The Knowledge Engineering Review, Vol. 28:1, 1–34. © Cambridge University Press, 2012
doi:10.1017/S0269888912000306
First published online 31 October 2012

An overview of fuzzy Description Logics for the Semantic Web

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Abstract

Information imprecision and uncertainty exist in many real world applications, and such information would be retrieved, processed, shared, reused, and aligned in the maximum automatic way possible. As a popular family of formally well-founded and decidable knowledge representation languages, fuzzy Description Logics (fuzzy *DLs*), which extend *DLs* with fuzzy logic, are very well suited to cover for representing and reasoning with imprecision and uncertainty. Thus, a requirement naturally arises in many practical applications of knowledge-based systems, in particular the Semantic Web, because *DLs* are the logical foundation of the Semantic Web. Currently, there have been lots of fuzzy extensions of *DLs* with Zadeh's fuzzy logic theory papers published, to investigate fuzzy *DLs* and more importantly serve as identifying the direction of fuzzy *DLs* study. In this paper, we aim at providing a comprehensive literature overview of fuzzy *DLs*, and we focus our attention on fuzzy extensions of *DLs* based on fuzzy set theory. Other relevant formalisms that are based on approaches like probabilistic theory or non-monotonic logics are covered elsewhere. In detail, we first introduce the existing fuzzy *DLs* (including the syntax, semantics, knowledge base, and reasoning algorithm) from the origin, development (from weaker to stronger in expressive power), some special techniques, and so on. Then, the other important issues on fuzzy *DLs*, such as reasoning, querying, applications, and directions for future research, are also discussed in detail. Also, we make a comparison and analysis.

1 Introduction

The Semantic Web is an extension of the current web in which the web information can be given well-defined semantic meaning, and thus enabling better cooperation between computers and people. From this point of view, we should find some methods that can describe the semantic meaning of the web. Fortunately, 'ontology' can do this. The core of the Semantic Web is 'ontology', which is a formal and explicit specification of a shared conceptualization and can enable semantic interoperability. With ontologies, it is possible to formally represent the knowledge of an application domain in terms of a set of vocabulary, the fundamental concepts in the domain, and relations between them. Over the past few years, several ontology definition languages for the Semantic Web have emerged, including RDF(S), OIL, DAML, DAML + OIL, and OWL (Berners-Lee *et al.*, 2001). Among them, OWL is the newly released standard recommended by W3C. As the Semantic Web expects, OWL has the reasoning nature because Description Logics (*DLs*; Baader *et al.*, 2003) are essentially the theoretical counterpart of OWL and play a crucial role in this context. *DL*s, which provide a logical reconstruction of object-centric and frame-based knowledge representation languages, are the subset of first-order logic that can provide sound and decidable reasoning support (Baader *et al.*, 2003).

https://doi.org/10.1017/S0269888912000306 Published online by Cambridge University Press

It is clear that *DLs* play a key role in the Semantic Web. As with the traditional crisp logic, any sentence in OWL, being asserted facts, or reasoning results, must be either true or false and nothing in between. However, most real-world domains contain uncertain knowledge and incomplete or imprecise information that is true only to a certain degree. Ontologies defined by these languages (e.g., OIL and OWL) thus cannot quantify the degree of the overlap or inclusion between two concepts, and also cannot support reasoning in which only partial information about a concept or individual in the domain can be obtained. Uncertainty becomes more prevalent when more than one ontology is involved, where it is often the case that a concept defined in an ontology can only find partial matches to one or more concepts in another ontology. To overcome the difficulty, existing ontology languages and traditional *DLs* need to be extended to be able to capture uncertain knowledge about the concepts, properties, and instances in the domain and to support reasoning with imprecise information. Along this direction, researchers in the past have attempted to apply different formalisms, such as fuzzy logic, rough set theory, Bayesian probability, and *ad hoc* heuristics, into ontology and *DLs* (Zadeh, 1965; Dubois & Prade, 2001; Hajek, 2006; Sanchez, 2006; Jiang *et al.*, 2009a).

In particular, the representation of fuzzy information has been addressed several decades ago by Zadeh (1965), and fuzzy set theory has been identified as a successful technique for modeling the imprecise and uncertain data in many application areas (Klir & Yuan, 1995), such as the database and the Semantic Web. Currently, there have been lots of fuzzy extensions of *DLs* with Zadeh's fuzzy logic theory papers published, but one only finds few comprehensive review papers of them. Only Lukasiewicz and Straccia (2008) and Straccia (2008) give an overview on several extensions of *DLs* with the probabilistic logic, possibilistic logic, and fuzzy logic. Regarding the fuzzy logic part, the authors pay more attention to the fuzzy *DL f-SHOIN(D)*, and then give a simple overview on other existing fuzzy *DLs*. Moreover, Bobillo (2008) surveys some contributions to the field of fuzzy ontologies (including the proposed definitions, some pointers to the most relevant applications, etc.), introduces his work fuzzy *DL f-SROIQ(D)*, and also gives an overview on other fuzzy *DLs*. However, the works (Bobillo, 2008; Lukasiewicz & Straccia, 2008; Straccia, 2008) have not given the detailed classifications of fuzzy *DLs* with fuzzy logic, and also the comparisons of different fuzzy *DLs*, the reasoning of fuzzy *DLs*, the querying of fuzzy *DLs*, and the detailed applications of fuzzy *DLs* were missed. On this basis, to investigate these issues and more importantly serve as identifying the direction of fuzzy *DLs* for the Semantic Web study, this paper aims at providing a comprehensive literature overview of fuzzy *DLs* for the Semantic Web. In our work, we will introduce fuzzy *DLs* from the origin, development (from weaker to stronger in expressive power), some special techniques, and so on. Moreover, the other important issues on fuzzy *DLs*, such as reasoning, querying, and applications, will also be discussed in detail. Also, we will make a comparison and analysis. Note that, however, it does not mean that this paper covers all publications in the research area and give complete descriptions.

In this paper, we review existing proposals to extend *DLs* and Semantic Web languages with the capability to handle imprecise and uncertain information. There are many ways of representing and dealing with imprecision and uncertainty. In particular, there are lots of existing Semantic Web languages and *DLs* dealing with imprecision and uncertainty that are based on fuzzy set theory. Therefore, in this paper, we restrict our attention to approaches based on fuzzy set theory for representing imprecise and uncertain information. We will not cover recent proposals for probabilistic extensions of *DLs*, and we will also not discuss non-monotonic and non-standard logics for representing uncertainty. In more detail, this paper will review a number of proposals for extending logical languages with fuzzy logic in the following aspects:

1. Approaches that extend *DLs* with fuzzy logic theory, which play as the theoretical counterpart of the Semantic Web languages.
2. Approaches that directly relate to fuzzy *DLs*, such as reasoning, querying, and applications of fuzzy *DLs*. Also, we make some comparisons and analyses.

In the first category, we cover fuzzy extensions of DLs, which are commonly accepted as being the formal basis of web ontology language (OWL). Even though most approaches only cover logics that

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are much weaker than OWL, the methods proposed can directly be applied to the corresponding subset of OWL without changes because *DLs* play as the theory counterpart of OWL. In particular, we will survey existing fuzzy *DLs* from the origin, development (from weaker to stronger in expressive power), some special techniques, and will simply discuss the following issues of a fuzzy *DL*:

- *Expressiveness* of the fuzzy *DL*;
- The *syntax*, *semantics*, and *knowledge base* (*KB*) of the fuzzy *DL*;
- *Reasoning algorithms* for the fuzzy *DL*;
- The *decidability* and *complexity* of the reasoning algorithms.

Indeed, the balance of expressive power and the computability of the fuzzy extension of *DLs* is a hot topic of the research. Generally speaking, the more expressive the *DL*, the higher computational complexity of it, so we should consider the balance of the two factors in a real application. At last, *in the second category*, *we also review a number of proposals that directly relate to fuzzy DLs*:

- *Reasoning* on fuzzy *DLs*, especially different reasoning techniques and fuzzy *DL* reasoners;
- *Querying* over fuzzy *DLs*;
- *The applications* of fuzzy *DLs*, including the fuzzy extensions of Semantic Web languages, the fuzzy Description Logic Programs (*DLPs*), some particular applications, etc. Also, we make some comparisons and analyses in our whole review.

The paper is structured as follows. In Section 2, we give some *background information on DLs and related formalisms* that are the basis for the logical languages used in the different approaches discussed later in the paper. We also provide a brief introduction to *fuzzy set theory* on which the fuzzy *DLs* are based. In the main parts of this paper (Sections 3–6), we survey the *different approaches for extending DLs* to represent the imprecise and uncertain information according to their expressive powers in Section 3; Section 4 investigates issues about *reasoning and querying of fuzzy DLs*, where we survey the reasoning techniques for fuzzy *DLs*, the existing reasoners and comparations of their expressive and reasoning powers, and the querying problems of fuzzy *DLs*; in Section 5, we discuss *the applications of fuzzy DLs* in detail, including the fuzzy extensions of Semantic Web languages, fuzzy *DLPs*, and some particular applications; finally, in Section 6, we conclude with a critical review of the state of the art and an analysis of directions for future research.

2 Description Logics and fuzzy sets theory

In this section, we first present some background information on *DLs* and related formalisms that are the basis for the logical languages used in the different approaches discussed in the later sections. Then, we provide a brief introduction to fuzzy sets theory on which the fuzzy *DLs* are based.

2.1 Description Logics

In the last decade, a substantial amount of work has been carried out in the context of *DLs*. *DLs* are a logical reconstruction of the so-called frame-based knowledge representation languages, with the aim of providing a simple well-established Tarski-style declarative semantics to capture the meaning of the most popular features of structured representation of knowledge. Nowadays, *DL*s have gained even more popularity due to their applications in the context of the Semantic Web (Baader *et al.*, 2003).

Generally, the recent researches on *DL*s can be divided into three categories:

- Introducing the *theoretical foundations* of *DL*s, addressing some of the most recent developments in theoretical research in the area;
- Focusing on the *implementation* of knowledge representation systems based on *DL*s, describing the basic functionality of a *DL* system, surveying the most influential knowledge representation systems based on *DL*s, and addressing specialized implementation techniques;
- Addressing the use of *DL*s and of *DL*-based systems in the design of several *applications* of practical interest.

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In the following statements, we mainly focus on the first category, especially the theoretical formalism of a *DL*, with respect to the balance between its expressive power and its computational complexity. Indeed, the tradeoff between the expressiveness of a *DL* language and its complexity of reasoning shows that a careful selection of language constructs is needed and that the reasoning services provided by the *DL* system are deeply influenced by the set of constructs. On this basis, we can characterize three different approaches to the implementation of reasoning services. The first can be referred to as *limited + complete* systems, which are designed by restricting the set of constructs in such a way that subsumption would be computed efficiently, possibly in polynomial time. The second approach can be denoted as *expressive + incomplete* systems, since the idea is to provide both an expressive language and efficient reasoning. The drawback is, however, that reasoning algorithms turn out to be incomplete in these systems. After some of the sources of incompleteness are discovered, often by identifying the constructs—or, more precisely, combinations of constructs—that would require an exponential algorithm to preserve the completeness of reasoning, the systems with complete reasoning algorithms are designed. Systems of this sort are therefore characterized as *expressive + complete*, they provide a test bed for the implementation of reasoning techniques developed in the theoretical investigations, and they play an important role in stimulating comparison and benchmarking with other systems.

In the following, we survey the languages of *DLs* according to their expressive power with the beginning of *AL* (i.e., *attributive language*; Baader *et al.*, 2003). The language *AL* has been introduced as a minimal language that is of practical interest, and the different *DL* languages are distinguished by the constructors they provide. In the sequel, we shall discuss various languages from the family of *AL*-languages. The other expressive languages from the extensions of this family are also introduced.

2.1.1 The basic Description Logic attributive language

Elementary descriptions are atomic concepts and atomic roles (also called concept names and role names), and complex descriptions can be built from them inductively with concept constructors and role constructors. In abstract notation, we use the letters A and B for atomic concepts, the letter R for atomic roles, and the letters C and D for concept descriptions.

Concept descriptions in *AL* are formed according to the following syntax rule:

$C, D \rightarrow \mathsf{T} \mid$	(universal concept)
$\bot \mid$	(bottom concept)
$A \mid$	(atomic concept)
$C \cap D \mid$	(intersection)
$\forall R.C \mid$	(value restriction)
$\exists R.\bot$	(limited existential quantification)

In *AL*, negation can only be applied to atomic concepts, and only the top concept is allowed in the scope of an existential quantification over a role. In order to define a formal semantics of *AL*-concepts, we consider an interpretations I that consist of a non-empty set Δ^I (the domain of the interpretation) and an interpretation function $\bullet^I$, which assigns to every atomic concept A, a set $A^I \subseteq \Delta^I$, and to every atomic role R, a binary relation $R^I \subseteq \Delta^I \times \Delta^I$. The interpretation function is extended to concept descriptions by the following inductive definitions:

$$\mathsf{T}^I = \Delta^I$$
$$\bot^I = \varnothing$$
$$(\neg A)^I = \Delta^I \backslash A^I$$
$$(C \cap D)^I = C^I \cap D^I$$
$$(\forall R.C)^I = \{a \in \Delta^I | \forall b.(a,b) \in R^I \rightarrow b \in C^I\}$$
$$(\exists R.\bot)^I = \{a \in \Delta^I | \exists b.(a,b) \in R^I\}$$

The above constructors can be used to build *DL KBs* for representing the knowledge of an application domain (the 'world') by first defining the relevant concepts of the domain (i.e., its

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terminology), and then using these concepts to specify properties of objects and individuals occurring in the domain (i.e., the world description; Baader *et al.*, 2003). Generally speaking, a *DL KB* comprises two components, the TBox and the ABox. The TBox introduces the terminology, that is, the vocabulary of an application domain, while the ABox contains assertions about named individuals in terms of this vocabulary.

In brief, a TBox T is a finite set of terminology axioms of the form $C \subseteq D$ or $C = D$. An interpretation I satisfies $C \subseteq D$ if $C^I \subseteq D^I$, and similarly for $C = D$. An interpretation I is a model of TBox T iff I satisfies all axioms in T. An ABox A is a finite set of assertions of the form $C(d)$ or $R(d_1, d_2)$. By the first kind, called concept assertions, one states that d belongs to (the interpretation) C, and by the second kind, called role assertions, one states that d_2 is a filler of the role R for d_1. An interpretation I is a model of ABox A iff I satisfies all assertions in A. Furthermore, an interpretation I satisfies a *DL* knowledge base *KB* if it satisfies all axioms and assertions in *KB*.

2.1.2 The family of attributive languages

We obtain more expressive languages if we add further constructors to *AL*. The following introduce several important constructors, which are often added to *AL*.

The *union* of concepts (indicated by the letter ***U***) is written as $C \cup D$, and interpreted as

$$(C \cup D)^I = C^I \cup D^I$$

Full existential quantification (indicated by the letter ***E***) is written as $\exists R.C$, and interpreted as

$$(\exists R.C)^I = \{a \in \Delta^I | \exists b.(a, b) \in R^I \wedge b \in C^I\}$$

Note that $\exists R.C$ differs from $\exists R.\bot$ in that arbitrary concepts are allowed to occur in the scope of the existential quantifier.

Number restrictions (indicated by the letter ***N***) are written as $\geqslant nR$ (at-least restriction) and as $\leqslant nR$ (at-most restriction), where n ranges over the non-negative integers. They are interpreted as

$$(\geqslant nR)^I = \{a \in \Delta^I | \#\{b | (a, b) \in R^I\} \geqslant n\}$$

and

$$(\leqslant nR)^I = \{a \in \Delta^I | \#\{b | (a, b) \in R^I\} \leqslant n\}$$

respectively, where '#{}' denotes the cardinality of a set. From a semantic view point, the coding of numbers in number restrictions is immaterial. However, for the complexity analysis of inferences it can matter whether a number n is represented in binary (or decimal) notation or by a string of length n, since binary (decimal) notation allows for a more compact representation.

The *negation* of arbitrary concepts (indicated by the letter ***C***, for 'complement') is written as $\neg C$, and interpreted as

$$(\neg C)^I = \Delta^I \backslash C^I$$

Extending *AL* by any subset of the above constructors yields a particular *AL*-language. We name each *AL*-language by a string of the form:

$$AL[U][E][N][C]$$

where a letter in the name stands for the presence of the corresponding constructor. For instance, *ALEN* is the extension of *AL* by full existential quantification and number restrictions. Moreover, since the union (U) and full existential quantification (E) can be expressed using negation (C), the letters UE in language names will be replaced by the letter C. For instance, we shall write *ALC* instead of *ALUE* and *ALCN* instead of *ALUEN*.

2.1.3 The more expressive Description Logics

There are several possibilities for extending *AL* in order to obtain a more expressive *DL*. The three most prominent are adding additional concept constructors, adding role constructors, and

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formulating restrictions on role interpretations. For these extensions, we introduce a naming scheme. Basically, each extension is assigned a letter or symbol:

- For *concept constructors*, the letters/symbols are written after the starting AL;
- For *role constructors*, we write the letters/symbols as superscripts;
- For *restrictions* on the interpretation of roles as subscripts.
- As an example, a DL called $ALCQI_{R+}$, extends AL with the concept constructors negation (C) and qualified number restrictions (Q), the role constructor inverse (I), and the restriction that some roles are transitive($_{R+}$).

Below, we start with the third case, since we need to refer to restrictions on roles when defining certain concept constructors.

Restrictions on role interpretations: These restrictions enforce the interpretations of roles to satisfy certain properties, such as functionality and transitivity. We consider these two prominent examples in more detail. Others would be symmetry or connections between different roles.

- *Functional roles*: Here one considers a subset N_F of the set of role names N_R, whose elements are called *features*. An interpretation must map features f to functional binary relations $f^I \subseteq \Delta^I \times \Delta^I$. AL extended with features is denoted by AL_f.
- *Transitive roles*: Here one considers a subset N_{R+} of N_R. Role names $R \in N_{R+}$ are called *transitive roles*. An interpretation must map transitive roles $R \in N_{R+}$ to transitive binary relations $R^I \subseteq \Delta^I \times \Delta^I$. AL extended with transitive roles is denoted by AL_{R+}.

All the DLs mentioned until now contain the concept constructors such as intersection ($C \cap D$) and value restriction ($\forall R.C$) as a common core. DLs that allow for intersection of concepts and existential quantification (but not value restriction) are collected in the *EL-family*. The only constructors available in EL are intersection of concepts ($C \cap D$) and existential quantification ($\exists R.\top$). Extensions of EL are again obtained by adding appropriate letters/symbols. Moreover, in order to avoid very long names for expressive DLs, the abbreviation S was introduced for ALC_{R+}, that is, DL that extends ALC by transitive roles. Prominent members of the S-family are SIN (which extends ALC_{R+} with number restrictions and inverse roles), $SHIF$ (which extends ALC_{R+} with role hierarchies, inverse roles, and number restrictions of the form $\leqslant 1R$), and $SHIQ$ (which extends ALC_{R+} with role hierarchies, inverse roles, and qualified number restrictions). Actually, DLs SIN, $SHIF$, and $SHIQ$ are somewhat less expressive than indicated by their name since the use of roles in number restrictions is restricted: roles that have a transitive sub-role must not occur in number restrictions.

2.1.4 Description Logics with data type representation

A drawback that all DLs introduced until now share is that all the knowledge must be represented on the abstract logical level. In many applications, one would like to be able to refer to concrete domains and predefined predicates on these domains when defining concepts, such as one wants to define the concept 'Minor' using the axiom $\text{Minor} = \text{Person} \cap \exists \text{age}.\leqslant_{18}$. To solve the problem, Baader and Hanschke (1991a) integrated concrete domains into concept languages, and the definition of *concrete domain* was given and a tableau-based algorithm for deciding consistency of $ALC(D)$-ABoxes for admissible D was introduced. The algorithm has an additional rule that treats existential predicate restrictions according to their semantics. The main new feature is that, in addition to the usual 'abstract' clashes, there may be concrete ones, that is, one must test whether the given combination of concrete predicate assertions is non-contradictory. This is the reason why we must require that the satisfiability problem for D is decidable. As described by Baader and Hanschke (1991b), the algorithm is not in *PSpace*. Using techniques similar to the ones employed for ALC it can be shown, however, that the algorithm can be modified such that it needs only polynomial space (Lutz, 1999), provided that the satisfiability procedure for D is in *PSpace*. In the presence of acyclic TBoxes, reasoning in $ALC(D)$ may become *NExpTime-hard* even for rather simple concrete domains with a polynomial satisfiability problem (Lutz, 2001).

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The more expressive *DLs* $SHN(D)^-$ (Haarslev & Möller, 2001) and $SHOQ(D)$ (Horrocks & Sattler, 2001), which can represent data information, are proposed. Although $SHN(D)^-$ and $SHOQ(D)$ are rather expressive, they also have a very serious limitation on data types, that is, they do not support customized data types. It has been pointed out that many potential users will not adopt them unless the limitation is overcome. Pan and Horrocks release a series of papers about data types to solve the problem (Pan & Horrocks, 2006; Pan, 2007). In the two papers, they summarize the limitations of OWL data typing and propose the data type approach. For example, $SHIQ(G)$ and $SHOQ(G)$ *DL*s presented by Pan (2007) and Pan and Horrocks (2006) can support user-defined data type and user-defined data type predicates.

2.2 Fuzzy set theory

In order to satisfy the need for modeling fuzzy information, the following several kinds of imperfect information have been extensively introduced into real-world applications (Klir & Yuan, 1995; Smets, 1997; Bosc *et al*., 2005; Ma & Yan, 2008):

- Inconsistency is a kind of semantic conflict, meaning the same aspect of the real world is irreconcilably represented more than once in a database or in several different databases. For example, the *age* of *George* is stored as 34 and 37 simultaneously.
- Imprecision is relevant to the content of an attribute value and means that a choice must be made from a given range (interval or set) of values without knowing which one to choose. For example, the *age* of *Michael* is a set {18, 19, 20, 21}.
- Vagueness is like imprecision but which is generally represented with linguistic terms. For example, the *age* of *John* is a linguistic value '*old*'.
- Uncertainty is related to the degree of truth of its attribute value, meaning that we can apportion some, but not all, of our belief to a given value or group of values. For example, the possibility that the *age* of *Chris* is 35 right now should be 0.98. The random uncertainty, described using probability theory, is not considered in the paper.
- Ambiguity means that some elements of the model lack complete semantics, leading to several possible interpretations.

Generally, several different kinds of imperfection can coexist with respect to the same piece of information. For example, the *age* of *John* is a set {18, 19, 20} and their possibilities are 0.70, 0.95, and 0.98, respectively. Imprecision, vagueness, and uncertainty are three types of imperfect information (Smets, 1997; Ma & Yan, 2008). Imprecision are essential properties of the information itself, whereas uncertainty is a property of the relation between the information and our knowledge about the world.

Currently, several approaches, such as probability, rough set, and fuzzy set theories, have been developed to handle uncertainty. There are some similarities and differences among these approaches. In brief, fuzzy logic and probabilistic logic are mathematically similar, and both have truth values ranging between 0 and 1, but conceptually distinct (Chen *et al*., 1999). The distinction between fuzzy set and probability theory has to do with the difference between the notions of probability and a degree of membership (Sanjaa & Tsoozol, 2007). Probability statements are about the likelihoods of outcomes: an event either occurs or does not, and you can choose on it. It interprets with 'degree of belief' in Bayesian theorem. In fuzziness, the concept 'degree of truth' is used, which one cannot choose and cannot say clearly whether an event occurs or not. That is, fuzzy logic and probability refer to different kinds of uncertainty, fuzziness is a type of deterministic uncertainty, and it measures the degree to which an event occurs, not whether it occurs. Probability arouses from the question whether or not an event occurs. Also, two theories of fuzzy sets and rough sets model different types of uncertainty (Yao, 1998). The theory of fuzzy sets is centered upon fuzzy information granulation, whereas rough set theory is focused on crisp information granulation. The rough set theory takes into consideration the indiscernibility between objects. The indiscernibility is typically characterized by an equivalence relation.

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Rough sets are the results of approximating crisp sets using equivalence classes. The fuzzy set theory deals with the ill-definition of the boundary of a class through a continuous generalization of set characteristic functions. The indiscernibility between objects is not used in fuzzy set theory. A fuzzy set may be viewed as a class with unsharp boundaries, whereas a rough set is a crisp set, which is coarsely described.

In particular, in the real-world applications, fuzzy set theory has been extensively introduced into database and information systems to deal with imprecision and uncertainty (Bosc *et al*., 2005; Ma & Yan, 2008). Also, in the Semantic Web, the fuzzy set theory has been considered as the main theory basis for fuzzy extensions to *DLs* to represent imprecise and uncertain information. Therefore, in this paper we will concentrate on *DLs* with fuzzy set theory. Here, we only give some basic knowledge of fuzzy set theory.

Fuzzy data is originally described as fuzzy set (Zadeh, 1965). A fuzzy set, say {0.6/18, 0.7/19, 0.8/20, 0.9/21} for the age of *Michael*, is more informative because it contains information imprecision (the age may be 18, 19, 20, or 21 and we do not know which one is true) and uncertainty (the degrees of truth of all possible age values are respectively 0.6, 0.7, 0.8, and 0.9) simultaneously.

Let U be a universe of discourse. A fuzzy value on U is characterized by a fuzzy set F in U. A membership function

$$\mu_F : U \to [0, 1]$$

is defined for the fuzzy set F, where $\mu_F(u)$, for each $u \in U$, denotes the degree of membership of u in the fuzzy set F. For example, $\mu_F(u) = 0.75$ means that u is 'likely' to be an element of F by a degree of 0.75. For ease of representation, a fuzzy set F over universe U is organized into a set of ordered pairs:

$$F = \{\mu_F(u_1)/u_1, \mu_F(u_2)/u_2, \ldots, \mu_F(u_\mathrm{n})/u_\mathrm{n}\}.$$

When U is an infinite set, then the fuzzy set F can be represented by

$$F = \int_{u \in U} \mu_F(u)/u$$

When the membership function $\mu_F(u)$ above is explained to be a measure of the possibility that a variable X has the value u in this approach, where X takes values in U, a fuzzy value is described by a possibility distribution π_X (Zadeh, 1978):

$$\pi_X = \{\pi_X(u_1)/u_1, \pi_X(u_2)/u_2, \ldots, \pi_X(u_n)/u_n\}.$$

Here, $\pi_X(u_i)$, $u_i \in U$, denotes the possibility that u_i is true. A fuzzy set is a representation of a concept while possibility distribution relates with the possibility of occurring a value within a distribution. Let π_X and F be the possibility distribution representation and the fuzzy set representation for a fuzzy value, respectively. It is clear that $\pi_X = F$ is true. Therefore, a fuzzy value on U can be characterized by a fuzzy set or a possibility distribution in U. Moreover, according to Zadeh, the membership function has to satisfy three well-known restrictions: for all $u \in U$ and for all fuzzy sets F, F' with respect to U:

$$\mu_{F \cap F}(u) = \min\{\mu_F(u), \mu_F(u)$$
$$\mu_{F \cup F}(u) = \max\{\mu_F(u), \mu_F(u)\}$$
$$\mu_{\overline{F}}(u) = 1 - \mu_F(u), \text{ where } \bar{F} \text{ is complement of } F \text{ in } U$$

For more concepts and operations about fuzzy sets and possibility distribution theory, one can refer to Zadeh (1965, 1978).

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3 Fuzzy extensions of Description Logics

Much work has been carried out toward combining fuzzy logic and *DLs* during the last decade. The following statements introduce almost all the fuzzy extensions to *DL*s (i.e., fuzzy *DLs*) from weaker to stronger in expressive power, including the family of *FALC* languages, the more expressive fuzzy *DLs*, and the tractable fuzzy *DLs*.

3.1 The family of FALC languages

The initial idea combining fuzzy logic and *DLs* was presented by Yen (1991), where a construct called *membership manipulators* was introduced, and a structural subsumption algorithm was also provided in order to perform reasoning. A later approach was presented by Tresp and Molitor (1998), and the authors also developed a tableaux calculus for the proposed fuzzy *DL* called ALC_{FM} (*ALC* extended with fuzzy set theory and the membership manipulator constructor). More importantly, the fuzzy extension of the basic *DL ALC*, that is, *FALC*, was presented by Straccia (1998), where the concept constructor membership manipulator was also included in the extended language. Furthermore, the complete tableau-based reasoning algorithm for *FALC* was provided by Straccia (2001). In particular, although both algorithms by Straccia (2001) and Tresp and Molitor (1998) are extensions of the usual tableau-based algorithm for *ALC*, they differ considerably. For example, the algorithm by Tresp and Molitor (1998) introduced numerical variables for the degrees, and produced a linear optimization problem, which must be solved in place of the usual clash test. In contrast, Straccia (2001) dealt with the membership degrees within his tableau-based algorithm. In addition, other fuzzy extensions of *ALC* were considered by Hölldobler *et al.* (2002, 2003), Liu and Chen (2006), etc.

In general, *the family of fuzzy ALC languages* can be summarized in Table 1.

In the following, we introduce the family of *FALC* languages, including *FALC*, ALC_{FM}, ALC_{FH}, ALC_{FLH}, and ALC_{FL} in detail.

3.1.1 The fuzzy Description Logic FALC

The *FALC* (Straccia, 1998, 2001) is considered as the basic fuzzy *DL*, thus we introduce its syntax, semantics, *KB*, and reasoning problems in detail.

Table 1 The family of *FALC* languages

Fuzzy *DLs*	Corresponding references	Mainly discussed issues
FALC	Straccia (1998, 2001), Bobillo *et al.* (2011) and Hajek (2005)	The fuzzy extension of the basic *DL ALC*. Its syntax, semantics, properties, and reasoning services
ALC_{FM}	Tresp and Molitor (1998)	Extending *ALC* with fuzzy set theory and the membership manipulator constructor. Its syntax, semantics, and a method for computing the degree of subsumption between two ALC_{FM}- concepts
ALC_{FH}	Hölldobler *et al.* (2002, 2003, 2004)	*FALC* plus linguistic hedges. Its syntax, semantics, and reasoning services
ALC_{FLH}	Hölldobler *et al.* (2005)	*FALC* plus linear linguistic hedges. Its syntax, semantics, and reasoning services
ALC_{FL}	Dinh-Khac *et al.* (2006)	An instance of *L-ALC* (*DLs* over lattices). Beside constructors of *L-ALC*, ALC_{FL} allows the modification by hedges. Its syntax, semantics, and reasoning services

DLs = Description Logics.

https://doi.org/10.1017/S0269888912000306 Published online by Cambridge University Press

DEFINITION 1 (syntax). *Let N_I, N_C, and N_R be three disjoint sets: N_I is a set of individual names, N_C is a set of fuzzy concept names, and N_R is a set of fuzzy role names. FALC-concepts are defined as follows (where $A \in N_C$, $R \in N_R$):*

$$C, D \rightarrow \bot | \top | A | \neg C | C \cap D | C \cup D | \exists R.C | \forall R.C$$

DEFINITION 2 (semantics). *The semantics of FALC is defined by a fuzzy interpretation $FI = \langle \Delta^{FI}, \bullet^{FI} \rangle$ where Δ^{FI} is a non-empty set and $\bullet^{FI}$ is a function which maps every $d \in N_I$ to an element $d^{FI} \in \Delta^{FI}$, maps every $A \in N_C$ into a function A^{FI}: $\Delta^{FI} \rightarrow [0, 1]$, and maps every $R \in N_R$ into a function R^{FI}: $\Delta^{FI} \times \Delta^{FI} \rightarrow [0, 1]$. Furthermore, for any FALC-concepts C and D, $R \in N_R$, and $d, d' \in \Delta^{FI}$, we have:*

$$\begin{gathered}
\top^{FI}(d) = 1 \\
\bot^{FI}(d) = 0 \\
(\neg C)^{FI}(d) = 1 - C^{FI}(d) \\
(C \cap D)^{FI}(d) = \min\{C^{FI}(d), D^{FI}(d)\} \\
(C \cup D)^{FI}(d) = \max\{C^{FI}(d), D^{FI}(d)\} \\
(\forall R.C)^{FI}(d) = \inf_{d' \in \Delta^{FI}}\{\max\{1 - R^{FI}(d, d'), C^{FI}(d')\}\} \\
(\exists R.C)^{FI}(d) = \sup_{d' \in \Delta^{FI}}\{\min\{R^{FI}(d, d'), C^{FI}(d')\}\}
\end{gathered}$$

With the introduction of fuzzy sets into the classical *ALC*, the form of the *KB* is changed accordingly:

DEFINITION 3 (knowledge base). *An FALC knowledge base K is composed of a* TBox *T and an* ABox *A:*

- *A* TBox *T is a finite set of terminology axioms of the form $C \subseteq D$ or $C = D$. An interpretation FI satisfies $C \subseteq D$ iff for any $d \in \Delta^{FI}$, $C^{FI}(d) \leqslant D^{FI}(d)$, and similarly for $C = D$. FI is a model of* TBox *T iff FI satisfies all axioms in T.*
- *An* ABox *A is a finite set of assertions of the form $\langle \alpha \ \infty \ n \rangle$. Here $\infty \in \{>, \geqslant, <, \leqslant\}$, $n \in [0, 1]$, α is either of the form d: C or (d_1, d_2): R. Especially, in order to give a uniform format of the* ABox*, we define: when $n = 1$, the form $\langle \alpha \geqslant 1 \rangle$ is equivalent to $\langle \alpha = 1 \rangle$. Concretely speaking, $\langle d$: $C \geqslant 1 \rangle$ means that d is determinately an individual of C; $\langle (d_1, d_2)$: $R \geqslant 1 \rangle$ means that (d_1, d_2) determinately has the relationship R. An interpretation FI satisfies $\langle d$: $C \infty n \rangle$ iff $C^{FI}(d^{FI}) \infty n$ and satisfies $\langle (d_1, d_2)$: $R \infty n \rangle$ iff $R^{FI}(d_1{}^{FI}, d_2{}^{FI}) \infty n$. FI is a model of* ABox *A iff FI satisfies all assertions in A.*
- *A fuzzy interpretation FI satisfies a knowledge base K if it satisfies all axioms and assertions in K.*

Now, we introduce the reasoning problems of *FALC*, including satisfiability, subsumption, consistency, entailment, and so on.

DEFINITION 4 (reasoning problems). *An FALC knowledge base K is* satisfiable *iff there exists a fuzzy interpretation FI, which satisfies all axioms and assertions in K. An FALC-concept C is* satisfiable *w.r.t. a* Tbox *T iff there exists some model FI of T for which there is some $d \in \Delta^{FI}$ such that $C^{FI}(d) = n$, and $n \in (0, 1]$. Let C and D be two FALC-concepts, we say that C is subsumed by D w.r.t. T if for every model FI of T it holds that, $\forall d \in \Delta^{FI}$, $C^{FI}(d) \leqslant D^{FI}(d)$. An FALC* Abox *$A$ is* consistent *w.r.t. T if there is a model FI of T, which is also a model of A. Furthermore, given an axiom or assertion ψ, an FALC knowledge base K* entails *ψ, written $K \models \psi$, iff all models of K also satisfy ψ. Moreover, given an FALC knowledge base K and a fuzzy assertion α, the* greatest lower bound *of α w.r.t. K is $glb(K, \alpha) = \sup\{n : K \models \alpha \geqslant n\}$, where $\sup\emptyset = 0$.*

The problem of concept satisfiability can be reduced to the problem of ABox consistency, and the problems of subsumption and entailment can be reduced to the problem of *KB* satisfiability (Straccia, 2001; Stoilos *et al.*, 2007). Furthermore, in the presence of only *simple TBoxes* (A Tbox *T* is called *simple* if it neither includes cyclic nor General Concept Inclusions (*GCIs*), that is, axioms are the form $A \subseteq C$ or $A \equiv C$, where A is a concept name that is never defined by itself either directly or indirectly, and A appears at most once at the left hand side.) (Baader *et al.*, 2003;

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Stoilos *et al.*, 2007), all the problems above can be reduced to ABox consistency w.r.t. an empty TBox. The reasoning algorithm and its decidability for *FALC* have been introduced in detail (Straccia, 2001; Bobillo *et al.*, 2011). Moreover, Hajek (2005) studied the problems of satisfiability, validity, and subsumption of *FALC*-concepts. Haarslev *et al.* (2006) introduced a generic framework, which allows representing several existing extensions of *DLs* with uncertainty in a uniformed manner, and used *FALC* as an example to illustrate how it can represent uncertainty and reasoning services in the frameworks considered.

3.1.2 The fuzzy Description Logic ALC_{FM}

In Tresp and Molitor (1998), an extension of *ALC* to many-valued logics called ALC_{FM} was proposed, where the construct called *membership manipulators* (Yen, 1991) also appeared, which are applied to concepts. Examples of manipulators could be '*mostly*', '*more or less*', or '*very*'. For example, if *Tall* is a concept (standing for the fuzzy set of all tall persons), then *VeryTall*, which is obtained by applying the manipulator *Very* to the concept *Tall*, is a new concept (standing for the fuzzy set of all *very tall* persons). Intuitively, the manipulators modify the membership degree functions of the concepts. In our example, the membership function for *VeryTall* should have its largest values at larger heights than the membership function for *Tall*. Formally, the semantics of manipulators is defined by a function that maps membership degree functions to membership degree functions (Tresp & Molitor, 1998).

Now, let us consider what kind of inference problems are of interest in this context (i.e., Yen, 1991; Tresp & Molitor, 1998). Yen (1991) considered crisp subsumption of fuzzy concepts, that is, given two concepts C, D defined in the fuzzy *DL*, the author was interested in the question whether $C^{FI}(d) \leqslant D^{FI}(d)$ for all fuzzy interpretations FI and $d \in \Delta^{FI}$. Thus, the subsumption relationship itself is not fuzzified. To this end, the author described a structural subsumption algorithm for a rather small fuzzy *DL*, which is almost identical to the subsumption algorithm for the corresponding classical *DL*. In contrast, Tresp and Molitor (1998) were interested in determining fuzzy subsumption between fuzzy concepts, that is, given concepts C, D, they want to know to which degree C is a subset of D. The complete reasoning algorithms can be found by Tresp and Molitor (1998) and Yen (1991), respectively.

3.1.3 The fuzzy Description Logics with hedges

The fuzzy extensions of *ALC* language were also considered in other approaches (Hölldobler *et al.*, 2002, 2003, 2004, 2005), where a concept constructor *hedges* (an algebraic approach to structures of sets of linguistic domains of linguistic truth variables (Cat Ho & Wechler, 1990)) were included in the extended languages. The fuzzy *DLs* ALC_{FH} (Hölldobler *et al.*, 2002, 2003, 2004) and ALC_{FLH} (Hölldobler *et al.*, 2005) were presented, respectively, where primitive concepts were modified by means of hedges. ALC_{FH} and ALC_{FLH} are strictly more expressive than *FALC* defined by Straccia (1998). Moreover, the authors showed that a linearly ordered set of hedges primitive concepts can be modified to any desired degree by prefixing them with appropriate chains of hedges. The authors also defined the decision procedures for the unsatisfiability problems in ALC_{FH} and ALC_{FLH}, and discussed truth bounds, expressivity as well as complexity issues.

In addition, in Dinh-Khac *et al.* (2006), a fuzzy linguistic *DL* called ALC_{FL} was proposed, which is an instance of *L-ALC* (*DLs* over lattices (Straccia, 2006d)). Beside constructors of *L-ALC*, ALC_{FL} allows the modification by hedges. Furthermore, the ambiguity of the hedge application in ALC_{FH} and ALC_{FLH} is solved in ALC_{FL}. In all these approaches, above tableaux decision procedures were presented for performing reasoning services.

In summary, the fuzzy extension to *ALC* can be used as a basis both for extending existing *DL*-based systems and for further research. In the latter case, there are several open points. For instance, it is not clear yet how to reason both in the case of fuzzy specialization of the general form $C \subseteq D$ and in the case that cycles are allowed in a fuzzy *KB*. Another interesting topic for further research concerns the semantics of fuzzy connectives. While for a huge number of

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proposals given in the literature their impact from a semantics point of view is well understood, the question how they impact from a computational complexity and algorithms point of view remains still open. For example, Meghini *et al.* (1997) proposed a preliminary fuzzy *DL*, which lacks reasoning algorithm, as a modeling tool for multimedia document retrieval.

3.2 The more expressive fuzzy Description Logics

In the following statements, we survey several more expressive fuzzy *DLs* according to the concept constructors or role restrictions that have been constrained on them, such as extending fuzzy *DLs* with number restrictions, inverse/transitive roles, fuzzy data type, fuzzy cut sets, and so on (see Table 2 in Section 3.2.4).

3.2.1 Fuzzy Description Logics with number restrictions

The number restrictions include the unqualified number restrictions N and qualified number restrictions Q. Currently, there have been several approaches for extending fuzzy ALC with number restrictions. In Li *et al.* (2005c), a fuzzy *DL* called *f-ALCN* was presented, and the authors provided reasoning for *f-ALCN* by using the idea of the reduction in order to annotate the concepts and roles of the crisp *DL ALCN*. Moreover, in Lu *et al.* (2005), an extended fuzzy *ALCN* and designed tableau algorithms for that were given. In addition, in Zhao *et al.* (2009), a fuzzy *DL* named fuzzy *ALCHIN* and an algorithm for consistency checking of *KBs* in the proposed language were proposed.

Approach toward more expressive *DL* was presented by Sánchez and Tettamanzi (2004), where the *DL* is called $ALCQ_F{}^+$. It includes *fuzzy quantifiers*, which allows for the definition of fuzzy quantifiers of the absolute and relative kind by means of piecewise linear functions on N and $Q \cap [0, 1]$, respectively, is a new novel idea for fuzzy *DL*s. Furthermore, in Sánchez and Tettamanzi (2006), the same authors introduced the reasoning procedures for $ALCQ_F{}^+$, where in order to reason about instances, the semantics of quantified expressions are defined based on recently developed measures of the cardinality of fuzzy sets. Moreover, a procedure was described to calculate the fuzzy satisfiability of a fuzzy assertion, which is a very important reasoning task. Besides, the procedure also considers several different cases and provides direct solutions for the most frequent types of fuzzy assertions. In addition, Sánchez and Tettamanzi (2006) defined *independence* of fuzzy assertions and obtained some results that speed up the calculation of fuzzy satisfiability in some (the most common) cases. More recently, Stoilos *et al.* (2008a) investigated on the problem of reasoning with *qualified cardinality restrictions* in fuzzy *DLs*, extending previous results on simple number restrictions, and thus presented a tableaux algorithm for the fuzzy *DL* *f-ALCIQ*. The other efforts for extending fuzzy *DLs* with number restrictions (such as f_{KD}-*SHIN* and fuzzy *SHOIQ*) can be found in Section 3.2.3.

The extensions with fuzzy quantifiers greatly increase the expressive power of *DLs*, but make reasoning hard. As a solution, Dragoni and Tettamanzi (2007) propose to use non-exact evolutionary algorithms to solve concept satisfiability. In fact, not just the reasoning, the semantics of qualified number restrictions is still open to discussion, as pointed by Bobillo and Straccia (2008a), where a new semantics is also introduced.

3.2.2 The more expressive fuzzy Description Logics with inverse roles/transitive roles/nominals

The fuzzy *DL FALC* and its some extensions above can effectively express fuzzy knowledge, but they provide limited representation and reasoning ability. For example, they only contain some simple operators, such as conjunction, intersection, negation, value restriction, and existential quantification, which cannot represent and reason on more complex fuzzy knowledge (e.g., the inverse of roles and the transitive roles). Therefore, some scholars have carried out researches on more expressive fuzzy *DLs*, such as f_{KD}-*SI* (Stoilos *et al.*, 2005a), f_{KD}-*SHIN* (Stoilos *et al.*, 2005b), *f-SHOIN* (Stoilos *et al.*, 2005c), *f-SHOIQ* (Stoilos *et al.*, 2006a), and *f-SROIQ* (Stoilos & Stamou, 2007) as follows.

https://doi.org/10.1017/S0269888912000306 Published online by Cambridge University Press

https://doi.org/10.1017/S0269888912000306 Published online by Cambridge University Press

Table 2 Some existing fuzzy *DLs* from weaker to stronger in expressive power

	Representation of fuzzy terminologies and concepts							
	f-TSL	f-ALC	ALC_{FH}	$ALCQ_F^+$	*f-ALCIQ*	*f-SHIN*	*f-SHOIN*	*f-SROIQ*
Syntax and semantics	Yen (1991)	Straccia (1998)	Hölldobler *et al.* (2002)	Sánchez and Tettamanzi (2004)	Stoilos *et al.* (2008a)	Stoilos *et al.* (2005b)	Stoilos *et al.* (2005c)	Stoilos and Stamou (2007) and Bobillo *et al.* (2007a)
Tableau algorithm	Yen (1991)	Straccia (2001)	Hölldobler *et al.* (2002)	Sánchez and Tettamanzi (2006a)	Stoilos *et al.* (2008a)	Stoilos *et al.* (2007)	Stoilos *et al.* (2005c)	
Decidability	Yen (1991)	Straccia (2001)	Hölldobler *et al.* (2002)	Sánchez and Tettamanzi (2006a)	Stoilos *et al.* (2008a)	Stoilos *et al.* (2007)	Stoilos *et al.* (2005c)	
	Representation of fuzzy data information							
	f-ALC(D)	*f-ALCF(D)*	*f-SHOIN(D)*	*f-SROIQ(D)*	*f-ALC(G)*			
Syntax and semantics	Straccia (2005b)	Bobillo and Straccia (2009b) and Straccia and Bobillo (2007)	Straccia (2005a)	Bobillo *et al.* (2009) and Bobillo and Straccia (2009c)	Wang and Ma (2008)			
Tableau algorithm	Straccia (2005b, 2005d)	Bobillo and Straccia (2009b) and Straccia and Bobillo (2007)			Wang and Ma (2008)			
Decidability	Straccia (2005b, 2005d)	Bobillo and Straccia (2009b) and Straccia and Bobillo (2007)			Wang and Ma (2008)			

DLs = Description Logics.

In order to represent and reason on more complex fuzzy knowledge, two very expressive fuzzy *DLs* f_{KD}-*SI* and f_{KD}-*SHIN* were presented. Stoilos *et al.* (2005a) proposed the fuzzy *DL* f_{KD}-*SI* by extending the *DL* language *SI* with fuzzy set theory. The combination of transitive and inverse roles can capture knowledge about part-whole relationships and aggregated objects. Both these properties fit well into the framework of knowledge-based multimedia processing where both part-whole relationships, as well as, imprecise and vague knowledge appear in applications like multimedia information retrieval and processing. In more detail, Stoilos and Stamou (2009) investigated the reasoning problems for fuzzy *SI* (*f-SI*) under arbitrary continuous fuzzy operators, and proved decidability of the fuzzy *DLs*, such as f_L-*SI*, f_P-*SI*. Furthermore, Stoilos *et al.* (2005b) provided an extension of the above f_{KD}-*SI* to an even more expressive fuzzy *DL*, namely f_{KD}-*SHIN*, which is also the extension of fuzzy *ALC* with transitive role axioms (*S*), inverse roles (*I*), role hierarchies (*H*), and number restrictions (*N*). More importantly, Stoilos *et al.* (2007) showed the semantics as well as sound, complete and terminating reasoning algorithms for both of the extended languages f_{KD}-*SI* and f_{KD}-*SHIN*, where the authors also investigated the applicability of *blocking* strategies, like dynamic blocking and pair-wise blocking. Moreover, in Gu *et al.* (2007), a fuzzy *DL* called *ef-SHIN* was introduced to support reasoning of fuzzy ontology.

In particular, the *RBox* will be added into the f_{KD}-*SHIN KB*, that is, an f_{KD}-*SHIN KB* consists of a *TBox*, an *ABox*, and an *RBox*, where the TBox and ABox have been introduced in Section 3.1.1. An f_{KD}-*SHIN* RBox is a finite set of fuzzy transitive role axioms of the form *Trans(R)* and fuzzy role inclusion axioms of the form $R \subseteq S$, where R, S are f_{KD}-*SHIN*-roles. A fuzzy interpretation *FI* satisfies *Trans(R)* if $\forall x, z \in \Delta^{FI}$, $R^{FI}(x, z) \geqslant sup_{y \in \Delta^{FI}} \{\min(R^{FI}(x, y), R^{FI}(y, z))\}$, while it satisfies $R \subseteq S$ if $\forall x, z \in \Delta^{FI}$, $R^{FI}(x, z) \leqslant S^{FI}(x, z)$. A fuzzy interpretation *FI* satisfies an f_{KD}-*SHIN* RBox iff it satisfies all fuzzy axioms in the RBox; in this case, we say that *FI* is a model of this RBox.

Moreover, a fuzzy extension of the corresponding *DL* of the ontology description language OWL DL, called *f-SHOIN(D)*, was developed by Straccia (2005a, 2006a), while no reasoning algorithms were given. Furthermore, in order to provide reasoning for fuzzy OWL web ontology language, in Stoilos *et al.* (2005c, 2010), the authors considered the *DL f-SHOIN* by discarding data types *D* constructor. The authors gave the syntax, semantics, and a tableau procedure for *f-SHOIN*, and also presented a reduction technique from fuzzy OWL to *f-SHOIN* to provide reasoning support for fuzzy OWL ontologies. In addition, Bobillo *et al.* (2006) presented a sound fuzzy extension of *SHOIN* including fuzzy nominals and fuzzy *GCIs*, and also presented a reasoning preserving procedure to reduce an *f-SHOIN KB* into a crisp one.

In addition, in Stoilos *et al.* (2006a), the syntax and semantics of fuzzy *SHOIQ* were presented and the properties of the semantics of transitivity, qualified cardinality restrictions, and reasoning capabilities were investigated. Furthermore, Stoilos and Stamou (2007) extended the current state-of-the-art on fuzzy extensions to Semantic Web languages by presenting the syntax and semantics of the fuzzy *SROIQ* as well as the abstract, XML syntax, and semantics of a fuzzy extension to OWL 1.1. Moreover, the authors also provided reasoning support for a fuzzy version of fuzzy *SROIQ* by extending well-known reduction techniques of fuzzy *DLs* to classical *DLs* for the additional axioms and constructors of fuzzy *SROIQ*.

3.2.3 Fuzzy Description Logics with fuzzy data type

In order that fuzzy *DLs* can represent and reason on the concrete knowledge in the real-world applications, several fuzzy *DLs* with *fuzzy concrete domains* (*D*) and *fuzzy data type group* (*G*) have been proposed as follows.

The Semantic Web is expected to process knowledge information and data information in an intelligent and automatic way. But recent research has shown that the OWL ontology language is very limited in representing fuzzy data information. To this end, Straccia (2005b, 2005c, 2005d) proposed a fuzzy *DL* called fuzzy *ALC(D)* (*ALC* extended with concrete domains), where the representation of concept membership functions and fuzzy modifiers are allowed, together with an inference procedure based on a mixture of a tableaux and bounded mixed integer programming.

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A reasoning algorithm for fuzzy *ALCF(D)* with *GCIs* and explicit membership functions was proposed by Bobillo and Straccia (2009b, 2009d), Bobillo and Straccia (2007), and Straccia and Bobillo (2007).

The more expressive *DL*, which can support fuzzy concrete domains was shown by Straccia (2005a), where the language is the fuzzy extension of *SHOIN(D)*, which is the corresponding *DL* of the ontology description language OWL DL. It shows that the representation and reasoning capabilities of fuzzy *SHOIN(D)* go clearly beyond classical *SHOIN(D)*. The following interesting features are considered in fuzzy *SHOIN(D)*: (i) concrete domains are fuzzy sets; (ii) fuzzy modifiers are allowed; and (iii) entailment and subsumption relationships may hold to some degree in the unit interval [0, 1]. A fuzzy concrete domain D is a pair $\langle \Delta_D, \phi_D \rangle$, where Δ_D is an interpretation domain and ϕ_D is the set of fuzzy domain predicates d with a predefined arity n and an interpretation d^D: ${\Delta_D}^n \rightarrow [0, 1]$. Furthermore, a fuzzy interpretation FI with respect to a fuzzy concrete domain D is a pair $FI = (\Delta^{FI}, {}^{FI})$ consisting of a non-empty set Δ^{FI} (called the domain), disjoint from Δ_D, and of a fuzzy interpretation function ^{FI} that assigns each concrete individual to an element in Δ_D; assigns each abstract individual to an element in Δ^{FI}; assigns each abstract concept C to a function C^{FI}: $\Delta^{FI} \rightarrow [0, 1]$; assigns each abstract role R to a function R^{FI}: $\Delta^{FI} \times \Delta^{FI} \rightarrow [0, 1]$; assigns each data type role T to a function T^{FI}: $\Delta^{FI} \times \Delta_D \rightarrow [0, 1]$; assigns each n-ary predicate p to the fuzzy relation p^D: ${\Delta_D}^n \rightarrow [0, 1]$, which means the relationship of data types $v_1, \ldots, v_n$ satisfies predicate p in a degree in [0, 1]. The detail reviews about fuzzy *SHOIN(D)* can be found in Lukasiewicz and Straccia (2008) and Straccia (2005a).

Moreover, in Bobillo *et al.* (2009), Bobillo and Straccia (2009c), and Bobillo (2008), the authors proposed the fuzzy extension of OWL 2 language, that is, fuzzy *SROIQ(D)*, and also presented a reasoning preserving procedure to obtain a crisp representation for the fuzzy *SROIQ(D)*. Based on the presentation of fuzzy *DLs* with fuzzy concrete domain, the fuzzy data information in the Semantic Web can be represented.

Furthermore, in order to support fuzzy customized data type information, a new kind of fuzzy *DL* called *F-ALC(G)* was proposed by Wang and Ma (2008), which can not only support the representation and reasoning of fuzzy concept knowledge, but also support fuzzy data information with customized fuzzy data types and customized fuzzy data type predicates.

3.2.4 The other fuzzy Description Logics with cut sets/fuzzy rough sets/fuzzy truth values

In the real-world applications, one wants to express using $\forall R_{0.9}.C_{0.6}$ the set of individuals that are related with degree 0.9 using role R with some individual that belongs to concept C with degree at least 0.6, that is, the fuzzy *DLs* allow for the cut sets of fuzzy concepts/roles.

To this end, by introducing the cut sets of fuzzy concepts and fuzzy roles as atomic concepts and atomic roles, and inheriting the concept and role constructors from *DLs*, Lu *et al.* (2006) and Li *et al.* (2005a) proposed a family of the so-called extended fuzzy *DLs* (*EFDLs*), which use α-cuts as atomic concept and roles. The definitions of syntax, semantics, reasoning tasks, and reasoning properties are given for *EFDLs*. In particular, *EFDLs* and *FALC* (Straccia, 1998) are fuzzy *DLs* with same concept constructors and different fuzzily methods, and the former supports more powerful presentation of fuzzy concepts. Furthermore, Lu *et al.* (2008a) presented sound and complete tableau algorithms for *EFDLs*, and proved the complexity is PSPACE-complete. Subsequently, based on the cut sets, several works also proposed different extension of *DL*s and reasoning algorithms for fuzzy *ALCH* (Kang *et al.*, 2005; Li *et al.*, 2005b), fuzzy *ALCN* (Li *et al.*, 2005c).

Furthermore, in order that the fuzzy *DLs* can support the expression of comparisons between fuzzy membership degrees, for example, it is a familiar description that '*John is taller than Tom*', which can be seemed as a comparison between two fuzzy membership degrees, it is necessary to extend fuzzy *DLs* with the ability of expressing comparison expressions. Toward this goal, in Lu *et al.* (2008b), the authors *EFDLs* with the fuzzy comparison cuts, and the extended language was called *FCDLs*. Also, the syntax, semantics, and reasoning algorithms for *FCDLs* were provided in detail.

Moreover, Bobillo and Straccia (2009a) and Jiang *et al.* (2009a, 2009b, 2009c) provided a simple solution to join fuzzy *DLs* and rough *DLs*, and studied how to combine fuzzy *DLs* with

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Table 3 The tractable fuzzy *DLs*

	Fuzzy *DLs*	Corresponding references	Mainly discussed issues
DL-Lite family	Fuzzy *DL-Lite*	Straccia (2006b)	Syntax, semantics, reasoning services, and query
	Fuzzy *DL-Lite*	Pan *et al.* (2007)	Syntax, semantics, reasoning services, query, a prototype implementation, and allow the users to specify threshold queries, which is not proposed in Straccia (2006b)
EL-family	Fuzzy *EL*	Vojtáš (2007, 2006) and Gurský *et al.* (2008)	Syntax, semantics, reasoning, and query
	Fuzzy EL^+, fuzzy EL^{++}	Stoilos *et al.* (2008b) and Mailis *et al.* (2008)	Syntax, semantics, reason, and classification algorithms
n-ary fuzzy *DLs*	Fuzzy *DLR-Lite*, *FDLR*, $f\text{-}DLR\text{-}Lite_{F,\cap}$	Straccia and Visco (2007), Zhang *et al.* (2008) and Cheng *et al.* (2008b)	Syntax, semantics, reasoning services, and query

DLs = Description Logics.

fuzzy rough sets, that is, fuzzy rough *DLs*. In particular, Bobillo and Straccia (2009a) presented a very expressive fuzzy rough extension of the *DL SROIQ(D)*, the logic behind the language OWL 2. In addition, Bobillo and Straccia (2009e) allowed fuzzy *DL* sentences to be qualified with fuzzy truth values, and, thus, allow expressions such as '*Tina is young is very true*' and '*Tina is young is almost true*'. The syntax, semantics, and reasoning algorithms for the extended languages were provided.

According to the statements above, some important existing fuzzy *DLs* can be summarized in Table 2. Notice that, however, it does not mean that Table 2 covers all publications in the research area and gives complete descriptions.

3.3 The tractable fuzzy Description Logics

In the area of *DLs*, another family of *DLs* cannot be ignored, that is, the *tractable DLs*, which are rich enough to capture significant ontology languages but keeping low complexity of reasoning. Some existing tractable fuzzy *DLs* can be summarized in Table 3.

The first kind of tractable *DLs* is *DL-Lite family*, which has the low complexity of reasoning, has received much attention in recent years. The detailed introduction about the *DL-Lite* family can be found in Calvanese *et al.* (2007). In order that the *DLs* of the *DL-Lite* family can handle the imprecise and uncertain information, fuzzy extensions to these *DLs* have started to receive attention. A fuzzy extension of *DL-Lite* called fuzzy *DL-Lite* was proposed by Straccia (2006b), where the syntax and semantics of fuzzy *DL-Lite* were defined, and the author also showed how to compute efficiently the top-k answers of a complex query over a huge set of instances in fuzzy *DL-Lite*. Furthermore, based on Straccia (2006b), Pan *et al.* (2007) proposed two new expressive query languages accompanied with query answering algorithms over fuzzy *DL-Lite*, and also developed a prototype implementation for querying fuzzy *DL-Lite* ontologies.

Moreover, as mentioned in Section 2.1, *DLs* that allow for intersection of concepts and existential quantification (but not value restriction) are collected in the *EL*-family, which can also be considered as the tractable *DLs*. Some fuzzy extensions to the *EL*-family languages are presented; Vojtáš (2006, 2007) presented a fuzzy extension of *EL* called fuzzy *EL*, and also reduced fuzzy *EL* to crisp *EL* for providing reasoning for the fuzzy *EL*. Gurský *et al.* (2008) also presented the model-theoretic semantics for fuzzy *EL*, which is the motivation of creating a model for

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fuzzy RDF. Furthermore, the fuzzy extensions of EL^+ and EL^{++} were investigated by Stoilos *et al.* (2008b) and Mailis *et al.* (2008), respectively. In Mailis *et al.* (2008), the authors provided the syntax and semantics together with a reasoning algorithm for the fuzzy concept subsumption problem of fuzzy EL^{++}.

In addition, in order to model relationships among more than two objects in some real world situations, various extensions of *DLs* with relations of arbitrary arity (such as *DLR*, DLR_{ifd}, and so on) have been proposed by Calvanese *et al.* (1997, 1998, 1999). On this basis, a fuzzy extension of *DL DLR-Lite* called *f-DLR-Lite* was introduced by Straccia and Visco (2007), where an ontology-mediated multimedia information retrieval system based on *f-DLR-Lite* was developed. A fuzzy extension of *DLR* called *FDLR* was proposed by Zhang *et al.* (2008) for representing and reasoning on fuzzy ER models, while reasoning algorithms for *FDLR* was missed. Moreover, a fuzzy *DL* called $f\text{-}DLR\text{-}Lite_{F,\cap}$ was proposed by Cheng *et al.* (2008b), where the fuzzy *DL* allows for the presence of *n*-ary relations and the occurrence of concept conjunction on the left land of inclusion axioms.

4 Reasoning and querying of fuzzy Description Logics

Reasoning and *querying* are the most important research lines in the area of *DLs*. The knowledge representation systems based on *DLs* provide their users with various reasoning capabilities and querying services. The former can deduce implicit knowledge from the explicitly represented knowledge, such as computing subsumption and checking satisfiability; and the latter can access and query the *KBs*, such as instance retrieval, realization, instantiation, and conjunctive query answering. In fact, both of them are not independent as mentioned by Calvanese *et al.* (2007). In the following, we will introduce the reasoning and querying of fuzzy *DLs*, respectively.

4.1 Reasoning on fuzzy Description Logics

One of the most important advantages of *DLs* is their reasoning ability. Therefore, for a fuzzy *DL*, how to *give its reasoning algorithm* and *implement its corresponding reasoner* have become the important tasks for researchers. The following will introduce the existing fuzzy *DL* reasoning techniques and fuzzy *DL* reasoners in detail.

4.1.1 Reasoning techniques for fuzzy Description Logics

So far, kinds of reasoning techniques for different fuzzy *DLs* have been proposed. Moreover, the reasoning complexity of some fuzzy *DL*s was investigated by Bonatti and Tettamanzi (2005) and the optimization techniques for fuzzy *DLs* were discussed by Simou *et al.* (2010). In general, the reasoning techniques can be summarized in three categories: *tableau-based reasoning technique*, *reasoning technique for GCIs*, and *reduction to crisp DLs*.

Besides, Keller and Heymans (2008) introduced a new class of inference procedures into fuzzy *DL*s reasoning, that is, the *fixpoint-based* decision procedure for deciding knowledge-based satisfiability in the fuzzy *DL FALC*.

4.1.1.1 The tableau-based reasoning technique. Most of the existing reasoning algorithms for fuzzy *DLs* (e.g., Straccia, 2001, 2005b; Li *et al.*, 2005c, 2006a; Sánchez & Tettamanzi, 2006; Stoilos *et al.*, 2008a, 2005a, 2007; Wang & Ma, 2008) are *tableau-based algorithms* as mentioned in Section 3, which have turned out to be very useful comparing with the early *structural subsumption algorithms* (Baader *et al.*, 2003). Here, we will not introduce the detailed procedures of tableau-based algorithms, please refer to Baader *et al.* (2003), Straccia (2001), Stoilos *et al.* (2007), *etc.*

Moreover, most of the tableau-based algorithms are given with respect to the *simple Tbox* (Stoilos *et al.*, 2007). A Tbox T is called *simple* if it neither includes cyclic nor *GCIs*, that is, axioms are the form $A \subseteq C$ or $A \equiv C$, where A is a concept name that is never defined by itself either directly or indirectly, and A appears at most once at the left hand side. Reasoning on fuzzy *DL*s

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with a *simple Tbox* can be transformed into reasoning on fuzzy *DL*s with an *empty Tbox* by a transformation called unfolding, or expansion (Nebel, 1990): Concept inclusion introductions $A \subseteq C$ can be replaced by concept equivalence introductions $A \equiv A' \cap C$, where A' is a new concept name, which stands for the qualities that distinguish the elements of A from the other elements of C. Subsequently, if C is a complex concept expression, which is defined in terms of concept names, defined in the Tbox, we replace their definitions in C. The detailed introduction about the expansion technique can be found in the works of Baader *et al.* (2003), Straccia (2001), and Stoilos *et al.* (2007).

4.1.1.2 The reasoning technique for General Concept Inclusions in fuzzy Description Logics. The *GCIs* (i.e., the Tbox in a fuzzy *DL KB* is a set of *GCI* axioms of the form $C \subseteq D$, where C and D are arbitrary concepts) are an important feature of *DLs*, for example, *GCIs* are necessary to represent domain and range constraints. The procedures to deal with *GCIs*, in the context of fuzzy *DLs*, have been recently developed. Some discussions about how to reason on *f-ALCH* with *GCIs* and how to provide a tableau for *FALC* with *GCIs* were given by Straccia (2004a) and Stoilos *et al.* (2006b), respectively. In particular, Stoilos *et al.* (2006b) pointed out that a major theoretical and computational limitation so far is the inability to deal with *GCIs*, and the authors also addressed this issue and developed a calculus for fuzzy *DLs* with *GCIs*. Moreover, the other approaches for reasoning with respect to simple and acyclic Tboxes were considered, for example, Li *et al.* (2006a) extended the fuzzy tableau of *f-SHI* (Stoilos *et al.*, 2005b) with an additional rule to handle with general and cyclic Tboxes in the language *f-SHI*. Interestingly, the technique used by Stoilos *et al.* (2006b) is different from that presented by Li *et al.* (2006a).

4.1.1.3 The reasoning technique of reducing to crisp Description Logics. To reason on fuzzy *DLs*, several approaches for reducing fuzzy *DLs* to classical *DLs* have been developed, so that reasoning on a fuzzy *DL KB* can be performed by using existing *DL* systems.

The first effort in this direction was presented by Straccia (2004a), where the authors presented a technique for reasoning on an *f-ALCH KB* by reduced *f-ALCH* to crisp *ALCH*. Li *et al.* (2005c, 2006b) also provided reasoning for fuzzy *DLs f-ALCN* and *f-ALCQ* using the idea of the reduction. Moreover, Bobillo *et al.* (2006) proposed a technique to be able to reduce an *f-SHOIN KB* to a crisp *SHOIN KB*. Vojtáš (2007) also used the idea of the reduction to reduce fuzzy *EL* to crisp *EL* in order to provide reasoning for fuzzy *EL*. In addition, Bobillo *et al.* (2007a) and Stoilos and Stamou (2007) provided reasoning support for fuzzy *SROIQ* by extending well-known reduction techniques of fuzzy *DLs* to classical *DLs*. Overall, by reducing fuzzy *DLs* to crisp *DLs*, reasoning in a fuzzy *DL KB* can be performed by using existing and optimized *DL* systems.

4.1.2 Fuzzy Description Logic reasoners

DLs serve as the theoretical counterpart of the Semantic Web and provide reasoning supports for it, and the *DL* reasoners are the basic supporting bodies for the Semantic Web coming into use. Therefore, in order to implement the automatic reasoning of fuzzy *DLs*, kinds of reasoners based on different fuzzy *DLs* have been put forward as will be introduced in the following.

In recent years, a series of reasoners, such as FaCT, RACER, and Pellet (Baader *et al.*, 2003), have been developed based on the classic *DLs*. These reasoners are very efficient and soundness for ontology reasoning. Pellet, for example, is an OWL-DL reasoner with good performance, extensive middleware, and a number of unique features. Also there are some other reasoners such as Jess, Jena, KOAN2, Cerebra, cwm, Euler, surnia, SWI, E-wallet, MSPASS (OWL Implementations, 2001), which are based on classic *DLs* and toward OWL. Most of classical reasoners mentioned above have been updated for several versions and have good reliability, robustness, and running efficiency. Some versions of them have been commercialized.

However, these classical reasoners cannot support the representation and reasoning of *fuzzy ontologies* and *fuzzy DLs*. With the extensive and in-depth application of intelligent systems, the processing of fuzzy, and uncertain information in the real world is becoming an essential part of

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intelligent systems (Straccia, 2001; Stoilos *et al.*, 2006c, 2007; Cimianoet *et al.*, 2008; Lukasiewicz & Straccia, 2008). It is imperative to develop fuzzy reasoners, which can process fuzzy ontologies and fuzzy *DLs*.

Table 4 lists the existing fuzzy reasoners that can support the processing of fuzzy and uncertain information. In Table 4, PDLP (Li *et al.*, 2006c) is a reasoner based on the probability *DL* for dealing with uncertain information, while the rest are used to deal with fuzzy information based on fuzzy *DLs*.

In summary, for the implementation of fuzzy reasoners, there are two strategies: one is to translate the fuzzy *DL* into the classic *DL* and then call the classic *DL* reasoner for reasoning, for example, DeLorean in Table 4 and the reasoning technology mentioned by Straccia (2004a). Another strategy is to directly implement fuzzy reasoners based on the tableau algorithm of the corresponding fuzzy *DL*, for example, the reasoners in Table 4 except for DeLorean and Soft-Facts. The latter strategy has the advantage that the optimized technology can be adopted according to the specific fuzzy *DL*, and their efficiency in the implementation can be enhanced.

Through comparison of the reasoners in Table 4, it can be found that these reasoners can support the particular fuzzy *DL*. The existing reasoners in Table 4 can support the representation and reasoning of fuzzy concept knowledge, fuzzy concrete domain, or fuzzy customized data type and predicates. The relationships of the reasoning capacity among parts of these fuzzy reasoners are shown in Figure 1. In Figure 1, **U** means human knowledge. The dotted circle, solid circle, and dotted and solid circles in Figure 1 denote FRESG reasoner and two fuzzy reasoners 1 and 2, respectively.

In Figure 1, **C** stands for *f-ALC* that all fuzzy reasoners can support; **E** stands for the *DLs* that support fuzzy concrete domain *D*, such as FuzzyDL in Table 4; **F** stands for the *DLs* that support fuzzy customized data type and fuzzy customized data type predicates *G*, such as FRESG in Table 4; **B** stands for the remaining part with the same reasoning abilities of reasoner 1 and reasoner 2 in addition to *f-ALC*; **A** and **D** stands for the unique part of reasoner 1 and 2 in reasoning capacity, respectively.

4.2 Querying over fuzzy Description Logics

With the emergence of a good number of large-scale domain ontologies or *KBs*, it is of particular importance to provide users with querying services. Therefore, *querying over DLs* from *DL-Lite* family to more expressive *DLs*, such as *SHOQ* and *SHIQ*, are extensively investigated (Cheng *et al.*, 2009b).

In the real world, however, there exists a great deal of uncertainty and imprecision. Thus, how to *query over fuzzy DLs* has started to receive much attention. A major problem fuzzy *DLs* have to face with is that, unlike classical *DLs*, where an answer to a query is a set of tuples ranked according to the degree they satisfy the query. This poses a new challenge in case the researchers have to deal with a huge amount of instances. The research on querying over fuzzy *DLs* can be summarized in Table 5.

To query over fuzzy *DLs*, some work focusing on *lightweight fuzzy DL languages* has been done. Straccia (2006b) first presented a fuzzy *DL* called fuzzy *DL-Lite* by extending *DL-Lite* with fuzzy set theory, and investigated how to compute efficiently the top-k answers of a complex query (i.e., conjunctive queries) over a huge set of instances in fuzzy *DL-Lite*. However, conjunctive query answering in fuzzy *DL-Lite* is quite similar to query answering in crisp *DL-Lite*. Furthermore, following Straccia's work (Straccia, 2006b), Pan *et al.* (2007, 2008) proposed two new expressive query languages accompanied with query answering algorithms over fuzzy *DL-Lite*, where the authors allowed the users to specify threshold queries, which are not presented by Straccia (2006b), and also presented a prototype implementation for querying fuzzy *DL-Lite* ontologies. In Cheng *et al.* (2008b), a conjunctive query algorithm over *f-DLR-Lite*$_{F,\cap}$ *KB* was proposed, and the authors showed that the query answering algorithm over the extended *DL* is still FOL reducible and shows polynomial data complexity.

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Table 4 The comparisons of the current fuzzy and probabilistic *DL* reasoners

		Features						
			Reasoning technology		Data type			
Fuzzy reasoners	Supporting *DLs*	'Standard' inference services	Tableau-based algorithm	Reducing to crisp *DLs*	*D*	*G*	DIG	Unique features
FiRE (Stoilos *et al*., 2006c)	f_{KD}-*SHIN*	√	√					Supporting graphical interface GUI
GURDL (Haarslev *et al*., 2007)	*f-ALC*	√	√					Proposing some techniques of optimization
DeLorean (Bobillo *et al*., 2007b)	*f-SHOIN*	√		√	√		√	Reducing *f-SHOIN* to *SHOIN* to solve
GERDS (Habiballa, 2007)	*f-ALC*	√	√					Adding role negation, top role, and bottom role
YADLR (Stasinos & Georgios, 2007)	*SLG algorithm*	√	√					Allowing to handle unknown degrees of truth in the fuzzy assertions of the knowledge base
FuzzyDL (Bobillo & Straccia, 2008b)	*f-SHIF*(*D*)	√	√		√			Supporting that the degree of a fuzzy assertion is a constant or a variable
PDLP (Li *et al*., 2006c)	*PDLP*	√	√					A probability *DL* reasoner, supporting to handle uncertain information
SoftFacts (Straccia, 2009c)	*SoftFacts*	√						An ontology-mediated top-k retrieval system
FRESG (Wang *et al*., 2009a)	*f-ALC(G)*	√	√			√		Supporting fuzzy data information

DLs = Description Logics.

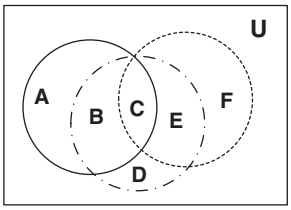


Figure 1 The relationships among existing fuzzy Description Logic reasoners

Table 5 Querying over fuzzy *DLs*

	Fuzzy *DLs*	Corresponding references	Mainly discussed issues
Querying over *lightweight* fuzzy *DLs*	Fuzzy *DL-Lite*	Straccia (2006b)	A reasoning algorithm to answer query in fuzzy *DL-Lite*
	Fuzzy *DL-Lite*	Pan *et al.* (2007, 2008)	Based on Straccia's work (Straccia 2006b), further investigate querying over fuzzy *DL-Lite*
	f-DLR-Lite$_{F,\cap}$	Cheng *et al.* (2008b)	A conjunctive query algorithm over *f-DLR-Lite*$_{F,\cap}$ knowledge base
Querying over *expressive* fuzzy *DLs*	Fuzzy *ALC*	Cheng *et al.* (2008a)	An algorithm for answering fuzzy conjunctive queries over fuzzy *ALC*, which allows the occurrence of both lower bound and the upper bound of threshold in a query atom
	Fuzzy *ALCN*	Cheng *et al.* (2009a)	An algorithm for answering expressive fuzzy conjunctive queries over fuzzy *ALCN*
	Fuzzy *ALCNR*	Mailis *et al.* (2007)	An algorithm for answering conjunctive queries over fuzzy *ALCNR*
	Fuzzy *SHIN*	Cheng *et al.* (2009b)	A tableau-based algorithm for deciding query entailment of fuzzy conjunctive queries w.r.t. fuzzy *SHIN* ontologies

DLs = Description Logics.

Moreover, *querying in other expressive fuzzy DLs* has been carried out to provide users with expressive querying services. Mailis *et al.* (2007) described fuzzy *CARIN*, a knowledge representation language combining fuzzy *DLs* with Horn rules, and also proposed a fuzzy version of existential entailment algorithm for answering conjunctive queries in fuzzy *ALCNR*. However, it allows only *positive role atoms* in a query, while the *negative atoms* are not touched on. On this basis, Cheng *et al.* (2008a, 2009a) presented the algorithms for answering expressive and fuzzy conjunctive queries, allowing in a query both *positive atoms* and *negative atoms*, over the relative expressive fuzzy *DLs*, namely *f-ALC* and *f-ALCN*. More recently, Cheng *et al.* (2009b) developed a tableau-based algorithm for deciding query entailment over *f-SHIN KBs*, where the query also allowed the occurrence of both lower bound and the upper bound of threshold in a query atom, and the authors proved that the algorithm for query entailment is co3NExpTime in the size of the *KB* and the query.

5 Applications

Like the classical *DLs* (Baader *et al.*, 2003), the fuzzy *DLs* have also been extensively applied to many application domains. In general, the applications about fuzzy *DLs* can be summarized in

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two categories: integrating fuzzy *DLs* and other techniques (such as ontology language OWL and fuzzy Logic Programs (fuzzy *LPs*)) to well represent and reason on fuzzy information (i.e., 1 and 2); applying fuzzy *DLs* to some particular domains such as information retrieval and image analysis (i.e., 3). In detail, the applications about fuzzy *DLs* including:

1. One of the most important applications of fuzzy *DLs* is the *fuzzy extensions of the Semantic Web languages*. Since *DLs* are the logical foundation of the Semantic Web and play a crucial role in the Semantic Web, recently there have been quite a few works on *DL*-based fuzzy ontology languages such as fuzzy OWL (Stoilos *et al.*, 2005c) and fuzzy SWRL (Pan *et al.*, 2006a).
2. To satisfy the requirement of the layered architecture of the Semantic Web (i.e., integrating the Rules and the Ontology layer), currently several researches have been concentrated on combining fuzzy *DLs* and fuzzy *LPs*, that is, *fuzzy DLPs*.
3. The fuzzy *DLs* have also been applied to *some particular application domains* such as information retrieval, medicine, and image analysis.

The applications of fuzzy *DLs* can be summarized in Table 6.

5.1 Fuzzy extensions of the Semantic Web languages

As mentioned in Section 1, *DLs* and ontologies play a key role in the Semantic Web, especially *DLs* that are the logic foundation of the Semantic Web. In order to deal with the fuzzy information, there have been kinds of fuzzy *DLs* as introduced in the previous sections. However, the Semantic Web languages such as OWL cannot represent the imprecise and uncertain information. Therefore, recently there have been quite a few works on *DL-based fuzzy ontology languages* as shown in Table 6, which have been proposed as formalisms capable of capturing and reasoning about imprecise and vague knowledge.

Extensions of OWL: Several works have been carried out toward combining fuzzy logic and the Semantic Web ontology. In order that the Web ontology languages can capture, represent, and reason on imprecise information directly, based on the fuzzy *DL f-SHOIN*, the OWL web ontology language was extended with the fuzzy set theory in Stoilos *et al.* (2005c, 2010), which is called *f-OWL*. The authors provided the syntax and semantics of *f-OWL*, as well as a reduction technique from *f-OWL* to *f-SHOIN* for providing reasoning support for *f-OWL* ontologies. Moreover, based on a fuzzy extension to OWL called *Fuzzy OWL*, Stoilos *et al.* (2006c) developed a reasoning platform, Fuzzy Reasoning Engine (FiRE), which lets *Fuzzy OWL* capture and reason about imprecise and uncertain knowledge.

In addition, Gao and Liu (2005) investigated how to extend OWL by fuzzy *DL*, and the authors extended existing OWL language by encoding fuzzy constructors, axioms and constraints, and mapped semantics of these new vocabularies to fuzzy *DLs*. Moreover, the authors also provided the translation rules from OWL to fuzzy OWL. More recently, based on the fuzzy *DL f-SROIQ(D)*, Bobillo and Straccia (2009c) investigated the fuzzy extension of OWL 2, and proposed an OWL ontology to represent some important features of fuzzy OWL 2 statements.

In more detail, the connections among fuzzy ontology, fuzzy *DLs*, and fuzzy OWL were introduced by Calegari and Ciucci (2007). Overall, extending OWL with fuzzy set theory can directly represent fuzzy ontology and resolve some fuzzy entailment tasks by constraint propagation calculus of fuzzy *DLs*.

Extensions of Web rule languages: As an important representation of knowledge, rules become a main design issue of the Semantic Web. Some Semantic Web rule languages, such as SWRL (Horrocks *et al.*, 2004), which is a Horn clause rules extension to OWL that overcomes many limitations of OWL, come into being. However, in the Semantic Web, there is much imprecise and uncertain knowledge, which cannot be represented by these rule languages. To this end, based on OWL DL, that is, the *DL SHOIN(D)*, Pan *et al.* (2006a) first advocated a fuzzy SWRL extension based on fuzzy sets, called *f-SWRL*, where the general form of fuzzy rules, the syntax, and the

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Table 6 The applications of fuzzy *DLs*

Applications				
Fuzzy extensions of Semantic Web languages		Fuzzy *DLPs* (fuzzy *DLPs*)		Some particular applications
Fuzzy extensions of OWL	Fuzzy extensions of Web rule languages	Positive fuzzy *DLPs*	Fuzzy normal/ disjunctive *DLPs*	Information retrieval: Meghini *et al.* (2001), Simou *et al.* (2008a, 2008b), Straccia and Visco (2007), Straccia (2010), Singh *et al.* (2004) and Stoilos *et al.* (2005d)
Sanchez and Yamanoi (2006), Stoilos *et al.* (2005c, 2010, 2006c), Gao and Liu (2005), Calegari and Ciucci (2007) and Bobillo and Straccia (2009c)	*f-SWRL*: Pan *et al.* (2006a), Wang *et al.* (2008); *f-NSWRL*: Wang *et al.* (2009c); f-if-then: Agarwal and Hitzler (2005); f-SW-if-then-RL: Wang *et al.* (2009b)	Straccia (2006c, 2004b, 2006e), Venetis *et al.* (2007) and Zhao and Boley (2008)	Lukasiewicz (2006) and Lukasiewicz and Straccia (2007a, 2007b)	Medicine: Molitor and Tresp (2000), D'Aquin *et al.* (2006) and Schlobach *et al.* (2007) Ontology mpping: Nováček and Smrž (2006), Ferrara *et al.* (2008) and Xu *et al.* (2005) Electronic market: Ragone *et al.* (2008a, 2008b, 2008c, 2007) and Agarwal and Lamparter (2005) Data modeling: Zhang *et al.* (2008, 2009) Semantic Web portals: Zhang *et al.* (2005) Semantic search engines: Li *et al.* (2007, 2006d) Image analysis: Dasiopoulou *et al.* (2008, 2009) and Vitucci *et al.* (2010) Decision making: Straccia (2009a) R&D project selection: Liu *et al.* (2004)

DLs = Description Logics; *DLP* = Description Logic Program.

semantics were investigated. And some examples of fuzzy rules were provided to illustrate the features of *f-SWRL*. Based on Pan *et al.* (2006a), another fuzzy extension of SWRL was proposed by Wang *et al.* (2008).

Subsequently, other fuzzy extensions of rule languages are presented. Wang *et al.* (2009c) proposed a fuzzy rule language, called *f-NSWRL*, which makes up for the insufficiency of *f-SWRL* in the respect of representing non-monotonicity. Agarwal and Hitzler (2005) showed how fuzzy membership functions and fuzzy IF-THEN rules can be modeled by means of an appropriate *DL* and how this can be employed for query answering. Moreover, by combining if-then rules with fuzzy OWL DL (i.e., the *DL f-SHOIN(D)*) in the framework of fuzzy sets, a Semantic Web rule language, called *f-SW-if-then-RL*, was presented by Wang *et al.* (2009b), which employs fuzzy if-then rules to represent and reason on imprecise and uncertain knowledge abundant in the context of the Semantic Web. In addition, to provide features for representing rule-based languages capable of handling uncertainty and imprecision, a general extension of RuleML (Rule Markup Language) was introduced by Technical group charter, FuzzyRuleML.

5.2 *Fuzzy extensions of Description Logic Programs*

In the Semantic Web, both aspects of structured and rule-based representation of knowledge are becoming of interest, thus integrating the Rules and the Ontology layer has become a key requirement for the layered architecture of the Semantic Web. Toward this goal, *DLPs* come into being (Levy & Rousset, 1998; Grosof *et al.*, 2003; Eiter *et al.*, 2004; Pan *et al.*, 2004). *DLPs* combine *DLs* and Logic Programs (*LPs*), where *DLs* capture the meaning of the most popular features of structured representation of knowledge, while *LPs* are powerful rule-based representation languages.

However, *DLPs* cannot represent imprecise knowledge. Therefore, currently several researches have been concentrated on extending *DLPs* to fuzzy *DLPs* in order to represent uncertain, vague, or imprecise knowledge. In general, *the fuzzy DLPs* can be classified into *the positive fuzzy DLPs*, *fuzzy normal DLPs*, and *fuzzy disjunctive DLPs*.

Straccia (2006c, 2004b) first combines *DLs*, *LPs*, and the management of uncertainty into a uniform framework. The author extends the *DL* called *L-ALC* (*DLs* over lattices) with rules and defines its syntax, semantics, and reasoning problems, where the management of uncertainty is based on so-called annotation terms. However, the approach presented by Straccia (2006c, 2004b) requires that the so-called annotation terms (see Kifer & Subrahmanian, 1992) are grounded, which makes the approach hardly feasible in practice. Furthermore, Straccia (2006e) integrates the management of imprecision into a highly expressive family of representation languages, called positive fuzzy Description Logic Programs (*positive fuzzy DLPs*), which are a combination of fuzzy *DLs* with fuzzy *LPs*, where the author considers fuzzy *ALC(D)* as the *DL* component. Also, the syntax, semantics, and reasoning of fuzzy *DLPs* are defined.

Moreover, significant research efforts focusing on fuzzy *DLPs* are carried out by Lukasiewicz. Differently from the work on positive fuzzy *DLPs* by Straccia, Lukasiewicz (2006) first presents an approach to fuzzy *DLPs* with default negation in rule bodies, called *fuzzy normal DLPs*, where the author presents an approach to fuzzy *DLPs* by integrating fuzzy *DLs* (e.g., *f-SHIF(D)* and *f-SHOIN(D)*) with non-monotonic *LPs* under the answer set semantics. Along this line of research, Lukasiewicz and Straccia (2007a) present a novel approach to tightly integrated fuzzy *DLPs*, called *fuzzy disjunctive DLPs*, which are a tight integration of fuzzy disjunctive programs under the answer set semantics with fuzzy generalizations of *SHIF(D)* and *SHOIN(D)*, and also the syntax, semantics, and reasoning of fuzzy *DLPs* are defined. Furthermore, Lukasiewicz and Straccia (2007b) give a polynomial reduction for certain fuzzy *DLPs* to tightly integrated disjunctive *DLPs*. In addition, Lukasiewicz (2005a, 2005b) and Lukasiewicz and Straccia (2009) also present probabilistic fuzzy *DLPs* for the Semantic Web, which allow for handling both probabilistic uncertainty and fuzzy vagueness in a uniform framework.

In addition, Zhao and Boley (2008) present a fuzzy extension to *DLPs*, called *fhDLP*, which not only combines *DL* with *LP*, as in *DLP*, but also supports uncertainty representation. Venetis *et al.* (2007)

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also extends *DLPs* to fuzzy *DLPs* by providing the mappings between fuzzy *DLs* (e.g., *f-SHOIN*) and fuzzy def-Horn rules (the fragment of Horn rules), in order to provide a complete framework for representing fuzziness in the Semantic Web.

5.3 Some particular applications of fuzzy Description Logics

Based on the high expressive power and effective reasoning service, the fuzzy *DLs* and existing fuzzy *DL* reasoners have also been applied to some particular application domains as shown in Table 6.

As mentioned by Lukasiewicz and Straccia (2008), fuzzy *DLs* were first proposed for logic-based *information retrieval* (Meghini *et al*., 2001). Meghini *et al*. (2001) dealt with retrieval and showed how the degree of relevance of a document to a query may be seen in terms of the fuzzy *DL* that underlies both the representation and query languages. In Straccia and Visco (2007), an ontology-mediated multimedia information retrieval system was presented based on the fuzzy *DL f-DLR-Lite*, which can handle hundreds of thousands of images. Simou *et al*. (2008a) proposed a methodology for semantic indexing and retrieval of images based on the fuzzy *DL f-SHIN*. The idea has also been adopted by Stoilos *et al*. (2005d), Singh *et al*. (2004), and Simou *et al*. (2008b) for information representation and retrieval.

Moreover, fuzzy *DLs* have also been applied to *medicine* as introduced by Molitor and Tresp (2000), where the authors investigated how to extend *DLs* to vague knowledge in medicine. D'Aquin *et al*. (2006) provided a use case in the medical domain, where fuzzy concrete domains are used to identify tumor regions in X-ray images. Schlobach *et al*. (2007) proposed a *DL* called Rough Description Logic (*RDL*) by extending traditional *DLs* based on rough-set semantics, and showed that Rough *DL*-based reasoning can be done in a realistic use case and that modeling vague knowledge helps to answer important questions in the design of clinical trials.

In addition, some researchers pointed out fuzzy *DLs* would contribute to *ontology mapping* (Xu *et al*., 2005; Nováček & Smrž, 2006; Ferrara *et al*., 2008). Ferrara *et al*. (2008) discussed the application of the fuzzy *DLs* theories to the problem of ontology mapping validation as a different way of handling mapping uncertainty with respect to probabilistic approaches. In Xu *et al*. (2005), the fuzzy *DLs* were used to resolve the mapping problem of fuzzy concepts between fuzzy ontologies.

Also, some researchers have shown that fuzzy *DLs* are very useful in other applications. For example, Zhang *et al*. (2008, 2009) studied *fuzzy data modeling* with fuzzy *DLs*, where they established the relationships between fuzzy data models (e.g., fuzzy ER/UML model) and fuzzy *DL KBs* in order to realize the automatic reasoning of data models. Ragone *et al*. (2007, 2008a, 2008b, 2008c) proposed a novel fuzzy *DL*-based approach to automate matchmaking in *e-marketplaces*, and in Agarwal and Lamparter (2005), fuzzy *DLs* have also been used to improve searching and comparing products in electronic markets. Li *et al*. (2006d, 2007) proposed an improved semantic search model through integrating inference and information retrieval, where a type-2 fuzzy *DL* was described and employed in the *semantic search engines*. In Dasiopoulou *et al*. (2008, 2009), a fuzzy *DL*-based reasoning framework was proposed for the extraction of enhanced image descriptions based on an initial set of graded annotations, generated through generic *image analysis* techniques, and Vitucci *et al*. (2010) built an *image analysis* system, which includes two modules, the vision module, and the semantic module implemented with the fuzzy reasoner FiRE (Stoilos *et al*., 2006c). Besides, fuzzy *DLs* have also been applied to other areas such as *Semantic Web portals* (Zhang *et al*., 2005), *context representation* (Bobillo *et al*., 2008), *decision making* (Straccia, 2009a), and *R&D project selection* (Liu *et al*., 2004).

6 Discussions and conclusions

Based on the introduction in the previous sections, it is shown that lots of fuzzy *DLs* and related techniques and applications have been investigated in order to handle fuzzy information in real-world applications. In the last, we provide a brief summarization to well gain a general

https://doi.org/10.1017/S0269888912000306 Published online by Cambridge University Press

understanding of the field. In order for *DLs* to represent and reason on fuzzy information, Yen (1991) first combined fuzzy logic and *DLs*. From then on, the incorporation of imprecise and uncertain information in *DLs* has increasingly received attention. In particular, after a fuzzy extension of the basic *DL ALC*, that is, *FALC*, was proposed by Straccia (1998), kinds of fuzzy *DLs* extending *FALC* were developed, such as the extensions of *FALC* with hedges (Hölldobler *et al*., 2003, 2004, 2005), number restrictions *N* (Lu *et al*., 2005; Li *et al*., 2005c; Zhao *et al*., 2009), and qualified cardinality restrictions *Q* (Sánchez & Tettamanzi, 2004; Stoilos *et al*., 2008a). Furthermore, Stoilos *et al*. (2005b) provided an extension of the *FALC* with transitive role axioms (*S*), inverse roles (*I*), role hierarchies (*H*), and number restrictions (*N*), and proposed a fuzzy *DL* named *f-SHIN*, which has a high expressive power and decidable reasoning procedure. More recently, a family of fuzzy *DLs* called tractable fuzzy *DLs* such as fuzzy *DL-Lite* (Straccia, 2006b) and fuzzy *EL* (Vojtáš, 2007, 2006) were proposed, which are rich enough to capture significant ontology languages but keeping low complexity of reasoning. Moreover, for representing fuzzy data type information, some fuzzy *DLs* with fuzzy concrete domains *D* and fuzzy data type group *G* were presented, such as fuzzy *ALC(D)* (Straccia, 2005b, 2005c, 2005d), fuzzy *ALCF(D)* (Bobillo & Straccia, 2007, 2009d), fuzzy *SHOIN(D)* (Straccia, 2005a, 2006a), fuzzy *SROIQ(D)* (Bobillo, 2008; Bobillo *et al*., 2009; Bobillo & Straccia, 2009c), and fuzzy *ALC(G)* (Wang & Ma, 2008). In addition, with the occurrence of many fuzzy *DLs*, kinds of reasoning and querying techniques for different fuzzy *DLs* (e.g., tableau-based reasoning technique (Straccia, 2001; Stoilos *et al*., 2007), reasoning technique for *GCIs* (Straccia, 2004a; Stoilos *et al*., 2006b), reasoning technique of reducing to crisp *DLs* (Straccia, 2004a; Li *et al*., 2005c; Bobillo, 2008), and querying over fuzzy *DLs* in Table 5) and fuzzy *DL* reasoners in Table 4 have been developed. Also, fuzzy *DLs* have been extensively applied to many application domains as shown in Table 6. The detailed introduction about fuzzy *DLs* can be found from Sections 3–5 in this paper, and for most recent research issues about fuzzy logic and more generally soft computing, in the *DLs*, ontologies, and the Semantic Web, one can refer to Ma (2006), Sanchez (2006b) and Shadbolt *et al*. (2006).

In this paper, we focus our attention on the recent research achievements on fuzzy extension approaches of *DLs* based on fuzzy set theory, and we provide a comprehensive literature overview of them, which can contribute to investigating the fuzzy *DLs* and more importantly serving as identifying the direction of fuzzy *DLs* study. We survey the existing fuzzy *DLs* from the origin, development (from weaker to stronger in expressive power), some special techniques, and so on. The important issues on fuzzy *DLs*, such as reasoning, querying, and applications, are discussed in detail. Also, we make a comparison and analysis in our whole review.

After reviewing most of the proposals of fuzzy extensions, it has been widely approved that fuzzy *DLs* could play a key role in the Semantic Web by serving as a mathematical framework for fuzzy knowledge representation and reasoning in applications. However, still the full potential of fuzzy *DLs* has not been exhaustively explored, we found that fuzzy *DLs* have been studied relatively enough regarding the theoretical side, but it is a paradox in the balance between the expressive power of the *DL* formalism and its computational complexity. Therefore, how to handle the balance and the development of tools and systems that would provide a flexible and efficient way to build and manage fuzzy knowledge will be a hot research topic. Besides, the researches on fuzzy extensions of *DLs* are still in a developing stage and the following issues may be important in order for fuzzy *DL* technologies to be more widely adoptable in the Semantic Web and other application domains:

- *The more expressive fuzzy DLs* may be needed to satisfy the requirement of applications, for example, extending the *DLs SHIQ*(*G*) and *SHOQ*(*G*) presented by Pan (2007)and Pan and Horrocks (2006)toward supporting the fuzzy user-defined data type and fuzzy user-defined data type predicate; extending fuzzy *DLs* toward supporting fuzzy spatial relations as shown by Straccia (2009b); extending fuzzy *DLs* by introducing Bayesian rules (Zhang *et al*., 2007). In all cases, the underlying principle is to remain as close as possible to classical logic for preserving computational efficiency as much as possible (Dubois *et al*., 2006).

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- *The balance* between expressivity and computation complexity of fuzzy *DLs*. As mentioned by Brachman and Levesque (1984), using expressive *DLs* to model complex knowledge definitely brings about higher complexity, while reducing reasoning complexity will lead to the limitation of expressive power. After prompting a reasoning algorithm to an expressive *DL* language, although the decidability of the algorithm is investigated, *the computational complexity* is still an open problem. Furthermore, *the optimal technologies* can be investigated, which can reduce the computational complexity of the algorithm.
- *The implementation of more expressive fuzzy DL reasoners*. As shown in this paper, fuzzy *DLs* have been investigated relatively enough regarding the theoretical side. Another important side is the development of tools and systems that would provide a flexible and efficient way to build and manage fuzzy knowledge. Although there have been several fuzzy *DL* reasoners as introduced in Section 4.1.2, most of them cannot support fuzzy data information, DIG standard interface, and so on. Therefore, to satisfy the need of applications, some more expressive fuzzy *DL* reasoners should be implemented and applied to the real world, and also the performance of them should be improved.
- *The query over more expressive fuzzy DLs* and *the complexity analysis for them*. Currently, most of researches on querying over fuzzy *DLs* focus on lightweight fuzzy ontology languages. Therefore, querying over even more expressive fuzzy *DLs* should be carried out, for example, fuzzy *DLs* additionally extended with nominals and data type groups, and so on. Moreover, there exist a few fuzzy query engines that can be applied in real world applications, only CARIN system (Mailis *et al*., 2007) and ONTOSEARCH2 (Pan *et al*., 2006b, 2007), which is a query engine for both *DL-Lite* and fuzzy *DL-Lite*. Thus, more fuzzy query engines should be exploited.
- *The application problems of fuzzy DLs* will become the important research lines in *DLs*. As introduced in this paper, currently, there have been a lot of fuzzy extensions of *DLs*, while few applications of them, thus how to apply them to the real world application domains will attract increasing attention, for example, fuzzy *DLs* for data mining, database modeling, fuzzy controlling, and so on.
- *The fuzzy DLPs* will be an interesting topic for future research. This is extremely important as the representation of vague and imprecise information in the Semantic Web, and gains more and more attention by the research community. Although there have been several researches on fuzzy *DLPs*, integrating more expressive fuzzy *DLs* into fuzzy *DLPs* is needed. Moreover, few researches focus on reasoning on fuzzy *DLPs*, thus investigating about reasoning algorithms for fuzzy *DLPs* and analyzing the computational complexity may attract much attention. In addition, the implementation for fuzzy *DLPs* will be desired to apply in practice.
- To overcome the identified shortcomings of OWL, such as expressivity issues and deficiencies in the definition of OWL species, OWL 2, an extension to and revision of OWL that is currently being developed within the W3C OWL Working Group (Cuenca Grauet *et al*., 2008). Following the current trend of research, in order to deal with the imprecise and uncertain information, *fuzzy extensions to OWL 2* will attract much attention.

Acknowledgments

The authors wish to thank the anonymous referees for their valuable comments and suggestions. The work is supported by the *National Natural Science Foundation of China* (60873010, 61073139) and the *Fundamental Research Funds for the Central Universities* (N090504005 and N090604012), and in part by the *Program for New Century Excellent Talents in University* (NCET-05-0288).

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