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A New Approach to Sequential Tidal Prediction

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Accurate tide prediction is required for safe marine navigation in shallow waters as well as for other marine operations. Traditionally, tide prediction was carried out using the harmonic method, which is based on the identification of the harmonic tidal constituents existing in the tidal record. Unfortunately, however, unless long tidal records are available at the tide gauges, some important tidal constituents may not be identified. This, in turn, deteriorates the accuracy of the tidal prediction. More recently, a sequential least-squares prediction method capable of using relatively short tidal records was developed. This method allows for the modifications and corrections of the original solution of the tidal constituents when new observations and parameters are included. Although it reduces the computation time significantly compared to the batch harmonic method, this method exhibits large residuals particularly when very short tidal records are used. To overcome the limitations of the sequential least-squares method, a neural network-based model is developed for sequentially predicting the tidal heights using tide data series collected at various tide gauges. A modular, three-layer feedforward neural network trained using the back-propagation algorithm is used for this purpose. Tide data from three tide gauges are used to validate the model. A comparison is made between the developed neural network model and the sequential least squares method for tidal prediction. It is shown that the accuracy level of the tidal prediction has improved by a factor of 5 when using the neural network model.

1. INTRODUCTION. Accurate tide information (current and predicted) is a key element for safe marine navigation in shallow waters as well as for other marine operations. Tidal prediction was conventionally carried out using various methods, with the harmonic method being the most common (Tianhang, 1991). With the harmonic method, the tide data series collected at a tide gauge is analysed to yield the amplitudes and the phase-lags of all the important harmonic tidal constituents with respect to those of the equilibrium tide. These tide constituents are then properly combined to predict the tidal heights at future times (Forrester, 1983). Depending on the number of constituent amplitudes and phase-lags taken in the prediction model, various accuracy levels can be expected. To be accurate, all the important constituents must be included in the prediction model, which requires long tidal records at the tide gauge, ideally spanning a period of more than 18·6 years. Unfortunately, a long data series is not always available, which may lead to either large residuals or unstable solution, as interference between neighbouring constituents is expected (Forrester, 1983; Tianhang, 1991). Tianhang developed a sequential least-squares prediction method, which allows for the modifications and

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corrections of the original solution of the tidal constituents by the addition of new observations and parameters. Although it reduces the computation time significantly compared to the batch method, this method exhibits large residuals particularly when short tidal records are used.

To overcome the above limitations, a neural network-based model for sequential tidal prediction is developed in this paper. The motivation behind the selection of neural networks lies in their ability to model highly non-linear functions and mapping through supervised learning by example. A modular, three-layer feedforward neural network trained using the back-propagation algorithm is used for this purpose. The design parameters of the model are the number of neural network inputs, outputs, and hidden nodes, as well as the structure of the feedforward network itself. Tide data from three tide gauges, namely Saint John, Yarmouth, and Charlottetown, are used to validate the model. A comparison is made between the developed neural network model and the sequential least-squares method. It is shown that the accuracy level of the tidal prediction has improved by a factor of 5 when using the neural network model.

2. TRADITIONAL TIDAL PREDICTION METHODS.

As mentioned above, tidal prediction was traditionally carried out by the harmonic method, which is based on the identification of the harmonic tidal constituents existing in the tidal record. The tidal observation equation can be written as (Forrester, 1983; Tianhang, 1991):

$$\begin{aligned} h(t_i) &= Z_0 + \sum_{j=1}^{n} H_j cos(\omega_j t + \theta_j - \phi_j) \\ &= Z_0 + \sum_{j=1}^{n} P_j cos(\omega_j t + \theta_j) + Q_j sin(\omega_j t + \theta_j) \end{aligned} \tag{1}$$

where $h(t_i)$ is the observed tidal height, t_i is the Greenwich mean time, $t \in \{t_1, t_2, \ldots, t_N\}$, N is the number of tidal records, Z_0 is an unknown parameter representing the mean water level, n is the number of the harmonic constituents, H_j and ϕ_j are the amplitude and phase-lag of the jth constituent, ω_j and θ_j are the equilibrium constituents and their Greenwich phases, and $P_j = H_j \cos\phi_j$, and $Q_j = H_j \sin\phi_j$. Equation (1) can be written in a more compact form for the entire tidal records as:

$$\boldsymbol{h} = \boldsymbol{A}\boldsymbol{x} \tag{2}$$

where $\boldsymbol{h} = [h(t_1), h(t_2), \ldots, h(t_N)]^{\mathrm{T}}$ is the N-1 vector of the observed tidal heights, $\boldsymbol{x} = [Z_0, P_1, Q_1, \ldots, P_n, Q_n]^{\mathrm{T}}$ is the $(2n+1)$-1 vector of the unknown parameters, and $\boldsymbol{A}$ is the design matrix given by:

$$\mathbf{A} = \begin{bmatrix} 1 & cos(\omega_1 t_1 + \theta_1) & \cdots & sin(\omega_n t_1 + \theta_n) \\ 1 & cos(\omega_1 t_2 + \theta_1) & \cdots & sin(\omega_n t_2 + \theta_n) \\ \vdots & \vdots & \cdots & \vdots \\ 1 & cos(\omega_1 t_N + \theta_1) & \cdots & sin(\omega_n t_N + \theta_n) \end{bmatrix}. \tag{3}$$

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The least-squares solution of the unknown parameters $\boldsymbol{x}$ can be obtained when $N > 2n+1$ as follows:

$$\hat{\boldsymbol{x}} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{h}. \tag{4}$$

The covariance matrix of the determined parameters $\hat{\boldsymbol{x}}$ can be estimated by applying the law of covariance propagation to (4). Once the least-squares solution for $\boldsymbol{x}$ is obtained, the predicted tidal heights $\hat{\boldsymbol{h}}(t_i)$ can be obtained at any desired time t_i from (1). Tianhang (1991) modified the above method to allow for the tidal prediction to be determined in a sequential manner. His algorithm accommodates the addition of new observations as well as new tidal constituents. To test his model, he predicted the tidal values over one day using approximately one-week of tidal records at three stations, namely Saint John, Yarmouth, and Halifax. The maximum residual error (the difference between predicted and true values) was approximately one metre, which, as would be expected, becomes larger with the increases in time.

3. TIDAL PREDICTION USING ARTIFICIAL NEURAL NETWORKS (ANN).

3.1. *What is a Neural Network?* Artificial Neural Networks, or simply neural networks, are computational models that imitate the human brain in performing a particular task (Haykin, 1999). They have the capability to solve complex problems through learning, or training, and then generalising the network outputs for other inputs. A neural network consists of processing elements, or neurons, that are massively interconnected. Each of the connecting links is characterised by its own weight, or strength. Figure 1 represents a block diagram of a simple model of a neuron showing the weights of the various links. An activation function, such as a sigmoid function or a hyperbolic tangent function, is applied to limit the amplitude of the neuron. The sigmoid function is an s-shaped function, which is used widely in the construction of the neural networks (Haykin, 1999). The logistic function represents an example of the sigmoid function, which is defined as:

$$\varphi(v) = [1 + \exp(-av)]^{-1} \tag{5}$$

where the parameter a represents the slope of the sigmoid function. Finally, an external bias, b_k, is applied to increase or lower the net input of the activation function. The neural network is trained to find the optimal values for the weights and the biases.

The above structure for a neuron k can be represented mathematically as:

$$v_k = \sum_{j=1}^{m} w_{kj}h_j + b_k = \sum_{j=0}^{m} w_{kj}h_j \tag{6}$$

$$y_k = \varphi(v_k) \tag{7}$$

where $h_0, h_1, h_2, \ldots, h_m$ are the input signals; v_k is the activation potential of neuron k; y_k is the output signal, and $w_{k0}, w_{k1}, w_{k2}, \ldots, w_{km}$ are the weights of neuron k. It should be noted in (6) that the values of $h_0 = +1$ and $w_{k0} = b_k$, respectively.

3.2. *Neural Network Structures and Training*. Artificial neural networks can be designed in various ways, depending on how the neurons are structured and the learning algorithms, or rules, used. Network architectures may be classified as

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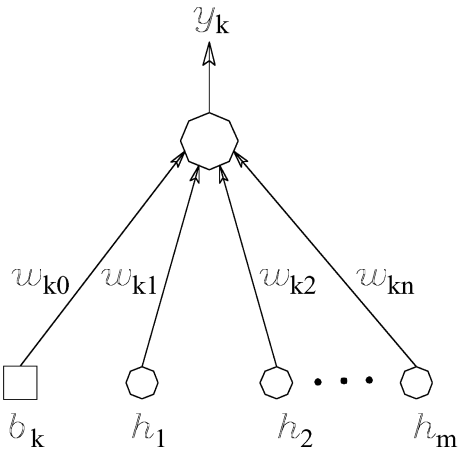


Figure 1. Simple neuron model.

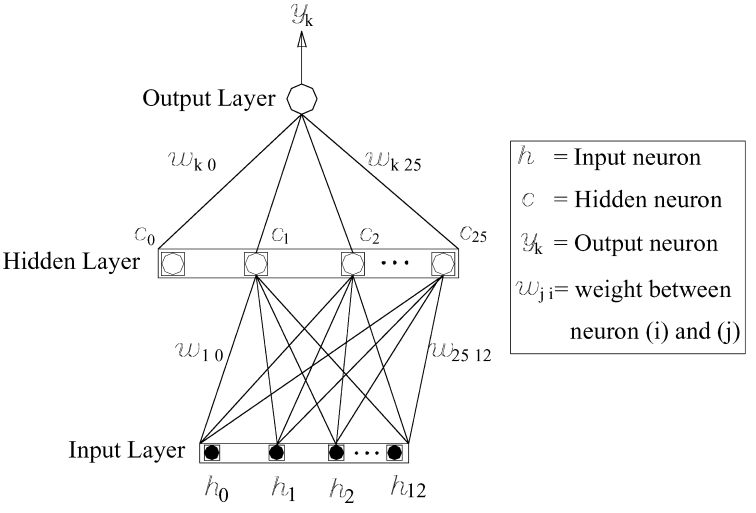


Figure 2. Three-Layer Feedforward Neural Network with the Structure [12-25-1].

single-layer feedforward, multi-layer feedforward, and recurrent networks (Haykin, 1999). Single-layer network is the simplest form of a layered network, which consists of an input layer of source neurons that are mapped onto an output layer of neurons. The computations are performed at the output layer only, which is why it is referred to as single-layer. Multi-layer feedforward networks, on the other hand, are characterised by the existence of additional one or more hidden layers containing a number of hidden neurons. The hidden layers help the network in extracting higher-order statistics (Haykin, 1999). Figure 2 shows an example of a fully connected three-layer feedforward network, which is referred to as 12-25-1 (12 source neurons, 25 hidden neurons, and one output neuron). With this architecture, the source neurons in the input layer (i.e., first layer) provide the input vector to the hidden layer (i.e., second layer). The output vector of the hidden layer is then used as input to the third layer

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(i.e., output layer), which outputs the overall network solution. Recurrent neural networks are similar to the feedforward networks, except that the former have at least one feedback loop. According to Schuh *et al.* (2002), feedforward networks have better prediction capabilities than recurrent networks. In our prediction model, we used the feedforward neural networks. In this case, the output signal at a neuron j (either a hidden neuron or an output node) can be written as:

$$y_j(n) = \varphi(v_j(n)) \tag{8}$$

where $v_j(n)$ is the activation potential of neuron j, which is defined by:

$$v_j(n) = \sum_{i=0}^{m} w_{ji}(n) y_i(n) \tag{9}$$

where m is the total number of inputs (without the bias) applied to neuron j; $w_{ji}(n)$ represents the weight connecting the output of neuron i to the input of neuron j at iteration n (n^{th} training example); and $y_i(n)$ is the output signal of neuron i (i.e., the input signal of neuron j). It should be clear that $y_i(n) = h_i(n)$, the i^{th} element in the input vector, if neuron j is in the first hidden layer.

3.3. *The Learning Process* (*backpropagation algorithm*). As stated above, neural networks solve complex problems through learning (i.e., training), and then generalising the network outputs for other inputs. Training the neural network is accomplished through iterative adjustments of the free parameters, i.e., the weights and bias, of the network until the optimal values are obtained. There exist various learning algorithms, which are fundamental to the design of neural networks. Of these, the backpropagation-learning algorithm is the most widely used for feedforward neural networks (Schuh *et al.*, 2002).

With the backpropagation-learning algorithm, the output signal of a neuron j, $y_j(n)$, is compared to a desired (target) output, $d_j(n)$. The error signal at the output of neuron j, $e_j(n)$, is defined as:

$$e_j(n) = d_j(n) - d_j(n) \tag{10}$$

where n represents the n^{th} training example (i.e., n^{th} pattern). The objective of the iterative adjustments is to make $y_j(n)$ as close as possible to $d_j(n)$, which can be achieved by minimizing a cost function (total instantaneous error energy over all neurons in the output layer) defined as:

$$E(n) = \frac{1}{2} \sum_{j \in C} e_j^2(n) \tag{11}$$

where C represents all neurons in the output layer. The weight correction $\Delta w_{ji}(n)$ can now be defined according to the delta rule as (Haykin, 1999):

$$\Delta w_{ji}(n) = -\eta \frac{fE(n)}{fw_{ji}(n)} = \eta \delta_j(n) y_i(n) \tag{12}$$

where η is the learning rate parameter; and $\delta_j(n)$ is the local gradient defined by:

$$\delta_j(n) = -\frac{fE(n)}{fv_j(n)} = e_j(n) \varphi_j'(v_j(n)) \tag{13}$$

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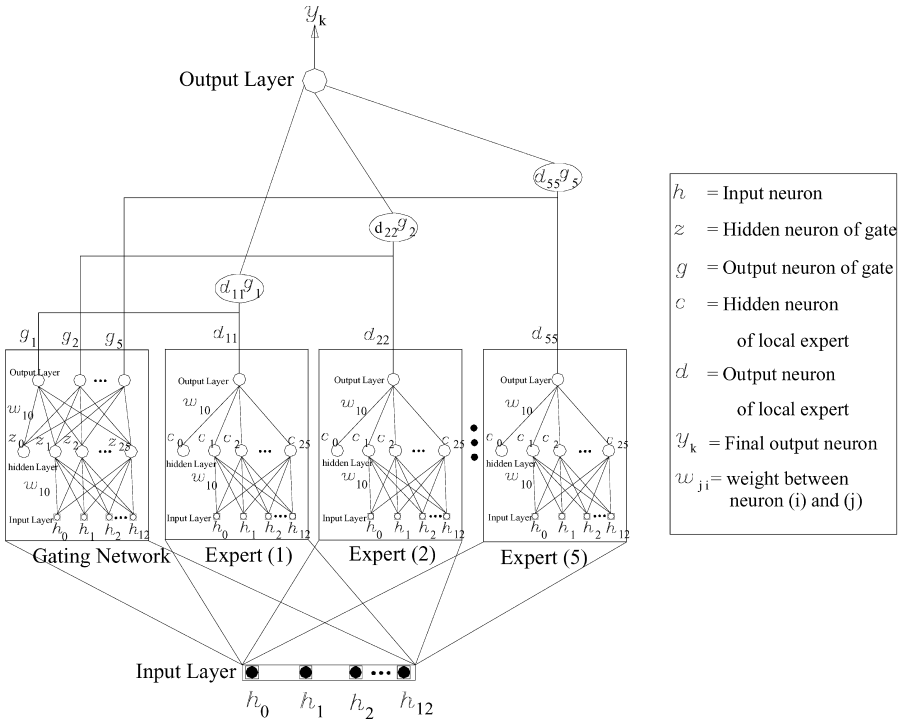


Figure 3. A modular neural network with the Structure [12-25-5-1].

where $\varphi_j'(v_j(n))$ is the derivative of the associated activation function. This means that for $\delta_j(n)$ to exist, the activation function must be continuous, which is satisfied by both the sigmoid and hyperbolic tangent functions presented above.

The selection of the learning rate parameter η affects the rate of learning of the neural network. The smaller the value of η is, the smaller the changes in the weights and network rate of learning. Smaller η values result in smaller changes to the weights in the network, and consequently slower rate of learning. If, on the other hand, the η values are too large, the network may become unstable (i.e., oscillatory) and the algorithm diverges. To overcome this problem, the generalised delta rule is used, which introduces an additional term to (12) known as the momentum constant (see Haykin, 1999 for details). The weights will be adjusted iteratively by presenting new epochs of training examples to the neural network. Unfortunately, there is no clear-cut criterion to decide when to stop the training, i.e., to consider that the backpropagation algorithm has converged (Haykin, 1999). If the training is not stopped at the right point, an over-fitting of the training data (i.e., model does not interpolate well between the points) might occur. One approach to address this problem is to create a test dataset, which tests the neural network for its generalisation performance (NeuralWare, 2001).

Under certain circumstances, for example when encountering a prediction problem, it might be better to use the modular neural networks (NeuralWare, 2001). A modular neural network has the capability of dividing a problem into sub-problems and resolving each sub-problem rather well. It consists of a group of back-propagation networks, sometimes referred to as ''local experts'', each has the same

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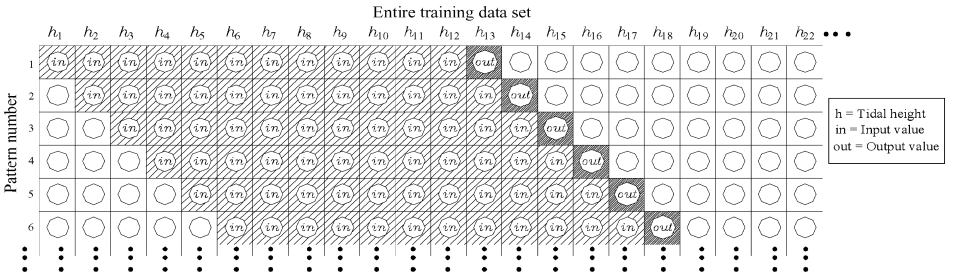


Figure 4. Training Patterns Used in the Neural Network Input.

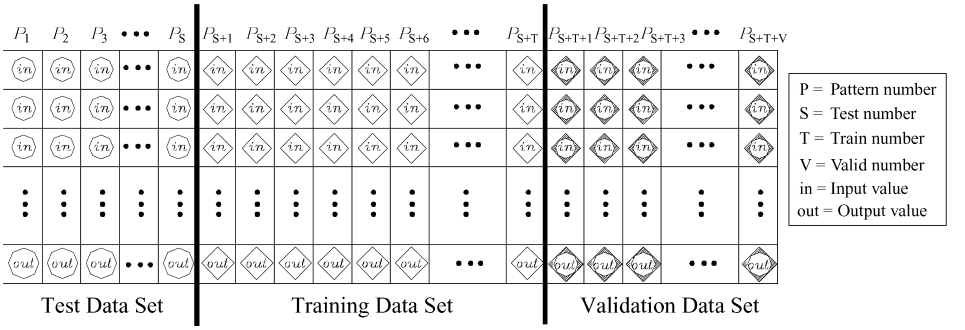


Figure 5. Selection of Testing, Training and Validation Datasets.

architecture. This group of networks compete to learn the various aspects of the problem, which is then controlled by a "gating network". The number of local experts is determined by the number of output neurons in the gating network. Figure 3 shows an example of a modular neural network, which is referred to as 12-25-5-1 (12 source neurons, 25 hidden neurons for both the local experts and the gating network, 5 "gating" output neurons, and one network output neuron).

4. RESULTS AND DISCUSSION. Hourly Tidal data from three tide gauges, namely Saint John, Yarmouth, and Charlottetown, were used to validate the neural network model. The structure of the neural network was built using the NeuralWare software (NeuralWare, 2001). Several tests were conducted to optimise the structure of the neural network for each of the three data series. It was concluded that the modular neural network with the structure [12-25-5-1] gives the best results, i.e., has the lowest root-mean-square (RMS) error. Initially, we used the variable time t as the only input to the network, which is usually followed in the literature (see, for example, El-Rabbany *et al.*, 2002). However, although the trained network represented the training dataset reasonably well, the network prediction was rather poor. Therefore, we followed another approach, which was suggested by Schuh *et al.* (2002). In this approach, the immediate past values of the tidal records are used as input to the network, while future values of the tidal records are used as the desired output. In the subsequent epochs, the training patterns were time-shifted as shown in Figure 4.

Each of the data series was divided into three datasets: training, testing and validation (Figure 5). The first 1000 hourly tidal records were assigned to the testing

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Table 1. Selection of Testing, Training and Validation Datasets.

Tidal station	Data span d/m/y–d/m/y	No. testing records (hrs.)	No. training records (hrs.)	No. valid. records (hrs.)	No. training iterations
Saint John	01/08/2000–31/12/2001	1000	10,432	1000	299,539
Yarmouth	01/03/1996–31/12/1997	1000	14,104	1000	339,863
Charlottetown	01/06/1997–31/10/1999	1000	19,192	1000	349,620

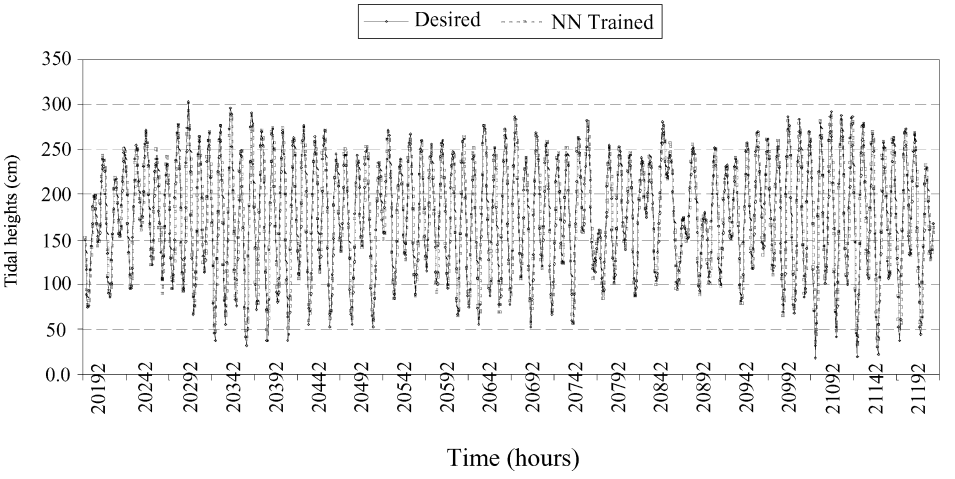


Figure 6. Predicted tidal heights versus desired (actual) values for Charlottetown.

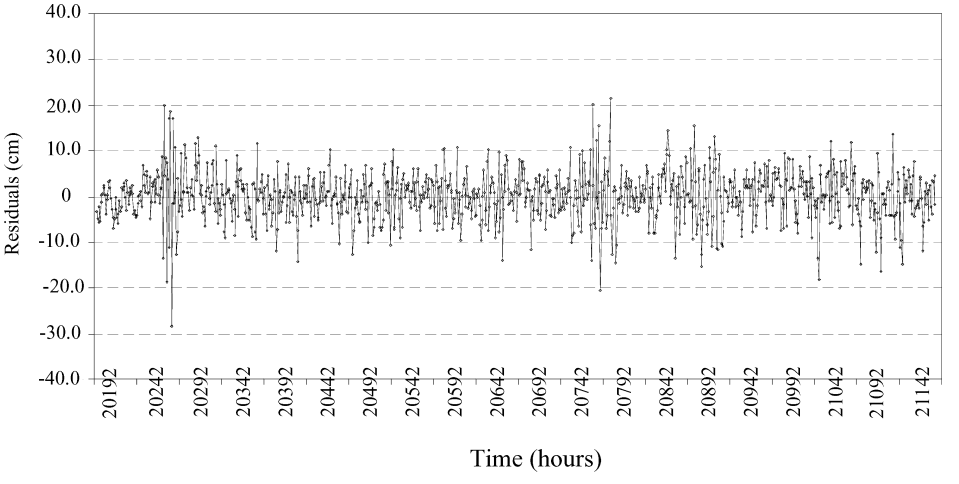


Figure 7. Prediction Errors for Charlottetown.

dataset, while the last 1000 hourly tidal records were assigned to the validation dataset. The training dataset was selected to represent the middle portion of the data series, which varied from one tide gauge to another (see Table 1 and Figure 5). The training was stopped based on testing the generalisation performance of the neural network using the testing dataset. Table 1 shows the number of iterations needed to

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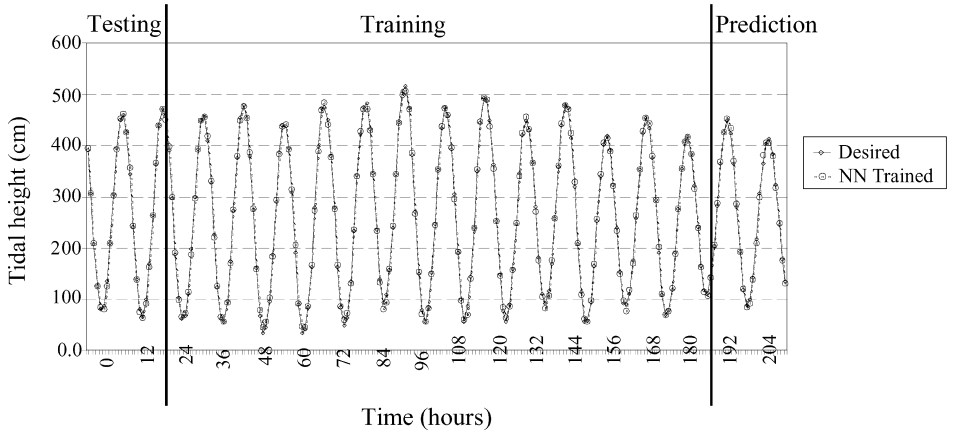


Figure 8. Predicted tidal heights versus desired (actual) values for Yarmouth [based on one-week tidal observations].

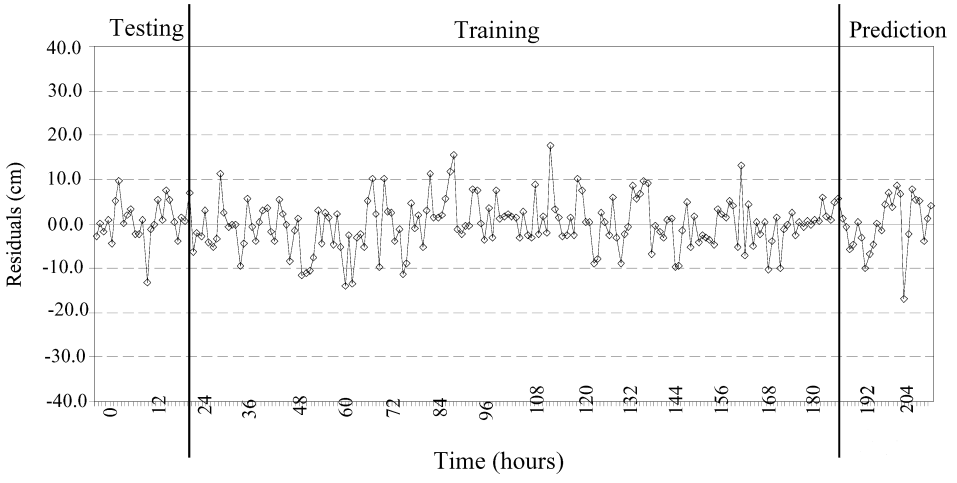


Figure 9. Prediction Errors for Yarmouth [based on one-week tidal observations].

train the network for each of the data series. After training and testing the network, we generalised the model to predict ahead the last 1000 values of the data series and compared the results with the observed tidal records. This was done in a sequential manner to emulate the real-time condition. Figures 6 and 7 show the predicted tidal heights versus the desired (i.e., actual) values, and the prediction errors (i.e., residuals) for Charlottetown. Similar results were obtained for the other two stations. It can be seen that the maximum prediction error is approximately 20 cm, with the bulk of the residuals within the ± 10 cm range. This shows that the developed neural network model has the capability to precisely predict the tidal values, even when long tidal records are not available.

To further validate our model, we compared it with the sequential least squares tidal prediction model developed by Tianhang (1991). To do this, we trained the network using a randomly selected one-week of tidal records at the three tide gauges

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mentioned above. Our goal was to sequentially predict the tidal values on the eighth day, i.e., the day which follows the one-week training. Figures 8 and 9 show the predicted tidal heights versus the desired (i.e., actual) values, and the prediction errors (i.e., residuals) for Yarmouth. Similar results were obtained for the other two stations. It can be seen that, despite the very short data span, the maximum prediction error is only 20 cm, which is five times lower than the sequential least squares tidal prediction model.

5. CONCLUSIONS. A sequential tidal prediction model using Artificial Neural Networks was developed in this paper. The modular neural network structure gave the best performance results (i.e., the minimum RMS), and therefore was used in predicting the tidal values. Hourly tide data, with varying lengths, from three tide gauges were used to verify the model. It is shown that the maximum prediction error is approximately 20 cm, with the bulk of the residuals fall within the ± 10 cm range. A performance comparison was made between the developed neural network model and the sequential least squares method for tidal prediction. Three randomly selected data series with one-week duration each were used for that purpose. It is shown that, despite the very short data span, the maximum prediction error is still 20 cm with the neural network model, which is five times lower than the sequential least squares model. This shows that the developed neural network model has the capability to precisely predict the tidal values in a sequential manner, even when long tidal records are not available.

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