Philosophy of Science (2025), **92**, 1327–1337
doi:10.1017/psa.2025.10146

PSA PHILOSOPHY OF SCIENCE ASSOCIATION
PHILOSOPHY OF SCIENCE

CONTRIBUTED PAPER

Quo Vadis Wheeler–DeWitt Time?: Challenging Emergent Time in Quantum Cosmology

Nicola Bamonti[1,2], Enrico Cinti[2,3] and Marco Sanchioni[4]

[1]Department of Philosophy, Scuola Normale Superiore, Piazza dei Cavalieri 7, 56126 Pisa, Italy, [2]Department of Philosophy, University of Geneva, 5, rue de Candolle, CH-1211 Genève 4, Switzerland, [3]Institute for Logic, Language, and Computation, University of Amsterdam, 1090 GL Amsterdam, The Netherlands and [4]Istituto Universitario Sophia, Via San Vito n.28, Loppiano, 50064 Figline e Incisa Valdarno (FI), Italia
Corresponding author: Nicola Bamonti; Email: nicola.bamonti@sns.it

(Received 08 January 2025; revised 19 April 2025; accepted 24 July 2025; first published online 02 September 2025)

Abstract

This paper challenges the notion of emergent time in quantum cosmology by examining the reconciliation of the timeless Wheeler–DeWitt equation with the Universe's dynamical evolution. We critically evaluate the analogy between the Wheeler–DeWitt and Klein–Gordon equations, highlighting challenges for the identification of an emergent time parameter. We conclude that refining this analogy may lead to a better understanding of emergent time in quantum cosmology, though it is still not free from complications.

1. Introduction

The problem of identifying time from a timeless description is one of the oldest and most fascinating puzzles in quantum gravity (QG) and, in particular, in quantum cosmology (QC). In this latter context, in particular, it takes the form of the problem of reconciling the apparent timelessness of the Wheeler–DeWitt (WdW) equation with the dynamical evolution of our Universe, particularly its (semi)classical description in terms of fields evolving over a smooth spacetime manifold.

Various strategies have been pursued in the attempt to resolve this problem, ranging from the use of a semiclassical approximation to justify the emergence of time in the semiclassical regime to different ways of identifying a time variable prior to quantisation, which then makes the quantum regime temporal (Isham 1993). The former strategy, realised by applying the WKB (Wentzel–Kramers–Brillouin) approximation to the WdW equation, has enjoyed significant popularity in the QC literature. Important discussions of this approach in the physics literature are Vilenkin (1989), Kuchař (2011), and Kiefer (2012); in the philosophical literature

© The Author(s), 2025. Published by Cambridge University Press on behalf of Philosophy of Science Association. This is an Open Access article, distributed under the terms of the Creative Commons Attribution licence (https://creativecommons.org/licenses/by/4.0/), which permits unrestricted re-use, distribution and reproduction, provided the original article is properly cited.

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press

Thébault (2021) and Huggett and Thébault (2023) are recent in-depth discussions of the conceptual foundations of this strategy.

The emergent time strategy is often justified with a formal analogy between the WdW equation and the Klein–Gordon (KG) equation: in particular, one claims that, in the semiclassical limit, the volume of the Universe α in the WdW equation is analogous to the time parameter in the KG equation. Hence, since upon taking two formally analogous equations and studying them in the appropriate regime, we find that the time parameter in KG is mapped to the α parameter in WdW, it is natural to view α as an emergent time in WdW, or so the argument goes.

In this paper, we challenge this widespread view in QC by first arguing that the choice of the parameter α is underdetermined, and there are multiple equally good candidates for time (section 2). We then discuss two possible strategies to isolate α as the "correct" time parameter, one based on the analogy with KG (section 3), and the other based on a form of functionalism put forward by Huggett and Thébault (2023) (section 4): both strategies are found to be unsatisfactory. Finally, we discuss how to make the analogy between KG and WdW precise (section 5). However, we point out that this analogy leads to a very different picture of time than we might have expected, and it is not even clear whether, in this form, the analogy is sufficient to identify *time* in WdW (section 6). We then conclude (section 7).

2. On the choice of α as time

To apply canonical QG à la WdW to cosmology, we employ *minisuperspace*. Minisuperspace arises from the possibility of restricting the general problem of defining QG to simpler, highly symmetric spacetimes, reducing the dynamics to a finite-dimensional problem.

The most relevant application of minisuperspace is to a homogeneous Universe, described by a class of different cosmologies known as the nine Bianchi models (Bianchi 1897). In such spaces, the 3-geometries $\{h_{ij}\}$ are equivalent to the three scale factors $\{a(t), b(t), c(t)\}$ of the anisotropic Universe, since the homogeneity condition makes the theory invariant under 3-diffeomorphisms. Consequently, the space of physical states is reduced to a finite-dimensional subspace of Wheeler's Superspace (the space of 3-geometries) and the WdW wavefunctional is replaced by a wavefunction.

The construction of the Hamiltonian representation of Bianchi models is usually performed by using the variables $(\alpha(t), \beta_{\pm}(t))$, known as *Misner variables* (Misner 1969): the variable $\alpha(t)$ is related to the spatial volume of the Universe, while $\beta_{\pm}$ represent the spatial anisotropies and correspond to the two physical degrees of freedom of gravity. It is worth noting that the physical states $\psi(t, \alpha, \beta_{\pm})$ are defined in the space of the volume and anisotropies of the Universe, which does not coincide with the physical spacetime, thus precluding any clear notion of causality.

In this new framework, the canonical formalism gives a weakly vanishing Hamiltonian, known as the Bianchi–Misner Hamiltonian H_{BM}, since it is a linear combination of first-class constraints. Therefore, as in the case of QG, the Universe's wavefunction ψ does not depend on the time coordinate t. The WdW equation for the Bianchi models is

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press

$$H_{\mathrm{BM}}|\psi\rangle = 2\chi e^{-3\alpha/2}\hbar^2\Big[\partial_\alpha^2 - \partial_{\beta_\pm}^2 + U_{\mathrm{B}}(\alpha,\beta_\pm)\Big]\psi(\alpha,\beta_\pm) = 0, \tag{1}$$

where $\chi = 8\pi G$ and $U_{\mathrm{B}}(\alpha,\beta_\pm)$ is known as the Bianchi potential. Consistently with the finite-dimensional reduction of the general theory, it represents a single equation defining a three-dimensional quantum system. One can see an immediate similarity with a KG equation with a varying mass, highlighting how the volume of the Universe, related to the variable α, has the features of good internal time, while the $\beta_\pm$ variables represent the spatial coordinates. Hence, the evolution of the quantum Universe resembles that of a relativistic particle moving in a three-dimensional space (although the causal structure of the minisuperspace has no direct physical meaning).

However, α is not unique in having the features of a good time parameter: as is well known in the literature, in certain cosmological models, it is also legitimate to choose, e.g., a scalar field as time parameter (Rovelli and Smolin 1994; Isham 1993; see also Bamonti 2023 for a review on scalar fields as reference frames). For instance, in the quantum FLRW (Friedmann–Lemaître–Robertson–Walker) cosmological model, it is possible to use the matter degrees of freedom as the physical clock of the theory.

Considering the Bianchi–Misner superHamiltonian (1) and taking the isotropic case, we obtain the WdW equation in Misner variables for the vacuum, flat FLRW cosmological model with zero cosmological constant:

$$2\chi e^{-3\alpha/2}\hbar^2[\partial_\alpha^2 + U_{\mathrm{B}}(\alpha)]\psi(\alpha) = 0. \tag{2}$$

Following Kiefer (1988), we take the matter content to be represented entirely by a homogeneous scalar field ϕ with a potential $V(\phi)$. In such a way, the WdW equation of the model is

$$\hat{H}_{\mathrm{FLRW}}|\psi\rangle = 2\chi e^{-3\alpha/2}\hbar^2\Big[\partial_\alpha^2 - \partial_\phi^2 + V(\alpha,\phi)\Big]\psi(\alpha,\phi) = 0, \tag{3}$$

where $V(\alpha,\phi) = e^{3\alpha}V(\phi)$. Under the inflationary assumption of a non-interacting field in the limit of the cosmological singularity (Kolb and Turner 1994), necessary for QC, the potential $V(\phi)$ dependent on the derivatives of the scalar field ϕ can be neglected. Consequently, we recover the two-dimensional KG-like equation

$$2\chi e^{-3\alpha/2}\hbar^2\Big[\partial_\alpha^2 - \partial_\phi^2\Big]\psi(\alpha,\phi) = 0, \tag{4}$$

which describes a free, massless particle, where both the variable α and the scalar field ϕ can play the role of time (Bamonti et al. 2022, and references therein). Hence, it is underdetermined which parameter plays the role of time in the (semi)classical regime since we have no straightforward way to decide which variable between ϕ and α should play this role.

In the next section we will discuss some attempts at establishing α as the correct emergent time parameter.

3. KG–WdW (dis)analogy

We can break the impasse discussed above as to which variable can be identified as time via the analogy between (4) and the standard massless KG equation:

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press

$$\left(\frac{c^2}{\hbar^2}\frac{\partial^2}{\partial t^2} - \hbar^2\nabla^2\right)\psi\left(t, \vec{x}\right) = 0. \tag{5}$$

Under this analogy, as discussed, e.g., in Kiefer and Singh (1991), the time parameter t maps to α in (4), while the matter degrees of freedom ϕ in (4) correspond to Minkowski 3-space $\vec{x}$ in (5). Hence, by appealing to this analogy we can break the symmetry between ϕ and α in (4), and identify α as the emergent time parameter.[1]

In what follows, however, we will show that this analogy is not straightforward.

The analogy between the KG and the WdW equations is fundamentally grounded in their structural similarity: both represent second-order differential equations that impose constraints on the wavefunction. This similarity lays the groundwork for the claim that the parameter α may fulfil an analogous role in each equation, despite the arguments of the preceding section. A crucial point for this analogy, in particular as concerns the temporal status of α, is that the *dynamics* of the two theories must be, in some sense, analogous. Since the temporal status of α emerges from dynamical considerations, as detailed in section 2, a failure in a dynamical analogy between WdW and KG would imply that the two theories are disanalogous in the regime relevant to justifying the use of α as time; hence, for α to count as time, we need a deep similarity in the *dynamical behaviour* of each theory.

In any quantum field theory, whether it describes gravitational or non-gravitational fields, the dynamic of the theory is determined by its inner product, as it underpins conservation laws and the unitary evolution of quantum states. Consequently, whether α can assume the role of time is the question of how analogous the inner products of the KG and WdW equations are. If these inner products were analogous, we would have a correspondence in the dynamics of both theories, thus validating the interpretation of α as describing time. However, a lack of analogy between the inner products suggests that the dynamics differ significantly between the two theories. This divergence would indicate that the role of time could be distinctly conceptualised within each theory, cautioning against a straightforward equivalence of the role played by α in the two theories. In this second case, the justification was merely on the formal structural similarities of the equations without a deeper dynamical analogy. To understand the status of the (dis)analogy of the inner product in the two theories, we follow Witten (2023).

The KG inner product between two wavefunctions ϕ_1 and ϕ_2, both satisfying the KG equation, is given by

$$\langle\phi_1|\phi_2\rangle_{\mathrm{KG}} = i\int_{\Sigma} d\Sigma^{\mu}\bar{\phi}_1\overleftrightarrow{\partial}_{\mu}\phi_2, \tag{6}$$

where Σ is any Cauchy hypersurface in a spacetime manifold M, $d\Sigma_{\mu}$ is the surface element over which the integral is performed, and $\bar{\phi}_1\overleftrightarrow{\partial}_{\mu}\phi_2 = \bar{\phi}_1\,\partial_{\mu}\phi_2 - \bar{\phi}_2\,\partial_{\mu}\phi_1$, with $\bar{\phi}$ the complex conjugate of ϕ.

[1] One might worry that, by changing the signature in, e.g., (5), one could exchange which parameter in (4) corresponds to the time coordinate t of (5), hence leading back to our original problem from section 2. However, it seems reasonable to demand that, if we change the signature of (5), we should similarly change the signature of (4) to respect the analogy between the two equations; doing this, however, ensures that in both signatures, t in (5) corresponds to α in (4).

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press

Defining an inner product for solutions to the WdW equation is significantly more involved than for the KG equation. Let us sketch the features of the WdW inner product that are most relevant to our discussion, though we will not give a complete construction. The general form of such an inner product is

$$\langle \phi_1 | \phi_2 \rangle_{\mathrm{WdW}} = \int_{\mathrm{Met}/3-\mathrm{Diff}} Dh \bar{\phi}_1(h) \prod_{\vec{x} \in S} \delta(\mathcal{H}(\vec{x})) \phi_2(h), \tag{7}$$

where the integral spans the space of 3-metrics (Met) on a hypersurface S modulo 3-diffeomorphisms (Met/3-Diff), also known as Wheeler's Superspace (see section 2). This space encompasses the 3-metric configurations considered equivalent under spatial diffeomorphism transformations, highlighting the gauge invariance intrinsic to General Relativity. The wavefunctions ϕ_1 and ϕ_2 are functions of the 3-metric h, reflecting the quantum gravitational states as configurations of the geometry itself. The presence of the delta functions $\delta(\mathcal{H}(\vec{x}))$ within the integral enforces the Hamiltonian constraint at every point $\vec{x}$ on the hypersurface S, ensuring that only configurations satisfying the WdW equation contribute to the inner product. This mechanism guarantees that the inner product is computed over "physical" states—those that are solutions to the Hamiltonian constraint $\mathcal{H}\psi = 0$.

This inner product also defines the equivalence relation

$$\psi(h) \equiv \psi(h) + \sum_i \mathcal{H}(\vec{x}_i) \chi_i(h), \tag{8}$$

which plays a crucial role in this context. It states that two quantum states are equivalent if their difference can be expressed as a sum of terms, each term being the application of the Hamiltonian operator $\mathcal{H}(\vec{x}_i)$ at various points $\vec{x}_i$ to arbitrary functions $\chi_i(h)$. This relation is a quantum analogue of the classical Hamiltonian constraint, extending its role from eliminating non-physical configurations to defining an equivalence class of quantum states. By integrating over (Met/3-Diff) and incorporating the product of delta functions, the inner product (7) inherently respects the equivalence relation, ensuring that the path integral quantisation framework accommodates diffeomorphism invariance.

This revised interpretation of the constraint operators underlying the inner product (7) carries a significant advantage: this formulation allows the WdW theory to be quantised using the BRST (Becchi–Rouet–Stora–Tyutin) formalism (Becchi et al. 1976; Henneaux and Teitelboim 1992). In BRST quantisation one has to introduce ghost fields, which transform like the generators of the gauge symmetry—diffeomorphisms in our framework—but with opposite statistics. Thus, in our case, we would have to introduce a new field $c^\mu(t, \vec{x})$ that transforms as the generator of diffeomorphisms but has a Fermi–Dirac statistics. Having introduced the ghost fields $c^\mu(t, \vec{x})$, one can then define the BRST charge Q as

$$Q = \int_\Sigma d^{D-1}x \sqrt{h} \big(c^0(t, \vec{x}) \mathcal{H}(\vec{x}) + c^i(t, \vec{x}) P_i(\vec{x}) + \cdots \big), \tag{9}$$

where c^0 is the temporal component of the ghost field $c^\mu(t, \vec{x})$ and is coupled with $\mathcal{H}$, while $c^i(t, \vec{x})$ is its spatial component, and it is coupled with $P_i(\vec{x})$. The BRST charge Q obeys $Q^2 = 0$ and one can thus define a *cohomology structure*, i.e., a space of states that satisfy $Q\psi = 0$ modulo the equivalence relation $\psi \equiv \psi + Q\chi$ for any other state χ.

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press

Cohomology is key in BRST quantisation because it defines the physical states of the theory: physical states belong to the cohomology generated by the BRST charge. This mathematical structure mirrors the Hamiltonian constraint in the WdW equation: the condition $Q\psi = 0$ gives both $\mathcal{H}\psi = 0$ and $P_i(\vec{x})\psi = 0$, which are, indeed, the traditional WdW constraints. Leaving aside the complex derivation of the inner product, one can rewrite the inner product (7) in the form

$$\langle \phi_1 | \phi_2 \rangle_{\mathrm{WdW}} = \int Dh_0 Dc^i D\bar{c}^j \bar{\phi}_1 (\det \Xi) \phi_2 . \tag{10}$$

Within this framework, the WdW inner product is expressed as a path integral over the metric h and ghost fields. A key aspect of this equation is the presence of $\det\Xi$, a determinant arising from the integration of additional ghost fields, analogous to the treatment of processes in quantum field theories like QCD.

Given equations (6) and (10) for KG and WdW respectively, we are now equipped to compare their inner products. Despite the structural similarity of the equations of the two theories, there are two crucial differences in their inner products:

- **Definiteness of the inner product:** It is well known that, unlike the positive-definite inner product found in non-relativistic quantum mechanics, the KG inner product can yield negative values for the norm of specific solutions. In fact, the inner product of the KG can be negative, zero, or positive, reflecting the indefinite nature of the norm in the relativistic context. However, Witten (2023) shows that the inner product of the (revised version) of the WdW theory is positive-definite (as it is supposed to be for any theory of gravity).
- **Status of the Hamiltonian constraint:** Another critical difference between KG and WdW lies in the status of the Hamiltonian constraint. In WdW, the constraint imposes $\mathcal{H}(\vec{x}) = 0$ for each point $\vec{x}$ in a Cauchy hypersurface Σ. However, the Hamilton constraint in KG theory imposes $\mathcal{H} = 0$, $\mathcal{H}$ being a single operator. Thus, the Hamilton constraint of the WdW theory can be associated with an *infinite family* of KG-like Hamiltonian constraints. The same crucial difference can be seen from another point of view: the KG inner product of equation (6) is defined on a codimension-1 hypersurface U, while the WdW inner product (10) is defined on a submanifold of infinite codimension.

4. Time and functionalism

Having seen how attempts at justifying the use of α as a time parameter in the semiclassical approximation based on an analogy with the Klein–Gordon equation fail, let us now move to consider a different approach, i.e., the functional strategy developed in Huggett and Thébault (2023). Without delving into unnecessary details, the point that Huggett and Thébault (2023) wish to make is that we can identify a time parameter by the role it plays in a certain physical theory, along the lines of how spacetime is analysed in a functionalist approach (Knox 2013; Knox 2019; Baker 2021).

In particular, Huggett and Thébault (2023) suggest that time should first be divided into two aspects: the *chrono-metric* structure and the *chrono-directed* structure; the first one encodes a relation of betweenness between events, including a temporal metric to define temporal distances between such events, while the second one encodes a

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press

directionality for temporal relations, i.e., it tells whether an event is before or after another event to which it is temporally related. For our purposes, it will be sufficient to discuss the status of the chrono-metric structure vis-à-vis emergent time, as this is already enough to see the problems that our discussion raises for this strategy to justify α as the time parameter.

Huggett and Thébault (2023) in general want to claim that any structure that plays the time role will satisfy the chrono-metric properties described above, i.e., it will fix a betweenness relation between events and duration for intervals between events. At the same time, (Huggett and Thébault 2023, 4) also emphasise that given a certain physical theory, we should be able to identify the time parameter not by checking that it satisfies certain roles but rather that it corresponds, in the appropriate regime where time indeed emerges, to the time parameter of a theory whose temporal structure we antecedently understand. The authors then suggest that such a role, in the case of the WdW equation, is indeed played by the parameter α.

From this discussion, it is immediate to see the problem for this strategy vis-à-vis our discussion in section 2. Our discussion above points out a fundamental ambiguity in what counts as a time parameter, in the sense that there are multiple parameters with a prima facie equally good claim to the title of time. Hence, the functionalist strategy that Huggett and Thébault (2023) use cannot resolve such a problem since the problem for us is not to identify a parameter playing the time role but rather that we have too many parameters playing such a role.

5. A good analogy between WdW and KG

Despite the crucial differences between the inner products of the KG and WdW theories analysed in section 3, an underlying analogy exists when considering the KG theory from an appropriate perspective.

We find common ground by reformulating the KG theory as a covariant theory on a one-dimensional worldline, effectively introducing a gravity-induced gauge symmetry akin to that found in the WdW theory. In this reformulation, the metric on the worldline is represented by $e^2(t)$, allowing the KG action to be rewritten to mirror the gauge symmetry considerations of the WdW discussion. This approach bridges the conceptual gap, demonstrating that the inner products of the two theories exhibit a crucial analogy rooted in their shared symmetry structures and gauge invariance principles.

In this framework, the KG action can be rewritten as

$$S = \frac{1}{2}\int_{\lambda} dt\left(-\frac{1}{e}\eta^{\mu\nu}\frac{dX_\mu}{dt}\frac{dX_\nu}{dt} - em^2\right), \tag{11}$$

where λ is an arbitrary coordinate parametrising the worldline. This action, invariant under reparametrisation of λ, captures the essence of KG's theory but incorporates a form of gauge symmetry through the dynamics of the one-dimensional worldline.

The equivalence of this reparametrised action to the KG theory becomes evident through the Euler–Lagrange equation for the metric field e, which yields the KG Hamiltonian constraint. This result is derived as follows:

$$0 = -\frac{\partial S}{\partial e} = \frac{1}{e^2}\eta^{\mu\nu}\frac{dX_\mu}{dt}\frac{dX_\nu}{dt} + m^2 = \eta^{\mu\nu}\Pi_\mu\Pi_\nu + m^2 = -\eta^{\mu\nu}\frac{\partial}{\partial^\mu}\frac{\partial}{\partial^\mu} + m^2, \tag{12}$$

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press

where the conjugate momenta Π_μ are defined as $\Pi_\mu = (1/e)\eta_{\mu\nu}dX^\nu/dt$, and, upon quantisation, these momenta become $\Pi_\mu = -i\partial/\partial X^\mu$.

By establishing a covariant version of the KG theory on the worldline, we can extend the application of BRST symmetry techniques—previously utilised in reformulating the WdW theory—to this context. This process necessitates introducing ghost fields associated with the theory's symmetry generators. Specifically, for the covariant KG theory, we introduce a ghost field c tied to infinitesimal reparametrisations of the worldline. Given the need for this ghost field to possess opposite statistics compared to the field e, c is defined as a "Grassmann number".

Without entering into unnecessary details of this derivation, it is possible to show that the inner product resulting from the BRST derivation aligns precisely with the KG inner product in (6). This derivation, revealing the structural analogy between the KG and WdW theories through BRST quantisation, is important in understanding how the ghost fields' integration, giving rise to the determinants, underpins both theories' inner products. Specifically, the KG's inner product $\overleftrightarrow{\partial}_\mu$ term and the WdW's $\det\Xi$ emerge as determinants resulting from integrating out ghost fields associated with their respective BRST symmetries. This structural similarity highlights a profound connection between the two theories, even as they manifest differently. The $\overleftrightarrow{\partial}_\mu$ in the KG theory and $\det\Xi$ in the WdW theory both function as ghost determinants but are tied to distinct BRST symmetries, reflective of their respective theories' underpinnings in relativistic quantum mechanics and QG. Consequently, these determinants contribute differently to the nature of each theory's inner product: $\overleftrightarrow{\partial}_\mu$ results in an indefinite inner product, aligning with the expectation for relativistic theories; on the other hand, $\det\Xi$ yields a positive-definite inner product, consistent with the requirements for QG theories.

Having identified a plausible analogy between WdW and KG, this can prima facie justify the identification of α as a time variable, modulo the caveats just highlighted. In the following section we will discuss some complications of this approach.

6. Where did time go?

We have seen in the previous section how one can justify using α as a time parameter, and that the analogy between the WdW and KG equations plays a crucial role. At the same time, such an analogy requires significant adjustments to justify the role of α as time. Let us briefly discuss some of the consequences and limitations of the argument in section 5.

First of all, however, let us emphasise what the argument of the previous section does indeed achieve. Insofar as it establishes a good formal analogy between the KG equation and the WdW equation, the argument of section 5 ensures that we can at least think of α as the right parameter when we look for something that formally satisfies some of the properties of a time parameter. So, at the very least, the underdetermination problem raised in section 2 is prima facie resolved by the discussion in section 5. However, the analogy of section 5 does not establish whether α behaves as we would expect time to behave, despite being formally analogous to the

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press

time parameter of the KG equation. For example, it is still the case that the Hamiltonian constraint of the WdW equation is pointwise. This fact implies that, as the Hamiltonian constraint is the origin of the alleged time flow parametrised by α, this flow is pointwise, not the kind of global flow we usually associate with time. In other words, the emergent time that we are defining from the WdW equation does not describe the flow of a three-dimensional surface along the time dimension but rather describes the flow of an infinite number of points, each flowing independently through their own time (fixed by the Hamiltonian constraint at that point).[2]

Another related but more fundamental problem with interpreting α as the time lies with how we obtained the analogy with the KG equation in BRST quantisation. In particular, we first moved to a worldline formulation of the theory to arrive at a BRST expression for the quantum KG equation. Then we noticed that in this context, there is a Hamiltonian constraint that enforces the invariance of dynamics under arbitrary reparametrisations of the coordinate λ, enforcing that this coordinate does not encode physically relevant information. This Hamiltonian constraint then, upon BRST quantisation, leads to an expression analogous to the WdW equation and hence allows for the analogy between the two equations, particularly with respect to time, to go through. However, what is unclear in this procedure is whether the Hamiltonian constraint on the worldline still encodes a time parameter. If this condition fails, the analogy cannot give us time in the WdW equation.

Let us briefly comment on why one might be sceptical regarding the temporal status of the Hamiltonian constraint in the KG equation.

As we mentioned above, the Hamiltonian constraint in this context enforces a kind of reparametrisation invariance on the worldline; as such, it does not seem to have any straightforward relation to physical time. One can relate it to time by noticing that the coordinate λ, parametrising the worldline, which extends only through time, keeps track of the time expired across two points on the worldline. However, this interpretation only works because, in the KG equation, we have a background spacetime that gives us a physical definition of the time parameter. If we did not have such a background, then the Hamiltonian constraint would have told us that the theory is invariant under a choice of coordinate on the worldline, which either has no relation to time or, at best, enforces the same kind of disappearance of time that we found problematic in the first place when looking at the WdW equation. In other words, by moving to BRST quantisation and fixing the analogy between KG and WdW, we have made KG more similar to WdW, and so harder to interpret, rather than simplifying the interpretation of WdW. In fact, without a background framework, KG would encounter issues similar to those of WdW. This is exactly the scenario we encounter with WdW, which lacks a spacetime background, thereby complicating the interpretation of α as time, despite the extensive analogy with KG. Indeed, once we have a well-defined analogy between KG and WdW, to identify time in KG we need to appeal to the spacetime background, which is the aspect in which the two theories are disanalogous. Hence, it is precisely in identifying the time parameter that the analogy between KG and WdW breaks down.

[2] A related notion is the idea that relativistic time is *many-fingered* (Anderson 2012).

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press

7. Conclusion

We examined the WdW equation's implications for the concept of time in QC, especially the challenge of defining an emergent time in a timeless Universe. Through a critical analysis of the analogy between the WdW and KG equations, we highlighted the difficulties in identifying a suitable time parameter in QC.

Our discussion underscores the complexity and ambiguity in conceptualising time in this domain, calling for a more sophisticated understanding of the nature of time and dynamics in the WdW context.

Indeed, the disanalogies in the inner products derived from KG and WdW underscore profound differences in their dynamics. While both theories exhibit structural similarities, the different nature of their inner products dictates fundamentally distinct dynamical behaviours. This disparity calls into question the straightforward use of analogy to justify treating the parameter α as time in QC.

However, our discussion also reveals a profound analogy between the two theories, emphasising the crucial role of quantising gauge symmetries. While not immediately decisive for the question of α's role, this important connection points to a rich vein of theoretical insight that could enhance our understanding of QG and relativistic quantum field theories. Exploring these implications will be left for future work.

In conclusion, our investigation into the WdW equation reveals that the quest for an emergent time in QC faces significant conceptual hurdles, suggesting the development of frameworks that can accommodate the peculiar dynamical features of QC and deepen our understanding of our Universe's quantum gravitational structure.

Acknowledgments. We thank the organisers and participants of PSA2024 for very helpful comments.

Funding Informations. EC's research has been supported by the SNSF project "Space, time, and causation in quantum gravity" and the NWA project "Is there space and time for experimental philosophy?". The other authors' research received no external funding.

Declarations. The authors declare that there is no conflict of interest and that no ethical approval was required for this study.

References

Anderson, Edward. 2012. "Problem of Time in Quantum Gravity". *Annalen der Physik* 524 (12):757–86. https://doi.org/10.1002/andp.201200147.

Baker, David John. 2021. "Knox's Inertial Spacetime Functionalism (and a Better Alternative)". *Synthese* 199 (Suppl 2):277–98. https://doi.org/10.1007/s11229-020-02598-z.

Bamonti, Nicola. 2023. "What Is a Reference Frame in General Relativity?" *Preprint.* https://doi.org/10.48550/ARXIV.2307.09338.

Bamonti, Nicola, Andrea Costantini, and Giovanni Montani. 2022. "Features of the Primordial Universe in $f(R)$f(R)-Gravity as Viewed in the Jordan Frame". *Classical and Quantum Gravity* 39 (17):175011. https://doi.org/10.1088/1361-6382/ac7694.

Becchi, Carlo, Alain Rouet, and Raymond Stora. 1976. "Renormalization of Gauge Theories". *Annals of Physics* 98 (2):287–321. https://doi.org/10.1016/0003-4916(76)90156-1.

Bianchi, Luigi. 1897. *Sugli spazi a tre dimensioni che ammettono un gruppo continuo di movimenti: memoria.* Tipografia della R. Accademis dei Lincei. Available at: https://media.accademiaxl.it/memorie/S3-VXI-1898/Bianchi267-352.pdf.

Henneaux, Marc and Claudio Teitelboim. 1992. *Quantization of Gauge Systems*. Princeton, NJ: Princeton University Press.

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press

Huggett, Nick and Karim P. Y. Thébault. 2023. "Finding Time for Wheeler–DeWitt Cosmology". *Preprint*. https://doi.org/10.48550/arXiv.2310.11072.

Isham, Chris J. 1993. "Canonical Quantum Gravity and the Problem of Time". In *Integrable Systems, Quantum Groups, and Quantum Field Theories*, edited by L. Alberto Ibort and Miguel A. Rodríquez, 157–287. Dordrecht: Springer. https://doi.org/10.1007/978-94-011-1980-1_6.

Kiefer, Claus. 1988. "Wave Packets in Minisuperspace". *Physical Review D* 38 (6):1761–72. https://doi.org/10.1103/physrevd.38.1761.

Kiefer, Claus. 2012. *Quantum Gravity*, 3rd ed. Oxford: Oxford University Press.

Kiefer, Claus and Tejinder P. Singh. 1991. "Quantum Gravitational Corrections to the Functional Schrödinger Equation". *Physical Review D* 44 (4):1067. https://doi.org/10.1103/PhysRevD.44.1067.

Knox, Eleanor. 2013. "Effective Spacetime Geometry". *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 44 (3):346–56. https://doi.org/10.1016/j.shpsb.2013.04.002.

Knox, Eleanor. 2019. "Physical Relativity from a Functionalist Perspective". *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 67:118–24. https://doi.org/10.1016/j.shpsb.2017.09.008.

Kolb, Edward W. and Michael S. Turner. 1994. *The Early Universe*. Boulder, CO: Westview Press.

Kuchař, Karel V. 2011. "Time and Interpretations of Quantum Gravity". *International Journal of Modern Physics D* 20 (supp01):3–86. https://doi.org/10.1142/s0218271811019347.

Misner, Charles W. 1969. "Quantum Cosmology. I". *Physical Review* 186 (5):1319–27. https://doi.org/10.1103/physrev.186.1319.

Rovelli, Carlo and Lee Smolin. 1994. "The Physical Hamiltonian in Nonperturbative Quantum Gravity". *Physical Review Letters* 72 (4):446–49. https://doi.org/10.1103/physrevlett.72.446.

Thébault, Karim P. Y. 2021. "The Problem of Time". In *The Routledge Companion to Philosophy of Physics*, edited by Eleanor Knox and Alastair Wilson, 386–400. Routledge. https://philsci-archive.pitt.edu/id/eprint/15795.

Vilenkin, Alexander. 1989. "Interpretation of the Wave Function of the Universe". *Physical Review D* 39 (4):1116–22. https://doi.org/10.1103/physrevd.39.1116.

Witten, Edward. 2023. "A Note on the Canonical Formalism for Gravity". *Advances in Theoretical and Mathematical Physics* 27 (1):311–80. https://doi.org/10.4310/atmp.2023.v27.n1.a6.

Cite this article: Bamonti, Nicola, Enrico Cinti, and Marco Sanchioni. 2025. "Quo Vadis Wheeler–DeWitt Time?: Challenging Emergent Time in Quantum Cosmology." *Philosophy of Science* 92 (5):1327–1337. https://doi.org/10.1017/psa.2025.10146

https://doi.org/10.1017/psa.2025.10146 Published online by Cambridge University Press