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Let Them Eat Pie: Addressing Sample Selection in Multiparty Elections

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Abstract

Elections are central to the study of politics. When studying parties' vote shares across districts, scholars are encouraged to use compositional-outcome models in order to test their theories about what factors shape the dynamics of electoral support. Existing compositional modeling approaches incorrectly deal with scenarios in which not all parties compete in every electoral district. Because unobserved factors that affect a party's decisions to contest a district are likely correlated with its performance in districts where the party fielded candidates, failing to account for partial contestation is likely to result in sample selection bias. Addressing sample selection in a compositional setting is challenging because the outcomes are in log-ratio form, and thus the errors often deviate from normality. To deal with these issues, we introduce a novel maximum likelihood approach which accounts for this type of sample selection and demonstrate through simulations that our method outperforms commonly used solutions, including the conventional Heckman correction. We illustrate the utility of our approach by analyzing the 2017 and 2019 U.K. parliamentary elections in English constituencies.

Keywords: sample selection; multiparty elections; compositional outcomes; maximum likelihood estimate

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1. Introduction

Elections are central to the study of politics. Scholars often analyze the vote shares of competing parties across districts to test theories about what influences election outcomes. It is well established that results from multiparty elections are compositional data: vote shares range from 0 to 1, and one party's gains/losses are offset by changes in votes for other parties (Katz and King 1999; Tomz, Tucker, and Wittenberg 2002). Despite this, few studies use compositional strategies to test the impact of predictors on the electoral performance of political parties. The main reason for this is the widespread phenomenon of *partial contestation*: some parties do not field candidates in every district. Existing compositional approaches to studying elections ignore that parties are rational actors which, when constrained by limited resources, strategically choose where to compete. Partial contestation is thus an example of sample selection, and failure to account for it is likely to result in biased and inconsistent estimators (Heckman 1979).

Addressing sample selection in compositional models presents unique challenges because outcomes are expressed in log-ratio form, and thus errors are typically assumed to follow an extreme value type-I (Gumbel) distribution (Train 2009, 34). This violates the crucial identification assumption of the classic selection approach proposed by Heckman (1979) that the errors are normally distributed. Under these

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conditions, the Heckman estimator is inefficient. A second identification assumption in the Heckman strategy is the exclusion restriction: at least one predictor in the sample selection equation (also known as an instrument) does not affect the outcome stage. Absent a valid instrument, the Heckman strategy results in severely biased estimates if the distributional assumption about the error terms is not met (Cook, Lee, and Newberger 2021; Sartori 2003; Wolfolds and Siegel 2019).

We introduce a novel approach that uses maximum likelihood estimation (MLE) to account for compositional trade-offs between parties' vote shares and sample selection due to partial contestation. MLE offers flexibility in modeling the distributional assumptions for identification and allows us to jointly model the selection process in which parties strategically decide the districts where they will compete and the outcome stage with the vote shares of parties that fielded candidates in a district. Using Monte Carlo experiments designed to reflect multiparty elections with partial contestation, we evaluate our approach against existing strategies to model compositional outcomes. Our analysis focuses on two key selection scenarios: one with a valid exclusion restriction and another in which it is violated. Across both settings, our proposed compositional model with selection (CMS) consistently outperforms competing approaches. Compositional methods that ignore partial contestation produce severely biased estimates, while the Heckman correction is inefficient under a valid exclusion and is both biased and inefficient without a valid instrument.

In the next section, we describe the existing approaches to studying elections and discuss the problem of partial contestation. We then present our strategy to account for sample selection, followed by sections on Monte Carlo experiments and an illustration of the utility of our approach analyzing the 2017 and 2019 elections in English constituencies. In the conclusion, we discuss the importance of correctly modeling sample selection in compositional outcomes and possible applications of our approach to other substantive domains.

2. Compositional models and partial contestation

Vote shares are inherently compositional data. If v_{ij} represents party j's vote share ($j = 1,2\ldots,J$) in electoral district i ($i = 1,2\ldots,N$), it follows that

$$0 \leq v_{ij} \leq 1 \ \forall j \text{ and } \sum_{j=1}^{J} v_{ij} = 1. \tag{1}$$

To estimate models with vote shares as the outcome, v_{ij} terms can be expressed as $J-1$ log-ratios to unbind them from a 0–1 scale to the real number line (Aitchison 1982). The denominator of each log-ratio is the vote share of an arbitrarily chosen baseline party ($j = 1$) and each numerator is v_{ij} of one of the remaining $J-1$ parties[1]:

$$s_{ij} = \log\left(\frac{v_{ij}}{v_{i1}}\right) \ \forall \ j \neq 1. \tag{2}$$

Katz and King (1999) introduced an additive logistic-t model to the study of election outcomes defined as s_{ij}. Tomz, Tucker, and Wittenberg (TTW; 2002) proposed a simpler approach in which they estimated the impact of predictors $\mathbf{x}_{ij}$ on s_{ij} using seemingly unrelated regressions (SURs; Zellner 1962). Estimated coefficients from the SUR equations, $\hat{\boldsymbol{\beta}}_j$, can be used for inferential hypothesis tests and to calculate quantities of interest such as predicted vote shares, $\hat{v}_{ij}$. The predictions ($\hat{s}_{ij}$) are translated back into $\hat{v}_{ij}$ using a multivariate logistic transformation (Philips, Rutherford, and Whitten 2016):

$$\begin{aligned} \hat{v}_{ij} &= \frac{\exp(\mathbf{x}'_{ij}\hat{\boldsymbol{\beta}}_j)}{1+\sum_{j=2}^{J}\exp(\mathbf{x}'_{ij}\hat{\boldsymbol{\beta}}_j)} \ \forall \ j \neq 1, \\ \hat{v}_{i1} &= \frac{1}{1+\sum_{j=2}^{J}\exp(\mathbf{x}'_{ij}\hat{\boldsymbol{\beta}}_j)}. \end{aligned} \tag{3}$$

[1]Compositional outcomes can also incorporate abstention by calculating v_{ij} terms for the entire electorate and including an additional category j for abstention (Horiuchi and Kang 2018; Moreira 2025; Moreira, Goidel, and Armstrong 2025).

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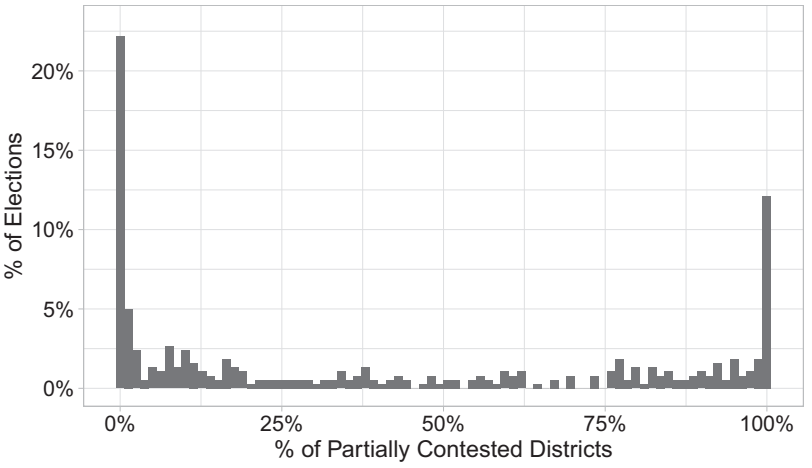


Figure 1. Contestation in 377 elections across 63 countries (1970–2019).
Note: Constituency-Level Elections Archive (CLEA).

A straightforward solution to modeling vote shares as compositional outcomes has thus been available for over two decades. Despite this, scholars seldom estimate compositional models to study elections (Kagalwala, Moreira, and Whitten 2024a). Even though most theories involve trade-offs across specific parties or ideological families of parties, the dominant approach is to analyze variation in the vote share of one party, typically the incumbent, against all other categories at the national or district level (e.g., Cavallaro, Pregliasco, and Vassallo 2018; Colantone and Stanig 2018; Duch and Stevenson 2005; Kayser and Peress 2012; Powell Jr. and Whitten 1993; Ward 2020). The most common explanation offered for why scholars choose not to model election results as compositions is the presence of partial contestation.

In Figure 1, we show that partial contestation is a common occurrence around the world.[2] In a sample of 377 national elections across 63 countries between 1970 and 2019, in only 80 elections (21.1%), all parties contested all districts. More than 50% of districts were partially contested in 40.1% of all cases (152 elections).[3]

Partial contestation creates two major problems. First, the categories of the compositional outcome vary across districts.[4] Second, partial contestation is an example of sample selection. To address the first issue, Katz and King recommended estimating the "effective vote": v_{ij} we would observe if all J parties had a candidate in all N districts (1999, 22). But as TTW noted, this insertion of a party into a race "awards that party more votes than it actually earned, at the expense of parties that truly competed" in that district (2002, 69). Instead, they proposed estimating separate SUR models for each pattern of contestation, a cumbersome strategy that can become unfeasible and leads to inconclusive analyses as the number of patterns of contestation increases and when there are few districts with a particular pattern of contestation. Kagalwala *et al.* (2024a) showed that researchers do not follow this approach; rather, when using compositional models to study elections, they employed some form of data modification to address partial contestation. The most common strategy used in political science is to limit the sample to fully contested districts.

[2]We created Figure 1 using data on all parties that received at least 5% of the national vote in elections with 50 or more districts. We merged candidates without a party into a single category.

[3]41 elections (10.8%) had all districts partially contested. This happens mostly in countries with regional parties.

[4]Related to this, researchers must choose how to treat parties that are not viable contenders—those that received a tiny percentage of overall votes in the country and did not win a legislative seat. In our empirical example, we classify those parties as a residual category and combine their votes into a j category to calculate s_{ij}s. This is similar to decisions made in the ParlGov project (Döring and Manow 2012).

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Other strategies to get around partial contestation also involve data modification. Pattie and Johnston (2009), for instance, replaced missing v_{ij} terms for parties that did not contest a district with a tiny value, 0.001 (0.1%), of the votes in the district:

$$s_{ij}^{*} = \begin{cases} \log\left(\frac{v_{ij}+0.001}{v_{i1}}\right) & \text{if } v_{ij} = 0 \\ s_{ij} & \text{if } v_{ij} > 0. \end{cases} \tag{4}$$

This small adjustment avoids zero values in Equation (2) that transforms v_{ij} into log ratios. The authors then use the SUR approach to estimate a single system of equations across all districts. However, this "tiny value" strategy "results in biased estimates, and the magnitude of the bias increases with the share of partially contested districts" (Kagalwala *et al.* 2024a, 7). This happens because the data adjustment in Equation (4) attenuates the relationship between a predictor of interest and s_{ij}. To address this, for each party that did not contest every district, Kagalwala *et al.* (2024a) included a dummy variable indicating whether party j contested district i. The dummy variable for j is then interacted with other predictors in j's log-ratio equation in the SUR. This "dummy variable" approach is equivalent to estimating an SUR model with only fully contested districts and avoids the pitfalls of the TTW strategy created by data limitations such as small samples when many constituencies are partially contested. However, this relatively simple strategy does not address the second problem that stems from partially contested elections: sample selection.

3. Accounting for sample selection in compositional data

Political parties do not randomly select the districts where they field candidates. Parties are collective entities seeking to maximize their chances of winning legislative seats while constrained by a finite amount of campaign resources (Boix 2009). Unobservable factors influencing their decision of where to compete are likely correlated with the party's expected performance. For example, a party's reputation and the quality of its candidate in a district should affect both its decision to contest the district and the vote share it receives when it does contest that district (e.g., Butler and Powell 2014; Cox and Katz 1996). This is a classic sample selection problem and failing to account for it results in biased parameter estimates (Heckman 1976, 1979). Heckman (1979) showed that such situations are analogous to an omitted variable problem and can be addressed using a two-stage estimator with selection and outcome equations.

In elections, the first stage models the likelihood that party j will contest district i, and we only observe v_{ij} if j chooses to contest i. This selection mechanism can be defined as

$$d_{ij}^{*} = \mathbf{z}_{ij}'\boldsymbol{\alpha}_j + \varepsilon_{ij}, \tag{5}$$

where d_{ij}^{*} is the latent utility of j contesting i, $\mathbf{z}_{ij}$ is a vector of predictors and ε_{ij} is the random error. Then, the selection stage we observe is

$$d_{ij} = \begin{cases} 1 & \text{if } d_{ij}^{*} \geq 0 \\ 0 & \text{if } d_{ij}^{*} < 0. \end{cases}$$

Using a single-outcome equation as a general case, the second stage consists of predictors that affect j's electoral performance in a given i. Let this outcome be

$$v_{ij} = \mathbf{x}_{ij}'\boldsymbol{\beta}_j + u_{ij}, \tag{6}$$

where v_{ij} is observed if $d_{ij} = 1$ and is otherwise missing. Heckman (1979) showed that failing to account for sample selection in Equation (6) leads to biased estimates ($\hat{\boldsymbol{\beta}}_j$). His two-step solution is:

1. Estimate Equation (5) with a probit model.
2. Estimate Equation (6) with OLS controlling for the inverse Mills ratio, $\frac{\phi(\mathbf{z}_{ij}'\hat{\alpha}_j)}{\Phi(\mathbf{z}_{ij}'\hat{\alpha}_j)}$, evaluated at the set of observables that influence the selection stage.

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This strategy, known as "Heckman correction," uses the inverse Mills ratio from the first stage to correct for sample selection in the second stage. This approach relies on two assumptions for identification: the error terms in both stages are bivariate normally distributed, and there is a valid exclusion restriction—that is, at least one explanatory variable in the selection equation is excluded from the outcome stage (Achen 1986; Cook *et al.* 2021; Sartori 2003; Wolfolds and Siegel 2019). Therefore, there must exist one variable in $\mathbf{z}_{ij}$ influencing d_{ij}^* that does not affect v_{ij}. When this exclusion restriction is violated, Heckman estimators are identified solely from distributional assumptions about the residuals, and the estimation procedure works "poorly in practice due to near collinearity" (Achen 1986, 99).[5]

The Heckman correction is inappropriate for compositional models derived from discrete choices (e.g., vote choices when parties partially contest districts) because they typically violate the assumption of normally distributed errors. Instead of a single outcome equation, the compositional nature of elections implies that the outcome stage is[6]

$$v_{ij} = \begin{cases} \frac{\exp(s_{ij})}{1+\sum_{j=2}^{J}(\exp(s_{ij}))} \ \forall j \neq 1 & \text{if } d_{ij} = 1 \\ \text{missing} & \text{if } d_{ij} = 0. \end{cases} \tag{7}$$

The log-transformation in Equation (2) imposes that the error terms in log-ratio equations are Gumbel distributed (Aitchison 1982; Train 2009).[7] The monotonic transformations of v_{ij} terms change the distribution of the random component (McFadden 1974, 111). This is similar to assumptions about error terms commonly made with multinomial logistic models: the stochastic components are independently and identically Gumbel distributed (Train 2009, 34).[8] Thus, accounting for sample selection by treating u_{ij} and ε_{ij} as jointly normal and using the inverse Mills ratio is no longer an adequate strategy. This will result in inefficiency and, without a valid instrument, in severe bias (Newey, Powell, and Walker 1990; Sartori 2003; Wolfolds and Siegel 2019).

To overcome this issue, we propose an MLE approach that accounts for sample selection in a compositional framework derived from a multinomial discrete choice process.[9] The conditional expectation of observing v_{ij} in Equation (7) is

$$\mathbb{E}[s_{ij}|\mathbf{x}'_{ij}, d_{ij} = 1] = \mathbf{x}'_{ij}\boldsymbol{\beta}_j + \mathbb{E}[u_{ij}|\varepsilon_{ij} \geq -\mathbf{z}'_{ij}\boldsymbol{\alpha}_j], \tag{8}$$

where $\mathbb{E}[u_{ij}|\varepsilon_{ij} \geq -\mathbf{z}'_{ij}\boldsymbol{\alpha}_j] \neq 0$.[10] If u_{ij} and ε_{ij} were jointly normal, then adding the inverse Mills ratio to the right-hand side of outcome equations could result in consistent parameter estimates (Heckman 1979). However, since compositional outcomes are transformed to log ratios, u_{ij} and ε_{ij} are assumed to

[5] To see this, note that if the same predictors in Equation (5) also affect the outcome in Equation (6), the second stage will be $v_{ij} = \mathbf{x}'_{ij}\boldsymbol{\beta}_j + \theta\frac{\phi(\mathbf{x}'_{ij}\hat{\alpha}_j)}{\Phi(\mathbf{x}'_{ij}\hat{\alpha}_j)} + u_{ij}$ (Sartori 2003, 115).

[6] For $j = 1$,

$$v_{i1} = \frac{1}{1+\sum_{j=2}^{J}(\exp(s_{ij}))}.$$

[7] The compositional v_{ij} is a multinomial transformation of the latent utilities of all voters in i, representing the average probability that a share of voters in i chooses j over all other parties (see Appendix A of the Supplementary Material).

[8] Our theory about the latent utility structure and vote-maximizing behavior informs our assumptions about the error distributions. We assume that latent vote shares follow a discrete process (multinomial). Assuming a Gumbel distribution for the errors is thus appropriate. However, assuming Gaussian distributions for the stochastic processes and using Heckman correction might be appropriate if the compositional data are not a function of a discrete-choice mechanism (e.g., cabinet portfolio and budgeting allocations). Our model allows for this type of flexibility.

[9] We are developing an R package to implement our CMS. In the meantime, practitioners can use the R function in our replication files following the instructions provided in Appendix E of the Supplementary Material.

[10] Appendix A of the Supplementary Material shows the mathematical derivation.

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follow a bivariate Gumbel distribution. Their marginal distributions are (Train 2009, 34)

$$f(w) = e^{-(w+\gamma+e^{-(w+\gamma)})}, \tag{9}$$

where γ is the Euler–Mascheroni constant. We can then estimate both stages simultaneously with the following likelihood function:

$$\mathcal{L}_j = \prod_{i=1}^{N} \mathbb{P}(\varepsilon_{ij} > -\mathbf{z}'_{ij}\boldsymbol{\alpha}_j)^{d_{ij}} f_{u|\varepsilon}(s_{ij} - \mathbf{x}'_{ij}\boldsymbol{\beta}_j | \varepsilon_{ij} > -\mathbf{z}'_{ij}\boldsymbol{\alpha}_j)^{d_{ij}} \mathbb{P}(\varepsilon_{ij} < -\mathbf{z}'_{ij}\boldsymbol{\alpha}_j)^{1-d_{ij}}, \tag{10}$$

where $\mathbb{P}(\varepsilon_{ij} > -\mathbf{z}'_{ij}\boldsymbol{\alpha}_j)^{d_{ij}}$ is the likelihood that j competes in district i, $f_{u|\varepsilon}(s_{ij} - \mathbf{x}'_{ij}\boldsymbol{\beta}_j | \varepsilon_{ij} > -\mathbf{z}'_{ij}\boldsymbol{\alpha}_j)^{d_{ij}}$ is the likelihood of observing v_{ji} given that j contested i and $\mathbb{P}(\varepsilon_{ij} < -\mathbf{z}'_{ij}\boldsymbol{\alpha}_j)^{1-d_{ij}}$ is the likelihood that j does not contest i. The log-likelihood is then[11]

$$\log(\mathcal{L}_j) = \sum_{i=1}^{N} d_{ij} \log\left(\int_{-\mathbf{z}'_{ij}\boldsymbol{\alpha}_j}^{\infty} f_{u\varepsilon}(s_{ij} - \mathbf{x}'_{ij}\boldsymbol{\beta}_j, \varepsilon_{ij}) \mathrm{d}\varepsilon_{ij} \right) + \sum_{i=1}^{N} (1 - d_{ij}) \log(F_\varepsilon(-\mathbf{z}'_{ij}\boldsymbol{\alpha}_j)), \tag{11}$$

where F_ε is ε_{ij}'s marginal CDF and $f_{u\varepsilon}$ is the joint density of u_{ij} and ε_{ij}. The CDF and PDF of the jointly Gumbel distributed u_{ij} and ϵ_{ij} are, respectively,

$$F(u, \varepsilon; m) = \exp\left[-\left(e^{-mu} + e^{-m\varepsilon}\right)^{\frac{1}{m}} \right] \tag{12}$$

$$f(u,\varepsilon;m) = f_{u\varepsilon} = F(u,\varepsilon;m) e^{-m(u+\varepsilon)} \left(e^{-mu} + e^{-m\varepsilon}\right)^{\frac{1}{m}-2} \cdot \left[m - 1 + \left(e^{-mu} + e^{-m\varepsilon}\right)^{\frac{1}{m}} \right], \tag{13}$$

where $m = \frac{1}{1-\rho} \in [1, \infty]$, ρ is the correlation between u_{ij} and ε_{ij}, and $m = 1$ when u_{ij} and ε_{ij} are independently distributed.[12] This approach (CMS) allows scholars to fully test hypotheses about trade-offs across parties while correcting for sample selection arising from strategic partial contestation.[13]

4. Simulations: Sample selection in compositional outcomes

To evaluate the efficacy of our proposed approach, we conduct a series of Monte Carlo experiments. We compare our CMS with the following four alternatives:

1. *Heckman correction*: Including an inverse Mills ratio for parties that partially contest electoral districts on the right-hand side of their respective log-ratio equations.

[11] Appendix B of the Supplementary Material shows the derivation of the log-likelihood.

[12] Although we assume that the errors are jointly Gumbel distributed, our MLE framework allows for other distributional assumptions based on substantive applications. In Appendix D of the Supplementary Material, we provide two examples: one where the DGP follows a jointly normal distribution and we assume extreme value type-I (D.5) and one in which we correctly assume a jointly normal distribution (D.6).

[13] Currently, our MLE approach cannot account for correlated errors across equations in the outcome stage. Greene (2017, 333) notes that efficiency gains from an approach that accounts for correlated errors across equations (SUR-GLS) compared to one that does not (OLS) depends on 1) the size of error correlations across equations and 2) the degree of similarity in the matrix of predictors across equations. The larger the error correlations, the greater the efficiency gains from SUR-GLS. The less similar the matrix of predictors, the greater the efficiency gains from SUR-GLS. In Appendix D.8 of the Supplementary Material, we assess how SUR-GLS compares to OLS. OLS is the closest comparison to CMS because it 1) is numerically equivalent to MLE and 2) does not account for error correlations. We find that the only efficiency trade-offs are for the estimates of parameters on predictors that differ across equations, and that these are relatively mild for the magnitude of error correlations found across equations in our empirical example. Figures S105–S107 in the Supplementary Material are particularly useful for seeing this, showing a difference of only 0.01% in mean squared error (MSE) for a +0.5 error correlation, the highest correlation we found in our empirical applications.

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2. *TTW*: Estimating an SUR model for only fully contested districts. Although TTW (2002) recommended estimating a separate SUR model for each pattern of contestation, restricting the sample to fully contested districts is the most common implementation of their strategy.
3. *Tiny value*: Adding a tiny value (0.01%) for parties that do *not* contest in a district and estimating SUR models.
4. *Tiny value with dummy variables*: Imputing a tiny value (0.01%) for parties that do not contest in a district *and* accounting for partial contestation by including dummy variables for parties that partially contest districts and interacting these with other predictors (Kagalwala *et al.* 2024a).

Our data-generating process (DGP) includes three parties $j \in \{A,B,C\}$ contesting elections in N districts ($i = 1,\ldots,N$).[14] B and C contest all N districts, and A fields candidates only in some districts. A's contestation is determined by the following equation:

$$d^*_{iA} = \alpha_{0A} + \alpha_{1A} z_{1i} + \alpha_{2A} z_{2i} + \varepsilon_{iA}, \tag{14}$$

where A contests i if $d^*_{iA} \geq 0$. We vary α_{0A} depending on partially contested districts such that $\alpha_{0A} = 0.57$, 0, or -0.53 when A contests 33%, 50%, or 66% of districts, respectively. $\alpha_{1A} = 0.8$ and $\alpha_{2A} = 0.2$. C is the baseline category in the compositional outcomes:

$$s_{ij} = \log\left(\frac{v_{ij}}{v_{iC}}\right) = \beta_{0j} + \beta_{1j} z_{1i} + \beta_{2j} z_{2i} + u_{ij}, \tag{15}$$

where $\beta_{0A} = 1$, $\beta_{1A} = 1.2$, $\beta_{0B} = 1$, $\beta_{1B} = -1$ and $\beta_{2A} = \beta_{2B} = 0$ for our initial set of experiments. Additionally, u_{iA} and ε_{iA} are jointly generated from mean-zero extreme value type-I distributions (McFadden 1974; Train 2009).[15] As per Heckman (1979), failing to account for the correlation between u_{iA} and ε_{iA} can bias the estimate for β_{1A}. z_{2i} influences sample selection, but is unrelated to the outcome, thus satisfying the exclusion restriction. In another scenario, we examine the consequences of violating the exclusion restriction by setting $\beta_{2A} = 1$ and estimating models without z_{2i} in the outcome stage.[16]

We vary the number of electoral districts, $N \in \{100, 200, 500, 800\}$. The correlation between ε_{iA} and u_{iA} (ρ) varies from 0.1 to 0.9. Finally, the probability that A will partially contest districts is set to 33%, 50%, or 66%, which means that A contests $\frac{2}{3}$, $\frac{1}{2}$, or $\frac{1}{3}$ of all districts in our simulations.[17] To assess how sample selection affects the performance of the relevant estimators, we present results for β_{1A}.[18]

4.1. Results: Valid exclusion restriction

Figure 2 shows results for bias across approaches when there is a selection process, z_{2i} is a valid instrument ($\beta_{2A} = 0$), and A does not contest 50% of the districts.[19] The four panels indicate different sample sizes, the horizontal axis is ρ ($\text{Cor}(\varepsilon_{iA}, u_{iA})$) and the vertical axis is the average bias. We find that the estimators for all three modeling strategies that ignore sample selection are severely biased: the TTW and dummy variable strategies systematically underestimate the effect and the magnitude of

[14] Appendix D.7 of the Supplementary Material shows results with six parties.

[15] We also investigate what happens when errors are jointly normal. While Heckman's correction is unbiased, the CMS returns slightly biased results (overestimates β_{1A} by 4%–8%), but performs much better than approaches that ignore sample selection (Appendix D.5 of the Supplementary Material). In the package we are developing, practitioners will be able to choose whether the errors are assumed to be jointly normally distributed (Appendix D.6 of the Supplementary Material) or to follow a bivariate Gumbel distribution.

[16] In this Monte Carlo experiment, we wrongly assume that z_{2i} is a valid instrument, and thus is wrongly omitted from the outcome equation. Since $\text{Cor}(z_{2i}, z_{1i}) = 0$, omitting z_{2i} in Equation (15) does not imply omitted variable bias.

[17] We also ran simulations with random coefficients, in which the effect of z_{1i} on the trade-off between parties B and C (β_{1B}) depends on whether A has a candidate in i: $\beta_{1B} = -1$ if $d_{iA} = 1$, otherwise $\beta_{1B} = 0.5$ (Appendix D.3 of the Supplementary Material).

[18] We show results for other parameters (α_{1A}, α_{2A} and β_{1B}) in Appendix D.4 of the Supplementary Material.

[19] Figures S20 and S30 in the Supplementary Material show bias when A does not contest 33% and 66% of districts.

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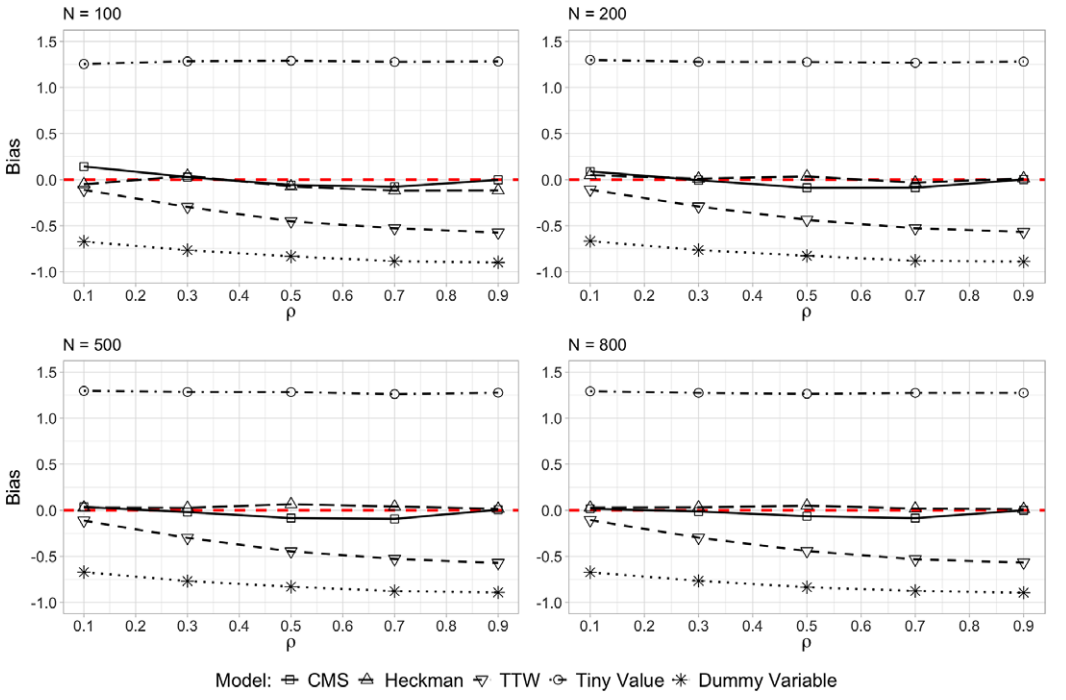


Figure 2. Bias in $\hat{\beta}_{1A}$—50% of districts partially contested.

this bias increases with ρ, while the tiny value strategy substantially overestimates the effect across all values of ρ. Both of the strategies that account for sample selection have minimal bias across the range of simulated circumstances.

Figure 3 presents the standard deviation of estimates from all five competing approaches. Across the board, the dummy variable and TTW strategies have the smallest standard deviation while the Heckman has the largest. The CMS and tiny value approaches perform much better than the Heckman and slightly worse than the other two. However, given the large amount of bias found for the TTW, dummy variable and tiny value strategies, the main comparison of interest in this figure is between the Heckman and CMS strategies.[20] Because ε_{iA} and u_{iA} are not normally distributed, the standard deviation of Heckman estimates is larger, especially in relatively small samples. This shows that the CMS is more efficient than the Heckman. In Appendix D of the Supplementary Material, we demonstrate that the Heckman strategy also results in overconfidence, higher MSE and lower statistical power than the CMS.[21] Our approach consistently outperforms the Heckman model.

4.2. Results: Invalid exclusion restriction

To assess how the competing approaches perform when the exclusion restriction is violated, we repeat the experiment from the previous section, but set $\beta_{2A} = 1$ such that z_{2i} is no longer a valid instrument in Equation (14). Figure 4 presents bias in the estimates of β_{1A} under this condition. The Heckman estimator is severely biased across all values of ρ, performing much worse than models that ignore

[20] Approaches that ignore selection perform much worse than the Heckman and CMS strategies on performance measures such as MSE that combine bias and efficiency. See results in Appendix D of the Supplementary Material.

[21] Following Hopkins *et al.* (2024), we compare the strategies across six performance statistics in Appendix D of the Supplementary Material: bias, MSE, standard deviation, overconfidence, coverage probability and statistical power.

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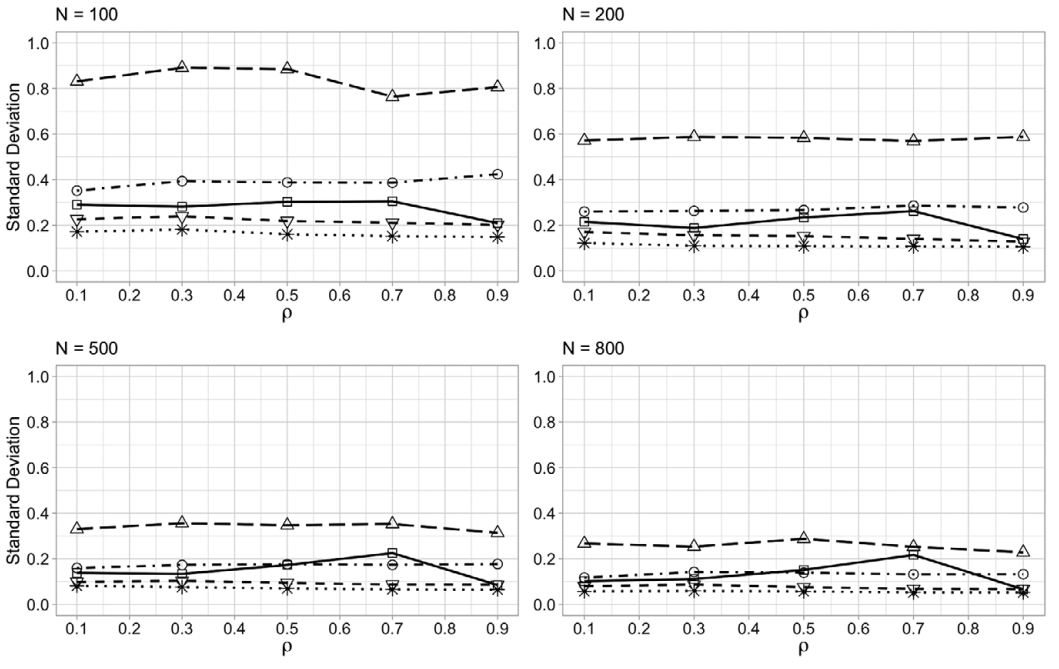


Figure 3. Standard deviation of $\hat{\beta}_{1A}$s—50% of districts partially contested.

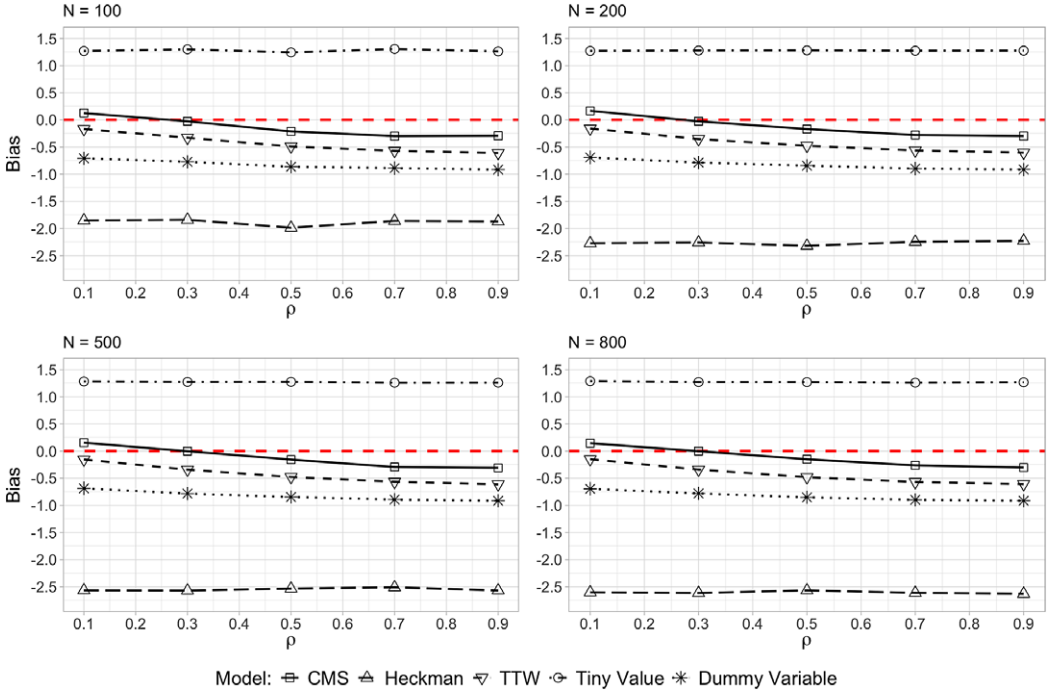


Figure 4. Bias in $\hat{\beta}_{1A}$—invalid instrument and 50% of partial contestation.

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sample selection; the magnitude of this bias worsens as the sample size increases. The Heckman estimator is biased because, without the exclusion restriction, the inverse Mills ratio is nearly collinear with other regressors, which makes the Heckman estimator potentially un-identifiable in practice (Honoré and Luojia 2024; Newey *et al.* 1990; Sartori 2003; Wolfolds and Siegel 2019).[22] In contrast, our CMS results in only slightly biased estimates when ρ is large, but it still outperforms alternatives that ignore sample selection.

In Appendix D of the Supplementary Material, we show that without a valid instrument, the Heckman approach also performs poorly in terms of efficiency, overconfidence, coverage and power, whereas the CMS performs well across all these performance statistics.[23] These results confirm that, under the distributional assumptions of error terms in compositional models derived from multinomial discrete choice processes, the Heckman correction is inappropriate if there is no valid instrument, a condition that is likely to happen in most empirical studies. Our CMS overcomes this limitation.

5. Application: Brexit and vote choice in England

We apply the CMS to analyze the English general elections in 2017 and 2019. Until the 2000s, elections in England yielded a suitable scenario to estimate compositional models as they revolved around two major parties (Labour and Conservatives) and a third force (Liberal Democrats, also known as Lib Dems), and all three parties fielded candidates in almost every district (Katz and King 1999; Tomz *et al.* 2002). This situation has drastically changed as new parties, particularly the Greens and the UK Independence Party (UKIP), gained traction and became more competitive in some districts.[24]

The 2017 and 2019 elections followed the Brexit referendum in 2016. National politics was reshaped by the division between "leavers" (i.e., supporters of a quick solution for the UK's departure from the European Union) and "remainers" (i.e., those who opposed Brexit). In both elections, the Conservatives were the party of the prime minister, held the majority of seats and were the main political force among the leavers.[25] During these elections, Brexit changed the competitive dynamics between parties (Fieldhouse *et al.* 2021). Rather than being viewed primarily as the incumbent party, the Conservatives were seen by most voters as the political force most able to implement Brexit. Hence, these two elections marked a transformation in long-term dynamics, such as economic voting and incumbency advantage, that had characterized English politics since the end of World War II (Kagalwala, Moreira, and Whitten 2024b). The leavers–remainers divide reduced the relevance of these previously stable voting patterns.

Partial contestation was high in 2017 and 2019 among small parties. Table 1 shows the frequency and the percentage of districts that parties did not contest.[26] The Greens did not have a candidate in about 15% of districts in both elections. UKIP contested 63% of districts in 2017, but only 48.75% in 2019. A residual category "Others"—comprised of parties that did not receive at least 1% of overall votes and did not win legislative seats—fielded candidates in 54.7% and 45% of districts in 2017 and 2019, respectively.[27]

[22] When the exclusion restriction is violated and the distributional error assumption is incorrect, our MLE approach is still identified, however, weakly, because it relies on the specification of error distribution. In Appendix D.9 of the Supplementary Material, we present results from a simulation where the errors are jointly normal, the exclusion restriction is violated and the estimated CMS assumes jointly Gumbel errors. We find that our CMS outperforms other approaches under these circumstances.

[23] See Figures S45–S47 in the Supplementary Material.

[24] UKIP and Brexit Party, the latter created in 2018, formed an electoral alliance in 2019, agreeing not to contest against one another in the same constituency. Hence, we treat them as one category in 2019 and refer to this alliance as UKIP.

[25] During the referendum, the Conservatives were neutral. After the "leave" victory, the party officially supported Brexit (Heath and Goodwin 2017; Prosser 2021).

[26] We exclude the constituency of the Speaker of the House of Commons, which is essentially uncontested. Because our models include a lagged dependent variable, we lose two districts in 2019 due to the Speaker changing shortly before the election. Hence, our data include 532 of 533 English districts in 2017 and 531 in 2019.

[27] As explained in Footnote 4, almost all elections have parties that are not viable competitors and thus cannot be included on their own in studies of results across districts.

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Table 1. Partial contestation by party and election

	2017		2019	
	Frequency	Percentage	Frequency	Percentage
Conservative	0	0%	0	0%
Labour	0	0%	0	0%
Liberal Democrats	2	0.37%	12	2.26%
Green	86	16.16%	78	14.68%
UKIP	197	37.03%	304	57.25%
Others	291	54.70%	239	45.01%
Constituencies	532		531	

Our CMS accounts for partial contestation as sample selection, allowing us to study how Brexit shaped trade-offs across all relevant parties. We use the Conservative Party as the baseline category of the log-ratios in Equation (2). Thus, the compositional outcome encompasses five log ratios, one for each $J-1$ category: the Labour Party, Liberal Democrats, Greens, UKIP and Others. For each of these parties, we estimate the following model specification:

$$\begin{aligned} d_{ijt} &= \alpha_{0j} + \boldsymbol{z}_{ijt}\boldsymbol{\alpha}_j + \varepsilon_{ijt}, \\ s_{ijt} &= \beta_{0j} + \phi_j s_{ijt-1} + \boldsymbol{x}_{ijt}\boldsymbol{\beta}_j + u_{ijt}, \end{aligned} \tag{16}$$

where the first equation models j's decision to contest district i in election t, and the second equation is the compositional outcome with $J-1$ log ratios. We estimate these equations jointly using the log-likelihood function in Equation (11).[28] In the selection stage, d_{ijt} is an indicator for whether j had a candidate in i, and $\boldsymbol{z}_{ijt}$ is a vector of the following district-level covariates:

- j's distance in vote share from the second place in the previous election ($t-1$),
- the difference in vote shares between the top two parties at $t-1$,
- the effective number of parties (ENP) at $t-1$,
- the percentage of votes for leaving the European Union from the 2016 referendum,[29]
- the percentage of residents claiming unemployment benefits in the district,
- the log of the electorate size,
- the share of the rural population,
- dummy variables that indicate Conservative and Labour Party incumbency,
- regional fixed effects,[30] and
- election fixed effects.

The first three variables in $\boldsymbol{z}_{ijt}$ are tactical factors that a party j may consider when deciding whether to contest a district. We expect that smaller parties are less likely to employ scarce resources in localities where a single party dominates the contest. This is captured by the distance between the top two parties at $t-1$. Similarly, j may avoid areas where its past electoral performances were poor and districts crowded with many contenders. We measure these factors with j's distance from the runner-up and the ENP, both measured at $t-1$. Because these first three variables are expected to affect parties' contestation

[28] Since the Labour and Conservative Parties contested all districts (Table 1), we do not estimate a selection stage in the trade-off between them.

[29] Data from the House of Commons Library. In total, there are observed results for 169 constituencies. For the remaining 481 districts, Chris Hanretty estimated the results. As a robustness check, we follow Adler and Ansell (2020) and Ansell *et al.* (2022), using the average percentage change in house prices as a proxy for Brexit support. See Appendix C.6 of the Supplementary Material.

[30] England's nine government office regions.

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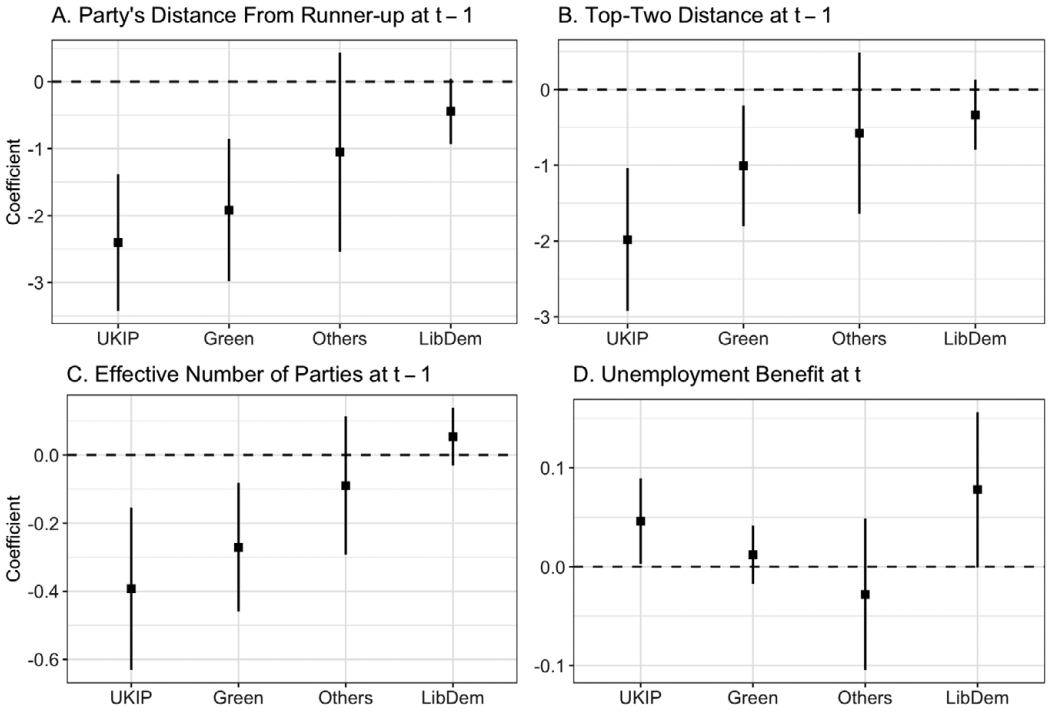


Figure 5. Coefficients in the selection stage.

decisions but not the vote share of parties in districts that they contest, these variables are present in $\boldsymbol{z}_{ijt}$ but not $\boldsymbol{x}_{ijt}$. Hence, they also play the role of meeting the exclusion restriction. The remaining variables in the list above are all expected to affect both the decision of parties about whether or not to contest a district and the percentages of support for parties that do contest them. Our equations for s_{ijt} also include the temporally lagged dependent variable, s_{ijt-1}, which both reflects our expectation of persistence over time and controls for unobservable common factors of a district that shape the trade-off between j and the baseline category (Keele and Kelly 2006).

In Figure 5, we display estimated coefficients for four predictors in the selection equations. Each panel shows the coefficient of each predictor for the partially contesting parties.[31] As expected, the tactical variables affect the likelihood that small parties will contest a district. As the difference between the top two parties in the previous elections increases, the Greens and UKIP are less likely to field candidates (Panel A). Similarly, as the distance between their previous vote and that of the runner-up (Panel B) increases, they are less likely to contest districts, and as the ENP (Panel C) at $t-1$ increases, they are less likely to field candidates. Finally, Panel D shows that covariates in the outcome stage can also affect a party's probability of fielding candidates. The unemployment variable is positively associated with the Liberals' and UKIP's decision to contest a district.

Figure S1 in the Supplementary Material shows the estimated coefficients for Brexit votes and unemployment benefits across the $J-1$ outcome equations.[32] Consistent with Kagalwala *et al.* (2024b), we find that support for Brexit increases the vote share of Conservatives to the detriment of parties that opposed leaving the European Union. Unemployment benefits, in turn, did not have the clear effect expected by economic voting theory.[33] These coefficients, though, are in log-ratio form. We can use

[31] Table S2 in the Supplementary Material shows full results.

[32] Figure S2 in the Supplementary Material shows that competing compositional approaches estimate different coefficients in equations for parties that did not contest all districts in our empirical example.

[33] As an indicator of deteriorating conditions, unemployment is expected to negatively affect the prime minister's party across most log ratios.

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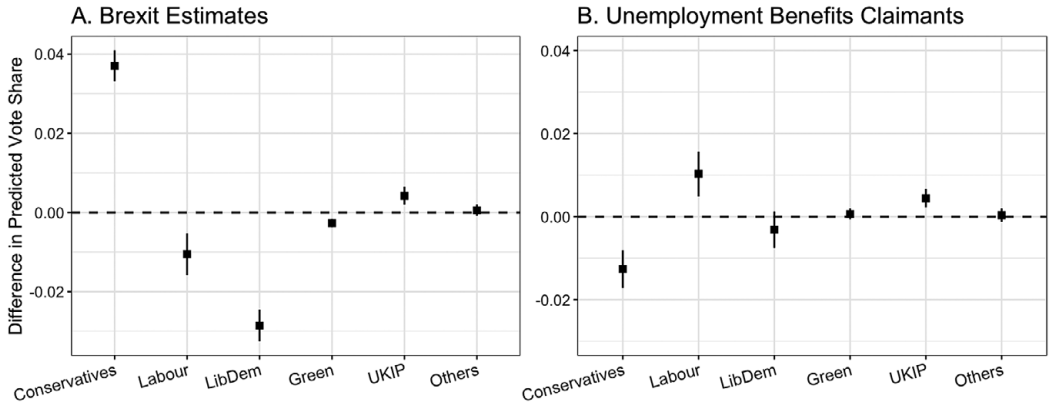


Figure 6. Change in vote share in response to +1SD shock.

them for hypothesis tests about the trade-offs between parties, assessing the direction of the relationship and its statistical significance.[34] To interpret the magnitude of these results, we transform them back into vote shares and calculate their associated confidence intervals using a non-parametric bootstrap in which we randomly sample the data 1,000 times with replacement. We calculate the difference in predicted vote share for each party after a one standard deviation increase (+1SD) holding all other variables at their observed values.[35]

Figure 6 presents the effect of a +1SD increase in Brexit votes (Panel A) and unemployment benefits (Panel B) on the predicted vote share for each party. This indicates that the Conservatives, as the major force supporting Brexit in 2017 and 2019, would win 3.7% of votes due to a +1SD increase in estimated votes for Brexit in 2016. UKIP's votes would also increase, but by a small degree, 0.4%. The Labour Party and Liberal Democrats would lose 1.1% and 2.8% of votes, respectively, from this increase in Brexit support. Panel B shows that the estimated changes in vote shares after a +1SD increase in unemployment benefits are mostly smaller −1% and −1.2% for the Labour and Conservatives, respectively. UKIP also electorally benefited from a shock in unemployment (0.4%). These results support the expectation that the Brexit debate dominated U.K. politics in the 2017 and 2019 elections.[36]

Table S2 in the Supplementary Material shows that the correlation between errors across stages is high (> 0.8) and statistically significant for all parties, demonstrating the necessity to account for sample selection: A party's decision to contest a district and its vote share when it does contest that district are functions of correlated unobservables.

6. Conclusion

In this article, we demonstrate that partial contestation constitutes a form of sample selection, which existing approaches to modeling election results fail to address. As a result, standard methods yield biased estimates. To address this, we introduce a novel MLE approach that jointly accounts for sample selection and the compositional nature of election results. Our approach yields mostly unbiased and efficient estimates, even if the exclusion restriction required by the traditional Heckman correction is violated.

In multiparty elections, outcomes are inherently interdependent—gains for one party necessarily entail losses for others. Compositional models are uniquely suited to capture these interrelationships and, most importantly, allow researchers to examine how shifts in support for one party influence the

[34]Negative values imply that a predictor benefits the baseline category at the expense of j in the numerator. Positive coefficients have the opposite interpretation.

[35]Table S1 in the Supplementary Material shows descriptive statistics.

[36]In Appendices C.2–C.5 of the Supplementary Material, we present a variety of model fit diagnostics, including in-sample and out-of-sample predictions.

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performances of others. This is particularly valuable in the study of vote switching and strategic voting. For example, rising support for radical parties does not uniformly affect all competitors (Colantone and Stanig 2018; Cremaschi *et al.* 2024); identifying the factors behind voters' departure from specific parties and their movement toward others lies at the core of electoral studies. Yet, partial contestation complicates the use of the standard compositional framework. Our proposed approach addresses this challenge.

Although elections provide a clear application, the utility of our approach extends to other substantive domains in which sample selection processes generate problems for models with compositional outcomes. These include analyses of government budget trade-offs, the allocation of portfolios among coalition partners and the share of attention that parties devote to particular issues in their manifestos (Abou-Chadi and Krause 2020; Cox 2021; Lipsmeyer, Philips, and Whitten 2023). In all such examples, correlated unobservables may simultaneously influence both the strategic decision for a category to be included and its observed share. Addressing sample selection in a compositional framework is thus critical for accurate and consistent inferences.

Our MLE approach is flexible, accommodating different error distributions depending on the application. We encourage practitioners to carefully consider the DGP and exclusion restriction and to interpret their results as contingent on these assumptions when estimating compositional models with selection. Without a valid instrument, the CMS relies solely on functional-form identification. Future work may develop a CMS with other methods for constructing joint distributions such as a copula approach.

Supplementary material. The supplementary material for this article can be found at https://doi.org/10.1017/pan.2026.10049.

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