x) = 0$ for all $x \in [0, 1]$, and on the segment $x = 0$, we have $t(p_1, 0) = 0$ for all $p_1 \in [0, 2]$. On the segment $x = 1$,

$$t(p_1, 1) = (p_1^2 - 4)^2(8(9 + 2p_1^2) - 56) = 16(p_1^2 - 4)^2(1 + p_1^2), \quad 0 \le p_1 \le 2. \quad (2.10)$$

Differentiating (2.10) with respect to p_1 gives

$$\frac{d}{dp_1}t(p_1, 1) = 32p_1(p_1 - 2)(p_1 + 2)(3p_1^2 - 2),$$

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so the critical points in $[0, 2]$ are $p_1 = 0$, $p_1 = 2$ and $p_1 = \sqrt{2/3}$. A simple computation shows that

$$t(0, 1) = 256, \quad t(2, 1) = 0, \quad t\left(\sqrt{\frac{2}{3}}, 1\right) = 16\left(1 + \frac{2}{3}\right)\left(4 - \frac{2}{3}\right)^2 = \frac{8000}{27}.$$

Therefore,

$$\max_{0 \le p_1 \le 2} t(p_1, 1) = \frac{8000}{27} \quad \text{attained at } p_1 = \sqrt{\frac{2}{3}}.$$

Combining the preceding estimates,

$$H(p_1, x, y) \le 960, \quad (p_1, x, y) \in D,$$

and, hence,

$$|H(3, 1)(f^{-1})| \le \frac{1}{36}.$$

To show sharpness of the inequality, we consider

$$p_0(z) = \frac{1 + z^3}{1 - z^3},$$

which is a Carathéodory function, that is, $\operatorname{Re} p_0(z) > 0$ for all $z \in \mathbb{D}$. Define a function f by the differential equation

$$1 + \frac{zf''(z)}{f'(z)} = p_0(z), \quad f(0) = 0, \ f'(0) = 1.$$

Then,

$$\frac{f''(z)}{f'(z)} = \frac{2z^2}{1 - z^3}.$$

Integrating from 0 to z gives

$$\log f'(z) = \int_0^z \frac{2\xi^2}{1 - \xi^3}\, d\xi = -\frac{2}{3}\log(1 - z^3),$$

and, hence,

$$f'(z) = (1 - z^3)^{-2/3}.$$

A further integration yields

$$f(z) = \int_0^z (1 - \xi^3)^{-2/3}\, d\xi.$$

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Using the binomial series expansion,

$$(1 - z^3)^{-2/3} = 1 + \tfrac{2}{3}z^3 + \tfrac{5}{9}z^6 + \cdots,$$

we obtain

$$f(z) = z + \tfrac{1}{6}z^4 + \tfrac{5}{63}z^7 + \cdots.$$

Let the inverse function be written as

$$f^{-1}(w) = w + b_2w^2 + b_3w^3 + b_4w^4 + b_5w^5 + \cdots.$$

Since the coefficients of z^2 and z^3 in $f(z)$ vanish, it follows that $b_2 = b_3 = 0$. Substituting the series for f^{-1} into the identity $f(f^{-1}(w)) \equiv w$ and comparing coefficients gives $b_4 = -1/6$ and $b_5 = 0$. This completes the proof. □

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VASUDEVARAO ALLU, Department of Mathematics, School of Basic Sciences, Indian Institute of Technology Bhubaneswar, Jatni Road, Argul, Bhubaneswar 752050, Odisha, India
e-mail: avrao@iitbbs.ac.in

SHOBHIT KUMAR, Department of Mathematics, School of Basic Sciences, Indian Institute of Technology Bhubaneswar, Jatni Road, Argul, Bhubaneswar 752050, Odisha, India
e-mail: a21ma09007@iitbbs.ac.in

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