10

Phase Shift Functions with Other Analytic Properties

The family of singular phase shift functions studied in Chapter 8 all have a simple pole associated with the exponential edge, i.e., they have pole singularities at $b_x = \pm c - i\pi\beta$, whereas those considered in Chapters 7 and 9 have essential singularities. A focus on these singularities is instructive, since it conveys information on how the points of stationary phase move at large momentum transfers.

For example, with the "Fermi" phase shift function

$$X(b_x) = \frac{A}{1 + \exp[(b - c)/\beta]}, \tag{10.1}$$

we will asymptotically (when $\exp[(b - c)/\beta] \to -1$) find

$$X(b_x) \sim \frac{-A\beta}{b_x - b_x^{\text{pole}}}, \quad b_x \to b_x^{\text{pole}}, \tag{10.2}$$

with

$$q = X'(b_x) \sim \frac{A\beta}{(b_x - b_x^{\text{pole}})^2}. \tag{10.3}$$

Thus, the derivatives will be more singular; they have a *double* pole. The points of the stationary phase will, as $q \to \infty$, asymptotically approach the pole singularity as the inverse square root of q:

$$b_x = b_x^{\text{pole}} \pm \sqrt{\frac{A\beta}{q}}. \tag{10.4}$$

For the case of a purely absorptive phase shift function, $A = i|A|$, the approach to the singularity is seen to be at an angle of $\pm\pi/4$, as we have seen in several examples in Chapter 8. This angle of $\pm\pi/4$ means that the period and exponential slope (determined by real and imaginary parts of b_x, respectively) will change with increasing momentum transfer at the same relative rate.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

This behavior is however not unique, as we will discuss in this chapter. The asymptotic behavior of the exponential slope of the differential cross section and the period of oscillation both depend on how the stationary point approaches the singularity. But they need not approach the singularity as $1/\sqrt{q}$, and they need not even approach it at the same rate.

10.1 Other Powers of the "Fermi" Phase Shift Functions

A simple modification of the "Fermi" phase shift function, in which we raise the denominator to some real power $p-1$,

$$\begin{aligned} X_p(b_x) &= \frac{A}{\{1+\exp[(b-c)/\beta]\}^{p-1}}, \quad \operatorname{Re} b_x > 0, \\ X_p(-b_x) &= X_p(b_x), \end{aligned} \tag{10.5}$$

will make the stationary point approach the singularity as

$$X'_p(b_x) \sim \frac{1}{(b_x - b_x^{\text{sing}})^p}, \tag{10.6}$$

with $b_x^{\text{sing}} = \pm c - i\pi$.

10.2 "Derivative-Based" Phase Shift Functions

The discussion is actually simpler in terms of the derivative of the phase shift function, rather than in terms of the phase shift function itself. However, in order to make sure the phase shift function has a reasonably well-defined extension, we will make use of the cut-off provided by the "Fermi" function. On the real axis, we shall study a generalization of the "Fermi" phase shift function:

$$\begin{aligned} X'(b_x) &= -\frac{(A/\beta)(b_x/c)^{1+2\nu}}{(1+\exp[(b_x-c)/\beta])^p}, \quad b_x > 0, \\ X'(-b_x) &= X'(b_x). \end{aligned} \tag{10.7}$$

Thus, we let the *derivative* have a singularity of order p, where p need not even be an integer. We allow the power ν to be zero or a positive integer in order that the phase shift function $X(b_x)$ be even. Asymptotically, for $q \to \infty$, the singularity of $X'(b_x)$ will be probed, where

$$X'(b_x) \sim -\frac{(A/\beta)\beta^p(1-i\pi\beta/c)^{1+2\nu}}{(b_x^{\text{sing}} - b_x)^p}, \quad \operatorname{Re} b_x > 0, \tag{10.8}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

with $b_x^{\text{sing}} = c - i\pi\beta$, and similarly the singularity at $\text{Re}\, b_x < 0$. It is convenient to introduce some notation for the last factor in the numerator of Eq. (10.8), stemming from the factor b_x/c in the numerator of $X'(b_x)$:

$$1 - i\pi\frac{\beta}{c} = \sqrt{1 + \left(\pi\frac{\beta}{c}\right)^2}\, e^{-i\eta} \equiv \xi e^{-i\eta}. \tag{10.9}$$

Equating now the expression Eq. (10.8) to the momentum transfer q, we find

$$b_x \simeq b_x^{\text{sing}} - \left(\frac{-A\xi e^{-i\eta}}{q\beta}\right)^{1/p}\beta, \tag{10.10}$$

where we have taken $\nu = 0$, but it is clear how this is modified for $\nu > 0$. It is also immediately clear that the power p controls how fast the point of stationary phase approaches the singularity as $q \to \infty$.

For simplicity of the discussion, we shall in this chapter only consider purely real or purely absorptive phase shift functions. Then, we note that a non-zero power ν can to some extent modify the phase of $b_x - b_x^{\text{sing}}$ in the complex plane. This, in turn, determines the relative rates of change of the period of oscillation versus the slope of the exponential envelope.

For $p = 1$, the approach is determined by $1/q$, and in the absence of the phase η, it would for an absorptive phase shift function ($A = i|A|$) be purely imaginary, i.e., only the exponential slope would change, not the period of oscillation:

$$b_x \simeq b_x^{\text{sing}} + \frac{i|A|}{q}. \tag{10.11}$$

For a more accurate analysis, keeping the last factor of Eq. (10.8), this approach would be rotated by the angle $-\eta$, which for $c = 3$ fm and $\beta = 0.5$ fm would amount to -27.6°, producing also some lengthening of the period.

For $p = 3$, on the other hand,

$$b_x = b_x^{\text{sing}} - \left(\frac{-A\xi e^{-i\eta}}{q\beta}\right)^{1/3}\beta, \tag{10.12}$$

which in the absorptive case becomes

$$b_x = b_x^{\text{sing}} + \exp\left[\frac{i}{3}\left(-\frac{\pi}{2} + 3\pi + 2n\pi - \eta\right)\right]\left(\frac{|A|\xi}{q\beta}\right)^{1/3}\beta. \tag{10.13}$$

As announced, the point of stationary phase approaches the asymptotic value much more slowly. For $\eta \to 0$, it would approach at an angle of $\pm 5\pi/6$ (for $n = 0$ in the above expression). For $n \neq 1$, there are additional solutions for the phase of $b_x - b_x^{\text{sing}}$, but those solutions b_x are not along the path of integration.

This chapter will illustrate some phenomena related to these other singularities.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

10.2.1 Soft-Exponential Phase Shift Function

We shall first discuss a phase shift function which is related to those phase shift functions with an exponential edge, but in analytical terms has a somewhat softer edge. As outlined above, we will define this in terms of its derivative. Let

$$f_1(b_x) \equiv \frac{-(A/\beta)(b_x/c)}{1+\exp[(b_x-c)/\beta]}, \quad b_x > 0, \tag{10.14}$$

and take

$$\begin{aligned} X'(b_x) &= f_1(b_x), \qquad b_x > 0, \\ X'(b_x) &= -f_1(b_x), \quad b_x < 0. \end{aligned} \tag{10.15}$$

In order to calculate the cross section, we also need the phase shift function itself. This can be obtained by integration:

$$\begin{aligned} X(b_x) &= -\int_{b_x}^{\infty} \mathrm{d}t\; X'(t) = -\int_{b_x}^{\infty} \mathrm{d}t\; f_1(t), \quad \mathrm{Re}\, b_x > 0, \\ X(-b_x) &= X(b_x). \end{aligned} \tag{10.16}$$

For b_x off the real axis, this will be an integral along a complex path, it can easily be expressed as a sum of two integrals, one along a path parallel to the imaginary axis, and one along the real axis. Another example will be based on

$$f_3(b_x) \equiv \frac{-(A/\beta)(b_x/c)^3}{1+\exp[(b_x-c)/\beta]}, \quad b_x > 0, \tag{10.17}$$

where $X'(b_x)$ and $X(b_x)$ are obtained by the same equations Eqs. (10.15 and 10.16), but with $f_3(b_x)$ substituted for $f_1(b_x)$. This gives a more flat phase shift function in the interior, for $|b_x| < c$, but more of a tail for $|b_x| > c$.

While the derivative of the "Fermi" phase shift function, given by Eq. (8.7), has a double pole at $b_x = \pm c - i\pi\beta$, the present one has only a single pole. As a result, with increasing momentum transfers, the stationary points will approach these singularities at a faster rate. The corresponding lengthening of the period of oscillation will set in at a lower value of momentum transfer, and will be faster. This is seen from the asymptotic relation

$$q = X'(b_x) \simeq \frac{A\xi e^{-i\eta}}{b_x - b_x^{\mathrm{pole}}}, \tag{10.18}$$

with $b_x^{\mathrm{pole}} = \pm c - i\pi\beta$. Inverting, one finds

$$b_x = b_x^{\mathrm{pole}} + \frac{A\xi e^{-i\eta}}{q}. \tag{10.19}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

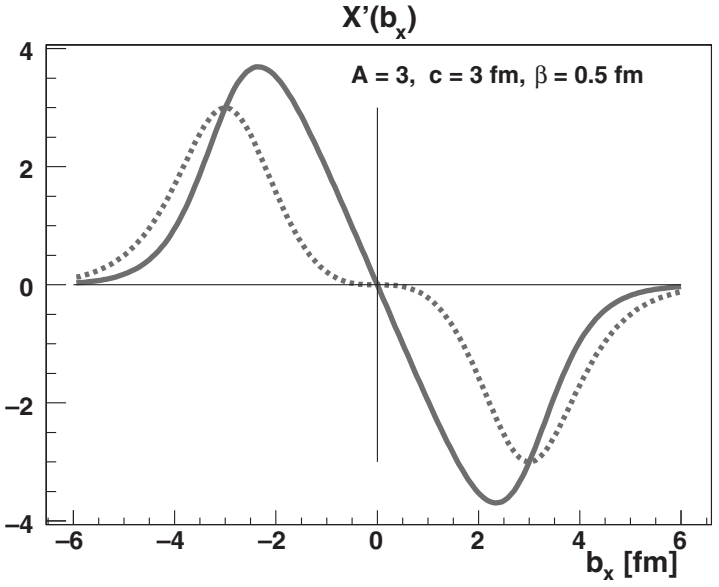


Figure 10.1 Derivatives of a phase shift function having a softer edge than the "Fermi" one. Solid curve: based on f_1, Eq. (10.15); dashed curve: based on f_3, Eq. (10.17).

For comparison, we recall that for the "Fermi" phase shift function discussed in Section 8.4, the approach to the pole singularity is like $1/\sqrt{q}$; see Eqs. (8.39) and (8.40).

For shape parameters $c = 3$ fm and $\beta = 0.5$ fm, together with a strength $A = 1$, we show in Figs. 10.1 and 10.2 this derivative $X'(b_x)$ and the phase shift function $X(b_x)$. While the region around $b_x = c$ (the edge) of the phase shift function is similar to the "Fermi" phase shift function, the interior is more peaked around $b_x = 0$.

Possible locations of points of stationary phase are given by the lines in the complex b_x-plane, where $X'(b_x)$ is real and positive. Such lines are identified in the two "maps" in Fig. 10.3. Here we display regions where $\mathrm{Re}\, X'(b_x) > 0$ (white) together with the complementary region where $\mathrm{Re}\, X'(b_x) < 0$ (shaded), as well as where $\mathrm{Im}\, X'(b_x) = 0$ (dashed lines). Both of these maps are based on derivatives $X'(b_x)$ having simple poles at $b_x = \pm c - i\pi\beta$ (indicated as "bullets", for $c = 3$ fm, $\beta = 0.5$ fm), one vanishes linearly at the origin, $X'(b_x) \sim b_x$ as $b_x \to 0$, the other in a cubic fashion, $X'(b_x) \sim b_x^3$ as $b_x \to 0$. The importance of these powers lies in the phase of the factor $(\xi e^{-i\eta})^{1+2\nu}$ present in the asymptotic expression Eq. (10.18). With the cubic factor of $f_3(b_x)$, we see that the stationary points approach the singularity practically parallel to the real axis. This means that

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use , available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

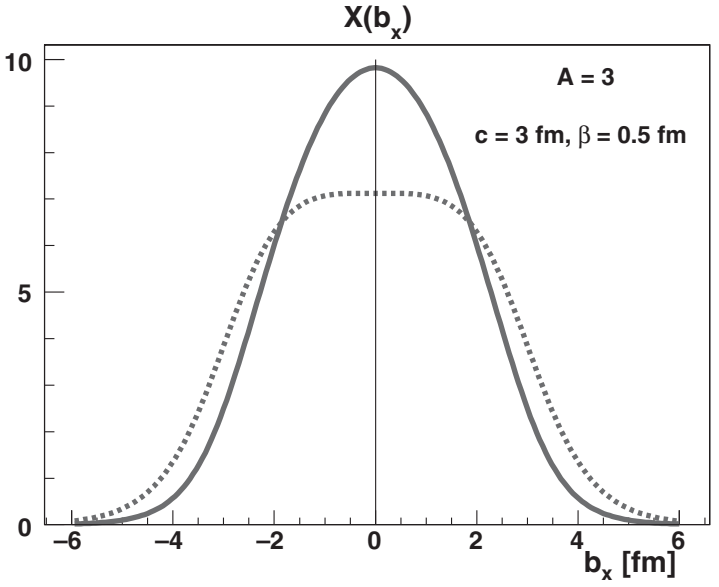


Figure 10.2 Phase shift functions $X(b_x)$ corresponding to the derivatives shown in Fig. 10.1.

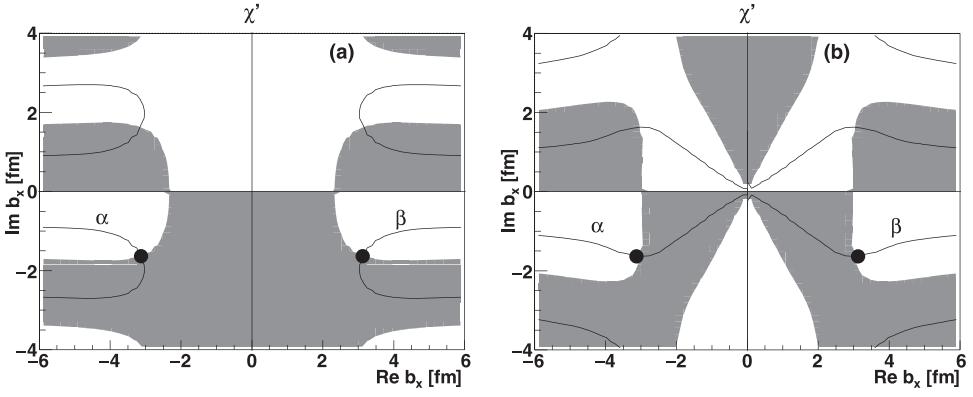


Figure 10.3 Regions of positive (white) and negative (shaded) values of $\mathrm{Re}\, X'(b_x)$, shown in the complex b_x-plane for the purely absorptive case. The figures are analogous to Fig. 7.2. Also indicated as dashed lines, are the locations where $\mathrm{Im}\, X'(b_x) = 0$. (a) $X'(b_x)$ determined by Eq. (10.14). (b) $X'(b_x)$ determined by Eq. (10.17).

asymptotically, the main change will be in the period of the oscillation, not so much in the exponential envelope.

For the purely absorptive case, with $A = 3i$, we obtain the differential cross section shown in Fig. 10.4. We here compare the differential cross section based

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use , available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

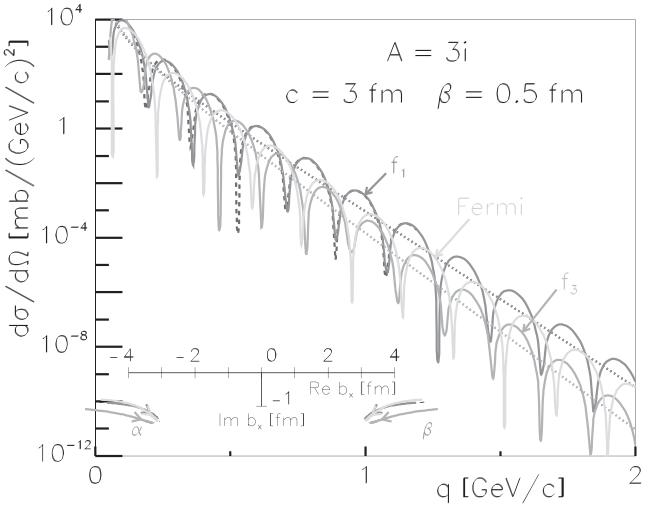


Figure 10.4 Differential cross section corresponding to the phase shift function whose derivative is given by Eq. (10.15). The different solid curves refer to f_1, f_3 and the "Fermi" phase shift function. Dashed (small q): numerical integration for the case of f_1. The inset shows the relevant trajectories of stationary points.

on the function f_1 (with $\nu = 0$) with that based on the function f_3 (with $\nu = 1$), for the same shape (c, β) and strength parameters. For a quantitative comparison of these differential cross sections, we can approximate the exponential envelope and the oscillations of the differential cross section as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \sim \exp\left(-\frac{q}{Q}\right) \times \cos^2\left(\frac{\pi q}{P} + \phi\right)^2 \tag{10.20}$$

and find the parameters Q (slope) and P (period) in different ranges of momentum transfer q. The results of such fits are quoted in Table 10.1. It should be noted that these quantities depend not only on the shape parameters c and β and the functional form, but also on the strength of the interactions, here taken as $|A| = 3$.

Because of the additional phase η, these numbers do not quite agree with the qualitative discussion above. But the rate of change (with q) is seen to be much faster than for the "Fermi" phase shift function.

10.2.2 Sharp-Exponential Phase Shift Function

As an example of a phase shift function with a higher-order pole near the surface, let its derivative be defined in terms of

$$g_1(b_x) \equiv \frac{-(A/\beta)(b_x/c)}{(1 + \exp[(b_x - c)/\beta])^3}, \quad b_x > 0, \tag{10.21}$$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

Table 10.1 *Exponential slopes Q and periods P, for phase shift functions derived from their derivatives, f_1 and f_3, and, for reference, the "Fermi" phase shift function. Two ranges of momentum transfer are considered (with the increment indicated in parenthesis).*

	0.1 GeV/c $< q <$ 0.5 GeV/c		1.5 GeV/c $< q <$ 1.9 GeV/c	
Model	Q	P	Q	P
"Fermi"	0.128	0.172	0.150 (+17%)	0.195 (+13%)
$\nu = 0$	0.129	0.161	0.168 (+30%)	0.193 (+20%)
$\nu = 1$	0.123	0.145	0.145 (+18%)	0.181 (+25%)

which is the same as the function $f_1(b_x)$ of Eq. (10.14), except for the higher power in the denominator. Note that this is *more* singular than the derivative of the "Fermi" phase shift function, that also here will serve as a reference. Again, we take

$$\begin{aligned} X'(b_x) &= g_1(b_x), \qquad b_x > 0, \\ X'(-b_x) &= -X'(b_x), \end{aligned} \tag{10.22}$$

and determine the phase shift function itself according to Eq. (10.16). We shall also compare with phase shift functions based on

$$g_3(b_x) \equiv \frac{-(A/\beta)(b_x/c)^3}{(1+\exp[(b_x-c)/\beta])^3}, \quad b_x > 0, \tag{10.23}$$

which is analogous to Eq. (10.17).

In Figs. 10.5 and 10.6 we show the derivative $X'(b_x)$ and $X(b_x)$, as defined by Eqs. (10.22) and (10.21) (solid) as well as by Eq. (10.23) (dashed). While the edges are similar, the interior is quite different.

Asymptotically, the points of stationary phase will be determined by

$$b_x \sim b_x^{\text{sing}} + \exp\left[\frac{i}{3}\left(-\frac{\pi}{2} + 3\pi + 2\pi n - \eta\right)\right]\left(\frac{|A|\xi}{q\beta}\right)^{1/3}, \tag{10.24}$$

$$b_x \sim b_x^{\text{sing}} + \exp\left[\frac{i}{3}\left(-\frac{\pi}{2} + 3\pi + 2\pi n - 3\eta\right)\right]\left(\frac{|A|\xi^3}{q\beta}\right)^{1/3}, \tag{10.25}$$

for g_1 and g_3, respectively.

In Fig. 10.7 we show the corresponding maps of possible trajectories of stationary points, analogous to Fig. 10.3 for the phase shift functions discussed in Section 10.2.1. Two cases are considered, $\nu = 0$ (Fig. 10.7(a) $X'(b_x) \sim b_x/\beta$ as $b_x \to 0$) and $\nu = 1$ (Fig. 10.7(b) $X'(b_x) \sim (b_x/\beta)^3$ as $b_x \to 0$). Each singularity here is surrounded by six lines (dashed) of vanishing $\text{Im}\, X'(b_x)$. Half of these are

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

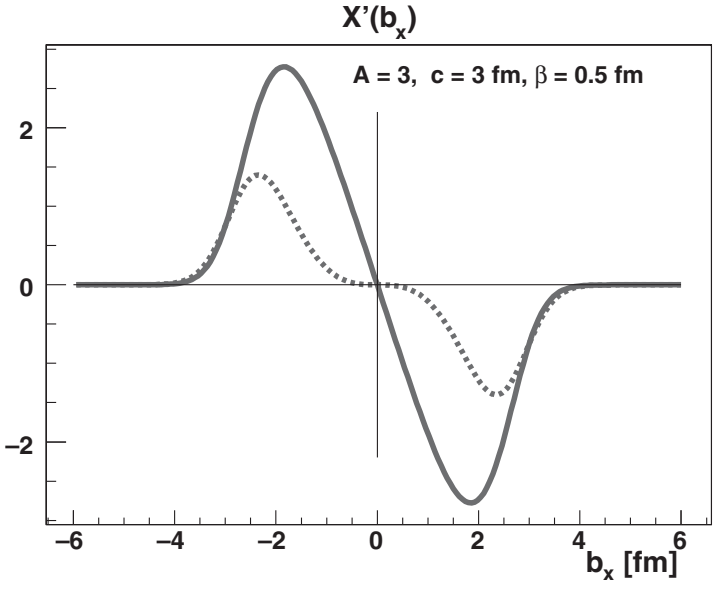


Figure 10.5 The derivative, Eq. (10.22), of phase shift functions based on $g_1(b_x)$ and $g_3(b_x)$. They are more singular than the "Fermi" one.

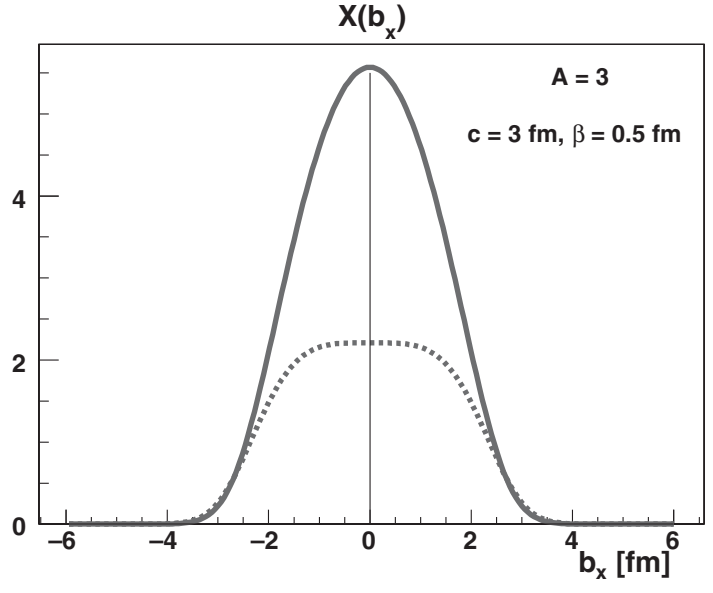


Figure 10.6 The phase shift functions $X(b_x)$ corresponding to the derivatives shown in Fig. 10.5.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use , available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

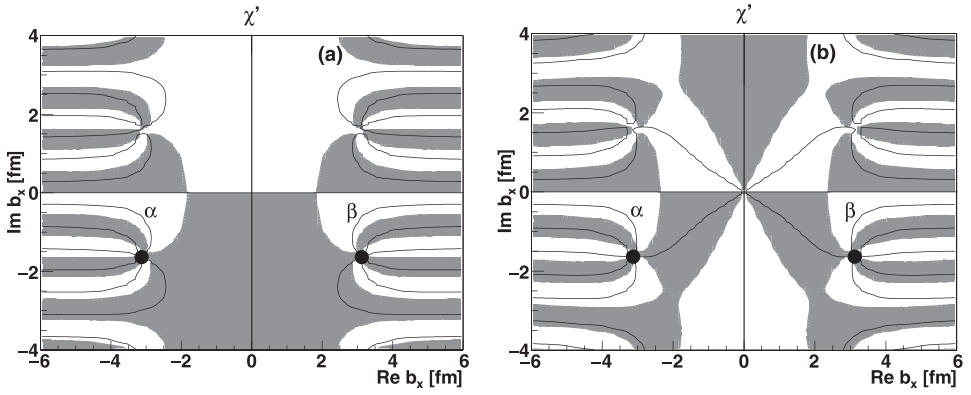


Figure 10.7 Regions of positive (white) and negative (shaded) values of $\mathrm{Re}\, X'(b_x)$, shown in the complex b_x-plane for the purely absorptive case. The figures are analogous to Fig. 10.3. Also indicated as dashed lines, are the locations where $\mathrm{Im}\, X'(b_x) = 0$. (a) $X'(b_x)$ determined by Eq. (10.21). (b) $X'(b_x)$ determined by Eq. (10.23).

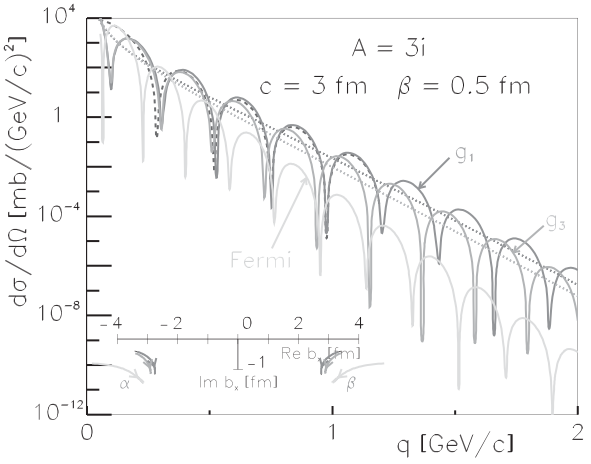


Figure 10.8 Differential cross section (denoted "g_1" and "g_3") corresponding to the sharp "Fermi" phase shift function whose derivatives are given by Eqs. (10.21) and (10.23).

in regions where $\mathrm{Re}\, X'(b_x) > 0$, but only one is actually on the path of integration. That is the one closest to the real axis.

Furthermore, in Fig. 10.8, we compare the differential cross sections of the present model, for both $g_1(b_x)$ and $g_3(b_x)$ with that obtained from the "Fermi" phase shift function, also shown in Figs. 8.10 and 8.12. The faster lengthening of the period is clearly seen in this figure. The inset shows the corresponding trajectories

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use , available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

Table 10.2 *Exponential slopes Q and periods P, for phase shift functions derived from their derivatives, g_1 and g_3, and, for reference, the "Fermi" phase shift function. Two ranges of momentum transfer are considered (with the increment indicated in parenthesis).*

	0.1 GeV/c $< q <$ 0.5 GeV/c		1.5 GeV/c $< q <$ 1.9 GeV/c	
Model	Q	P	Q	P
"Fermi"	0.128	0.172	0.150 (+17%)	0.195 (+13%)
$\nu = 0$	0.194	0.220	0.214 (+10%)	0.224 (+2%)
$\nu = 1$	0.168	0.218	0.180 (+7%)	0.212 (−3%)

of stationary points. They exhibit indeed a faster approach to the pole singularities, as compared with the "Fermi" phase shift function.

The fact that the differential cross section for the "Fermi" phase shift function is more steep, is reflected in Q being smaller for that case. On the other hand, the sharp "Fermi" phase shift function gives longer periods, reflected in a larger value of P. The faster approach to the pole singularities is reflected in P being larger than for the "Fermi" distribution already at low values of q, see Table 10.2.

10.2.3 Other Powers

As a generalization of the shapes defined by Eqs. (10.14) and (10.21), let us consider

$$p_1(b_x) \equiv \frac{-(A/\beta)(b_x/c)}{(1+\exp[(b_x-c)/\beta])^p}, \quad b_x > 0, \tag{10.26}$$

for p not necessarily an integer. We take the derivative of the phase shift function to be

$$\begin{aligned} X'(b_x) &= p_1(b_x), \qquad b_x > 0, \\ X'(-b_x) &= -X'(b_x), \end{aligned} \tag{10.27}$$

and determine the phase shift function itself according to Eq. (10.16). In analogy with Eqs. (10.17) and (10.23), we shall also compare with phase shift functions with a higher power in the numerator,

$$p_3(b_x) \equiv \frac{-(A/\beta)(b_x/c)^3}{(1+\exp[(b_x-c)/\beta])^p}, \quad b_x > 0. \tag{10.28}$$

For $p = 1$, we recover the "soft exponential" phase shift functions discussed in Section 10.2.1, whereas for $p = 3$ we recover the "sharp exponential" phase shift functions discussed in Section 10.2.2. For $p = 2$ we obtain a phase shift function

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use , available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

whose derivative has the same singularities as the "Fermi" distribution. However, the derivative of the "Fermi" distribution will differ by an additional exponential factor, as well as a different interpretation of the normalization factor χ_0.

For Eq. (10.8), with $X'(b_x) = q$ and for purely absorptive interactions, the approach to the singularity in the fourth quadrant for arbitrary powers p and ν is given by

$$b_x = b_x^{\text{sing}} + \beta \exp\left\{\frac{i}{p}\left[-\frac{\pi}{2} + p\pi + 2\pi n - \eta(1+2\nu)\right]\right\}\left(\frac{|A|}{q\beta}\right)^{1/p} \xi^{(1+2\nu)/p},$$
$$n = 0, 1, 2, \ldots, p-1. \tag{10.29}$$

For the singularities in the fourth quadrant, this can be written schematically as

$$b_x = b_x^{\text{sing}} + \beta\left(\frac{q_0}{q}\right)^{1/p} e^{i\varphi}. \tag{10.30}$$

The envelope of the differential cross section will then depend on q as

$$Q = -\operatorname{Im} b_x^{\text{sing}} - \beta\left(\frac{q_0}{q}\right)^{1/p} \sin\varphi, \tag{10.31}$$

whereas the period will vary as

$$P = \operatorname{Re} b_x^{\text{sing}} + \beta\left(\frac{q_0}{q}\right)^{1/p} \cos\varphi, \tag{10.32}$$

Some powers and the corresponding phases φ are collected in Table 10.3.

10.3 Additional Phases

It is now clear that a variety of asymptotic approaches to the singularities are possible. Another set of approaches is provided by multiplying the derivative $X'(b_x)$

Table 10.3 *Approach of the stationary points to the singularity in the fourth quadrant, for some powers p and phase φ, as defined by Eq. (10.30).*

p	φ
3	$5\pi/6 - \eta(1+2\nu)/3$
2	$-\pi/4 - \eta(1+2\nu)/2$
1	$\pi/2 - \eta(1+2\nu)$
1/2	$\pi - 2\eta(1+2\nu)$
1/3	$3\pi/2 - 3\eta(1+2\nu)$

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

of Eq. (10.7) by some function which is real and positive on the real axis, but which acquires a different phase in the neighborhood of the singularities. An example of such a factor is $\exp[(b_x - c)/\beta]$. This is obviously real and positive on the real axis, but approaches -1 in the neighborhood of the singularity. Mathematically, this can be described by replacing q by $-q$ in Eq. (10.10). This amounts to changing φ of Table 10.3 by π/p, and is in fact how one recovers the approach $\varphi = \pi/4$ for the "Fermi" phase shift function in the case $p = 2$.

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use , available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 14 Nov 2026 at 09:57:48, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781316219256.013