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ASTIN Bulletin

Volume **38** (1), 2008

Asimit, A. V. & Jones, B. L. *Asymptotic Tail Probabilities for Large Claims Reinsurance of a Portfolio of Dependent Risks.* 147-159. We consider a dependent portfolio of insurance contracts. Asymptotic tail probabilities of the ECOMOR and LCR reinsurance amounts are obtained under certain assumptions about the dependence structure.

Barrieu, P. & Scandolo, G. *General Pareto Optimal Allocations and Applications to Multi-period Risks.* 105-136. In this paper, we consider the problem of Pareto optimal allocation in a general framework, involving preference functionals defined on a general real vector space. The optimization problem is equivalent to a modified sup-convolution of the different agents' preference functionals. The results are then applied to a multi-period setting and some further characterization of Pareto optimality for an allocation is obtained for expected utility for processes.

Boratynska, A. *Posterior Regret G-Minimax Estimation of Insurance Premium in Collective Risk Model.* 277-291. The collective risk model for the insurance claims is considered. The objective is to estimate a premium which is defined as a functional H specified up to an unknown parameter (the expected number of claims). Four principles of calculating a premium are applied. The Bayesian methodology, which combines the prior knowledge about a parameter with the knowledge in the form of a random sample is adopted. Two loss functions (the square-error loss function and the asymmetric loss function LINEX) are considered. Some uncertainty about a prior is assumed by introducing classes of priors. Considering one of the concepts of robust procedures the posterior regret G-minimax premiums are calculated, as an optimal robust premiums. A numerical example is presented.

Chan, J. S. K., Choy, S. T. B. & Makov, U. E. *Robust Bayesian Analysis of Loss Reserves Data Using the Generalized-t Distribution.* 207-230. This paper presents a Bayesian approach using Markov chain Monte Carlo methods and the generalized-t (GT) distribution to predict loss reserves for the insurance companies. Existing models and methods cannot cope with irregular and extreme claims and hence do not offer an accurate prediction of loss reserves. To develop a more robust model for irregular claims, this paper extends the conventional normal error distribution to the GT distribution which nests several heavy-tailed distributions including the Student-t and exponential power distributions. It is shown that the GT distribution can be expressed as a scale mixture of uniforms (SMU) distribution which facilitates model implementation and detection of outliers by using mixing parameters. Different models for the mean function, including the log-ANOVA, log-ANCOVA, state space and threshold models, are adopted to analyze real loss reserves data. Finally, the best model is selected according to the deviance information criterion (DIC).

Couret, J. & Venter, G. *Using Multi-dimensional Credibility to Estimate Class Frequency Vectors in Workers Compensation.* 73-85. The US workers compensation system is different from those in many countries, but it is reinsured in the world-wide market and so has

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international impact. From its origin in the early 20th century it has been a laboratory for actuarial credibility techniques. In recent years deductibles have been increasing, so that fairly high excess coverage is now commonplace. This puts growing emphasis on estimation of the percentage of loss that is excess of high deductibles. A key element of the excess percentage is the frequency of loss by injury type. Fatalities and permanent disabilities cost more than other injury types, so when they have high relative frequency, more of the claims cost arises from large losses. The vector of claim frequency by injury type can be estimated by class of business using multi-dimensional credibility techniques. Historically the fraction of costs excess of various retentions has been calculated for large groups of classes (hazard groups) and not individual classes. We show, by testing a hold-out sample, that credibility estimation by class does produce additional information in comparison to a widely-used seven-hazard-group system.

Dickson, D. C. M. *Some Explicit Solutions for the Joint Density of the Time of Ruin and the Deficit at Ruin.* 259-276. Using probabilistic arguments we obtain an integral expression for the joint density of the time of ruin and the deficit at ruin. For the classical risk model, we obtain the bivariate Laplace transform of this joint density and invert it in the cases of individual claims distributed as Erlang(2) and as a mixture of two exponential distributions. As a consequence, we obtain explicit solutions for the density of the time of ruin.

Englund, M., Guillén, M., Gustafsson, J., Nielsen, L. H. & Nielsen, J. P. *Multivariate Latent Risk: A Credibility Approach.* 137-146. We investigate a concept of multivariate pricing, which includes claim history for more than one line of business and is a generalization of the Bühlmann-Straub model. The multivariate credibility model is extended to allow for the age of claims to influence the estimation of future claims. The model is applied to data from a portfolio of commercial lines of business.

Falin, G. I. *On the Optimal Pricing of a Heterogeneous Portfolio.* 161-170. We apply simple geometrical arguments to show that well-known approaches to determine the premium in insurance contract minimize a weighted squared differences both between the individual premiums and the individual claims and between the total premiums for classes of homogeneous risks and total claims from these blocks of business.

Frostig, E. *On Risk Model with Dividends Payments Perturbed by a Brownian Motion: An Algorithmic Approach.* 183-206. Assume that an insurance company pays dividends to its shareholders whenever the surplus process is above a given threshold. In this paper we study the expected amount of dividends paid, and the expected time to ruin in the compound Poisson risk process perturbed by a Brownian motion. Two models are considered: In the first one the insurance company pays whatever amount exceeds a given level b as dividends to its shareholders. In the second model, the company starts to pay dividends at a given rate, smaller than the premium rate, whenever the surplus up-crosses the level b. The dividends are paid until the surplus down-crosses the level a, $a < b$. We assume that the claim sizes are phase-type distributed. In the analysis we apply the multidimensional Wald martingale, and the multidimensional Asmussesn and Kella martingale.

Kraft, H. & Steffensen, M. *Optimal Consumption and Insurance: A Continuous-time Markov Chain Approach.* 231-257. Personal financial decision making plays an important role in modern finance. Decision problems about consumption and insurance are in this article modelled in a continuous-time multi-state Markovian framework. The optimal solution is derived and studied. The model, the problem, and its solution are exemplified by two special cases: In one model the individual takes optimal positions against the risk of dying; in another model the individual takes optimal positions against the risk of losing income as a consequence of disability or unemployment.

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LI, S. & LU, Y. *The Decompositions of the Discounted Penalty Functions and Dividends-penalty Identity in a Markov-modulated Risk Model.* 53-71. In this paper, we study the expected discounted penalty functions and their decompositions in a Markov-modulated risk process in which the rate for the Poisson claim arrivals and the distribution of the claim amounts vary in time depending on the state of an underlying (external) Markov jump process. The main feature of the model is the flexibility modeling the arrival process in the sense that periods with very frequent arrivals and periods with very few arrivals may alternate. Explicit formulas for the expected discounted penalty function at ruin, given the initial surplus, and the initial and terminal environment states, are obtained when the initial surplus is zero or when all the claim amount distributions are from the rational family. We also investigate the distributions of the maximum surplus before ruin and the maximum severity of ruin. The dividends-penalty identity is derived when the model is modified by applying a barrier dividend strategy.

MACK, T. *The Prediction Error of Bornhuetter/Ferguson.* 87-103. Together with the Chain Ladder (CL) method, the Bornhuetter/Ferguson (BF) method is one of the most popular claims reserving methods. Whereas a formula for the prediction error of the CL method has been published already in 1993, there is still nothing equivalent available for the BF method. On the basis of the BF reserve formula, this paper develops a stochastic model for the BF method. From this model, a formula for the prediction error of the BF reserve estimate is derived. Moreover, the model gives important advice on how to estimate the parameters for the BF reserve formula. E.g. it turns out that the appropriate BF development pattern is different from the CL pattern. This is a nice add-on as it makes BF to a standalone reserving method which is fully independent from CL. The other parameter required for the BF reserve is the well-known initial estimate for the ultimate claims amount. Here the stochastic model clearly shows what has to be meant with 'initial'. In order to apply the formula for the prediction error, the actuary must assess his uncertainty about both sets of parameters, about the development pattern and about the initial ultimate claims estimates. But for both, much guidance can be drawn from the estimates itself and from the run-off data given. Finally, a numerical example shows how the resulting prediction error compares to the one of the CL method.

MILIDONIS, A. & GRACE, M. F. *Tax-deductible Pre-event Catastrophe Loss Reserves.* 13-51. After Hurricane Andrew the U.S. Congress entertained proposals to allow insurers to employ tax-deferred loss reserves. Interest was strong at first, but as the events receded interest waned. However, after the most recent severe hurricane seasons the proposals are again being discussed. In this paper we examine the institution of catastrophe loss reserves in a stylized model of insurance provisions. First, we find that the benefits of the tax-deferred loss reserves depend on the actuarial assumptions regarding the expected loss distribution. Second, we make the first attempt at estimating the change in consumer behavior and the social welfare implications for permitting tax deferred loss reserves. In sum, we find under specific circumstances there are large welfare gains for allowing the tax deferral of reserves.

PELSSER, A. *On the Applicability of the Wang Transform for Pricing Financial Risks.* 171-181. In an arbitrage-free economy, it is well-known that financial risks can be priced using equivalent martingale measures. We establish in this paper that, for general stochastic processes, the Wang Transform does not lead to a price which is consistent with the arbitrage-free price. Based on these results we must conclude that the Wang Transform cannot be a universal framework for pricing financial and insurance risks.

PORTEOUS, B. T. & TAPADAR, P. *The Impact of Capital Structure on Economic Capital and Risk Adjusted Performance.* 341-380. The impact that capital structure and capital asset allocation have on financial services firm economic capital and risk adjusted performance is considered. A stochastic modelling approach is used in conjunction with banking and insurance examples.

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It is demonstrated that gearing up Tier 1 capital with Tier 2 capital can be in the interests of bank Tier 1 capital providers, but may not always be so for insurance Tier 1 capital providers. It is also shown that, by allocating a bank or insurance firm's Tier 1 and Tier 2 capital to higher yielding, more risky assets, risk adjusted performance can be enhanced. These results are particularly pertinent with the advent of the new Basel 2 and Solvency 2 risk based capital initiatives, for banks and insurers respectively.

YOW, S. & SHERRIS, M. *Enterprise Risk Management, Insurer Value Maximisation, and Market Frictions.* 293-339. Enterprise risk management has become a major focus for insurers and reinsurers. Capitalization and pricing decisions are recognized as critical to firm value maximization. Market imperfections including frictional costs of capital such as taxes, agency costs, and financial distress costs are an important motivation for enterprise risk management. Risk management reduces the volatility of financial performance and can have a significant impact on firm value maximization by reducing the impact of frictional costs. Insurers operate in imperfect markets where demand elasticity of policyholders and preferences for financial quality of insurers are important determinants of capitalization and pricing strategies. In this paper, we analyze the optimization of enterprise or firm value in a model with market imperfections. A realistic model of an insurer is developed and calibrated. Frictional costs, imperfectly competitive demand elasticity, and preferences for financial quality are explicitly modelled and implications for enterprise risk management are quantified.

ZHANG, Y. *Allocation of Capital between Assets and Liabilities.* 1-11. We propose a capital allocation method for insurance companies. The amount of capital is directly related to the default risk. The expected value of default can be distributed among the liabilities based on the rule of asset payoff at the time of default. We derive a capital allocation scheme from this allocation of the expected default. Assets, liabilities, and other risky items on the balance sheet are treated in a uniform framework. The insurer's capital is allocated among all these risk contributors. The allocated capitals are given in closed-form formulas, which have straightforward interpretations and are easy to compute. Connections with other allocation methods are also discussed.

ASTIN BULLETIN

Volume **38** (2), 2008

AVANZI, B. & GERBER, H. U. *Optimal dividends in the dual model with diffusion.* 653-667. In the dual model, the surplus of a company is a Lévy process with sample paths that are skip-free downwards. In this paper, the aggregate gains process is the sum of a shifted compound Poisson process and an independent Wiener process. By means of Laplace transforms, it is shown how the expectation of the discounted dividends until ruin can be calculated, if a barrier strategy is applied, and how the optimal dividend barrier can be determined. Conditions for optimality are discussed and several numerical illustrations are given. Furthermore, a family of models is analysed where the individual gain amount distribution is rescaled and compensated by a change of the Poisson parameter.

AVRAM, F. & USABEL, M. *The Gerber-Shiu expected discounted penalty-reward function under an affine jump-diffusion model.* 461-481. We provide a unified analytical treatment of first passage problems under an affine state-dependent jump-diffusion model (with drift and volatility depending linearly on the state). Our proposed model, that generalizes several previously studied cases, may be used for example for obtaining probabilities of ruin in

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the presence of interest rates under the rational investment strategies proposed by Berk & Green (2004).

Bauer, D., Kling, A. & Russ, J. *A universal pricing framework for guaranteed minimum benefits in variable annuities.* 621-651. Variable Annuities with embedded guarantees are very popular in the US market. There exists a great variety of products with both, guaranteed minimum death benefits (GMDB) and guaranteed minimum living benefits (GMLB). Although several approaches for pricing some of the corresponding guarantees have been proposed in the academic literature, there is no general framework in which the existing variety of such guarantees can be priced consistently. The present paper fills this gap by introducing a model, which permits a consistent and extensive analysis of all types of guarantees currently offered within Variable Annuity contracts. Besides a valuation assuming that the policyholder follows a given strategy with respect to surrender and withdrawals, we are able to price the contract under optimal policyholder behavior. Using both, Monte-Carlo methods and a generalization of a finite mesh discretization approach, we find that some guarantees are overpriced, whereas others, e.g. guaranteed annuities within guaranteed minimum income benefits (GMIB), are offered significantly below their risk-neutral value.

Cheung, E. C. K. *Dividend Moments in the Dual Risk Model: Exact and Approximate Approaches.* 399-422. In the classical compound Poisson risk model, it is assumed that a company (typically an insurance company) receives premium at a constant rate and pays incurred claims until ruin occurs. In contrast, for certain companies (typically those focusing on invention), it might be more appropriate to assume expenses are paid at a fixed rate and occasional random income is earned. In such cases, the surplus process of the company can be modelled as a dual of the classical compound Poisson model, as described in Avanzi *et al.* (2007). Assuming further that a barrier strategy is applied to such a model (i.e., any overshoot beyond a fixed level caused by an upward jump is paid out as a dividend until ruin occurs), we are able to derive integro-differential equations for the moments of the total discounted dividends as well as the Laplace transform of the time of ruin. These integro-differential equations can be solved explicitly assuming the jump size distribution has a rational Laplace transform. We also propose a discrete-time analogue of the continuous-time dual model and show that the corresponding quantities can be solved for explicitly leaving the discrete jump size distribution arbitrary. While the discrete-time model can be considered as a stand-alone model, it can also serve as an approximation to the continuous-time model. Finally, we consider a generalization of the so-called Dickson–Waters modification in optimal dividends problems by maximizing the difference between the expected value of discounted dividends and the present value of a fixed penalty applied at the time of ruin.

Furman, E. & Landsman, Z. *Economic capital allocations for non-negative portfolios of dependent risks.* 601-619. In this paper we explore the problem of economic capital allocations in the context of non-negative multivariate (insurance) risks possessing a dependence structure. We derive a general result and illustrate it with a number of useful examples. One such example, for instance, develops explicit expressions for the discussed economic capital decomposition rule when the underlying portfolio consists of dependent compound Poisson risks.

Gisler, A. & Wüthrich, M. V. *Credibility for the chain ladder reserving method.* 565-600. We consider the chain ladder reserving method in a Bayesian set up, which allows for combining the information from a specific claims development triangle with the information from a collective. That is, for instance, to consider simultaneously own company specific data and industry-wide data to estimate the own company's claims reserves. We derive Bayesian estimators and credibility estimators within this Bayesian framework. We show that the credibility estimators are exact Bayesian in the case of the exponential dispersion family with its natural conjugate priors. Finally, we make the link to the classical chain ladder method and

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we show that using non-informative priors we arrive at the classical chain ladder forecasts. However, the estimates for the mean square error of prediction differ in our Bayesian set up from the ones found in the literature. Hence, the paper also throws a new light upon the estimator of the mean square error of prediction of the classical chain ladder forecasts and suggests a new estimator in the chain ladder method.

GOLUBIN, A. Y. *Optimal Insurance and Reinsurance Policies in the Risk Process.* 383-397. The paper examines a classical risk model where both insurance and reinsurance policies are chosen by the insurer in order to minimize the expected maximal loss. We show that the optimal control problem reduces to a static case. We found that the optimal reinsurance is excess of loss reinsurance and describe the set optimal insurance policies. Such a policy providing the minimal variance of the risk left with insured turns out to be a combination of stop loss and deductible policies. The results are illustrated by two numerical examples.

GOLUBIN, A. Y. *Pareto optimality and equilibrium in an insurance market.* 441-459. The concept of economic equilibrium under uncertainty is applied to a model of insurance market where, in distinction to the classic Borch's model of a reinsurance market, risk exchanges are allowed between the insurer and each insured only, not among insureds themselves. Conditions characterizing an equilibrium are found. A variant of the conditions, based on the Pareto optimality notion and involving risk aversion functions of the agents, is derived. An existence theorem is proved. Computation of the market premiums and optimal indemnities is illustrated by an example with exponential utility functions.

KORN, R. & WIESE, A. *Optimal investment and bounded ruin probability: constant portfolio strategies and mean–variance analysis.* 423-440. We study the continuous-time portfolio optimization problem of an insurer. The wealth of the insurer is given by a classical risk process plus gains from trading in a risky asset, modelled by a geometric Brownian motion. The insurer is not only interested in maximizing the expected utility of wealth but is also concerned about the ruin probability. We thus investigate the problem of optimizing the expected utility for a bounded ruin probability. The corresponding optimal strategy in various special classes of possible investment strategies will be calculated. For means of comparison we also calculate the related mean–variance optimal strategies.

LU, L., MACDONALD, A. & WATERS, H. R. *Sampling distributions of critical illness insurance premium rates: breast and ovarian cancer.* 527-542. Evaluating the risk of disorders in long-term insurance often relies on rates of onset estimated from quite small epidemiological studies. These estimates can carry considerable uncertainty, hence so may functions of them, such as a premium rate. In the case of genetic disorders, where it may be required to demonstrate the reliability of genetic information as a risk factor, such uncertainty may be material. Epidemiological studies publish their results in a variety of forms and it is rarely easy to estimate the sampling distribution of a premium rate without access to the original data. We found a large study of breast and ovarian cancer that cited relative risks of breast and ovarian cancer onset, with confidence intervals, in 10-year age groups. We obtained critical illness premium rates and their sampling distributions by parametric bootstrapping, and investigated the effect of possible patterns of sampling correlations. We found that this study provides ample statistical evidence that known BRCA1 or BRCA2 mutations, or a typical family history of breast or ovarian cancer, are reliable risk factors, but the sampling covariances of the relative risks could be important at some ages and terms. Studies that cite only standard errors of parameter estimates erect a small but awkward barrier between the models they describe, and some important actuarial questions.

MALAMUD, S., TRUBOWITZ, E. & WÜTHRICH, M. V. *Market consistent pricing of insurance products.* 483-526. We present the first step in a program to develop a comprehensive, unified

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equilibrium theory of asset and liability pricing. We give a mathematical framework for pricing insurance products in a multiperiod financial market. This framework reflects classical economic principles (like utility maximization) and generates pricing algorithms for non-hedgeable insurance risks.

ULM, E. R. *Analytic solution for return of premium and rollup guaranteed minimum death benefit options under some simple mortality laws.* 543-563. Much attention has been focused recently on the issue of valuing guaranteed minimum death benefits embedded in annuity contracts. These benefits resemble a sequence of put options and their value should obey a differential equation similar to the Black–Scholes equation for simple put options. This paper derives a number of analytic solutions to this equation for a number of simple mortality laws.

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AUSTRALIAN ACTUARIAL JOURNAL

Volume **14** (1), 2008

BUTT, A. *Effect of AASB 119 'Employee benefits' on Defined Benefit Superannuation Costs recognised by Sponsoring employers.* 97-170. A comprehensive financial and superannuation projection model is developed to investigate the impact of the introduction of AASB 119 on employer sponsors of defined benefit superannuation funds. A model Plan is projected forward 25 to 30 years. It is found that closing a plan to new entrants can lead to surpluses greater than the value of future benefits, which in turn leads to distortion of the AASB 119 outcomes through aggregation of actuarial losses relative to the balance sheet asset, when these losses are recognised in retained earnings. If the plan is open to new entrants, expense items under AASB 119 have lower volatility than employer contributions, as long as the minimum actuarial losses and gains are recognised as expenses and revenues or if all actuarial losses and gains are recognised in retained earnings. It is also found that funding methods and salary inflation assumptions have a significant impact on results.

MARTIN, G. *2008 presidential address — what is an actuary?: Discussion — presidential address.* 1-96. The presidential address is presented in the nature of a chairman's address. It covers a high-level overview of 2007 achievements, a high level overview of the key strategic and business foci for 2008, and some thoughts on some issues relevant to our outlook. The issues discussed can be collected under the question of "What is an actuary?"

AUSTRALIAN ACTUARIAL JOURNAL

Volume **14** (2), 2008

LI, J. *On the use of MCMC simulation for stochastic reserving.* 227-271. In this paper we explore the use of Markov chain Monte Carlo (MCMC) simulation in stochastic reserving

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under Australian regulatory environment. We apply two sets of Bayesian models within the framework of generalised linear models (GLMs) to claim run-off data (gross of reinsurance) of two lines of business and use the software WinBUGS (Bayesian Inference Using Gibbs Sampling) to carry out MCMC simulation. This simulation procedure is capable if producing the entire distribution and is a useful tool for determining different percentiles for risk margin, security loading, and risk management purposes. While the previous papers on Bayesian MCMC reserving generally do not examine the residuals after model fitting, here we adopt formal model criticism procedures to refine and validate the model structures tested. We also perform convergence and autocorrelation tests on the simulated samples. We find that our final models are able to incorporate particular features of the data and produce reasonable results in general. We also compare our estimates with the industry risk margin levels. Keywords: Bayesian, MCMC, WinBUGS, outstanding claims liability, risk margin, percentile.

Li, S. *The moments of the present value of total dividends in the compound binomial model under a constant dividend barrier and stochastic interest rates.* 175-192. We study the moments of the present value of total dividend payments in the compound binomial risk model in the presence of a constant dividend barrier. In the evaluation of the present value of dividends, the interest rates are assumed to follow a Markov chain with finite state space. A system of difference equations with certain boundary conditions for the moments of the present value of total dividend payments prior to ruin is derived and solved. Explicit results can be obtained when the claim sizes are rationally distributed or the claim-size distributions have finite support. Numerical results are given for two special claim-size distributions: the geometric distribution and the degenerate distribution at 2. Keywords: compound binomial risk model; discounted dividend payments; stochastic interest rate; Markov chain; time of ruin.

Neuhaus, W. *Risk equalisation by mixed schemes.* 193-226. This paper shows how a risk equalisation scheme using risk-based capitation (RBC) can be mollified by adding partial equalisation of health insurers' differential utilisation rates. A risk equalisation scheme that combines RBC with partial equalisation of the 'utilisation component' is called a 'mixed scheme'. Besides introducing the reader to mixed equalisation schemes, the aim of this paper is twofold. Firstly, to illustrate by a simple, made-up example how a mixed scheme responds to self-selection effects more efficiently than a RBC scheme. Secondly, to outline how a mixed scheme could be designed in practice, using only statistics that are available in the Australian private health insurance market. Keywords: health insurance, community rating, risk equalisation.

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Geneva Risk and Insurance Review

Volume **33** (1), 2008

Doskeland, T. M. & Nordahl, H. A. *Intergenerational effects of guaranteed pension contracts.* 19-46. In this paper we show that there exist intergenerational cross-subsidization effects in guaranteed interest rate life and pension contracts as the different generations partially share the same reserves. Early generations build up bonus reserves, which are left with the company at expiry of the contract. These bonus reserves function partly as a subsidy of later generations, such that the latter earn a risk-adjusted return above the risk-free rate. Furthermore, we show that this subsidy may be large enough to explain why late generations buy guaranteed interest rate products, which otherwise would not have been part of the optimal portfolio allocation.

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KARNI, E. *On optimal insurance in the presence of moral hazard.* 1-18. This paper expounds on the importance of identifiability of subjective probabilities in agency theory with moral hazard. An application to insurance is examined. Keywords: moral hazard, agency theory, subjective probabilities.

PAPON, T. *The effect of pre-commitment and past-experience in insurance choices: an experimental study.* 47-73. This paper reports results from an experimental study that investigates insurance behaviours in low-probability, high-loss risk situations. The study reveals that insurance behaviours may depend on the length of the commitment period of insurance policies, namely the period during which individuals commit themselves to maintain the same insurance decisions. The results of this study also seem to support the predictions of the Dual Theory concerning the demand for co-insurance policies, that is to say the preference of individuals for extreme (null or full) levels of insurance coverage. This study also suggests that prior risk occurrences influence subsequent insurance choices. The paper provides a new possible explanation about the puzzling fact that people usually fail to obtain insurance against disaster-type risks such as natural disasters, even when premiums are close to actuarially fair levels.

GENEVA RISK AND INSURANCE REVIEW

Volume **33** (2), 2008

CROCKER, K. J. & SNOW, A. *Background risk and the performance of insurance markets under adverse selection.* 137-160. Background risk can influence the performance of insurance markets that must deal with adverse selection when applicants are risk vulnerable, since they are more averse to bearing the insurable risk as a result of their exposures to background risk. We show that background risk always results in a lower deductible for the incentive constrained contract, and that a broader range of markets attains the stable sequential equilibrium cross-subsidized pair of separating contracts. We conclude that background risk always improves the performance of markets for coverage against (insurable) foreground risks that must deal with adverse selection. We also find, however, that these improvements are never sufficient to offset the cost to insureds of bearing the background risk.

DENUIT, M. & FROSTIG, E. *First-order mortality basis for life annuities.* 75-89. Mortality improvements pose a challenge for the life annuity business. For the management of such portfolios, it is important to forecast future mortality rates. Standard models for mortality forecasting assume that the force of mortality at age x in calendar year t is of the form $\exp(x + xt)$, where the dynamics of the time index t is described by a random walk with drift. Starting from such a best estimate of future mortality (called second-order mortality basis in actuarial science), the paper explains how to determine a conservative life table serving as first-order mortality basis. The idea is to replace the stochastic projected life table with a deterministic conservative one, and to assume mutual independence for the remaining life times. The paper then studies the distribution of the present value of the payments made to a closed group of annuitants. It turns out that De Pril–Panjer algorithm can be used for that purpose under first-order mortality basis. The connection with ruin probabilities is briefly discussed. An inequality between the distribution of the present value of future annuity payments under first-order and second-order mortality basis is provided, which allows to link value-at-risk computed under these two sets of assumptions. A numerical example performed on Belgian mortality statistics illustrates how the approach proposed in this paper can be implemented in practice.

LE COURTOIS, O. & QUITTARD-PINON, F. *Fair valuation of participating life insurance contracts with jump risk.* 106-136. The purpose of this article is to value participating life insurance

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contracts when the linked portfolio is modeled by a jump-diffusion. More precisely, this process has a Brownian component and a compound Poisson one, where the jump size is driven by a double exponential distribution. Specifically here, the bankruptcy risk of the insurance company is considered. Thus, market and credit risks are taken into account. A quasi-closed-form formula is obtained in fair value for the price of the considered life insurance contract. This allows us to investigate the impact of strategic parameters as well as structural ones, as is shown in the numerical section of this paper. In particular, we study the impact on the contract of the volatility, jump intensity, jump asymmetry, company leverage, guaranteed rate, participation rate and level of the default barrier, and comment on how they are likely to increase the probability of early default of the issuer.

Santerre, R. E. *Which Price is Right: Load or Premium?* 90-105. This paper uses national time series data for the United States to investigate whether changes in the premium or loading fee offer a better explanation for variations in the percentage of the population with private health insurance from 1960 to 2004. The empirical results suggest that premium provides a better measure of price when estimating the demand for health insurance at the extensive margin. The empirical analysis also indicates that the aggregate short-run price and income elasticities of demand for health insurance are fairly close at −0.19 and 0.27, respectively. One implication is that the percentage of the population with private health insurance in the United States should continue to decline in the future if real premiums persistently grow significantly faster than the overall economy.

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Insurance: Mathematics & Economics

Volume **42** (1), 2008

Avram, F., Palmowski, Z. & Pistorius, M. *A two-dimensional ruin problem on the positive quadrant.* 227-234. In this paper we study the joint ruin problem for two insurance companies that divide between them both claims and premia in some specified proportions (modeling two branches of the same insurance company or an insurance and re-insurance company). Modeling the risk processes of the insurance companies by Cramér–Lundberg processes we obtain the Laplace transform in space of the probability that either of the insurance companies is ruined in finite time. Subsequently, for exponentially distributed claims, we derive an explicit analytical expression for this joint ruin probability by explicitly inverting this Laplace transform. We also provide a characterization of the Laplace transform of the joint ruin time.

Barone, L. *Bruno de Finetti and the case of the critical line's last segment.* 359-377. The anticipatory views of Bruno de Finetti on portfolio theory, set out by the author in a 1940 article, have recently been discovered by Mark Rubinstein and reviewed by Harry Markowitz. This paper analyzes the crucial parts of de Finetti's paper and discusses the controversial issue of the critical line's last segment, i.e. the segment that leads to the minimum-variance efficient portfolio. Markowitz [Markowitz, Harry M., 2006. De Finetti scoops Markowitz. In: A Literature postscript. *J. Invest. Manag.* **4** (3), 3-18 (special issue)] derives the criterion for the last segment to lie on one of the boundaries of the set of legitimate portfolios in two (and n) dimensions and Pressacco [Pressacco, Flavio, 2005. De Finetti, Markowitz e la congettura dell'ultimo segmento. Rendiconti per gli Studi Economici Quantitativi, Università Cà Foscari di Venezia] shows a necessary and sufficient condition for the last segment to lie inside the

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legitimate set in three dimensions when the correlations are uniformly positive. This paper revises the terms of the problem and completes the analysis. Keywords: Probability of default; Reinsurance; Mean–variance criterion; Correlation; Critical line algorithm.

Brokate, M., Klüppelberg, C., Kostadinova, R., Maller, R. & Seydel, R. C. *On the distribution tail of an integrated risk model: A numerical approach.* 101-106. We consider an insurance risk process with the possibility to invest the capital reserve into a portfolio consisting of a risky asset and a riskless asset. The stock price is modelled by an exponential Lévy process and the riskless interest rate is assumed to be constant. We aim at the risk assessment of the integrated risk process in terms of a high quantile or the far out distribution tail. We indicate an application to an optimal investment strategy of an insurer. Keywords: Exponential Lévy process; Finite difference method; Integrated insurance risk process; Integrated risk management; Optimal investment strategy; Value-at-Risk; Partial integro-differential equation; Tail behaviour.

Brückner, K. *Quantifying the error of convex order bounds for truncated first moments.* 261-270. The concepts of convex order and comonotonicity have become quite popular in risk theory, essentially since Kaas *et al.* [Kaas, R., Dhaene, J. & Goovaerts, M. J., 2000. Upper and lower bounds for sums of random variables. *Insurance: Math. Econ.* **27**, 151-168] constructed bounds in the convex order sense for a sum S of random variables without imposing any dependence structure upon it. Those bounds are especially helpful, if the distribution of S cannot be calculated explicitly or is too cumbersome to work with. This will be the case for sums of lognormally distributed random variables, which frequently appear in the context of insurance and finance.

In this article we quantify the maximal error in terms of truncated first moments, when S is approximated by a lower or an upper convex order bound to it. We make use of geometrical arguments; from the unknown distribution of S only its variance is involved in the computation of the error bounds. The results are illustrated by pricing an Asian option. It is shown that under certain circumstances our error bounds outperform other known error bounds, e.g. the bound proposed by Nielsen and Sandmann [Nielsen, J. A. & Sandmann, K., 2003. Pricing bounds on Asian options. *J. Financ. Quant. Anal.* **38**, 449-473]. Keywords: Convex order; Truncated first moments; Stop-loss-premiums; Sums of random variables; Asian options.

Buch, A. & Dorfleitner, G. *Coherent risk measures, coherent capital allocations and the gradient allocation principle.* 235-242. The gradient allocation principle, which generalizes the most popular specific allocation principles, is commonly proposed in the literature as a means of distributing a financial institution's risk capital to its constituents. This paper is concerned with the axioms defining the coherence of risk measures and capital allocations, and establishes results linking the two coherence concepts in the context of the gradient allocation principle. The following axiom pairs are shown to be equivalent: positive homogeneity and full allocation, subadditivity and "no undercut", and translation invariance and riskless allocation. Furthermore, we point out that the symmetry property holds if and only if the risk measure is linear. As a consequence, the gradient allocation principle associated with a coherent risk measure has the properties of full allocation and "no undercut", but not symmetry unless the risk measure is linear. The results of this paper are applied to the covariance, the semi-covariance, and the expected shortfall principle. We find that the gradient allocation principle associated with a nonlinear risk measure can be coherent, in a suitably restricted setting. Keywords: Risk capital allocation; Gradient allocation principle; Coherent risk measures; Coherent capital allocations.

Burgert, C. & Rüschendorf, L. *Allocation of risks and equilibrium in markets with finitely many traders.* 177-188. The optimal risk allocation problem, equivalently the optimal risk

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sharing problem, in a market with n traders endowed with risk measures $1, \ldots, n$ is a classical problem in insurance and mathematical finance. This problem however only makes sense under a condition motivated from game theory which is called Pareto equilibrium. There are many situations of practical interest, where this condition does not hold. This is the case if the risk measures are based on essential different views towards risk. In this paper we introduce and analyze a meaningful extension of the optimal risk allocation (risk sharing) problem without assuming the equilibrium condition. The main point of this is to introduce a suitable and well motivated restriction on the class of admissible allocations which prevents effects of artificial 'risk arbitrage'. As a result we obtain a new coherent risk measure which describes the inherent risk which remains after using admissible risk exchange in an optimal way.

Consiglio, A. & Giovanni, D. De. *Evaluation of insurance products with guarantee in incomplete markets.* 332-342. Life insurance products are usually equipped with minimum guarantee and bonus provision options. The pricing of such claims is of vital importance for the insurance industry. Risk management, strategic asset allocation, and product design depend on the correct evaluation of the written options. Also regulators are interested in such issues since they have to be aware of the possible scenarios that the overall industry will face. Pricing techniques based on the Black & Scholes paradigm are often used, however, the hypotheses underneath this model are rarely met.

To overcome Black & Scholes limitations, we develop a stochastic programming model to determine the fair price of the minimum guarantee and bonus provision options. We show that such a model covers the most relevant sources of incompleteness accounted in the financial and insurance literature. We provide extensive empirical analyses to highlight the effect of incompleteness on the fair value of the option, and show how the whole framework can be used as a valuable normative tool for insurance companies and regulators. Keywords: Life insurance; Policies with minimum guarantee; Option pricing; Incomplete markets.

Courtois, C. & Denuit, M. *Convex bounds on multiplicative processes, with applications to pricing in incomplete markets.* 95-100. Extremal distributions have been extensively used in the actuarial literature in order to derive bounds on functionals of the underlying risks, such as stop-loss premiums or ruin probabilities, for instance. In this paper, the idea is extended to a dynamic setting. Specifically, convex bounds on multiplicative processes are derived. Despite their relative simplicity, the extremal processes are shown to produce reasonably accurate bounds on option prices in the classical trinomial model for incomplete markets. Keywords: Convex order; Extremal distributions; Incomplete market; Risk-neutral martingales; Option pricing; Trinomial model.

Delong, L., Gerrard, R. & Haberman, S. *Mean–variance optimization problems for an accumulation phase in a defined benefit plan.* 107-118. In this paper we deal with contribution rate and asset allocation strategies in a pre-retirement accumulation phase. We consider a single cohort of workers and investigate a retirement plan of a defined benefit type in which an accumulated fund is converted into a life annuity. Due to the random evolution of a mortality intensity, the future price of an annuity, and as a result, the liability of the fund, is uncertain. A manager has control over a contribution rate and an investment strategy and is concerned with covering the random claim. We consider two mean–variance optimization problems, which are quadratic control problems with an additional constraint on the expected value of the terminal surplus of the fund. This functional objectives can be related to the well-established financial theory of claim hedging. The financial market consists of a risk-free asset with a constant force of interest and a risky asset whose price is driven by a Lévy noise, whereas the evolution of a mortality intensity is described by a stochastic differential equation driven by a Brownian motion. Techniques from the stochastic control theory are applied in order to find optimal strategies. Keywords: Lévy diffusion financial market;

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Stochastic mortality intensity process; Hamilton–Jacobi–Bellman equation; Feynman–Kac representation.

Eisele, K.-T. *Recursions for multivariate compound phase variables.* 65-72. We show how to generalize the result given in [Eisele, K.-Th., 2006. Recursions for compound phase distributions. *Insurance: Math. Econ.* **38**, 149-156] to the multivariate case, i.e. we find a Panjer-like recursion principle for the distribution of a multivariate compound phase variable. Recursion formulas and procedures for the bivariate case are given in detail. We give a possible application for agricultural risks and calculate concrete examples via a VB-program. Keywords: Discrete multivariate phase variables; Recursions for multivariate compound variables.

Furman, Edward & Zitikis, R. *Weighted premium calculation principles.* 459-465. A prominent problem in actuarial science is to define, or describe, premium calculation principles (pcp's) that satisfy certain properties. A frequently used resolution of the problem is achieved via distorting (e.g., lifting) the decumulative distribution function, and then calculating the expectation with respect to it. This leads to coherent pcp's. Not every pcp can be arrived at in this way. Hence, in this paper we suggest and investigate a broad class of pcp's, which we call weighted premiums, that are based on weighted loss distributions. Different weight functions lead to different pcp's: any constant weight function leads to the net premium, an exponential weight function leads to the Esscher premium, and an indicator function leads to the conditional tail expectation. We investigate properties of weighted premiums such as ordering (and in particular loading), invariance. In addition, we derive explicit formulas for weighted premiums for several important classes of loss distributions, thus facilitating parametric statistical inference. We also provide hints and references on non-parametric statistical inferential tools in the area. Keywords: Weighted transform; Weighted distribution; Weighted premium calculation principle; Loaded premium; Distorted premium; Esscher's premium; Kamps's premium; Conditional tail expectation; Tail variance premium; Stochastic ordering.

Gaillardetz, P. *Valuation of life insurance products under stochastic interest rates.* 212-226. In this paper, we introduce a consistent pricing method for life insurance products whose benefits are contingent on the level of interest rates. Since these products involve mortality as well as financial risks, we present an approach that introduces stochastic models for insurance products through stochastic interest rate models. Similar to Black *et al*. [Black, Fisher, Derman, Emanuel, Toy, William, 1990. A one-factor model of interest rates and its application to treasury bond options. *Financ. Anal. J.* **46** (January-February), 33-39], we assume that the premiums and volatilities of standard insurance products are given exogenously. We then project insurance prices to extract underlying martingale probability structures. Numerical examples on variable annuities are provided to illustrate the implementation of this method.

Gatzert, N. & Schmeiser, H. *The influence of corporate taxes on pricing and capital structure in property–liability insurance.* 50-58. A change in the corporate tax level can have a significant impact on rate making and capital structure for insurance companies. The purpose of this paper is to study this effect on competitive equity–premium combinations for different asset and liability models while retaining a fixed safety level. This is a crucial consideration as a change in the tax rate leads, in general, to a different risk of insolvency. Hence, fixing the safety level serves to isolate the effect of taxes without shifting the insurer's risk situation whenever taxes are varied. The model framework includes stochastic assets as well as stochastic claims costs. We further compare the results for liability models with and without a jump component. Insurance rate making is conducted using option pricing theory. Keywords: Property–liability insurance; Corporate taxation; Contingent claims valuation.

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Gerber, H. U., Shiu, E. S. W. & Smith, N. *Methods for estimating the optimal dividend barrier and the probability of ruin.* 243-253. In applications of collective risk theory, complete information about the individual claim amount distribution is often not known, but reliable estimates of its first few moments may be available. For such a situation, this paper develops methods for estimating the optimal dividend barrier and the probability of ruin. In particular, two De Vylder approximations are explained, and the first and second order diffusion approximations are examined. For several claim amount distributions, the approximate values are compared numerically with the exact values. The De Vylder and diffusion approximations can be adapted to the more general situation where the aggregate claims process is a Lévy process with nonnegative increments. Keywords: Optimal dividend barrier; Ruin probability; Lévy processes; Subordinators; De Vylder approximations; Diffusion approximations; Lundberg function; Lundberg's fundamental equation; Gamma processes.

Gómez-Déniz, E., Sarabia, J. M. & Calderín-Ojeda, E. *Univariate and multivariate versions of the negative binomial-inverse Gaussian distributions with applications.* 39-49. In this paper we propose a new compound negative binomial distribution by mixing the p negative binomial parameter with an inverse Gaussian distribution and where we consider the reparameterization $p = \exp(-)$. This new formulation provides a tractable model with attractive properties which make it suitable for application not only in the insurance setting but also in other fields where overdispersion is observed. Basic properties of the new distribution are studied. A recurrence for the probabilities of the new distribution and an integral equation for the probability density function of the compound version, when the claim severities are absolutely continuous, are derived. A multivariate version of the new distribution is proposed. For this multivariate version, we provide marginal distributions, the means vector, the covariance matrix and a simple formula for computing multivariate probabilities. Estimation methods are discussed. Finally, examples of application for both univariate and bivariate cases are given. Compound negative binomial distribution; Inverse Gaussian distribution; Overdispersion; Recursion; Premium.

Hainaut, D. & Devolder, P. *Mortality modelling with Lévy processes.* 409-418. This paper addresses the modelling of human mortality by the aid of doubly stochastic processes with an intensity driven by a positive Lévy process. We focus on intensities having a mean reverting stochastic component. Furthermore, driving Lévy processes are pure jump processes belonging to the class of a-stable subordinators. In this setting, expressions of survival probabilities are inferred, the pricing is discussed and numerical applications to actuarial valuations are proposed. Keywords: Stochastic mortality; Longevity risk; Lévy processes.

Han, Z. & Gau, W.-C. *Estimation of loss reserves with lognormal development factors.* 289-395. This paper uses a development technique to estimate the loss reserve in a classical run-off triangle setting. Closed-form solutions for unbiased estimates of reserves and their corresponding standard errors can be obtained by assuming lognormal distributions of the development factors. The technique is applied to the Bornhuetter–Ferguson method [Bornhuetter, R. L., Ferguson, R. E., 1972. The actuary and IBNR. *Proc. Casualty Actuarial Soc.* **59**, 181-195] and to two previously studied data sets. Keywords: Development factors; Lognormal distribution; Unbiased estimation.

Horneff, W. J., Maurer, R. H., Mitchell, O. S. & Dus, I. *Following the rules: Integrating asset allocation and annuitization in retirement portfolios.* 396-408. Financial advisers have developed standardized payout strategies to help Baby Boomers manage their money in their golden years. Prominent among these are phased withdrawal plans offered by mutual funds including the "self-annuitization" or default rules encouraged under US tax law, and fixed payout annuities offered by insurers. Using a utility-based framework, and taking account of stochastic capital markets and uncertain lifetimes, we first evaluate these rules on a stand-

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alone basis for a wide range of risk aversion. Next, we permit the consumer to integrate these standardized payout strategies at retirement and compare the results. We show that integrated strategies can enhance retirees' well-being by 25%-50% for low/moderate levels of risk aversion when compared to full annuitization at retirement. Finally, we examine how welfare changes if the consumer is permitted to switch to a fixed annuity at an optimal point after retirement. This affords the retiree the chance to benefit from the equity premium when younger, and exploit the mortality credit in later life. For moderately risk-averse retirees, the optimal switching age lies between 80 and 85.

JONG, F. DE. *Pension fund investments and the valuation of liabilities under conditional indexation.* 1-13. This paper reviews the investment policy of collective pension plans. We focus on funds with a collective Defined Contribution character. We suggest two reasons to invest in equities: the lack of a well-developed market in index-linked bonds, and deliberate deviations from the Defined Benefit nature of the plan. Furthermore, this paper assesses the value of limited or conditional indexation options found in many plans.

JUMARIE, G. *Stock exchange fractional dynamics defined as fractional exponential growth driven by (usual) Gaussian white noise. Application to fractional Black–Scholes equations.* 271-287. Stock exchange dynamics of fractional order are usually modeled as a non-random exponential growth process driven by a fractional Brownian motion. Here we propose to use rather a non-random fractional growth driven by a (standard) Brownian motion. The key is the Taylor's series of fractional order where $Ea(.)$ denotes the Mittag–Leffler function, and is the so-called modified Riemann–Liouville fractional derivative which we introduced recently to remove the effects of the non-zero initial value of the function under consideration. Various models of fractional dynamics for stock exchange are proposed, and their solutions are obtained. Mainly, the Itô's lemma of fractional order is illustrated in the special case of a fractional growth with white noise. Prospects for the Merton's optimal portfolio are outlined, the path probability density of fractional stock exchange dynamics is obtained, and two fractional Black–Scholes equations are derived. This approach avoids using fractional Brownian motion and thus is of some help to circumvent the mathematical difficulties so involved. Keywords: Fractional Gaussian noises; Fractional stochastic differential equation; Fractional exponential growth; Fractional Brownian motion; Path probability density; Fractional Black–Scholes equation.

KALLENBERG, W. C. M. *Modelling dependence.* 127-146. A new way of choosing a suitable copula to model dependence is introduced. Instead of relying on a given parametric family of copulas or applying the other extreme of modelling dependence in a nonparametric way, an intermediate approach is proposed, based on a sequence of parametric models containing more and more dependency aspects. In contrast to a similar way of thinking in testing theory, the method here, intended for estimating the copula, often requires a somewhat larger number of steps. One approach is based on exponential families, another on contamination families. An extensive numerical investigation is supplied on a large number of well-known copulas. The method based on contamination families is recommended. A Gaussian start in this approximation looks very promising. Keywords: Copula; Legendre polynomials; Exponential family; Contamination family; Nonlinear correlation.

KASSBERGER, S., KIESEL, R. & LIEBMANN, T. *Fair valuation of insurance contracts under Lévy process specifications.* 419-433. The valuation of options embedded in insurance contracts using concepts from financial mathematics (in particular, from option pricing theory), typically referred to as fair valuation, has recently attracted considerable interest in academia as well as among practitioners. The aim of this article is to investigate the valuation of participating and unit-linked life insurance contracts, which are characterized by embedded rate guarantees and bonus distribution rules. In contrast to the existing literature, our

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approach models the dynamics of the reference portfolio by means of an exponential Lévy process. Our analysis sheds light on the impact of the dynamics of the reference portfolio on the fair contract value for several popular types of insurance policies. Moreover, it helps to assess the potential risk arising from misspecification of the stochastic process driving the reference portfolio. Keywords: Embedded options; Interest rate guarantees; Model risk; Participating contracts; Risk-neutral valuation; Unit-linked contracts.

Kolev, N. & Paiva, D. *Random sums of exchangeable variables and actuarial applications.* 147-153. In this paper we study the accumulated claim in some fixed time period, skipping the classical assumption of mutual independence between the variables involved. Two basic models are considered: Model 1 assumes that any pair of claims are equally correlated which means that the corresponding square-integrable sequence is exchangeable one. Model 2 states that the correlations between the adjacent claims are the same. Recurrence and explicit expressions for the joint probability generating function are derived and the impact of the dependence parameter (correlation coefficient) in both models is examined. The Markov binomial distribution is obtained as a particular case under assumptions of Model 2. Keywords: Correlation coefficient; Collective risk model; Exchangeability; Homogeneous Markov chain; Joint probability generating function; Random sums.

Landriault, D. *Constant dividend barrier in a risk model with interclaim-dependent claim sizes.* 31-38. The risk model with interclaim-dependent claim sizes proposed by Boudreault *et al.* [Boudreault, M., Cossette, H., Landriault, D. & Marceau, E., 2006. On a risk model with dependence between interclaim arrivals and claim sizes. *Scand. Actur. J.*, 265-285] is studied in the presence of a constant dividend barrier. An integro-differential equation for some Gerber–Shiu discounted penalty functions is derived. We show that its solution can be expressed as the solution to the Gerber–Shiu discounted penalty function in the same risk model with the absence of a barrier and a combination of two linearly independent solutions to the associated homogeneous integro-differential equation. Finally, we analyze the expected present value of dividend payments before ruin in the same class of risk models. An homogeneous integro-differential equation is derived and then solved. Its solution can be expressed as a different combination of the two fundamental solutions to the homogeneous integro-differential equation associated to the Gerber–Shiu discounted penalty function Keywords: Risk model; Interclaim-dependent claim sizes; Constant dividend barrier; Integro-differential equation; Defective renewal equation; Gerber–Shiu discounted penalty function; Expected discounted dividend payments.

Loubergé, H. & Watt, R. *Insuring a risky investment project.* 301-310. In the standard model for insurance demand, the risk is totally exogenous and the insurance premium is paid for out of riskless wealth. This model yields results that are mostly in contradiction to everyday observation and have been used to question the pertinence of expected utility theory on which the model is based. For some years now, several papers have made attempts to provide foundations for a theory of insurance demand that leads to less provocative comparative statics results. In these papers, the risk for which coverage is sought becomes endogenous, and the decision to purchase insurance is made simultaneously with the decision on how much to invest in insurable assets. All these papers use a standard financial investment framework. This paper offers a contribution to this literature by using a slightly different framework: the case of a firm exposed to an insurable risk affecting return on a real investment project. The model is kept simple by using a two-state environment. It yields results that are both more complete and more general than results in previous work with the same motivation. Keywords: Insurance coverage; Risk aversion; Normality; Giffen good; Actuarially fair premium.

Lu, L., Macdonald, A. & Wekwete, C. *Premium rates based on genetic studies: How reliable are they?* 319-331. Underwriting the risk of rare disorders in long-term insurance often relies

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on rates of onset estimated from quite small epidemiological studies. These estimates can have considerable sampling uncertainty and any function based upon them, such as a premium rate, is also an estimate subject to uncertainty. This is particularly relevant in the case of genetic disorders, because the acceptable use of genetic information may depend on establishing its reliability as a measure of risk. The sampling distribution of a premium rate is hard to estimate without access to the original data, which is rarely possible. From two studies of adult polycystic kidney disease (APKD) we obtain, not the original data, but the cases and exposures used for Kaplan–Meier estimates of the survival probability. We use three resampling methods with these data, namely: (a) the standard bootstrap; (b) the weird bootstrap; and (c) simulation of censored random lifetimes. Rates of onset were obtained from each simulated sample using kernel-smoothed Nelson–Aalen estimates, hence critical illness insurance premium rates for a mutation carrier or a member of an affected family. From 10,000 such samples we estimate the sampling distributions of the premium rates, finding considerable uncertainty. Very careful consideration should be given before using small-sample epidemiological data to deal with insurance problems. Keywords: Adult polycystic kidney disease; Bootstrapping; Critical illness insurance; Kernel smoothing; Nelson–Aalen estimate.

Ludkovski, Michael & Young, V. R. *Indifference pricing of pure endowments and life annuities under stochastic hazard and interest rates.* 14-30. We study indifference pricing of mortality contingent claims in a fully stochastic model. We assume both stochastic interest rates and stochastic hazard rates governing the population mortality. In this setting we compute the indifference price charged by an insurer that uses exponential utility and sells k contingent claims to k independent but homogeneous individuals. Throughout we focus on the examples of pure endowments and temporary life annuities. We begin with a continuous-time model where we derive the linear pdes satisfied by the indifference prices and carry out extensive comparative statics. In particular, we show that the price-per-risk grows as more contracts are sold. We then also provide a more flexible discrete-time analog that permits general hazard rate dynamics. In the latter case we construct a simulation-based algorithm for pricing general mortality-contingent claims and illustrate with a numerical example.

Luo, S., Taksar, M. & Tsoi, A. *On reinsurance and investment for large insurance portfolios.* 434-444. We consider a problem of optimal reinsurance and investment for an insurance company whose surplus is governed by a linear diffusion. The company's risk (and simultaneously its potential profit) is reduced through reinsurance, while in addition the company invests its surplus in a financial market. Our main goal is to find an optimal reinsurance–investment policy which minimizes the probability of ruin. More specifically, in this paper we consider the case of proportional reinsurance, and investment in a Black–Scholes market with one risk-free asset (bond, or bank account) and one risky asset (stock). We apply stochastic control theory to solve this problem. It transpires that the qualitative nature of the solution depends significantly on the interplay between the exogenous parameters and the constraints that we impose on the investment, such as the presence or absence of shortselling and/or borrowing. In each case we solve the corresponding Hamilton–Jacobi–Bellman equation and find a closed-form expression for the minimal ruin probability as well as the optimal reinsurance–investment policy. Keywords: Ruin probability; Stochastic control; Black–Scholes model; Hamilton–Jacobi–Bellman equation; Proportional reinsurance.

Malinovskii, V. K. *Adaptive control strategies and dependence of finite time ruin on the premium loading.* 81-94. The paper is devoted to risk theory insight into the problem of asset–liability and solvency adaptive management. Two adaptive control strategies in the multiperiodic insurance risk model composed of chained classical risk models are introduced and their performance in terms of probability of ruin is examined. The analysis is based on an explicit expression of the probability of ruin within finite time in terms of Bessel functions.

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The dependence of that probability on the premium loading, either positive or negative, is a basic technical result of independent interest. Multiperiodic insurance risk model; Equity–solvency balance; Adaptive control; Classical risk model; Finite time ruin probabilities; Bessel functions; Premium loading.

MENONCIN, F. *The role of longevity bonds in optimal portfolios.* 343-358. We study the optimal consumption and portfolio for an agent maximizing the expected utility of his intertemporal consumption in a financial market with: (i) a riskless asset, (ii) a stock, and (iii) a bond as a derivative on the stochastic interest rate, and (iv) a longevity bond whose coupons are proportional to the population (stochastic) survival rate. With a force of mortality instantaneously uncorrelated with the interest rate (but not necessarily independent), we demonstrate that the wealth invested in the longevity bond must be taken from the ordinary bond and the riskless asset proportionally to the duration of the two bonds. This result is valid for both a complete and an incomplete financial market. Keywords: Longevity risk; Dynamic programming.

NEUENSCHWANDER, D. *Retrieval of Black–Scholes and generalized Erlang models by perturbed observations at a fixed time.* 453-458. S-stable laws on the real line (more generally on Hilbert spaces), associated with some non-linear transformations (so-called "shrinking operations"), were introduced in [Jurek, Z. J., 1977. Limit distributions for truncated random variables. In: *Proc. 2nd Vilnius Conference on Probability and Statistics*, June 28-July 3, 1977. In: *Abstracts of Communications*, vol. 3, pp. 95-96; Jurek, Z. J., 1979. Properties of s-stable distribution functions. *Bull. Acad. Polon. Sci. Sér. Math.* **XXVII** (1), 135-141; Jurek, Z. J., 1981. Limit distributions for sums of shrunken random variables. *Dissertations Math.* vol. CLXXXV]. In [Jurek, Z. J. & Neuenschwander, D., 1999. S-stable laws in insurance and finance and generalization to nilpotent Lie groups. *J. Theoret. Probab.* **12** (4), 1089-1107], the authors interpreted s-stable motions on the real line as limits of total amount of claims processes (up to a deterministic premium) of a portfolio of excess-of-loss reinsurance contracts and showed that they led to Erlang's model or to Brownian motion. In [Neuenschwander, D., 2000b. On option pricing in models driven by iterated integrals of Brownian motion. In: *Mitt. SAV 2000*, Heft 1, pp. 35-39], we considered stochastic integrals whose integrand and integrator are both independent Brownian motions, thus modelling a stochastic volatility; as a result we got an analogue of the Black–Scholes formula in this model, confirming a result of Hull and White [Hull, J. & White, A., 1987. The pricing of options on assets with stochastic volatility. *J. Finance* **XLII** (2), 281-300]. In the present paper, we will look at a common generalization of these processes, namely s-stable motions on the real line perturbed by a stochastic integral whose integrand and integrator are both (not necessarily independent) s-stable motions. The main result will be that if we can observe the distribution of such so-perturbed s-stable motions (together with the values of the perturbing processes) at time $t = 1$, then we can identify the whole model (including the perturbation) among all models with Lévy processes perturbed by an iterated stochastic integral of two Lévy processes (in the gaussian case) resp. among all models with a compound Poisson process with drift perturbed by an iterated stochastic integral of two compound Poisson processes (in the completely non-gaussian case if the perturbing processes have no drift) without knowing anything about the history or about its distribution during $0 = t < 1$. This applies, e.g., to a situation where several assets obey the same model and one can estimate the distribution at time one by looking at the values of all these assets at time $t = 1$.

Interestingly enough, it will be convenient to treat the whole matter in the algebraic framework of the so-called Heisenberg group. This is a concept coming in fact from quantum mechanics and is in a certain sense the simplest non-commutative Lie group.

NIU, L. *Some stability results of optimal investment in a simple Lévy market.* 445-452. We investigate some investment problems of maximizing the expected utility of the terminal

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wealth in a simple Lévy market, where the stock price is driven by a Brownian motion plus a Poisson process. The optimal investment portfolios are given explicitly under the hypotheses that the utility functions belong to the HARA, exponential and logarithmic classes. We show that the solutions for the HARA utility are stable in the sense of weak convergence when the parameters vary in a suitable way. Keywords: Lévy process; Optimal investment; Martingale method; HARA; Weak convergence.

PAN, M., WANG, R. & WU, X. *On the consistency of credibility premiums regarding Esscher principle.* 119-126. In this paper, we investigate the problems of convergence of experience-based ratemakings regarding the Esscher principle. In addition to the Bayes and the classical credibility premiums, we suggest a new credibility formula for the Esscher premium. Then we show the convergence of the Bayes and the newly defined credibility premiums towards the individual premium and point out that the classical credibility premium does not generally converge to the individual premium by presenting a sufficient and necessary condition under which the classical credibility Esscher premium converges to the individual premium. A simulation study is carried out to illustrate the theoretical conclusions. Keywords: Esscher premium; Credibility premium; Loss function; Convergence.

PITSELIS, G. *Robust regression credibility: The influence function approach.* 288-300. In classical credibility theory we assume that the vector of claims conditionally on has independent components with identical means. However, this assumption is sometimes unrealistic. To relax this condition Hachemeister (Hachemeister, C. A., 1975. Credibility for regression models with application to trend. In: Kahn, P. (Ed.), *Credibility, Theory and Applications. Academic Press*, New York) introduced regressors. The presence of large claims can perturb the credibility premium estimation. The lack of robustness of regression credibility estimators, as well as the fairness of tariff evaluation, led to the development of this paper. Our proposal is to apply robust statistics to the regression credibility estimation by using the robust influence function approach of M-estimators. Keywords: Robust regression credibility; M-estimators; Influence function; Portfolio unbiasedness; Pure robust estimation.

PSARRAKOS, G. & KONSTADINOS, P. *Tail bounds for the joint distribution of the surplus prior to and at ruin.* 163-176. For the classical risk model with Poisson arrivals, we study the (bivariate) tail of the joint distribution of the surplus prior to and at ruin. We obtain some exact expressions and new bounds for this tail, and we suggest three numerical methods that may yield upper and lower bounds for it. As a by-product of the analysis, we obtain new upper and lower bounds for the probability and severity of ruin. Many of the bounds in the present paper improve and generalise corresponding bounds that have appeared earlier. For the numerical bounds, their performance is also compared against bounds available in the literature. Keywords: Surplus prior to ruin; Deficit at ruin; Ruin probability; Renewal equation.

WANG, G. & WU, R. *The expected discounted penalty function for the perturbed compound Poisson risk process with constant interest.* 59-64. Abstract
In this paper, we consider the Gerber–Shiu expected discounted penalty function for the perturbed compound Poisson risk process with constant force of interest. We decompose the Gerber–Shiu function into two parts: the expected discounted penalty at ruin that is caused by a claim and the expected discounted penalty at ruin due to oscillation. We derive the integral equations and the integro-differential equations for them. By solving the integro-differential equations we get some closed form expressions for the expected discounted penalty functions under certain assumptions. Keywords: Brownian motion; Constant force of interest; Gerber–Shiu expected discounted penalty function; Time of ruin.

WÜTHRICH, M. V. *Prediction error in the chain ladder method.* 378-388. We define a chain ladder model which allows for the study of three different error types: (a) diversifiable process

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error, (b) non-diversifiable process error, and (c) parameter estimation error. The model is based on the classical stochastic chain ladder model introduced by Mack [Mack, T., 1993. Distribution-free calculation of the standard error of chain ladder reserve estimates. *Astin Bull.* **23** (2), 213-225]. In order to clearly distinguish the different sources of prediction uncertainty, we have to slightly modify that classical chain ladder model. Keywords: Claims reserving; Chain ladder method; Mean square error of prediction.

ZHANG, M.-H. *Modelling total tail dependence along diagonals.* 73-80. An approach to modelling total tail dependence beyond the main diagonals is proposed. The concept introduced combines the principal and minor diagonals to describe total extreme dependence. A framework is introduced for the measurement of total tail dependence under model mixture. Illustrations are presented using empirical data on stock market indices and exchange rates. An extension is provided to the multivariate case and total tail dependence is considered for model mixtures. Keywords: Copula; Dependence structure; Model mixture; Quantitative risk; Total tail dependence.

ZHOU, C., WU, C., ZHANG, S. & HUANG, X. *An optimal insurance strategy for an individual under an intertemporal equilibrium.* 255-260. In this paper, we discuss how a risk-averse individual under an intertemporal equilibrium chooses his/her optimal insurance strategy to maximize his/her expected utility of terminal wealth. It is shown that the individual's optimal insurance strategy actually is equivalent to buying a put option, which is written on his/her holding asset with a proper strike price. Since the cost of avoiding risk can be seen as a risk measure, the put option premium can be considered as a reasonable risk measure. Jarrow [Jarrow, R., 2002. Put option premiums and coherent risk measures. *Math. Finance* **12**, 135-142] drew this conclusion with an axiomatic approach, and we verify it by solving the individual's optimal insurance problem. Keywords: Optimal insurance strategy; Put option; Expected utility.

ZHU, J. & YANG, H. *Ruin theory for a Markov regime-switching model under a threshold dividend strategy.* 311-318. In this paper, we study a Markov regime-switching risk model where dividends are paid out according to a certain threshold strategy depending on the underlying Markovian environment process. We are interested in these quantities: ruin probabilities, deficit at ruin and expected ruin time. To study them, we introduce functions involving the deficit at ruin and the indicator of the event that ruin occurs. We show that the above functions and the expectations of the time to ruin as functions of the initial capital satisfy systems of integro-differential equations. Closed form solutions are derived when the underlying Markovian environment process has only two states and the claim size distributions are exponential. Keywords: Markov regime-switching; Dividend; Ruin probability; Deficit at ruin; Integro-differential equation.

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BERMÚDEZ, L. L., PÉREZ, J. M., AYUSO, M., GÓMEZ, E. & VÁZQUEZ, F. J. *A Bayesian dichotomous model with asymmetric link for fraud in insurance.* 779-786. Standard binary models have been developed to describe the behavior of consumers when they are faced with two choices. The classical logit model presents the feature of the symmetric link function. However, symmetric links do not provide good fits for data where one response is much more frequent than the other (as it happens in the insurance fraud context). In this paper, we use an asymmetric or skewed logit link, proposed by Chen *et al.* [Chen, M., Dey, D. & Shao, Q., 1999. A new skewed link model for dichotomous quantal response data. *J. Amer. Statist.*

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Assoc. **94** (448), 1172-1186], to fit a fraud database from the Spanish insurance market. Bayesian analysis of this model is developed by using data augmentation and Gibbs sampling. The results show that the use of an asymmetric link notably improves the percentage of cases that are correctly classified after the model estimation. Keywords: Bayesian statistics; Logit model; Gibbs sampling; Automobile insurance; Fraud.

BOUCHER, J.-P. & DENUIT, M. *Credibility premiums for the zero-inflated Poisson model and new hunger for bonus interpretation.* 727-735. The purpose of this paper is to explore and compare the credibility premiums in generalized zero-inflated count models for panel data. Predictive premiums based on quadratic loss and exponential loss are derived. It is shown that the credibility premiums of the zero-inflated model allow for more flexibility in the prediction. Indeed, the future premiums not only depend on the number of past claims, but also on the number of insured periods with at least one claim. The model also offers another way of analysing the hunger for bonus phenomenon. The accident distribution is obtained from the zero-inflated distribution used to model the claims distribution, which can in turn be used to evaluate the impact of various credibility premiums on the reported accident distribution. This way of analysing the claims data gives another point of view on the research conducted on the development of statistical models for predicting accidents. A numerical illustration supports this discussion. Keywords: Credibility; Count data; Quadratic loss; Exponential loss; Zero-inflated models; Number of accidents; Hunger for bonus.

CHAN, Y. & LI, H. *Tail dependence for multivariate* t*-copulas and its monotonicity.* 763-770. The tail dependence indexes of a multivariate distribution describe the amount of dependence in the upper right tail or lower left tail of the distribution and can be used to analyse the dependence among extremal random events. This paper examines the tail dependence of multivariate *t*-distributions whose copulas are not explicitly accessible. The tractable formulas of tail dependence indexes of a multivariate *t*-distribution are derived in terms of the joint moments of its underlying multivariate normal distribution, and the monotonicity properties of these indexes with respect to the distribution parameters are established. Simulation results are presented to illustrate the results. Keywords: Tail dependence; Copula; Multivariate *t*-distribution; Inverse gamma distribution; Regularly varying distribution; Orthant dependence.

CHEUNG, K. C. *Improved convex upper bound via conditional comonotonicity.* 651-655. Comonotonicity provides a convenient convex upper bound for a sum of random variables with arbitrary dependence structure. Improved convex upper bound was introduced via conditioning by Kaas *et al.* [Kaas, R., Dhaene, J. & Goovaerts, M., 2000. Upper and lower bounds for sums of random variables. *Insurance: Math. Econ.* **27**, 151-168]. In this paper, we unify these results in a more general context using the concept of conditional comonotonicity. We also construct an approximating sequence of convex upper bounds with nice convergence properties. Keywords: Comonotonicity; Conditional comonotonicity; Convex order; Regular conditional distribution.

CHRISTIANSEN, M. C. *A sensitivity analysis concept for life insurance with respect to a valuation basis of infinite dimension.* 680-690. A sensitivity analysis concept is introduced for prospective reserves of individual life insurance contracts as deterministic mappings of the actuarial assumptions interest rate, mortality probability, disability probability, etc. Upon modeling these assumptions as functions on a real time line, the prospective reserve is here a mapping with infinite dimensional domain. Inspired by the common idea of interpreting partial derivatives of first order as local sensitivities, a generalized gradient vector approach is introduced in order to allow for a sensitivity analysis of the prospective reserves as functionals on a function space. The capability of the concept is demonstrated with an example Keywords: Life insurance; Variations in the technical basis; Prospective reserve; Sensitivity analysis; Generalized gradient vector.

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CHRISTIANSEN, M. C. *A sensitivity analysis of typical life insurance contracts with respect to the technical basis.* 787-796. In [Christiansen, M. C., 2007. A sensitivity analysis concept for life insurance with respect to a valuation basis of infinite dimension. *Insurance: Math. Econ.* doi:10.1016/j.insmatheco.2007.07.005] a sensitivity analysis concept was introduced for the prospective reserve of individual life insurance contracts as functional of the technical basis parameters such as interest rate, mortality probability, disability probability, et cetera. On the basis of that concept, the present paper gives in addition the sensitivities of the premium level. Applying these approaches, an extensive sensitivity analysis is carried out: A study of the basic life insurance contract types 'pure endowment insurance', 'temporary life insurance', 'annuity insurance' and 'disability insurance' identifies their diverse characteristics, in particular their weakest points concerning fluctuations of the technical basis. An investigation of combinations of these insurance contract types shows what synergy effects can be expected by creating insurance packages. Keywords: Life insurance; Variations in the technical basis; Prospective reserve; Premium level; Sensitivity analysis; Insurance packages.

DENUIT, M. *Comonotonic approximations to quantiles of life annuity conditional expected present value.* 831-838. In large portfolios, the risk borne by annuity providers (insurance companies or pension funds) is basically driven by the randomness in the future mortality rates. To fix the ideas, we adopt here the standard Lee–Carter framework, where the future forces of mortality are decomposed in a log-bilinear way. This paper aims to provide accurate approximations for the quantiles of the conditional expected present value of the payments to the annuity provider, given the future path of the Lee–Carter time index. Mortality is stochastic while the discount factors are derived from a zero-coupon yield curve and are assumed to be deterministic. Numerical illustrations based on Belgian mortality (general population and insurance market statistics) show that the accuracy of the approximations proposed in this paper is remarkable, with relative difference less than 1% for most probability levels. Keywords: Life annuity; Mortality projection; Lee–Carter model; Comonotonicity; Risk measures.

DHAENE, J., HENRARD, L., LANDSMAN, Z., VANDENDORPE, A. & VANDUFFEL, S. *Some results on the CTE-based capital allocation rule.* 855-863. Tasche [Tasche, D., 1999. Risk contributions and performance measurement. Working paper, Technische Universität München] introduces a capital allocation principle where the capital allocated to each risk unit can be expressed in terms of its contribution to the conditional tail expectation (CTE) of the aggregate risk. Panjer [Panjer, H. H., 2002. Measurement of risk, solvency requirements and allocation of capital within financial conglomerates. Institute of Insurance and Pension Research, University of Waterloo, Research Report 01-15] derives a closed-form expression for this allocation rule in the multivariate normal case. Landsman and Valdez [Landsman, Z. & Valdez, E., 2002. Tail conditional expectations for elliptical distributions. *North American Actuarial J.* **7** (4)] generalize Panjer's result to the class of multivariate elliptical distributions. In this paper we provide an alternative and simpler proof for the CTE-based allocation formula in the elliptical case. Furthermore, we derive accurate and easy computable closed-form approximations for this allocation formula for sums that involve normal and lognormal risks.

DREES, H. & MÜLLER, P. *Fitting and validation of a bivariate model for large claims.* 638-650. We consider an extended version of a model proposed by Ledford and Tawn [Ledford, A. W., Tawn, J. A., 1997. Modelling dependence within joint tail regions. *J. R. Stat. Soc.* **59** (2), 475-499] for the joint tail distribution of a bivariate random vector, which essentially assumes an asymptotic power scaling law for the probability that both the components of the vector are jointly large. After discussing how to fit the model, we devise a graphical tool that analyzes the differences between certain empirical probabilities and model based estimates of the same probabilities. The asymptotic normality of these differences allows the construction of statistical tests for the model assumption. The results are applied to claims of a Danish fire

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insurance and to medical claims from US health insurances. Keywords: Asymptotic normality; Bivariate tail estimation; Dependent catastrophic risks; Extreme value theory; Model validation.

EGAMI, M. & YOUNG, V. R. *Indifference prices of structured catastrophe (CAT) bonds.* 771-778. We present a method for pricing structured CAT bonds based on utility indifference pricing. The CAT bond considered here is issued in two distinct notes called tranches, specifically senior and junior tranches each with its own payment schedule. Our contributions to the literature of CAT bond pricing are two-fold. First, we apply indifference pricing to structured CAT bonds. We find a price for the senior tranche as a relative indifference price, that is, relative to the price of the junior tranche. Alternatively, one could take the approach that the senior tranche is priced first and the price of the junior tranche is relative to that. Second, instead of simply supposing that the "not-issue-a-CAT-bond" strategy of the reinsurer is to do nothing, we suppose that the reinsurer reduces its risk by reinsuring proportionally less claims. We assume that the reinsurance claims follow a (Poisson) jump–diffusion process. Keywords: Catastrophe (CAT) bond; Structured derivative security; Indifference price; Exponential utility; Jump–diffusion; Reinsurance strategy.

FOUNTAIN, R. L., HERMAN, J. R. JR & RUSTVOLD, D. L. *An application of Kendall distributions and alternative dependence measures: SPX vs. VIX.* 469-472. Most of the recently-defined notions of positive or negative dependence rely upon a variety of orderings of bivariate random vectors. These orderings are generally partial orders, and thus there are many pairs of random vectors which are not comparable. By using a weakened version of stochastic domination and the concepts of Kendall distributions and metacopulas, an entirely new class of orderings, in which the comparability issue is resolved, has been recently created. Each ordering in this class can be used to construct a measure of dependence. A detailed example will be given, using data from the Standard & Poor's 500 index and Chicago Board of Trades index for implied volatility. Keywords: Kendall distributions; Dependence measures; Copulas.

GATZERT, N. *Asset management and surplus distribution strategies in life insurance: An examination with respect to risk pricing and risk measurement.* 839-849. In this paper, we investigate the impact of different asset management and surplus distribution strategies in life insurance on risk-neutral pricing and shortfall risk. In general, these feedback mechanisms affect the contract's payoff and hence directly influence pricing and risk measurement. To isolate the effect of such strategies on shortfall risk, we calibrate contract parameters so that the compared contracts have the same market value and same default-value-to-liability ratio. This way, the fair valuation method is extended since, in addition to the contract's market value, the default put option value is fixed. We then compare shortfall probability and expected shortfall and show the substantial impact of different management mechanisms acting on the asset and liability side. Keywords: Life insurance; Asset management; Surplus distribution; Fair valuation; Lower partial moments.

GERSTNER, T., GRIEBEL, M., HOLTZ, M., GOSCHNICK, R. & HAEP, M. *A general asset–liability management model for the efficient simulation of portfolios of life insurance policies.* 704-716. New regulations and a stronger competition have increased the importance of stochastic asset–liability management (ALM) models for insurance companies in recent years. In this paper, we propose a discrete time ALM model for the simulation of simplified balance sheets of life insurance products. The model incorporates the most important life insurance product characteristics, the surrender of contracts, a reserve-dependent bonus declaration, a dynamic asset allocation and a two-factor stochastic capital market. All terms arising in the model can be calculated recursively which allows an easy implementation and efficient simulation. Furthermore, the model is designed to have a modular organization which

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permits straightforward modifications and extensions to handle specific requirements. In a sensitivity analysis for sample portfolios and parameters, we investigate the impact of the most important product and management parameters on the risk exposure of the insurance company and show that the model captures the main behaviour patterns of the balance sheet development of life insurance products Keywords: Asset–liability management; Participating policies; Numerical simulation.

Gómez-Déniz, E. *A generalization of the credibility theory obtained by using the weighted balanced loss function.* 850-854. In this paper an alternative to the usual credibility premium that arises for weighted balanced loss function is considered. This is a generalized loss function which includes as a particular case the weighted quadratic loss function traditionally used in actuarial science. From this function credibility premiums under appropriate likelihood and priors can be derived. By using weighted balanced loss function we obtain, first, generalized credibility premiums that contain as particular cases other credibility premiums in the literature and second, a generalization of the well-known distribution free approach in [Bühlmann, H., 1967. Experience rating and credibility. *Astin Bull.* **4** (3), 199-207]. Keywords: Loss function; Credibility premium; Net; Esscher; Distribution free approach.

Goovaerts, M. J. & Laeven, R. J. A. *Actuarial risk measures for financial derivative pricing.* 540-547. We present an axiomatic characterization of price measures that are superadditive and comonotonic additive for normally distributed random variables. The price representation derived involves a probability measure transform that is closely related to the Esscher transform, and we call it the Esscher–Girsanov transform. In a financial market in which the primary asset price is represented by a stochastic differential equation with respect to Brownian motion, the price mechanism based on the Esscher–Girsanov transform can generate approximate-arbitrage-free financial derivative prices. Keywords: Derivative pricing; Incomplete markets; Stochastic ordering; Esscher transform; Girsanov's theorem; Comonotonicity; Equivalent martingale measure; Feynman–Kac integration.

Guerra, M. & De Lourdes Centeno, M. *Optimal reinsurance policy: The adjustment coefficient and the expected utility criteria.* 529-539. This paper is concerned with the optimal form of reinsurance from the ceding company point of view, when the cedent seeks to maximize the adjustment coefficient of the retained risk. We deal with the problem by exploring the relationship between maximizing the adjustment coefficient and maximizing the expected utility of wealth for the exponential utility function, both with respect to the retained risk of the insurer. Assuming that the premium calculation principle is a convex functional and that some other quite general conditions are fulfilled, we prove the existence and uniqueness of solutions and provide a necessary optimal condition. These results are used to find the optimal reinsurance policy when the reinsurance premium calculation principle is the expected value principle or the reinsurance loading is an increasing function of the variance. In the expected value case the optimal form of reinsurance is a stop-loss contract. In the other cases, it is described by a nonlinear function. Keywords: Optimal reinsurance; Risk; Stop loss; Ruin probability; Adjustment coefficient; Premium principles; Exponential utility function.

Hari, N., De Waegenaere, A., Melenberg, B. & Nijman, T. E. *Estimating the term structure of mortality.* 492-504. In modeling and forecasting mortality the Lee–Carter approach is the benchmark methodology. In many empirical applications the Lee–Carter approach results in a model that describes the log central death rates by means of linear trends. However, due to the volatility in (past) mortality data, the estimation of these trends, and, thus, the forecasts based on them, might be rather sensitive to the sample period employed. We allow for time-varying trends, depending on a few underlying factors, to make the estimates of the future trends less sensitive to the sampling period. We formulate our model in a state-space framework, and use the Kalman filtering technique to estimate it. We

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illustrate our model using Dutch mortality data. Keywords: Mortality forecasting; State-space modeling; Parameter risk.

Hári, N., De Waegenaere, A., Melenberg, B. & Nijman, T. E. *Longevity risk in portfolios of pension annuities.* 505-519. We analyze the importance of longevity risk for the solvency of portfolios of pension annuities. We distinguish two types of mortality risk. Micro-longevity risk quantifies the risk related to uncertainty in the time of death if survival probabilities are known with certainty, while macro-longevity risk is due to uncertain future survival probabilities. We use a generalized two-factor Lee–Carter mortality model to produce forecasts of future mortality rates, and to assess the relative importance of micro- and macro-longevity risk for funding ratio uncertainty. The results show that if financial market risk is fully hedged so that uncertainty in future lifetime is the only source of uncertainty, pension funds are exposed to a substantial amount of risk. Systematic and non-systematic deviations from expected survival imply that, depending on the size of the portfolio, buffers that reduce the probability of underfunding to 2.5% at a 5-year horizon have to be of the order of magnitude of 7% to 8% of the initial value of the liabilities. Keywords: Longevity risk; Mortality projection; Pension fund; Funding ratio; Solvency buffer.

Hoevenaars, R. P. M. M. & Ponds, E. H. M. *Valuation of intergenerational transfers in funded collective pension schemes.* 578-593. This paper applies contingent claim analysis to value pension contracts for real-life collective pension plans with intergenerational risk sharing and offering DB-like benefits. We rewrite the balance sheet of such a pension fund as an aggregate of embedded generational options. This implies that a pension fund is a zero-sum game in value terms, so any policy change inevitably leads to value transfers between generations. We explore intergenerational value transfers that may arise from a plan redesign or from changes in funding policy and risk sharing rules. We develop a stochastic framework which accounts for time-varying investment opportunities and computes the embedded generational options. Changes in the values of the generational options enable us to evaluate the impact of policy modifications in the pension contract with respect to intergenerational transfers and redistribution. We find that a switch to a less risky asset mix is beneficial to elderly members at the expense of younger members who lose value. A reallocation of risk bearing from a plan with flexible contributions and fixed benefits to a plan with fixed contributions and flexible benefits leads to value redistribution from older plan members to younger ones. Keywords: Value-based ALM; Generational accounting; Embedded options; Intergenerational risk sharing; Funded defined-benefit plans; Public finance; Social security reform.

Ibragimov, R. & Walden, J. *Portfolio diversification under local and moderate deviations from power laws.* 594-599. This paper analyzes portfolio diversification for nonlinear transformations of heavy-tailed risks. It is shown that diversification of a portfolio of convex functions of heavy-tailed risks increases the portfolio's riskiness if expectations of these risks are infinite. In contrast, for concave functions of heavy-tailed risks with finite expectations, the stylized fact that diversification is preferable continues to hold. The framework of transformations of heavy-tailed risks includes many models with Pareto-type distributions that exhibit local or moderate deviations from power tails in the form of additional slowly varying or exponential factors. The class of distributions under study is therefore extended beyond the stable class. Keywords: Heavy-tailed risks; Nonlinear transformations; Portfolios; Diversification; Riskiness; Value at risk; Risk bounds; Pareto-type distributions; Power laws; Local and moderate deviations; Domain of attraction of stable distributions.

Kim, B., Kim, H.-S. & Kim, J. *A risk model with paying dividends and random environment.* 717-726. We consider a discrete time risk model where dividends are paid to insureds and the claim size has a discrete phase-type distribution, but the claim sizes vary according to an

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underlying Markov process called an environment process. In addition, the probability of paying the next dividend is affected by the current state of the underlying Markov process. We provide explicit expressions for the ruin probability and the deficit distribution at ruin by extracting a QBD (quasi-birth-and-death) structure in the model and then analyzing the QBD process. Numerical examples are also given. Keywords: Ruin probability; Dividend; Environment process; Deficit distribution; QBD (quasi-birth-and-death) process; Phase-type distribution.

KLÜPPELBERG, C. & KOSTADINOVA, R. *Integrated insurance risk models with exponential Lévy investment.* 560-577. We consider an insurance risk model for the cashflow of an insurance company, which invests its reserve into a portfolio consisting of risky and riskless assets. The price of the risky asset is modeled by an exponential Lévy process. We derive the integrated risk process and the corresponding discounted net loss process. We calculate certain quantities as characteristic functions and moments. We also show under weak conditions stationarity of the discounted net loss process and derive the left and right tail behavior of the model. Our results show that the model carries a high risk, which may originate either from large insurance claims or from the risky investment. Keywords: Continuous time perpetuity; Discounted net loss process; Exponential Lévy process; Generalized Ornstein–Uhlenbeck process; Integrated insurance risk process; Integrated risk management; Stochastic recurrence equations; Tail behavior.

LIN, Y. & COX, S. H. *Securitization of catastrophe mortality risks.* 628-637. Securitization with payments linked to explicit mortality events provides a new investment opportunity to investors and financial institutions. Moreover, mortality-linked securities provide an alternative risk management tool for insurers. As a step toward understanding these securities, we develop an asset pricing model for mortality-based securities in an incomplete market framework with jump processes. Our model nicely explains opposite market outcomes of two existing pure mortality securities.

LOISEL, S., MAZZA, C. & RULLIÈRE, D. *Robustness analysis and convergence of empirical finite-time ruin probabilities and estimation risk solvency margin.* 746-762. We consider the classical risk model and carry out a sensitivity and robustness analysis of finite-time ruin probabilities. We provide algorithms to compute the related influence functions. We also prove the weak convergence of a sequence of empirical finite-time ruin probabilities starting from zero initial reserve toward a Gaussian random variable. We define the concepts of reliable finite-time ruin probability as a Value-at-Risk of the estimator of the finite-time ruin probability. To control this robust risk measure, an additional initial reserve is needed and called Estimation Risk Solvency Margin (ERSM). We apply our results to show how portfolio experience could be rewarded by cut-offs in solvency capital requirements. An application to catastrophe contamination and numerical examples are also developed. Keywords: Finite-time ruin probability; Robustness; Solvency II; Reliable ruin probability; Asymptotic normality; Influence function; Estimation Risk Solvency Margin (ERSM).

MALINOVSKII, V. K. *Risk theory insight into a zone-adaptive control strategy.* 656-667. The main purpose of this paper is a risk theory insight into the problem of asset–liability and solvency adaptive management. In the multiperiodic insurance risk model composed of chained classical risk models, a zone-adaptive control strategy, essentially similar to that applied in Directives [Directive 2002/13/EC of the European Parliament and of the Council of 5 March 2002, Brussels, 5 March 2002], is introduced and its performance is examined analytically. That examination was initiated in [Malinovskii, V. K., 2006b. Adaptive control strategies and dependence of finite time ruin on the premium loading. *Insurance: Math. Econ.* (in press)] and is based on the application of the explicit expression for the finite-time ruin probability in the classical risk model. The result of independent interest in the paper is the

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representation of that finite-time ruin probability in terms of asymptotic series, as time increases. Keywords: Multiperiodic insurance process; Equity; Solvency; Adaptive control; Classical risk model.

Nelson, R. B., Quesada-Molina, J. J., Rodriguez-Lallena, J. A. & Ubeda-Flores, M. *On the construction of copulas and quasi-copulas with given diagonal sections.* 473-483. We study a method, which we call a copula (or quasi-copula) diagonal splice, for creating new functions by joining portions of two copulas (or quasi-copulas) with a common diagonal section. The diagonal splice of two quasi-copulas is always a quasi-copula, and we find a necessary and sufficient condition for the diagonal splice of two copulas to be a copula. Applications of this method include the construction of absolutely continuous asymmetric copulas with a prescribed diagonal section, and determining the best-possible upper bound on the set of copulas with a particular type of diagonal section. Several examples illustrate our results. Keywords: Bounds; Copulas; Diagonal sections; Distribution functions; Quasi-copulas.

Pitts, S. M. & Politos, K. *Approximations for the moments of ruin time in the compound Poisson model.* 668-679. In the classical risk model with Poisson arrivals, we study a functional approach which can be used to obtain new approximation formulae for the moments of the time to ruin. We explain how establishing differentiability of a functional, in appropriate function spaces, may lead to approximations for these moments. We consider various choices for the function spaces, which are suitable both for heavy-tailed and light-tailed claim-size distributions. The results are illustrated by some particular examples. Keywords: Compound Poisson model; Probability of ruin; Time of ruin; Moments of the time to ruin; Fréchet differentiability.

Renshaw, A. E. & Haberman, S. *On simulation-based approaches to risk measurement in mortality with specific reference to Poisson Lee–Carter modelling.* 797-816. This paper provides a comparative study of simulation strategies for assessing risk in mortality rate predictions and associated estimates of life expectancy and annuity values in both period and cohort frameworks. Keywords: Poisson modelling; Over-dispersion; Joint modelling; Negative binomial modelling; Mortality projections; Mortality statistics; Simulated risk.

Sanguesa, C. *Error bounds in approximations of random sums using gamma-type operators.* 484-491. In this work we deal with approximations of compound distributions, that is, distribution functions of random sums. More specifically, we obtain a discrete compound distribution by replacing each summand in the initial random sum by a discrete random variable whose probability mass function is related to a well-known inversion formula for Laplace transforms [cf. Feller, W., 1971. *An Introduction to Probability Theory and its Applications*, vol. II, second edn. Wiley, New York]. Our aim is to show the advantages that this method has in the context of compound distributions. In particular we give accurate error bounds for the distance between the initial random sum and its approximation when the individual summands are mixtures of gamma distributions. Keywords: Random sum; Compound distribution; Laplace transform; Kolmogorov distance; Error bound; Positive linear operator; Gamma mixture.

Sheldon Lin, X. & Sendova, K. P. *The compound Poisson risk model with multiple thresholds.* 617-627. In this paper we consider a multi-threshold compound Poisson risk model. A piecewise integro-differential equation is derived for the Gerber–Shiu discounted penalty function. We then provide a recursive approach to obtain general solutions to the integro-differential equation and its generalizations. Finally, we use the probability of ruin to illustrate the applicability of the approach Compound Poisson model; Expected discounted penalty function; Integro-differential equation; Multiple threshold strategy; Probability of ruin.

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Sun, J., Frees, E. W. & Rosenberg, M. A. *Heavy-tailed longitudinal data modeling using copulas.* 817-830. In this paper, we consider "heavy-tailed" data, that is, data where extreme values are likely to occur. Heavy-tailed data have been analyzed using flexible distributions such as the generalized beta of the second kind, the generalized gamma and the Burr. These distributions allow us to handle data with either positive or negative skewness, as well as heavy tails. Moreover, it has been shown that they can also accommodate cross-sectional regression models by allowing functions of explanatory variables to serve as distribution parameters. The objective of this paper is to extend this literature to accommodate longitudinal data, where one observes repeated observations of cross-sectional data. Specifically, we use copulas to model the dependencies over time, and heavy-tailed regression models to represent the marginal distributions. We also introduce model exploration techniques to help us with the initial choice of the copula and a goodness-of-fit test of elliptical copulas for model validation. In a longitudinal data context, we argue that elliptical copulas will be typically preferred to the Archimedean copulas. To illustrate our methods, Wisconsin nursing homes utilization data from 1995 to 2001 are analyzed. These data exhibit long tails and negative skewness and so help us to motivate the need for our new techniques. We find that time and the nursing home facility size as measured through the number of beds and square footage are important predictors of future utilization. Moreover, using our parametric model, we provide not only point predictions but also an entire predictive distribution. Keywords: Healthcare costs; Predictive modeling.

Tsanakas, A. *Risk measurement in the presence of background risk.* 520-528. A distortion-type risk measure is constructed, which evaluates the risk of any uncertain position in the context of a portfolio that contains that position and a fixed background risk. The risk measure can also be used to assess the performance of individual risks within a portfolio, allowing for the portfolio's re-balancing, an area where standard capital allocation methods fail. It is shown that the properties of the risk measure depart from those of coherent distortion measures. In particular, it is shown that the presence of background risk makes risk measurement sensitive to the scale and aggregation of risk. The case of risks following elliptical distributions is examined in more detail and precise characterisations of the risk measure's aggregation properties are obtained. Keywords: Risk measures; Background risk; Capital allocation; Portfolio management; Elliptical distributions.

Vandendorpe, A., Ho, N.-D., Vanduffel, S. & Van Dooren, P. *On the parameterization of the CreditRisk+ model for estimating credit portfolio risk.* 736-745. The CreditRisk+ model is one of the industry standards for estimating the credit default risk for a portfolio of credit loans. The natural parameterization of this model requires the default probability to be apportioned using a number of (non-negative) factor loadings. However, in practice only default correlations are often available but not the factor loadings. In this paper we investigate how to deduce the factor loadings from a given set of default correlations. This is a novel approach and it requires the non-negative factorization of a positive semi-definite matrix which is by no means trivial. We also present a numerical optimization algorithm to achieve this. Keywords: Default correlation; Aggregation; Factorization; Panjer's recursion.

Vereda, L., Lopes, H. & Fukuda, R. *Estimating VAR models for the term structure of interest rates.* 548-559. In this paper we follow the work of Evans and Marshall and propose new approaches for modelling the joint development of macro variables and the returns of government bond yields of several maturities. The models are estimated and compared with other forecasting schemes previously proposed in the literature, especially those relying on univariate, VAR and error correction methods. The models are then used to judge the hypothesis that the information content of macro variables and the term structure of interest rates as a whole help improving forecasting performance. Our main conclusion is quite simple: if one is interested in computing short-term forecasts, then there is no significant

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improvement in incorporating information other than the one already present in past observations of the yield at hand; however, if one worries about long-term forecasts (which is frequently the case with pension insurance companies), then the information content of macro variables and the term structure can improve forecasting performance. Keywords: Term structure of interest rates; Econometrics; Macroeconomic variables.

YOUNG, V. R. *Pricing life insurance under stochastic mortality via the instantaneous Sharpe ratio.* 691-703. We develop a pricing rule for life insurance under stochastic mortality in an incomplete market by assuming that the insurance company requires compensation for its risk in the form of a pre-specified instantaneous Sharpe ratio. Our valuation formula satisfies a number of desirable properties, many of which it shares with the standard deviation premium principle. The major result of the paper is that the price per contract solves a linear partial differential equation as the number of contracts approaches infinity. One can represent the limiting price as an expectation with respect to an equivalent martingale measure. Via this representation, one can interpret the instantaneous Sharpe ratio as a market price of mortality risk. Another important result is that if the hazard rate is stochastic, then the risk-adjusted premium is greater than the net premium, even as the number of contracts approaches infinity. Thus, the price reflects the fact that systematic mortality risk cannot be eliminated by selling more life insurance policies. We present a numerical example to illustrate our results, along with the corresponding algorithms. Keywords: Stochastic mortality; Pricing; Life insurance; Sharpe ratio; Non-linear partial differential equations; Market price of risk; Equivalent martingale measures.

ZHOU, Q., WU, W. & WANG, Z. *Cooperative hedging with a higher interest rate for borrowing.* 609-616. The paper studies the cooperative hedging problem of contingent claims in an incomplete financial market. Firstly we give the characterization of the optimal cooperative hedging strategy for the Black–Scholes model and the Volatility Jump model explicitly, then we consider the problem of cooperative hedging for the multi-agent case in a market with a higher borrowing interest rate. By the results of concave and linear backward stochastic differential equations, we give the optimal cooperative hedging strategy in our model Keywords: Hedging; Neyman Pearson lemma; Stochastic volatility; Backward stochastic differential equation.

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ALBRECHTER, H., BADESCU, A. & LANDRIAULT, D. *On the dual risk model with tax payments.* 1086-1094. In this paper, we study the dual risk process in ruin theory (see e.g. Cramér, H. 1955. Collective Risk Theory: A Survey of the Theory from the Point of View of the Theory of Stochastic Processes. Ab Nordiska Bokhandeln, Stockholm, Takacs, L. 1967. Combinatorial methods in the Theory of Stochastic Processes. Wiley, New York and Avanzi, B., Gerber, H. U. & Shiu, E. S. W., 2007. Optimal dividends in the dual model. *Insurance: Math. Econ.* **41**, 111-123) in the presence of tax payments according to a loss-carry forward system. For arbitrary inter-innovation time distributions and exponentially distributed innovation sizes, an expression for the ruin probability with tax is obtained in terms of the ruin probability without taxation. Furthermore, expressions for the Laplace transform of the time to ruin and arbitrary moments of discounted tax payments in terms of passage times of the risk process are determined. Under the assumption that the inter-innovation times are (mixtures of) exponentials, explicit expressions are obtained. Finally, we determine the critical surplus level at which it is optimal for the tax authority to start collecting tax payments.

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Bai, L. & Guo, J. *Optimal proportional reinsurance and investment with multiple risky assets and no-shorting constraint.* 968-975. In this paper, the basic claim process is assumed to follow a Brownian motion with drift. In addition, the insurer is allowed to invest in a risk-free asset and n risky assets and to purchase proportional reinsurance. Under the constraint of no-shorting, we consider two optimization problems: the problem of maximizing the expected exponential utility of terminal wealth and the problem of minimizing the probability of ruin. By solving the corresponding Hamilton–Jacobi–Bellman equations, explicit expressions for their optimal value functions and the corresponding optimal strategies are obtained. In particular, when there is no risk-free interest rate, the results indicate that the optimal strategies, under maximizing the expected exponential utility and minimizing the probability of ruin, are equivalent for some special parameter. This validates Ferguson's longstanding conjecture about the relation between the two problems. Keywords: Exponential utility; Hamilton–Jacobi–Bellman equation; Optimal strategy; Probability of ruin; Proportional reinsurance.

Baumann, R. T. & Muller, H. H. *Pension funds as institutions for intertemporal risk transfer.* 1000-1012. A continuous time overlapping generation model is used to analyse defined-contribution pension plans. Without intergenerational risk transfer between employees the optimal investment strategy results from the Merton model. Introducing intergenerational risk transfer leads to an increase in the risk tolerance of future employees and allows us to improve their anticipated expected utility resulting from accrued retirement benefits. Of course, this leads to a risk of temporary underfunding. But even for an underfunded pension plan one can guarantee that in the long run, the median of the funding ratio exceeds one. Keywords: Portfolio choice; Pension finance; Defined-contribution pension fund; Intergenerational risk transfer; Overlapping generation model.

Blake, D., Dowd, K. & Cairns, A. J. G. *Longevity risk and the Grim Reaper's toxic tail: the survivor fan charts.* 1062-1066. This paper uses survivor fan charts to illustrate the prospective density functions of future male survival rates. The fan charts are based on a version of the Cairns–Blake–Dowd model of male mortality that provides a good fit to recent mortality data for England and Wales. They indicate that although none of us can escape the Grim Reaper, survivorship uncertainty is greatest for males aged a little over 90, confirming that there exists a 'toxic tail' for those institutions, such as annuity and pension providers, which are obliged to make payments to them for as long as they live. We also find that taking account of uncertainty in the parameters of the underlying mortality model leads to major increases in estimates of the widths of the fan charts. Keywords: Cairns–Blake–Dowd model; Mortality; Fan charts; Longevity risk; Survival rates; Toxic tail.

Borovkov, K. A. & Dickson, D. S. M. *On the ruin time distribution for a Sparre Andersen process with exponential claim sizes.* 1104-1108. We derive a closed-form (infinite series) representation for the distribution of the ruin time for the Sparre Andersen model with exponentially distributed claims. This extends a recent result of Dickson *et al.* [Dickson, D. C. M., Hughes, B. D. & Zhang, L., 2005. The density of the time to ruin for a Sparre Andersen process with Erlang arrivals and exponential claims. *Scand. Actuar. J.*, 358-376] for such processes with Erlang inter-claim times. The derivation is based on transforming the original boundary crossing problem to an equivalent one on linear lower boundary crossing by a spectrally positive Lévy process. We illustrate our result in the cases of gamma, mixed exponential and inverse Gaussian inter-claim time distribution Keywords: Sparre Andersen model; Time of ruin; Exponential claims.

Chen, A. N. *Loss analysis of a life insurance company applying discrete-time risk-minimizing hedging strategies.* 1035-1049. The present paper investigates the net loss of a life insurance company issuing equity-linked pure endowments in the case of periodic premiums. Due to the

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untradability of the insurance risk which affects both the in- and outflow side of the company, the issued insurance claims cannot be hedged perfectly. Furthermore, we consider an additional source of incompleteness caused by trading restrictions, because in reality the hedging of the contingent claims is more likely to occur at discrete times. Based on Møller [Møller, T., 1998. Risk-minimizing hedging strategies for unit-linked life insurance contracts. *Astin Bull.* **28**, 17-47], we particularly examine the situation, where the company applies a time-discretized risk-minimizing hedging strategy. Through an illustrative example, we observe numerically that only a relatively small reduction in ruin probabilities is achieved with the use of the discretized originally risk-minimizing strategy because of the accumulated extra duplication errors caused by discretizing. However, the simulated results are highly improved if the hedging model instead of the hedging strategy is discretized. For this purpose, Møller's [Møller, T., 2001. Hedging equity-linked life insurance contracts. *North Amer. Actuarial J.* **5** (2), 79-95] discrete-time (binomial) risk-minimizing strategy is adopted. Keywords: Net loss; Discrete-time risk-minimizing hedging strategies; Pure endowment equity-linked life insurance; Ruin probability.

Chen, X., Deelstra, G., Dhaene, J. & Vanmaele, M. *Static super-replicating strategies for a class of exotic options.* 1067-1085. In this paper, we investigate static super-replicating strategies for European-type call options written on a weighted sum of asset prices. This class of exotic options includes Asian options and basket options among others. We assume that there exists a market where the plain vanilla options on the different assets are traded and hence their prices can be observed in the market. Both the infinite market case (where prices of the plain vanilla options are available for all strikes) and the finite market case (where only a finite number of plain vanilla option prices are observed) are considered. We prove that the finite market case converges to the infinite market case when the number of observed plain vanilla option prices tends to infinity. We show how to construct a portfolio consisting of the plain vanilla options on the different assets, whose pay-off super-replicates the pay-off of the exotic option. As a consequence, the price of the super-replicating portfolio is an upper bound for the price of the exotic option. The super-hedging strategy is model-free in the sense that it is expressed in terms of the observed option prices on the individual assets, which can be e.g. dividend paying stocks with no explicit dividend process known. This paper is a generalization of the work of Simon *et al.* [Simon, S., Goovaerts, M. & Dhaene, J., 2000. An easy computable upper bound for the price of an arithmetic Asian option. *Insurance: Math. Econ.* **26** (2-3), 175-184] who considered this problem for Asian options in the infinite market case. Laurence and Wang [Laurence, P. & Wang, T. H., 2004. What's a basket worth? *Risk Mag.* **17**, 73-77] and Hobson *et al.* [Hobson, D., Laurence, P. & Wang, T. H., 2005. Static-arbitrage upper bounds for the prices of basket options. *Quant. Fin.* **5** (4), 329-342] considered this problem for basket options, in the infinite as well as in the finite market case. As opposed to Hobson *et al.* [Hobson, D., Laurence, P. & Wang, T. H., 2005. Static-arbitrage upper bounds for the prices of basket options. *Quant. Fin.* **5** (4), 329-342] who use Lagrange optimization techniques, the proofs in this paper are based on the theory of integral stochastic orders and on the theory of comonotonic risks. Keywords: Static super-replicating strategies; Stochastic ordering; Comonotonicity; Exotic options.

Costabile, M., Massabo, I. & Russo, E. *A binomial model for valuing equity-linked policies embedding surrender options.* 873-886. The computation of the fair periodical premiums for equity-linked policies in a Cox–Ross–Rubinstein (CRR) [Cox, J. C., *et al.*, 1979. Option pricing: A simplified approach. *J. Financial Economics* **7**, 229-263] evaluation framework is computationally complex. In fact, despite we assume that the equity value evolves according to a CRR lattice, the dynamics of the reference fund made up of equities of the same kind is described by a non-recombining tree since, at each contribution date, a constant contribution is added to the fund value. We propose to overcome this problem by selecting representative values among all the effective reference fund values. Then, the fair periodical premiums for

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equity-linked policies embedding a surrender option and a minimum guarantee are computed following the usual backward-induction scheme coupled with linear interpolation. Keywords: Equity-linked; Binomial algorithms; Discrete-time models.

COULIBALY, I. & LEFEVRE, C. *On a simple quasi-Monte Carlo approach for classical ultimate ruin probabilities.* 935-942. This note discusses a simple quasi-Monte Carlo method to evaluate numerically the ultimate ruin probability in the classical compound Poisson risk model. The key point is the Pollaczek–Khintchine representation of the non-ruin probability as a series of convolutions. Our suggestion is to truncate the series at some appropriate level and to evaluate the remaining convolution integrals by quasi-Monte Carlo techniques. For illustration, this approximation procedure is applied when claim sizes have an exponential or generalized Pareto distribution. Keywords: Risk theory; Compound Poisson model; Ultimate ruin probability; Numerical integration; Quasi-Monte Carlo methods; Low-discrepancy sequences; Monte Carlo simulation.

COUSIN, A. & LAURENT, J.-P. *Comparison results for exchangeable credit risk portfolios.* 1118-1127. This paper is dedicated to risk analysis of credit portfolios. Assuming that default indicators form an exchangeable sequence of Bernoulli random variables and as a consequence of de Finetti's theorem, default indicators are Binomial mixtures. We can characterize the supermodular order between two exchangeable Bernoulli random vectors in terms of the convex ordering of their corresponding mixture distributions. Thus we can proceed to some comparisons between stop-loss premiums, CDO tranche premiums and convex risk measures on aggregate losses. This methodology provides a unified analysis of dependence for a number of CDO pricing models based on factor copulas, multivariate Poisson and structural approaches. Keywords: Default risk; CDOs; Factor copulas; Multivariate Poisson; Structural models; Stochastic orders.

CRANE, G. & HOEK, J. VAN DER. *Using distortions of copulas to price synthetic CDO's.* 903-908. This paper demonstrates how to use distorted Gaussian copula functions to produce a heavy tailed portfolio loss distribution in the context of synthetic Collateralized Debt Obligations (CDOs). Distortion functions have not previously been used in this area. Hence, we demonstrate that it is possible to simulate realistic tranche prices by incorporating distorted copula functions within a well established CDO pricing system, such as that of JP Morgan. Furthermore, we only require a single dependence parameter for the entire portfolio rather than one per tranche. Thus, we are providing practitioners with a simpler and more flexible alternative to current CDO pricing methods.

FILIPOVIC, D. & VOGELPOTH, N. *A note on the Swiss solvency test risk measure.* 897-902. In this paper we examine whether the Swiss Solvency Test risk measure is a coherent measure of risk as introduced in Artzner *et al.* [Artzner, P., Delbaen, F., Eber, J. M. & Heath, D., 1999. Coherent measures of risk. *Math. Finance* **9**, 203-228; Artzner, P., Delbaen, F., Eber, J. M., Heath, D. & Ku, H., 2004. Coherent multiperiod risk adjusted values and Bellman's principle. Working Paper. ETH Zurich]. We provide a simple example which shows that it does not satisfy the axiom of monotonicity. We then find, as a monotonic alternative, the greatest coherent risk measure which is majorized by the Swiss Solvency Test risk measure. Keywords: Multiperiod risk measure; Swiss solvency test; Target capital.

FREHEN, R. G. P., HOEVENAARS, R. P. M. M., PALM, F. C. & SCHOTMAN, P. C. *Regret aversion and annuity risk in defined contribution pension plans.* 1050-1061. The high value of the implicit option to choose a retirement date at which interest rates are particularly high and life annuities relatively cheap, leads to the possibility to introduce regret aversion in the retirement investment decision of defined contribution plan participants. As a remedy for regret aversion in retirement investment decisions, this paper develops and prices a lookback

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option on a life annuity contract. We determine a closed-form option value under the restriction that the option holder invests risklessly during the time to maturity of the option and without the guarantee that the exact amount of retirement wealth is converted into a life annuity at retirement. Thereafter the investment restriction is relaxed and the guarantee of exact conversion is imposed and the option is priced via Monte Carlo simulations in an economic environment with a stochastic discount factor. Option price sensitivities are determined via the pricing of alternative options. We find that the price of a lookback option, with a maturity of three years, amounts to 8%-9% of the wealth at the option issuance date. The option price is highly sensitive to the exercise price of the option, i.e. pricing alternative options (e.g. Asian) substantially lowers the price. Time to maturity and interest rate volatility are other important option price drivers. Asset allocation decisions and initial interest rates hardly affect the option price. Keywords: Lookback option; Life annuity; Annuity risk; Defined contribution; Stochastic discount factor.

Gao, J. *Stochastic optimal control of DC pension funds.* 1159-1164. In this paper, we study the portfolio problem of a pension fund manager who wants to maximize the expected utility of the terminal wealth in a complete financial market with the stochastic interest rate. Using the method of stochastic optimal control, we derive a non-linear second-order partial differential equation for the value function. As it is difficult to find a closed form solution, we transform the primary problem into a dual one by applying a Legendre transform and dual theory, and try to find an explicit solution for the optimal investment strategy under the logarithm utility function. Finally, a numerical simulation is presented to characterize the dynamic behaviour of the optimal portfolio strategy. Keywords: Defined-contribution pension plans; Optimal investment strategy; Stochastic optimal control; Legendre transform; Hamilton–Jacobi–Bellman equation.

Gebizlioglu, O. L. & Yagci, B. *Tolerance intervals for quantiles of bivariate risks and risk measurement.* 1022-1027. This paper considers joint distributions of order statistics for risk variables and their concomitants for actuarial risk analysis under dependence. With this purpose, bivariate integral transformations are performed and some examples are presented using copulas, the FGM copulas in particular. Quantiles of the distributions concerned are discussed and their tolerance intervals are constructed. Risk measures such as VaR in the set up of the tolerance intervals are included in the discussions. Keywords: Bivariate risks; Quantiles; Tolerance intervals; Farlie–Gumbel–Morgenstern family of distributions; Copulas; Order statistics; Concomitants; Probability integral transform.

He, L. & Liang, Z. *Optimal financing and dividend control of the insurance company with proportional reinsurance policy.* 976-983. We consider the optimal control problem of the insurance company with proportional reinsurance policy. The management of the company controls the reinsurance rate, dividends payout as well as the equity issuance processes to maximize the expected present value of the dividends minus the equity issuance until the time of bankruptcy. This is the first time that the financing process in an insurance model has been considered, which is more realistic. To find the solution of the mixed singular-regular control problem, we firstly construct two categories of suboptimal models, one is the classical model without equity issuance, the other never goes bankrupt by equity issuance. Then we identify the value functions and the optimal strategies corresponding to the suboptimal models depending on the relationships between the coefficients. Keywords: Optimal dividends control; Optimal financing control; Proportional reinsurance; HJB equation; Transaction cost; Regular-singular control.

Hua, L. & Cheung, K. C. *Stochastic orders of scalar products with applications.* 865-872. In this paper, we study stochastic orders of scalar products of random vectors. Based on the study of Ma [Ma, C., 2000. Convex orders for linear combinations of random variables. *J.*

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Statist. Plann. Inference **84**, 11-25], we first obtain more general conditions under which linear combinations of random variables can be ordered in the increasing convex order. As an application of this result, we consider the scalar product of two random vectors which separates the severity effect and the frequency effect in the study of the optimal allocation of policy limits and deductibles. Finally, we obtain the ordering of the optimal allocation of policy limits and deductibles when the dependence structure of the losses is unknown. This application is a further study of Cheung [Cheung, K. C., 2007. Optimal allocation of policy limits and deductibles. *Insurance: Math. Econ.* **41**, 382-391]. Keywords: Weak majorization; Arrangement increasing function; Stochastic orders; Increasing convex order; Comonotonicity; Scalar product.

Kijima, M. & Muromachi, Y. *An extension of the Wang transform derived from Buhlmann's economic permium principle for insurance risk*. 887-896. It is well known that the Wang transform [Wang, S. S., 2002. A universal framework for pricing financial and insurance risks. *Astin Bull.* **32**, 213-234] for the pricing of financial and insurance risks is derived from Bühlmann's economic premium principle [Bühlmann, H., 1980. An economic premium principle. *Astin Bull.* **11**, 52-60]. The transform is extended to the multivariate setting by [Kijima M., 2006. A multivariate extension of equilibrium pricing transforms: The multivariate Esscher and Wang transforms for pricing financial and insurance risks, *Astin Bull.* **36**, 269-283]. This paper further extends the results to derive a class of probability transforms that are consistent with Bühlmann's pricing formula. The class of transforms is extended to the multivariate setting by using a Gaussian copula, while the multiperiod extension is also possible within the equilibrium pricing framework. Keywords: Bühlmann's equilibrium price; Wang transform; Gaussian copula; non-central t distribution.

Kotter, M. & Bauerle, N. *The periodic risk model with investment*. 962-967. We consider a periodic risk model with the possibility of investing into a risky asset, given by a geometrical Brownian motion. The aim is to maximize the adjustment coefficient of the risk process. It is shown that the optimal investment strategy only depends on the averaged data of the model and is constant over time. Thus maximizing the adjustment coefficient is a very weak optimization criterion. Keywords: Ruin probability; Adjustment coefficient; Optimal investment; Periodic environment.

Lokka, A. & Zervos, M. *Optimal dividend and issuance of equity policies in the presence of proportional costs*. 954-961. We consider three optimisation problems faced by a company that can control its liquid reserves by paying dividends and by issuing new equity. The first of these problems involves no issuance of new equity and has been considered by several authors in the literature. The second one aims at maximising the expected discounted dividend payments minus the expected discounted costs of issuing new equity over all strategies associated with positive reserves at all times. The third problem has the same objective as the second one, but with no constraints on the reserves. Assuming proportional issuance of equity costs, we derive closed form solutions and we completely characterise the optimal strategies. We also provide a relationship between the three problems.

Nordahl, H. A. *Valuation of life insurance surrender and exchange options*. 909-919. In this paper I analyze two American-type options related to life and pension insurance contract. I use Monte Carlo simulations combined with the Longstaff and Schwartz approach for the valuation of American options to find the value of a typical surrender option. I find that the values may be much lower than previously indicated. This reduction of value is due to a different treatment of bonuses, limiting the customers' ability to forecast the return of their policies. The numerical results show that the value may be higher than the corresponding surrender option. Keywords: Surrender options; Exchange options; Life and pension insurance.

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Oliviera, A. & Pitacco, E. *Assessing the cost of capital for longevity risk.* 1013-1021. The cost of capital is a key element of the embedded value methodology for the valuation of a life business. Further, under some solvency approaches (in particular, the Swiss Solvency Test and the developing Solvency 2 project) assessing the cost of capital constitutes a step in determining the required capital allocation. Whilst the cost of capital is usually meant as a reward for the risks encumbering a given life portfolio, in actuarial practice the relevant parameter has been traditionally chosen, at least to some extent, inconsistently with such risks. The adoption of market-consistent valuations has then been advocated to reach a common standard. A market-consistent value usually acknowledges a reward to shareholders' capital as long as the market does, namely if the risk is systematic or undiversifiable. When dealing with a life annuity portfolio (or a pension plan), an important example of systematic risk is provided by the longevity risk, i.e. the risk of systematic deviations from the forecasted mortality trend. Hence, a market-consistent approach should provide appropriate valuation tools. In this paper we refer to a portfolio of immediate life annuities and we focus on longevity risk. Our purpose is to design a framework for a valuation of the portfolio which is market-consistent, and therefore based on a risk-neutral argument, while involving some of the basic items of a traditional valuation, viz best estimate future flows and allocated capital. This way, we try to reconcile the traditional with a market-consistent (or risk-neutral) approach. This allows us, in particular, to translate the results obtained under the risk-neutral approach in terms of a properly redefined embedded value. Keywords: Risk reward; Embedded value; Market-consistent valuation; Risk-neutral setting; Life annuities.

Petrichev, K. & Thorp, S. *The private value of public pensions.* 1138-1145. As individual retirement savings accounts replace public pensions and defined benefit schemes, more retirees will decumulate using commercial income streams rather than public or corporate annuities. Here we use an approximation to the retirement income problem [Huang, H., Milevsky, M. A. & Wang, J., 2004. Ruined moments in your life: How good are the approximations? *Insurance: Math. Econ.* **34**, 421-447] to compute the cost of replicating a public real life annuity (the Australian Age Pension) using commercial decumulation products. We treat the public pension as a phased withdrawal plan, matching insurance and payment features, and back out the stochastic present value of the plan under an arbitrarily small ruin probability. To reproduce the pension payment with 99% certainty, a male retiree needs 3.6 times the current average retirement savings account balance, and a female retiree needs more than 10 times the average female account balance. At 95% certainty, required wealth falls by around 25%. We measure separately the impact of gender, investment strategy, retirement age and management fees on this valuation. Keywords: Social security; Longevity risk; Phased withdrawal; Stochastic present value.

Siu, T. K. *A game theoretic approach to option valuation under Markovian regime-switching models.* 1146-1158. In this paper, we consider a game theoretic approach to option valuation under Markovian regime-switching models, namely, a Markovian regime-switching geometric Brownian motion (GBM) and a Markovian regime-switching jump-diffusion model. In particular, we consider a stochastic differential game with two players, namely, the representative agent and the market. The representative agent has a power utility function and the market is a "fictitious" player of the game. We also explore and strengthen the connection between an equivalent martingale measure for option valuation selected by an equilibrium state of the stochastic differential game and that arising from a regime switching version of the Esscher transform. When the stock price process is governed by a Markovian regime-switching GBM, the pricing measures chosen by the two approaches coincide. When the stock price process is governed by a Markovian regime-switching jump-diffusion model, we identify the condition under which the pricing measures selected by the two approaches are identical. Keywords: Option valuation; Regime switching; Stochastic differential game; Esscher transform; Jump-diffusion model; Power utility.

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Sordo, M. A. *Characterizations of classes of risk measures by dispersive orders.* 1028-1034. In this paper, a class C1 of risk measures, which generalizes the class of risk measures for the right-tail deviation suggested by Wang [Wang, S., 1998. An actuarial index of the right-tail risk. *North Amer. Actuarial J.* **2**, 88-101], is characterized in terms of dispersive order. If dispersive order does not hold, unanimous comparisons are still possible by restricting our attention to a subclass C2C1 and then the criterion is the excess-wealth order. Sufficient conditions for stochastic equivalence of excess-wealth ordered random variables are derived in terms of some particular measures of C2. Keywords: Dispersive order; Excess-wealth order; Risk measures; Gini mean difference; Wang's right-tail deviation.

Vandaele, N. & Vanmaele, M. *A locally risk-minimizing hedging strategy for unit-linked life insurance contracts in a Levy process financial market.* 1128-1137. In [Riesner, M., 2006. Hedging life insurance contracts in a Lévy process financial market. *Insurance: Math. Econ.* **38**, 599-608] the (locally) risk-minimizing hedging strategy for unit-linked life insurance contracts is determined in an incomplete financial market driven by a Lévy process. The considered risky asset is not a martingale under the original measure and therefore, a change of measure to the minimal martingale measure is performed. The goal of this paper is to show that the risk-minimizing hedging strategy under the new martingale measure which is found in the paper cited above is not the locally risk-minimizing strategy under the original measure. Finally, the real locally risk-minimizing strategy is derived and a relationship between the number of risky assets held in the proposed portfolio cited in the above-mentioned paper and the one proposed here is given. Keywords: Unit-linked; Local risk-minimization; Hedging strategy; Lévy process.

Vanduffel, S., Shang, Z., Henrard, L., Dhaene, J. & Valdez, E. A. *Analytic bounds and approximations for annuities and Asian options.* 1109-1117. Even in case of the Brownian motion as most natural rate of return model it appears too difficult to obtain analytic expressions for most risk measures of constant continuous annuities. In literature the so-called comonotonic approximations have been proposed but these still require the evaluation of integrals. In this paper we show that these integrals can sometimes be computed, and we obtain explicit approximations for some popular risk measures for annuities. Next, we show how these results can be used to obtain fully analytic expressions for lower and upper bounds for the price of a continuously sampled European-style Asian option with fixed exercise price. These analytic lower bound prices are as sharp as those from [Rogers, L. C. G. & Shi, Z., 1995. The value of an Asian option. *J. Appl. Probab.* **32**, 1077-1088], if not sharper, but in contrast do not require any longer the evaluation of a two-dimensional or a one-dimensional integral. Keywords: Asian option; Brownian motion; Analytic; Closed-form; Lower bound; Annuity; Fixed strike; Continuously sampled.

Xie, S., Li, Z. & Wang, S. *Continuous-time portfolio selection with liability: mean–variance model and stochastic LQ approach.* 943-953. In this paper we formulate a continuous-time mean–variance portfolio selection model with multiple risky assets and one liability in an incomplete market. The risky assets' prices are governed by geometric Brownian motions while the liability evolves according to a Brownian motion with drift. The correlations between the risky assets and the liability are considered. The objective is to maximize the expected terminal wealth while minimizing the variance of the terminal wealth. We derive explicitly the optimal dynamic strategy and the mean–variance efficient frontier in closed forms by using the general stochastic linear-quadratic (LQ) control technique. Several special cases are discussed and a numerical example is also given. Keywords: Portfolio selection; Asset–liability management; Continuous-time; Mean–variance model; Stochastic linear-quadratic control.

Yang, H. & Zhang, Z. *Gerber-Shiu discounted penalty function in a Sparre Andersen model with multi-layer dividend strategy.* 984-991. This paper studies a Sparre Andersen model in

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which the inter-claim times are generalized Erlang(n) distributed. We assume that the premium rate is a step function depending on the current surplus level. A piecewise integro-differential equation for the Gerber–Shiu discounted penalty function is derived and solved. Finally, to illustrate the solution procedure, explicit expression for the Laplace transform of the time to ruin is given when the inter-claim times are generalized Erlang(2) distributed and the claim amounts are exponentially distributed. Keywords: Multi-layer dividend strategy; Integro-differential equation; Gerber–Shiu discounted penalty function; Time to ruin.

YANG, S. S., YUEH, M.-L. & TANG, C.-H. *Valuation of interest rate guarantee embedded in defined contribution pension plans.* 920-934. In this research, we derive the valuation formulae for a defined contribution pension plan associated with the minimum rate of return guarantees. Different from the previous studies, we work on the rate of return guarantee which is linked to the d-year spot rate. The payoffs of interest rate guarantees can be viewed as a function of the exchange option. By employing Margrabe's [Margrabe, W., 1978. The value of an option to exchange one asset for another. *Journal of Finance* **33**, 177-186] option pricing approach, we derive general pricing formulae under the assumptions that the interest rate dynamics follow a single-factor HJM (1992) [Heath. D. *et al*., 1992. Bond pricing and the term structure of interest rates: a new methodology for contingent claims valuation. *Econometrica* **60**, 77-105] interest rate model and the asset prices follow a geometric Brownian motion. The volatility of the forward rates is assumed to be exponentially decaying. The formula is explicit for valuing maturity guarantee (type-I guarantee). For multi-period guarantee (type-II guarantee), the analytical formula only exists when the guaranteed rate is the one-year spot rate. The accuracy of the valuation formulae is illustrated with numerical analysis. We also investigate the effect of mortality and the sensitivity of key parameters on the value of the guarantee. We find that type-II guarantee is much more costly than the type-I guarantee, especially with a long duration policy. The closed form solution provides the advantage in valuing pension guarantees.

ZHANG, J. & GUEGAN, D. *Pricing bivariate option under GARCH processes with time-varying copula.* 1095-1103. This paper develops a method for pricing bivariate contingent claims under General Autoregressive Conditionally Heteroskedastic (GARCH) process. As the association between the underlying assets may vary over time, the dynamic copula with time-varying parameter offers a better alternative to any static model for dependence structure and even to the dynamic copula model determined by dynamic dependence measure. Therefore, the proposed method proves to play an important role in pricing bivariate options. The approach is illustrated with one type of better-of-two-markets claims: call option on the better performer of Shanghai and Shenzhen Stock Composite Indexes. Results show that the option prices obtained by the time-varying copula model differ substantially from the prices implied by the static copula model and even the dynamic copula model derived from the dynamic dependence measure. Moreover, the empirical work displays the advantages of the suggested method. Keywords: Call-on-max option; GARCH process; Kendall's tau; Copula; Dynamic Copula; Time-varying parameter.

ZHOU, C. & WU, C. *Optimal insurance under the insurer's risk constraint.* 992-999. In this paper, we impose the insurer's risk constraint on Arrow's optimal insurance model. The insured aims to maximize his/her expected utility of terminal wealth, under the constraint that the insurer wishes to control the expected loss of his/her terminal wealth below some prespecified level. We solve the problem, and it is shown that when the insurer's risk constraint is binding, the solution to the problem is not linear, but piecewise linear deductible. Moreover, it can be shown that the insured's optimal expected utility will increase if the insurer increases his/her risk tolerance. Keywords: Optimal insurance; Expected loss; Risk constraint.

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ALBRECHER, H. & THONHAUSER, S. *Optimal dividend strategies for a risk process under force of interest*. 134-149. In the classical Cramér–Lundberg model in risk theory the problem of maximizing the expected cumulated discounted dividend payments until ruin is a widely discussed topic. In the most general case within that framework it is proved [Gerber, H. U., 1968. Entscheidungskriterien fuer den zusammengesetzten *Poisson-prozess. Schweiz. Aktuarver. Mitt.* **1**, 185-227; Azcue, P. & Muler, N., 2005. Optimal reinsurance and dividend distribution policies in the Cramér–Lundberg model. *Math. Finance* **15** (2) 261-308; Schmidli, H., 2008. Stochastic Control in Insurance. Springer] that the optimal dividend strategy is of band type. In the present paper we discuss this maximization problem in a generalized setting including a constant force of interest in the risk model. The value function is identified in the set of viscosity solutions of the associated Hamilton–Jacobi–Bellman equation and the optimal dividend strategy in this risk model with interest is derived, which in the general case is again of band type and for exponential claim sizes collapses to a barrier strategy. Finally, an example is constructed for Erlang(2)-claim sizes, in which the bands for the optimal strategy are explicitly calculated.

BADESCU, A. M. & KULPERGER, R. J. *GARCH option pricing: A semiparametric approach*. 69-84. Option pricing based on GARCH models is typically obtained under the assumption that the random innovations are standard normal (normal GARCH models). However, these models fail to capture the skewness and the leptokurtosis in financial data. We propose a new method to compute option prices using a nonparametric density estimator for the distribution of the driving noise. We investigate the pricing performances of this approach using two different risk neutral measures: the Esscher transform pioneered by Gerber and Shiu [Gerber, H. U., Shiu, E. S. W., 1994a. Option pricing by Esscher transforms (with discussions). *Trans. Soc. Actuar.* **46**, 99-91], and the extended Girsanov principle introduced by Elliot and Madan [Elliot, R. J., Madan, D. G., 1998. A discrete time equivalent martingale 9 measure. *Math. Finance* **8**, 127-152]. Both measures are justified by economic arguments and are consistent with Duan's [Duan, J.-C., 1995. The GARCH option pricing model. *Math. Finance* **5**, 13-32] local risk neutral valuation relationship (LRNVR) for normal GARCH models. The main advantage of the two measures is that one can price derivatives using skewed or heavier tailed innovations distributions to model the returns. An empirical study regarding the European Call option valuation on S&P500 Index shows: (i) under both risk neutral measures our semiparametric algorithm performs better than the existing normal GARCH models if we allow for a leverage effect, and (ii) the pricing errors when using the Esscher transform are quite small even though our estimation procedure is based only on historical return data. Keywords: Option pricing; GARCH; Extended Girsanov principle; Esscher transform; Kernel density estimator; Semiparametric pricing.

BARBARIN, J. *Heath–Jarrow–Morton modelling of longevity bonds and the risk minimization of life insurance portfolios*. 41-55. This paper has two parts. In the first, we apply the Heath–Jarrow–Morton (HJM) methodology to the modelling of longevity bond prices. The idea of using the HJM methodology is not new. We can cite Cairns *et al*. [Cairns, A. J., Blake, D. & Dowd, K., 2006. Pricing death: framework for the valuation and the securitization of mortality risk. *Astin Bull.* **36** (1), 79-120], Miltersen and Persson [Miltersen K. R., Persson S. A., 2005. Is mortality dead? Stochastic force of mortality determined by arbitrage? Working Paper, University of Bergen] and Bauer [Bauer D., 2006. An arbitrage-free family of longevity bonds. Working Paper, Ulm University]. Unfortunately, none of these papers properly defines the prices of the longevity bonds they are supposed to be studying. Accordingly, the main contribution of this section is to describe a coherent theoretical setting in which we can properly define these longevity bond prices. A second objective of this section is to describe a

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more realistic longevity bonds market model than in previous papers. In particular, we introduce an additional effect of the actual mortality on the longevity bond prices, that does not appear in the literature. We also study multiple term structures of longevity bonds instead of the usual single term structure. In this framework, we derive a no-arbitrage condition for the longevity bond financial market. We also discuss the links between such HJM based models and the intensity models for longevity bonds such as those of Dahl [Dahl M., 2004. Stochastic mortality in life insurance: Market reserves and mortality-linked insurance contracts, *Insurance: Math. Econ.* **35** (1) 113-136], Biffis [Biffis E., 2005. Affine processes for dynamic mortality and actuarial valuations. *Insurance: Math. Econ.* **37**, 443-468], Luciano and Vigna [Luciano E. and Vigna E., 2005. Non mean reverting affine processes for stochastic mortality. ICER working paper], Schrager [Schrager D. F., 2006. Affine stochastic mortality. *Insurance: Math. Econ.* **38**, 81-97] and Hainaut and Devolder [Hainaut D., Devolder P., 2007. Mortality modelling with Lévy processes. *Insurance: Math. Econ.* (in press)], and suggest the standard pricing formula of these intensity models could be extended to more general settings. In the second part of this paper, we study the asset allocation problem of pure endowment and annuity portfolios. In order to solve this problem, we study the "risk-minimizing'' strategies of such portfolios, when some but not all longevity bonds are available for trading. In this way, we introduce different basis risks. Keywords: Term structure model; No arbitrage; Martingale measure; Hedging strategies; Marked point process; Jump-diffusion.

Berry-Stölzle, T. R. *The impact of illiquidity on the asset management of insurance companies.* 1-14. This paper investigates optimal asset management strategies for property and casualty insurance companies in illiquid markets. Using a cash-flow based liquidation model of an insurance company, we consider the effects of permanent and temporary price impact as well as commonality in price impact. Focusing on the interaction of a single large investor with the financial market makes the main results generally applicable for any institutional investor with stochastic future liabilities and restrictions on short-sales and financial leverage. Our analysis reveals a clear diversification benefit in illiquid markets apart from the one introduced by Markowitz [Markowitz, H., 1952. *Portfolio selection. J. Financ.* **7**, 77-91]. In the presence of commonality, cash-flow matching is shown to be the optimal strategy for a large investor. Keywords: Liquidity; Optimal liquidation; Asset liability management; Portfolio diversification; Cash-flow matching.

Cai, J., Tan, K. S., Weng, C. & Zhang, Y. *Optimal reinsurance under VaR and CTE risk measures.* 185-196. Let X denote the loss initially assumed by an insurer. In a reinsurance design, the insurer cedes part of its loss, say $f(X)$, to a reinsurer, and thus the insurer retains a loss $If(X) = X - f(X)$. In return, the insurer is obligated to compensate the reinsurer for undertaking the risk by paying the reinsurance premium. Hence, the sum of the retained loss and the reinsurance premium can be interpreted as the total cost of managing the risk in the presence of reinsurance. Based on a technique used in [Müller, A. & Stoyan, D., 2002. Comparison Methods for Stochastic Models and Risks. In: *Willey Series in Probability and Statistics*] and motivated by [Cai, J. & Tan, K. S., 2007. Optimal retention for a stop-loss reinsurance under the VaR and CTE risk measure. *Astin Bull.* **37** (1), 93-112] on using the value-at-risk (VaR) and the conditional tail expectation (CTE) of an insurer's total cost as the criteria for determining the optimal reinsurance, this paper derives the optimal ceded loss functions in a class of increasing convex ceded loss functions. The results indicate that depending on the risk measure's level of confidence and the safety loading for the reinsurance premium, the optimal reinsurance can be in the forms of stop-loss, quota-share, or change-loss. Keywords: Value-at-risk (VaR); Conditional tail expectation (CTE); Ceded loss; Retained loss; Increasing convex function; Expectation premium principle; Stop-loss reinsurance; Quota-share reinsurance; Change-loss reinsurance.

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Chen, Z., Vetzal, K. & Forsyth, P. A. *The effect of modelling parameters on the value of GMWB guarantees.* 165-173. In this article, an extensive study of the no-arbitrage fee for Guaranteed Minimum Withdrawal Benefit (GMWB) variable annuity riders is carried out. The value of the GMWB guarantee increases substantially when taking into account the separation of mutual fund fees and the fees earmarked for hedging the guarantee. In addition, the fee is also adversely affected if the underlying risky asset follows a jump diffusion process. We also explore the effects of various modelling assumptions on the optimal withdrawal strategy of the contract holder, as well as the impact on the guarantee value of sub-optimal withdrawal behaviour. Our general conclusions are that only if several unrealistic modelling assumptions are made is it possible to obtain GMWB fees in the same range as is normally charged. In all other cases, it would appear that typical fees are not enough to cover the cost of hedging these guarantees. Keywords: Variable annuities; Pensions; GMWB guarantees.

Gourieroux, C. & Monfort, A. *Quadratic stochastic intensity and prospective mortality tables.* 174-184. We consider a quadratic stochastic intensity model with a Gaussian autoregressive factor, derive explicit formulas for predictive mortality tables and recursive updating formulas are also provided. We also explain how to use appropriately the Kalman filter to estimate the parameters of the model and to approximate the values of the underlying factor. This methodology is applied to French human mortality tables. Keywords: Mortality table; Life insurance; Longevity risk; Mortality linked securities (MLS); Affine model; Quadratic model; Kalman filter.

Hao, X. & Tang, Q. *A uniform asymptotic estimate for discounted aggregate claims with subexponential tails.* 116-120. In this paper we study the tail probability of discounted aggregate claims in a continuous-time renewal model. For the case that the common claim-size distribution is subexponential, we obtain an asymptotic formula, which holds uniformly for all time horizons within a finite interval. Then, with some additional mild assumptions on the distributions of the claim sizes and inter-arrival times, we further prove that this formula holds uniformly for all time horizons. In this way, we significantly extend a recent result of Tang [Tang, Q., 2007. Heavy tails of discounted aggregate claims in the continuous-time renewal model. *J. Appl. Probab.* **44** (2), 285-294]. Keywords: Asymptotics; Poisson process; Renewal process; Subexponential distribution; Uniformity.

Hashorva, E. *Tail asymptotic results for elliptical distributions.* 158-164. Let (X, Y) be a bivariate elliptical random vector with associated random radius in the Gumbel max-domain of attraction. In this paper we obtain a second order asymptotic expansion of the joint survival probability and the conditional probability, for x, y large. Keywords: Elliptical random vectors; Exact asymptotics; Second order bounds; Gumbel max-domain of attraction; Conditional distribution; Conditional quantile function; Joint survival probability.

Hoermann, G. & Russ, J. *Enhanced annuities and the impact of individual underwriting on an insurer's profit situation.* 150-157. We analyze the effect of enhanced annuities on an insurer engaging in individual underwriting. We use a frailty model for heterogeneity of the insured population and model individual underwriting by a random variable that positively correlates with the corresponding frailty factor. For a given annuity portfolio, we analyze the effect of the quality of the underwriting on the insurer's profit/loss situation and the impact of adverse selection effects. Keywords: Enhanced annuity; Individual underwriting; Adverse selection.

Hua, L. & Cheung, K. C. *Worst allocations of policy limits and deductibles.* 93-98. In the literature, orderings of optimal allocations of policy limits and deductibles were established with respect to a policyholder's preference. However, from the viewpoint of an insurer, the orderings are not enough for the purpose of pricing. In this paper, by applying the

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equivalent utility premium principle, we study worst allocations of policy limits and deductibles for an insurer, which give rise to the maximum fair premiums. Closed-form solutions are derived. Then we present a result concerning the optimality in a general risk-sharing scheme, by which we obtain optimal allocations for policyholders directly from worst allocations for an insurer. Several results in Cheung [Cheung, K. C., 2007. Optimal allocation of policy limits and deductibles. *Insurance: Math. Econ.* **41**, 382-391] are generalized here. Keywords: Stochastic orders; Principle of equivalent utility; Majorization order; Comonotonicity.

Lau, J. W. & Siu, T. K. *On option pricing under a completely random measure via a generalized Esscher transform.* In this paper, we develop an option valuation model when the price dynamics of the underlying risky asset is governed by the exponential of a pure jump process specified by a shifted kernel-biased completely random measure. The class of kernel-biased completely random measures is a rich class of jump-type processes introduced in [James, L. F., 2005. Bayesian Poisson process partition calculus with an application to Bayesian Lévy moving averages. *Ann. Statist.* **33**, 1771-1799; James, L. F., 2006. Poisson calculus for spatial neutral to the right processes. *Ann. Statist.* **34**, 416-440] and it provides a great deal of flexibility to incorporate both finite and infinite jump activities. It includes a general class of processes, namely, the generalized Gamma process, which in its turn includes the stable process, the Gamma process and the inverse Gaussian process as particular cases. The kernel-biased representation is a nice representation form and can describe different types of finite and infinite jump activities by choosing different mixing kernel functions. We employ a dynamic version of the Esscher transform, which resembles an exponential change of measures or a disintegration formula based on the Laplace functional used by James, to determine an equivalent martingale measure in the incomplete market. Closed-form option pricing formulae are obtained in some parametric cases, which provide practitioners with a convenient way to evaluate option prices. Keywords: Option pricing; Kernel-biased completely random measures; Generalized Gamma processes; Esscher transform; Laplace functionals.

Nielsen, P. H. & Steffensen, M. *Optimal investment and life insurance strategies under minimum and maximum constraints.* 15-28. We derive optimal strategies for an individual life insurance policyholder who can control the asset allocation as well as the sum insured (the amount to be paid out upon death) throughout the policy term. We first consider the problem in a pure form without constraints (except nonnegativity on the sum insured) and then in a more general form with minimum and/or maximum constraints on the sum insured. In both cases we also provide the optimal life insurance strategies in the case where risky-asset investments are not allowed (or not taken into consideration), as in basic life insurance mathematics. The optimal constrained strategies are somewhat more complex than the unconstrained ones, but the latter can serve to ease the understanding and implementation of the former. Keywords: Life insurance; Asset allocation; Optimization.

Paulsen, J., Lunde, A. & Skaug, H. J. *Fitting mixed-effects models when data are left truncated.* 121-133. Damage sizes, i.e. all damages occurring to a policy and not only those that are reported to an insurance company, are modelled as a linear mixed model. Only those damages that are larger than their deductibles are reported to the company, and this fact should be taken into account when analyzing such data. In statistical terms, the problem is to make inference in a linear mixed model with left truncated data. Estimation methods based on a Monte Carlo simulation of the likelihood are proposed, and extensive simulations to evaluate the quality of the methods are reported. The proposed methods are then used to analyze claimsizes for some marine insurance data, where shipowners represent random effects and technical data about the ships represent fixed effects. Keywords: Deductibles; Random effects; Likelihood estimation; Monte Carlo techniques; Prediction.

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Robert, C. Y. & Segers, J. *Tails of random sums of a heavy-tailed number of light-tailed terms.* 85-92. The tail of the distribution of a sum of a random number of independent and identically distributed nonnegative random variables depends on the tails of the number of terms and of the terms themselves. This situation is of interest in the collective risk model, where the total claim size in a portfolio is the sum of a random number of claims. If the tail of the claim number is heavier than the tail of the claim sizes, then under certain conditions the tail of the total claim size does not change asymptotically if the individual claim sizes are replaced by their expectations. The conditions allow the claim number distribution to be of consistent variation or to be in the domain of attraction of a Gumbel distribution with a mean excess function that grows to infinity sufficiently fast. Moreover, the claim number is not necessarily required to be independent of the claim sizes. Keywords: Compound distribution; Collective risk model; Consistent variation; Heavy-tailed distributions; Light-tailed distributions; Large deviations; Random sum.

Stamos, M. Z. *Optimal consumption and portfolio choice for pooled annuity funds.* 56-68. This paper presents the optimal continuous time dynamic consumption and portfolio choice for pooled annuity funds. A pooled annuity fund constitutes an alternative way to protect against mortality risk compared to purchasing a life annuity. The crucial difference between the pooled annuity fund and purchase of a life annuity offered by an insurance company is that participants of a pooled annuity fund still have to bear some mortality risk while insured annuitants bear no mortality risk at all. The population of the pool is modelled by employing a Poisson process with time-dependent hazard-rate. It follows that the pool member's optimization problem has to account for the stochastic investment horizon and for jumps in wealth which occur if another pool member dies. In case the number of pool members goes to infinity analytical solutions are provided. For finite pool sizes the solution of the optimization problem is reduced to the numerical solution of a set of ODEs. A simulation and welfare analysis show that pooled annuity funds insure very effectively against longevity risk even if their pool size is rather small. Only very risk averse investors or those without access to small pools are more inclined to pay a risk premium to access private life annuity markets in order to lay off mortality risk completely. As even families constitute such small pools the model provides theoretical justification for the low empirical annuity demand. Keywords: Dynamic consumption and portfolio choice; Mortality risk; Pooling; Insurance; Jump risk; HJB; Finite differences.

Yan, J., Liu, J. J. & Li, K. X. *Threshold control of mutual insurance with limited commitment.* 108-115. We study the optimal premium policy of mutual insurance when the charged premium cannot be higher than a preset rate. We provide a complete solution to the problem and use numerical simulations to illustrate how the optimal premium policy responds to changes of outside factors. The results are useful for mutual insurance firms to design premium policies and can be used to test the behavior of these firms in empirical studies. Keywords: Stochastic programming; Mutual insurance; Threshold control; Limited commitment; Diffusion process.

Zaglauer, K. & Bauer, D. *Risk-neutral valuation of participating life insurance contracts in a stochastic interest rate environment.* 29-40. Over the last years, the valuation of life insurance contracts using concepts from financial mathematics has become a popular research area for actuaries as well as financial economists. In particular, several methods have been proposed of how to model and price participating policies, which are characterized by an annual interest rate guarantee and some bonus distribution rules. However, despite the long terms of life insurance products, most valuation models allowing for sophisticated bonus distribution rules and the inclusion of frequently offered options assume a simple Black–Scholes setup and, more specifically, deterministic or even constant interest rates. We present a framework in which participating life insurance contracts including predominant kinds of guarantees and

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options can be valuated and analyzed in a stochastic interest rate environment. In particular, the different option elements can be priced and analyzed separately. We use Monte Carlo and discretization methods to derive the respective values. The sensitivity of the contract and guarantee values with respect to multiple parameters is studied using the bonus distribution schemes as introduced in [Bauer, D., Kiesel, R., Kling, A. & Ruß, J., 2006. Risk-neutral valuation of participating life insurance contracts. *Insurance: Math. Econ.* **39**, 171-183]. Surprisingly, even though the value of the contract as a whole is only moderately affected by the stochasticity of the short rate of interest, the value of the different embedded options is altered considerably in comparison to the value under constant interest rates. Furthermore, using a simplified asset portfolio and empirical parameter estimations, we show that the proportion of stock within the insurer's asset portfolio substantially affects the value of the contract. Keywords: Participating life insurance contracts; Risk-neutral valuation; Stochastic interest rates; Embedded options.

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Biagini, F., Bregman, Y. & Meyer-Brandis, T. *Pricing of catastrophe insurance options written on a loss index with reestimation.* 214-222. We propose a valuation model for catastrophe insurance options written on a loss index. This kind of options distinguishes between a loss period [0, $T1$], during which the catastrophes may happen, and a development period [$T1$, $T2$], during which losses entered before $T1$ are reestimated. Here we suppose that the underlying loss index is given by a time inhomogeneous compound Poisson process before $T1$ and that losses are reestimated by a common factor given by an exponential time inhomogeneous Lévy process after $T1$. In this setting, using Fourier transform techniques, we are able to provide analytical pricing formulas for catastrophe options written on this kind of index.

Bolancé, C., Guillen, M. & Pinquet, J. *On the link between credibility and frequency premium.* 209-213. This paper questions the equidistribution assumption for the random effects in a frequency risk model. Two models are presented, which use parametric and nonparametric links between the variance of the random effect and frequency risk. They are estimated on a Spanish automobile insurance portfolio, for which a decreasing link is obtained. Conclusions are drawn for credibility and bonus-malus coefficients.

Brito, M. & Freitas, A. C. M. *Edgeworth expansion for an estimator of the adjustment coefficient.* 203-208. We establish an Edgeworth expansion for an estimator of the adjustment coefficient R, directly related to the geometric-type estimator for general exponential tail coefficients, proposed in [Brito, M. & Freitas, A. C. M., 2003. Limiting behaviour of a geometric-type estimator for tail indices. *Insurance: Math. Econ.* **33**, 211-226]. Using the first term of the expansion, we construct improved confidence bounds for R. The accuracy of the approximation is illustrated using an example from insurance (cf. [Schultze, J. & Steinebach, J., 1996. On least squares estimates of an exponential tail coefficient. *Statist. Dec.* **14**, 353-372]).

Chen, Z. & Hu, T. *Asset proportions in optimal portfolios with dependent default risks.* 223-226. In this note, we consider the dependent default risk model of factor type. The dependence between the returns of assets is driven by default indicators. Sufficient conditions on the dependence structure of default indicators and on the utility function are investigated which enable one to order the optimal amount invested in each asset. We thus complement one result in [Cheung, K. C. & Yang, H., 2004. Ordering optimal proportions in the asset allocation problem with dependent default risks. *Insurance: Math. Econ.* **35**, 595-609].

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Escudero, L. F. & Ortega, E.-M. *Actuarial comparisons for aggregate claims with randomly right-truncated claims.* 255-262. In this note, we consider an extension of the largest claims reinsurance treaty (LCR) with random upper thresholds for the claim sizes, that we call retention levels. The Laplace transform order for insurer's aggregate claims is obtained assuming dependence among the random retention levels. Different results about the influence of dependence on the insurer total claim amount are also given including the connections with LCR and the case of combination with quota-share. Algebraic bounds for the insurer aggregate claims are obtained when a common fixed threshold is considered.

Furman, E. & Zitikis, R. *Weighted risk capital allocations.* 263-269. By extending the notion of weighted premium calculation principles, we introduce weighted risk capital allocations, explore their properties, and develop computational methods. When achieving these goals, we find it particularly fruitful to relate the weighted allocations to general Stein-type covariance decompositions, which are of interest on their own.

Gerber, H. U. & Smith, N. *Optimal dividends with incomplete information in the dual model.* 227-233. In [Gerber, H. U., Shiu, E. S. W. & Smith, N., 2008. Methods for estimating the optimal dividend barrier and the probability of ruin. *Insurance: Math. Econ.* **42** (1), 243-254], methods were analyzed for estimating the optimal dividend barrier (in the sense of de Finetti). In particular, De Vylder approximations and diffusion approximations are discussed. These methods are useful when only the first few moments of the claim amount distribution are known. The purpose of this paper is to examine these and other methods (such as the gamma approximations and the gamproc approximations) in the dual model, see [Avanzi, B., Gerber, H. U. & Shiu, E. S., 2007. Optimal dividends in the dual model. *Insurance: Math. Econ.* **41** (1), 111-123]. The dual model is obtained if the roles of premiums and claims are exchanged. In other words, the company has random gains, which constitute a compound Poisson process, and expenses occur continuously at a constant rate. The approximations can easily be implemented, and their accuracy is surprisingly good. Several numerical illustrations enhance the paper.

Kulenko, N. & Schmidli, H. *Optimal dividend strategies in a Cramér–Lundberg model with capital injections.* 270-278. We consider a classical risk model with dividend payments and capital injections. Thereby, the surplus has to stay positive. Like in the classical de Finetti problem, we want to maximise the discounted dividend payments minus the penalised discounted capital injections. We derive the Hamilton–Jacobi–Bellman equation for the problem and show that the optimal strategy is a barrier strategy. We explicitly characterise when the optimal barrier is at 0 and find the solution for exponentially distributed claim sizes.

Luciano, E., Spreeuw, J. & Vigna, E. *Modelling stochastic mortality for dependent lives.* 234-244. Stochastic mortality, i.e. modelling death arrival via a jump process with stochastic intensity, is gaining an increasing reputation as a way to represent mortality risk. This paper is a first attempt to model the mortality risk of couples of individuals, according to the stochastic intensity approach. Dependence between the survival times of the members of a couple is captured by an Archimedean copula. We also provide a methodology for fitting the joint survival function by working separately on the (analytical) marginals and on the (analytical) copula. First, we provide a sample-based calibration for the intensity, using a time-homogeneous, non mean-reverting, affine process: this gives the marginal survival functions. Then we calibrate and select the best fit copula according to the Wang and Wells [Wang, W. & Wells, M. T., 2000b. Model selection and semiparametric inference for bivariate failure-time data. *J. Amer. Statis. Assoc.* **95**, 62-72] methodology for censored data. By coupling the calibrated marginals with the best fit copula, we obtain a joint survival function, which incorporates the stochastic nature of mortality improvements. We apply the

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methodology to a well known insurance data set, using a sample generation. The best fit copula turns out to be one listed in [Nelsen, R. B., 2006. An Introduction to Copulas, Second ed. In: Springer Series], which implies not only positive dependence, but dependence increasing with age.

Psarrakos, G. *Tail bounds for the distribution of the deficit in the renewal risk model.* 197-202. We obtain upper and lower bounds for the tail of the deficit at ruin in the renewal risk model, which are (i) applicable generally; and (ii) based on reliability classifications. We also derive two-side bounds, in the general case where a function satisfies a defective renewal equation, and we apply them to the renewal model, using the function u introduced by [Psarrakos, G. & Politis, K., 2007. A generalisation of the Lundberg condition in the Sparre Andersen model and some applications (submitted for publication)]. Finally, we construct an upper bound for the integrated function and an asymptotic result when the adjustment coefficient exists.

Puustelli, A., Koskinen, L. & Luoma, A. *Bayesian modelling of financial guarantee insurance.* 245-254. In this paper we model the claim process of financial guarantee insurance, and predict the pure premium and the required amount of risk capital. The data used are from the financial guarantee system of the Finnish statutory pension scheme. The losses in financial guarantee insurance may be devastating during an economic depression (i.e., deep recession). This indicates that the economic business cycle, and in particular depressions, must be taken into account in modelling the claim amounts in financial guarantee insurance. A Markov regime-switching model is used to predict the frequency and severity of future depression periods. The claim amounts are predicted using a transfer function model where the predicted growth rate of the real GNP is an explanatory variable. The pure premium and initial risk reserve are evaluated on the basis of the predictive distribution of claim amounts. Bayesian methods are applied throughout the modelling process. For example, estimation is based on posterior simulation with the Gibbs sampler, and model adequacy is assessed by posterior predictive checking. Simulation results show that the required amount of risk capital is high, even though depressions are an infrequent phenomenon.

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Alba, E. De. *Government-provided annuities under insolvency risk.* 377-385. This paper seeks to determine whether governments should intervene in the private annuity market by directly providing public insurance in the form of annuities when both the government and the insurance companies could default. It is found that, although the government could default, intervening by means of an annuity can improve social welfare if the insurance companies could default and the expected return on the public annuity is greater than the rate of return on a risk-free bond. We also find that, under actuarially fair pricing, the government should provide more in terms of a public annuity than the optimal amount of the annuity that the individual purchases in the private market if the government is less likely to default on the public annuity than an insurance company would in the case of a private annuity.

Asimit, A. V. & Jones, B. L. *Dependence and the asymptotic behavior of large claims reinsurance.* 407-411. We consider an extension of the classical compound Poisson risk model, where the waiting time between two consecutive claims and the forthcoming claim are no longer independent. Asymptotic tail probabilities of the reinsurance amount under ECOMOR and LCR treaties are obtained. Simulation results are provided in order to illustrate this.

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Biard, R., Lefevre, C. & Loisel, S. *Impact of correlation crises in risk theory: Asymptotics of finite-time ruin probabilities for heavy-tailed claim amounts when some independence and stationarity assumptions are relaxed.* 412-421. In the renewal risk model, several strong hypotheses may be found too restrictive to model accurately the complex evolution of the reserves of an insurance company. In the case where claim sizes are heavy-tailed, we relax the independence and stationarity assumptions and extend some asymptotic results on finite-time ruin probabilities, to take into account possible correlation crises like the one recently bred by the sub-prime crisis: claim amounts, in general assumed to be independent, may suddenly become strongly positively dependent. The impact of dependence and non-stationarity is analyzed and several concrete examples are given.

Bolance, C., Guillen, M., Pelican, E. & Vernic, R. *Skewed bivariate models and nonparametric estimation for the CTE risk measure.* 386-393. In this paper, we illustrate the use of the Conditional Tail Expectation (CTE) risk measure on a set of bivariate real data consisting of two types of auto insurance claim costs. Several continuous bivariate distributions (normal, lognormal, skew-normal with the alternative log-skew-normal) are fitted to the data. Besides, a bivariate nonparametric transformed kernel estimation is presented. CTE formulas are given for all these, and numerical results on the real data are discussed and compared.

Boyle, P., Imai, J. & Tan, K. S. *Computation of optimal portfolios using simulation-based dimension reduction.* 327-338. This paper describes a simple and efficient method for determining the optimal portfolio for a risk averse investor. The portfolio selection problem is of long standing interest to finance scholars and it has obvious practical relevance. In a complete market the modern procedure for computing the optimal portfolio weights is known as the martingale approach. Recently, alternative implementations of the martingale approach based on Monte Carlo methods have been proposed. These methods use Monte Carlo simulation to compute stochastic integrals. This paper examines the efficient implementation of one of these methods due to [Cvitanic, J., Goukasian, L. & Zapatero, F. 2003. Monte Carlo computation of optimal portfolios in complete markets. *J. Econom. Dynam. Control* **27**, 971-986]. We explain why a naive application of the quasi-Monte Carlo method to this problem is often only marginally more efficient than the classical Monte Carlo method. Using the dimension reduction technique of [Imai, J. & Tan, K. S., 2007. A general dimension reduction method for derivative pricing. *J. Comput. Financ.* **10** (2), 129-155] it is possible to significantly reduce the effective dimension of the problem. The paper shows why the proposed technique leads to a dramatic improvement in efficiency.

Boyle, P. P. & Tian, W. *The design of equity-indexed annuities.* 303-315. There is a rich variety of tailored investment products available to the retail investor in every developed economy. These contracts combine upside participation in bull markets with downside protection in bear markets. Examples include equity-linked contracts and other types of structured products. This paper analyzes these contracts from the investor's perspective rather than the issuer's using concepts and tools from financial economics. We analyze and critique their current design and examine their valuation from the investor's perspective. We propose a generalization of the conventional design that has some interesting features. The generalized contract specifications are obtained by assuming that the investor wishes to maximize end of period expected utility of wealth subject to certain constraints. The first constraint is a guaranteed minimum rate of return which is a common feature of conventional contracts. The second constraint is new. It provides the investor with the opportunity to outperform a benchmark portfolio with some probability. We present the explicit form of the optimal contract assuming both constraints apply and we illustrate the nature of the solution using specific examples. The paper focusses on equity-indexed annuities as a representative type of such contracts but our approach is applicable to other types of equity-linked contracts and structured products.

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CHANG, C. W., CHANG, J. S. K. & LU, W. *Pricing catastrophe options in discrete operational time.* 422-430. We employ a doubly-binomial process as in Gerber [Gerber, H. U., 1988. Mathematical fun with the compound binomial process. *ASTIN Bull.* **18**, 161-168] to discretize and generalize the continuous "randomized operational time" model of Chang *et al.* ([Chang, C. W., Chang, J. S. K. & Yu, M. T., 1996. Pricing catastrophe insurance futures call spreads: A randomized operational time approach. *J. Risk Insurance* **63**, 599-616] and CCY hereafter) from a complete-market continuous-time setting to an incomplete-market discrete-time setting, so as to price a richer set of catastrophe (CAT) options. For futures options, we derive the equivalent martingale probability measures by benchmarking to the shadow price of a bond to span arrival uncertainty, and the underlying futures price to span price uncertainty. With a time change from calendar time to the operational transaction-time dimension, we derive CCY as a limiting case under risk-neutrality when both calendar-time and transaction-time intervals shrink to zero. For a cash option with non-traded underlying loss index, we benchmark to the market reinsurance premiums to span claim uncertainty, and with a time change to claim time, we derive the cash option price as a binomial sum of claim-time binomial Asian option prices under the martingale measures.

CHEN, P., YANG, H. & YIN, G. *Markowitz's mean–variance asset-liability management with regime switching: A continuous-time model.* 456-465. This paper considers an asset-liability management (ALM) problem under a continuous-time Markov regime-switching model. By adopting the techniques of [Zhou, X. Y. & Yin, G., 2003. Markowitz's mean–variance portfolio selection with regime switching: A continuous-time model. *SIAM J. Control Optim.* **42**, 1466-1482], we investigate the feasibility, obtain the optimal strategy, delineate the efficient frontier, and establish the associated mutual fund theorem.

CHEN, R.-R., CHENG, Z. & LIU, B. *Estimation and evaluation of the term structure of credit default swaps: An empirical study.* 339-349. Chen, Cheng, Fabozzi and Liu [Chen, Ren-Raw, Cheng, Xiaolin, Fabozzi, Frank, Liu, Bo, 2008. An explicit, multi-factor credit default swap pricing model with correlated factors. *J. Financial Quantitative Anal.* **43** (1), 123-160] provide an explicit solution to the value of the credit default swap when the interest rate and the hazard rate are correlated. They also provide empirical evidence to support the model with transaction prices. In this paper, we extend their empirical work to study the term structure of CDS spreads by using a matrix CDS dataset from J. P. Morgan Chase. Matrix data contain interpolated prices based on traders' expectations, which are often criticized as being "unreal". However, the benefit of this matrix dataset is that it contains the entire credit spread curves, which allows us to understand the cross-sectional variation of the credit risk. The empirical results show that the parameters of the model are highly significant and it captures most of the cross-sectional as well as time series variation.

CHEUNG, K. C. *Characterization of comonotonicity using convex order.* 403-406. It is well known that if a random vector with given marginal distributions is comonotonic, it has the largest sum with respect to the convex order. In this paper, we prove that the converse is also true, provided that each marginal distribution is continuous.

COSSETTE, H., MARCEAU, E. & MARRI, F. *On the compound Poisson risk model with dependence based on a generalized Farlie–Gumbel–Morgenstern copula.* 444-455. In this paper we consider an extension to the classical compound Poisson risk model in which we introduce a dependence structure between the claim amounts and the interclaim time. This structure is embedded via a generalized Farlie–Gumbel–Morgenstern copula. In this framework, we derive the Laplace transform of the Gerber–Shiu discounted penalty function. An explicit expression for the Laplace transform of the time of ruin is given for exponential claim sizes.

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Detemple, J. & Rindisbacher, M. *Dynamic asset liability management with tolerance for limited shortfalls.* 281-294. A dynamic asset allocation problem in the presence of liabilities is considered. The fund manager has von Neumann–Morgenstern preferences with terminal utility function defined over the excess of liquid wealth over a minimum liability coverage tolerated and intermediate utility function defined over dividends, the excess of expenditures over liability cash flows. Preferences incorporate a parameter controlling the tolerance for a shortfall in the funding ratio at the terminal date. The optimal asset allocation rule is derived and its sensitivity with respect to the parameters of the model is analyzed.

Dicesare, J. & McLeish, D. *Simulation of jump diffusions and the pricing of options.* 316-326. We present importance sampling and acceptance–rejection simulation methods for one dimensional diffusions. This effectively reduces the computation of many path functionals of general diffusions to a similar computation for the Brownian bridge. We use this approach to efficiently obtain Monte Carlo prices of path-dependent derivative securities such as Barrier and Look-back options for a CEV jump-diffusion model.

Gzyl, H. & Mayoral, S. *Determination of risk pricing measures from market prices of risk.* 437-443. A new insurance provider or a regulatory agency may be interested in determining a risk measure consistent with observed market prices of a collection of risks. Using a relationship between distorted coherent risk measures and spectral risk measures, we provide a method for reconstructing distortion functions from the observed prices of risk. The technique is based on an appropriate application of the method of maximum entropy in the mean, which builds upon the classical method of maximum entropy.

He, L., Hou, P. & Liang, Z. *Optimal control of the insurance company with proportional reinsurance policy under solvency constraints.* 474-479. This paper considers the optimal control problem of the insurance company with proportional reinsurance policy under solvency constraints. The management of the company controls the reinsurance rate and dividends payout processes to maximize the expected present value of the dividend until the time of bankruptcy. This is a mixed singular-regular control problem. However, the optimal dividend payout barrier may be too low to be acceptable. The company may be prohibited to pay dividend according to external reasons because this low dividend payout barrier will result in bankruptcy soon. Therefore, some constraints on the insurance company's dividend policy will be imposed. One reasonable and normal constraint is that if b is the minimum dividend barrier, then the bankrupt probability should not be larger than some predetermined e within the time horizon T. This paper is to work out the optimal control policy of the insurance company under the solvency constraints.

Hsu, J. C. & Schwartz, E. S. *A model of R&D valuation and the design of research incentives.* 350-367. We develop a real options model of R&D valuation that takes into account the uncertainty in the quality (or efficacy) of the research output, the time and cost to completion, and the market demand for the R&D output. The model is then applied to study the problem of pharmaceutical under-investment in R&D for vaccines to treat diseases affecting the developing regions of the world. To address this issue, world organizations and private foundations are willing to sponsor vaccine R&D, but there is no consensus on how to administer the sponsorship effectively. Different research incentive contracts are examined using our valuation model. Their effectiveness is measured in the following five dimensions: expected cost to the sponsor, probability of development success, consumer surplus generated, expected number of successful vaccinations and expected cost per person successfully vaccinated. We find that, in general, purchase commitment plans (pull subsidies) are more effective than cost subsidy plans (push subsidies). Moreover, we find that a hybrid subsidy plan constructed from a purchase commitment combined with a sponsor research cost-sharing subsidy is the most effective.

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JIANG, J. & TANG, Q. *Reinsurance under the LCR and ECOMOR treaties with emphasis on light-tailed claims.* 431-436. Suppose that, over a fixed time interval of interest, an insurance portfolio generates a random number of independent and identically distributed claims. Under the LCR treaty the reinsurance covers the first l largest claims, while under the ECOMOR treaty it covers the first l-1 largest claims in excess of the lth largest one. Assuming that the claim sizes follow an exponential distribution or a distribution with a convolution-equivalent tail, we derive some precise asymptotic estimates for the tail probabilities of the reinsured amounts under both treaties.

KWON, H.-S. & JONES, B. L. *Applications of a multi-state risk factor/mortality model in life insurance.* 394-402. Mortality rates are known to depend on socio-economic and behavioral risk factors, and actuarial calculations for life insurance policies usually reflect this. It is typically assumed, however, that these risk factors are observed only at policy issue, and the impact of changes that occur later is not considered. In this paper, we present a discrete-time, multi-state model for risk factor changes and mortality. It allows one to more accurately describe mortality dynamics and quantify variability in mortality. This model is extended to reflect health status and then used to analyze the impact of selective lapsation of life insurance policies and to predict mortality under reentry term insurance.

SARABIA, J. M. & GUILLEN, M. *Joint modelling of the total amount and the number of claims by conditionals.* 466-473. In the risk theory context, let us consider the classical collective model. The aim of this paper is to obtain a flexible bivariate joint distribution for modelling the couple (S, N), where N is a count variable and $S = X1 + XN$ is the total claim amount. A generalization of the classical hierarchical model, where now we assume that the conditional distributions of $S \mid N$ and belong to some prescribed parametric families, is presented. A basic theorem of compatibility in conditional distributions of the type S given N and N given S is stated. Using a known theorem for exponential families and results from functional equations new models are obtained. We describe in detail the extension of two classical collective models, which now we call Poisson–Gamma and the Poisson–Binomial conditionals models. Other conditionals models are proposed, including the Poisson–Lognormal conditionals distribution, the Geometric-Gamma conditionals model and a model with inverse Gaussian conditionals. Further developments of collective risk modelling are given.

SIU, T. K., YANG, H. & LAU, J. W. *Pricing currency options under two-factor Markov-modulated stochastic volatility models.* 295-302. This article investigates the valuation of currency options when the dynamic of the spot Foreign Exchange (FX) rate is governed by a two-factor Markov-modulated stochastic volatility model, with the first stochastic volatility component driven by a lognormal diffusion process and the second independent stochastic volatility component driven by a continuous-time finite-state Markov chain model. The states of the Markov chain can be interpreted as the states of an economy. We employ the regime-switching Esscher transform to determine a martingale pricing measure for valuing currency options under the incomplete market setting. We consider the valuation of the European-style and American-style currency options. In the case of American options, we provide a decomposition result for the American option price into the sum of its European counterpart and the early exercise premium. Numerical results are included.

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BERRY-STÖLZLE, T. R. *Evaluating liquidation strategies for insurance companies.* 207-230. In this article, we examine liquidation strategies and asset allocation decisions for property and casualty insurance companies for different insurance product lines. We propose a cash-flow-based liquidation model of an insurance company and analyze selling strategies for a portfolio with liquid and illiquid assets. Within this framework, we study the influence of different bid-ask spread models on the minimum capital requirement and determine a solution set consisting of an optimal initial asset allocation and an optimal liquidation strategy. We show that the initial asset allocation, in conjunction with the appropriate liquidation strategy, is an important tool in minimizing the capital committed to cover claims for a predetermined ruin probability. This interdependence is of importance to insurance companies, stakeholders, and regulators.

BIN, O., KRUSE, J. B. & LANDRY, C. E. *Flood Hazards, Insurance Rates, and Amenities: Evidence From the Coastal Housing Market.* 63-82. This study employs the hedonic property price method to examine the effects of flood hazard on coastal property values. We utilize Geographic Information System data on National Flood Insurance Program flood zones and residential property sales from Carteret County, North Carolina. Our results indicate that location within a flood zone lowers property value. Price differentials for flood risk and the capitalized value of flood insurance premiums are roughly equivalent — both exhibiting a nonlinear relationship in flood probability. Our results support the conclusion that flood zone designation and insurance premiums convey risk information to potential buyers in the coastal housing market.

BROWNE, M. J. & SCHMIT, J. T. *Litigation Patterns in Automobile Bodily Injury Claims 1977-1997: Effects of Time and Tort Reforms.* 83-100. This study uses data from the Insurance Research Council to investigate changes in the use of attorneys and in the filing of legal claims to resolve automobile third-party bodily injury claims between 1977 and 1997. We find results consistent with the general public perception that the use of attorneys and the filing of legal claims have increased over the study period. In addition, we find evidence that tort reforms enacted by the states have slowed the rates of increase in the use of attorneys and in the filing of legal claims to resolve automobile insurance claim disputes.

CHEN, X., DOERPINGHAUS, H., LIN, B.-X. & YU, T. *Catastrophic Losses and Insurer Profitability: Evidence From 9/11.* 39-62. We examine the effects of 9/11 on the insurance industry, hypothesizing a short-run claim effect, resulting from insufficient premium ex ante for catastrophic losses, and a long-run growth effect, resulting from ex post insurance supply reductions and risk updating. Following Yoon and Starks (1995) we use short- and long-run abnormal forecast revisions to measure both effects, analyzing them as a function of firm-specific characteristics. We find that firm type, loss estimates, reinsurance use, and tax position are important determinants of the short-run position. Firm type, loss estimates, financial strength, underwriting risk, and reinsurance are key determinants of the firm's long-run position.

CLARK, S. P., COGGIN, T. D. & NEALE, F. R. *Mean Reversion in Net Discount Ratios: A Study in the Context of Fractionally Integrated Models.* 231-247. This article introduces a new alternative to the ongoing debate about stationarity and mean reversion of the net discount ratio. Modeling the net discount ratio as a fractionally integrated (I(d)) process, we apply recently developed frequency domain estimation procedures and find evidence that the net discount ratio is an $I(d)$ process with $1/2 = d < 1$. Although nonstationary, such series behave like stationary processes in one interesting respect; they are mean-reverting. We present

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results from a simulation experiment suggesting that the finding of a nonstationary, but mean-reverting net discount ratio generally supports the validity of current practice in estimating economic damages in personal injury litigation. Moreover, if recognized and accounted for, the presence of long memory in the net discount ratio even offers the potential to significantly improve forecasts of the present value of future earnings.

Fei, W. & Schlesinger, H. *Precautionary Insurance Demand With State-Dependent Background Risk*. 1-16. This article considers a zero-mean background risk that is uncorrelated with insurable losses, but is not necessarily statistically independent. In particular, the size of the background risk can vary in different insurable-loss states. We show how a prudent individual will buy either more insurance or less insurance than with no background risk, depending on the relative size of the background risk in the loss states vis-á-vis the no-loss states. If we consider two individuals, with one more risk averse than the other, we need to compare the intensities of their precautionary motives, in addition to their measures of risk aversion, before we can determine who buys more insurance coverage in the presence of the state dependent background risk.

Lai, G. C., McNamara, M. J. & Yu, T. *The Wealth Effect of Demutualization: Evidence From the U.S. Property-Liability and Life Insurance Industries*. 125-144. This study examines the wealth effect of demutualization initial public offerings (IPOs) by investigating underpricing and postconversion long-run stock performance. Our results suggest that there is more "money left on the table" for demutualized insurers than for non-demutualized insurers. We show that higher underpricing for demutualized firms can be explained by greater market demand, market sentiment, and the size of the offering. Further, contrary to previous research reporting an average underperformance of industrial IPOs, we show that demutualization IPOs outperform non-IPO firms with comparable size and book-to-market ratios and non-demutualized insurers. We present evidence that the outperformance in stock returns is mainly attributable to improvement in post-demutualization operating performance and demand at the time of the IPOs. The combined results of underpricing and long-term performance suggest that the wealth of policyholders who choose stock rather than cash or policy credits is not harmed by demutualization. Stockholders who purchase demutualized company shares either during or after the IPO have earned superior returns. Our findings are consistent with the efficiency improvement hypothesis.

Picard, P. *Natural disaster insurance and the equity-efficiency trade-off*. 17-38. This article investigates the role of private insurance in the prevention and mitigation of natural disasters. We characterize the equity-efficiency trade-off faced by the policymakers under imperfect information about individual prevention costs. It is shown that a competitive insurance market with actuarial rate making and compensatory tax-subsidy transfers is likely to dominate regulated uniform insurance pricing rules or state-funded assistance schemes. The model illustrates how targeted tax cuts on insurance contracts can improve the incentives to prevention while compensating individuals with high prevention costs. The article highlights the complementarity between individual incentives through tax cuts and collective incentives through grants to the local jurisdictions where risk management plans are enforced.

Seog, S. H. *Informational cascade in the insurance market*. 145-165. We develop an informational cascade model based on Bikhchandani, Hirshleifer, and Welch (1992) with applications to the insurance market. We investigate the existence of cascades and the effects of public information on cascades. We apply the results to insurance markets to explain how catastrophic events may lead to demand increases, how loss shocks may lead to insurance cycles, and how the heterogeneity of policyholders affects the choice of limited tort auto insurance in Pennsylvania.

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Tse, W.-M., Chang, E. C., Li, L. K. & Mok, H. M. K. *Pricing and hedging of discrete dynamic guaranteed funds.* 167-192. We derive a risk-neutral pricing model for discrete dynamic guaranteed funds with geometric Gaussian underlying security price process. We propose a dynamic hedging strategy by adding a gamma factor to the conventional delta. Simulation results demonstrate that, when hedging discretely, the risk-neutral gamma-adjusted-delta strategy outperforms the dynamic delta hedging strategy by reducing the expected hedging error, lowering the hedging error variability, and improving the self-financing possibility. The discrete dynamic delta-only hedging not only causes potential overcharge to clients but also could be costly to the issuers. We show that a naive application of continuous-time hedging formula to a discrete-time hedging setting tends to worsen these possibilities.

Wen, M.-M., Martin, A. D., Lai, G. & O'Brien, T. J. *Estimating the cost of equity for property–liability insurance companies.* 101-124. Due to the highly skewed and heavy-tailed distributions associated with the insurance claims process, we evaluate the Rubinstein-Leland (RL) model for its ability to improve the cost of equity estimates of insurance companies because of its distribution-free feature. Our analyses show that there is as large as a 94-basis-point difference in the estimated cost of insurance equity between the RL model and the capital asset pricing model (CAPM) for the sample of property–liability insurers with more severe departures from normality. In addition, consistent with our hypotheses, significant differences in the market risk estimates are found for insurers with return distributions that are asymmetrically distributed, and for small insurers. Third, we find significant performance improvements from using the RL model by showing smaller values of excess return of the expected return of the portfolio to the model return for a portfolio of insurers with returns that are more skewed and for a portfolio of small insurers. Finally, our panel data analysis shows the differences in the market risk estimates are significantly influenced by firm size, degree of leverage, and degree of asymmetry. The implication is that insurers should use the RL model rather than the CAPM to estimate its cost of capital if the insurer is small (assets size is less than $2,291 million), and/or its returns are not symmetrical (the value of skewness is greater than 0.509 or less than −0.509).

Zweifel, P. & Auckenthaler, C. *On the feasibility of insurers' investment policies.* 193-206. This article calls attention to a difficulty with insurers' investment policies that seems to have been overlooked so far. There is the distinct possibility that insurers cannot satisfy the demands of different stakeholders in terms of expected returns and volatility. While using the capital asset pricing model as the benchmark, this article distinguishes two groups of stakeholders that impose additional constraints. One is "income security" in the interest of current beneficiaries and older workers; the other is "predictability of contributions" in the interest of contributing younger workers and sponsoring employers. It defines the conditions for which the combination of these constraints results in a lack of feasibility of investment policy. Minimum deviation from the capital market line is proposed as the performance benchmark in these situations.

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Bali, T. G. & Theodossiou, P. *Risk measurement performance of alternative distribution functions.* 411-437. This paper evaluates the performance of three extreme value distributions, i.e., generalized Pareto distribution (GPD), generalized extreme value distribution (GEV), and Box-Cox-GEV, and four skewed fat-tailed distributions, i.e., skewed generalized error distribution (SGED), skewed generalized *t* (SGT), exponential generalized beta of the second kind (EGB2), and inverse hyperbolic sign (IHS) in estimating conditional and unconditional value at risk (VaR) thresholds. The results provide strong evidence that the SGT, EGB2, and

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IHS distributions perform as well as the more specialized extreme value distributions in modeling the tail behavior of portfolio returns. All three distributions produce similar VaR thresholds and perform better than the SGED and the normal distribution in approximating the extreme tails of the return distribution. The conditional coverage and the out-of-sample performance tests show that the actual VaR thresholds are time varying to a degree not captured by unconditional VaR measures. In light of the fact that VaR type measures are employed in many different types of financial and insurance applications including the determination of capital requirements, capital reserves, the setting of insurance deductibles, the setting of reinsurance cedance levels, as well as the estimation of expected claims and expected losses, these results are important to financial managers, actuaries, and insurance practitioners.

Boyer, M. M. & Gobert, K. *Dynamic prevention in short-term insurance contracts.* 289-312. This article looks at the dynamic properties of insurance contracts when insurers have a better technology at preventing catastrophic losses than the insured. When the prevention technology is irreversible and its benefits last for all future periods although its cost is borne in the period in which it is made, a hold-up problem occurs because the insured can change insurer after his initial insurer has invested in prevention. Investment in prevention is then delayed compared to the first best outcome. When the audit cost must be incurred by the insured when he wants to change insurer, the incumbent insurer has an informational advantage so that he can keep his client over the entire investment horizon, even though long-term contracts are not possible. This does not avoid the delay in investment, however.

Dhaene, J., Laeven, R. J. A., Vanduffel, S., Darkiewicz, G. & Goovaerts, M. J. *Can a coherent risk measure be too subadditive?* 365-386. We consider the problem of determining appropriate solvency capital requirements for an insurance company or a financial institution. We demonstrate that the subadditivity condition that is often imposed on solvency capital principles can lead to the undesirable situation where the shortfall risk increases by a merger. We propose to complement the subadditivity condition by a regulator's condition. We find that for an explicitly specified confidence level, the Value-at-Risk satisfies the regulator's condition and is the "most efficient" capital requirement in the sense that it minimizes some reasonable cost function. Within the class of concave distortion risk measures, of which the elements, in contrast to the Value-at-Risk, exhibit the subadditivity property, we find that, again for an explicitly specified confidence level, the Tail-Value-at-Risk is the optimal capital requirement satisfying the regulator's condition.

Hau, A. *When is a coinsurance-type insurance policy inferior or even giffen?* 343-364. This article derives the necessary and sufficient conditions for a coinsurance-type insurance policy covering a particular risk to be inferior and to be Giffen. Mossin's decreasing absolute risk aversion assumption for insurance to be inferior is avoided. The result generalizes Hoy and Robson and Briys, Dionne, and Eeckhoudt's results to the case with a continuum of states and relaxes their assumption of constant relative risk aversion. It is shown that knowledge about the distribution of risk can be used to relax assumptions on an utility function for a coinsurance-type insurance policy to be inferior and to be Giffen.

Kaluszka, M. & Okolewski, A. *An extension or Arrow's results on optimal reinsurance contract.* 275-288. We consider the problem of finding reinsurance policies that maximize the expected utility, the stability and the survival probability of the cedent for a fixed reinsurance premium calculated according to the maximal possible claims principle. We show that the limited stop loss and the truncated stop loss are the optimal contracts.

Li, J. *A remark on 'A shortcut way of pricing default risk through zero-utility principle'.* 517-519. This remark studies Tibiletti's bargaining condition and shows that, for risk neutral buyers or

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the default loss are small relative to the buyer's size, there exists a more shortcut bargaining condition.

OUTREVILLE, J. F. *Foreign affiliates of the largest insurance groups:: location-specific advantages.* 463-491. This article has two objectives. The first is the documentation of the relative importance of the largest insurance or reinsurance companies in the world and changes that may have occurred in the past 15 years. The second objective is to identify some of the factors that may explain the increased internationalization and most-favored locations of the world's largest insurance groups in transition and developing economies. The results of this study have important implications. First, they indicate that as expected, location-specific factors such as the size of a market, human capital, and good governance do provide an explication of the internationalization of insurance groups. Second, they also show that other factors, such as cultural distance, regulatory barriers, and competitiveness have a significant impact on the choice of countries.

PENALVA-ZUASTI, J. *A study of the interaction of insurance and financial markets:: efficiency and full insurance coverage.* 313-342. The first contribution of this article is to provide a framework, a model together with a corresponding equilibrium notion, suitable for the study of the interaction between insurance and dynamic financial markets. Our central result is that in equilibrium risk-averse agents purchase full insurance coverage, despite unfair insurance prices. We identify three conditions that explain this result: (1) insurance contracts are priced competitively, (2) financial prices include a risk premium only for undiversifiable risk, and (3) financial markets are effectively complete. An implication is that in this model disasters can be insured by fully assessable stock insurance companies.

POWELL, L. S., SOMMER, D. W. & ECKLES, D. L. *The role of internal capital markets in financial intermediaries:: evidence from insurer groups.* 439-461. We exploit the transparency of internal capital markets (ICMs) within insurance groups to investigate the activity and efficiency of ICMs within insurance groups. Specifically, we compare the relationship between internal capital transfers and investment to that between capital from other sources and investment. The ability to track the actual ICM transactions allows for more direct analysis of ICM activity than most previous studies. Consistent with theory, we find evidence that ICMs play a significant role in the investment behavior of affiliated insurers. We then use these detailed data to execute a more direct test of ICM efficiency than currently exists in the literature. Consistent with ICM efficiency, results suggest that capital is allocated to subsidiaries with the best expected performance.

PURCAL, S. & PIGGOTT, J. *Explaining low annuity demand:: An optimal portfolio application to Japan.* 493-516. Using an optimizing financial planning model in the tradition of Merton and Richard we explore how individuals should determine their life insurance and annuity choices, given uncertainty about investment returns and mortality. Both consumption and bequests appear as arguments in the individual's preference function. The model explicitly recognizes the existence of social security in retirement, and of loadings on insurance premiums, due to administration costs in the life insurance and annuities markets. The model sheds light on the reasons for the thinness of voluntary life annuity markets worldwide. The relative importance of pre-existing annuitization through social security, the role of bequests, and premium loadings are quantitatively assessed within a single optimizing framework. Results are presented for a model specification calibrated to Japan.

VALDEZ, E. A. & HAMADA, M. *CAPM and option pricing with elliptically contoured distributions.* 387-409. This article offers an alternative proof of the capital asset pricing model (CAPM) when asset returns follow a multivariate elliptical distribution. Empirical studies continue to demonstrate the inappropriateness of the normality assumption for modeling asset

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returns. The class of elliptically contoured distributions, which includes the more familiar Normal distribution, provides flexibility in modeling the thickness of tails associated with the possibility that asset returns take extreme values with nonnegligible probabilities. As summarized in this article, this class preserves several properties of the Normal distribution. Within this framework, we prove a new version of Stein's lemma for this class of distributions and use this result to derive the CAPM when returns are elliptical. Furthermore, using the probability distortion function approach based on the dual utility theory of choice under uncertainty, we also derive an explicit form solution to call option prices when the underlying is log-elliptically distributed. The Black–Scholes call option price is a special case of this general result when the underlying is log-normally distributed.

Weiss, M. A. & Qiu, J. *The journal of risk and insurance: A 75-year historical perspective.* 253-274. This research provides a comprehensive historical analysis of articles published in The Journal of Risk and Insurance over the 75-year period from 1932 to 2006. Historical statistics are provided including the number of articles, number of authors per article, geographic location of contributors, leading contributors, author affiliation (industry or academic), the proportion of articles that are theoretical and empirical, and topics covered. Statistics relating to the entire 75-year period are provided as well as breakdowns by decade. The results indicate that the contributors to The Journal of Risk and Insurance have become more international over time, average article length and average number of authors per article have increased over time, and empirical articles appear more frequently than theoretical articles.

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Antonio, K. & Beirlant, J. *Issues in Claims Reserving and Credibility: A Semiparametric Approach With Mixed Models.* 643-676. Using the statistical methodology of semi-parametric regression and its connection with mixed models, this article revisits smoothing models for loss reserving and credibility. Apart from the flexibility inherent to all semiparametric methods, advantages of the semiparametric approach developed here are threefold. First, a Bayesian implementation of these smoothing models is relatively straightforward and allows simulation from the full predictive distribution of quantities of interest. Second, because the constructed models have an interpretation as (generalized) linear mixed models ((G)LMMs), standard statistical theory and software for (G)LMMs can be used. Third, more complicated data sets, dealing, for example, with quarterly development in a reserving context, heavy tails, semi-continuous data, or extensive longitudinal data, can be modeled within this framework.

Brockett, P. L., Golden, L. L., Guillen, M., Nielsen, J. P., Parner, J. & Perez-Martin, A. M. *Survival Analysis of a Household Portfolio of Insurance Policies: How Much Time Do You Have to Stop Total Customer Defection?* 713-737. Customer-side influences on insurance have been relatively ignored in the literature. Using the household as the unit of analysis, this article focuses on the behavior of households having multiple policies of different types with the same insurance company, and who cancel their first policy. How long after the household's cancellation of the first policy does the insurer have to retain the customer and avoid customer defection on all policies to the competition? And, what customer characteristics are associated with customer loyalty? Using logistic regression and survival analysis techniques, an assessment is made of the probability of total customer withdrawal, and the length of time between first cancellation and subsequent customer withdrawal. Using a European database spanning 54 months of household multiple policyholder behavior, the results show that cancellation of one policy is a very strong indicator that other household policies will be canceled. Further, the insurer can have time to react to retain the customer after the first

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cancellation, however, this time is significantly dependent on the method used to contact the company, household demographics, and the nature of the household's insurance policy portfolio. Surprisingly, core customers having three or more policies in addition to the canceled policy are more vulnerable to total defection on all policies than noncore customers. Further, the potential customer repelling effects of premium increases seem to wear out after 12 months. Strategic implications of the results are presented.

Doerpinghaus, H. I., Schmit, J. T. & Jia-Hsing Ye, J. *Age and Gender Effects on Auto Liability Insurance Payouts.* 527-550. We examine the relationship between claimant demographic characteristics (specifically, gender, age, and marital status) and the relative size of automobile third-party settlements. We present three possible theories to explain differences in payouts associated with gender and age: variations in risk attitudes, variations in negotiating costs, and discrimination. Results of empirical testing are consistent with differences in settlement amounts, particularly with respect to gender. These differences are examined and discussed along with suggestions for future research.

Elango, B., Ma, Y.-L. & Pope, N. *An Investigation Into the Diversification–Performance Relationship in the U.S. Property–Liability Insurance Industry.* 567-591. This article investigates the relationship between product diversification and firm performance in the U.S. property–liability insurance industry using data over the 1994 through 2002 time period. Using various measures of product diversification and firm performance, we find that the extent of product diversification shares a complex and nonlinear relationship with firm performance. Our findings suggest that performance benefits associated with product diversification are contingent upon an insurer's degree of geographic diversification. Robustness tests using subsamples and market returns for public firms show consistent results.

Emms, P. & Haberman, S. *Income Drawdown Schemes for a Defined-Contribution Pension Plan.* 739-761. In retirement a pensioner must often decide how much money to withdraw from a pension fund, how to invest the remaining funds, and whether to purchase an annuity. These decisions are addressed here by introducing a number of income drawdown schemes, which are relevant to a defined-contribution personal pension plan. The optimal asset allocation is defined so that it minimizes the expected loss of the pensioner as measured by the performance of the pension fund against a benchmark. Two benchmarks are considered: a risk-free investment and the price of an annuity. The fair-value income drawdown rate is defined so that the fund performance is a martingale under the objective measure. Annuitization is recommended if the expected fair-value drawdown rate falls below the annuity rate available at retirement. As an illustration, the annuitization age is calculated for a Gompertz mortality distribution function and a power law loss function.

Gatzert, N. & Schmeiser, H. *Assessing the Risk Potential of Premium Payment Options in Participating Life Insurance Contracts.* 691-712. Most life insurance contracts embed the right to stop premium payments during the term of the contract (paid-up option). Thereby, the contract is not terminated but continues with reduced benefits and often provides the right to resume premium payments later, thus increasing the previously reduced benefits (resumption option). In our analysis, we start with a basic contract with two standard options, namely, an interest rate guarantee and annual surplus participation. Next, in addition to the features of the basic contract, a paid-up and resumption option is included in the framework. The valuation process is not based on assumptions about a particular policyholders' exercise strategy but instead assesses the risk potential from the insurer's viewpoint by providing an upper bound for any possible exercise behavior. This approach provides important information to the insurer about the potential hazard of offering the paid-up and resumption option. Further, the approach allows an analysis of the impact of guaranteed interest rate, annual surplus participation, and investment volatility on the values of the premium payment options.

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GERKE, W., MAGER, F., REINSCHMIDT, T. & SCHMIEDER, C. *Empirical Risk Analysis of Pension Insurance: The Case of Germany*. 763-784. The book reserve system is the most widespread method of financing occupational pension plans in Germany. The pension liabilities are mutually insured by the Pensions-Sicherungs-Verein VVaG (PSVaG) against bankruptcy. The PSVaG recently stated that the insurance system needed to be reformed. In the future, risk-adjusted premiums as foreseen for the newly established Pension Protection Fund in the United Kingdom could become feasible. We perform a credit portfolio analysis to determine the risk profile of the PSVaG. The magnitude of a tail risk event suggests that under the current financing system it can only be smoothed out over decades. Under an expected loss pricing plan insurance premiums would vary greatly. In a marginal risk contribution approach the variation of the premiums would be less pronounced.

GUTTER, M. S. & HATCHER, C. B. *Racial Differences in the Demand for Life Insurance*. 677-689. The objective of this article is to measure racial differences in the proportion of human capital that households protect with life insurance. Using the 2004 Survey of Consumer Finances data, racial differences in two stages of the process are tested, where it is assumed that households must decide both whether or not to purchase life insurance and how much of their human capital to insure (if they decide to purchase). Among married and cohabitating households, we find that, controlling for demographics and other factors, there is little difference in life insurance ownership between black and white households but that white households insure a larger proportion of their human capital than black households.

HSU, S., LIN, C. & YANG, Y. *Integrating Neural Networks for Risk-Adjustment Models*. 617-642. This article demonstrates the possibility of an alternative approach for risk-adjustment models. In the proposed model the risk characteristics of the beneficiary's health within the same cohort classified by Self-Organizing Map network are highly homogeneous, whereas the numbers of individuals within each cohort remain sufficient to allow further investigation of the causal effect from clustered data. A comparison of different models by the 10-fold cross-validation reveals that the performance improvement in the proposed integration model is both significant and stable across the estimation and validation sampling.

NAM, J., WANG, J. & ZHANG, G. *Managerial Career Concerns and Risk Management*. 785-809. We present a dynamic model of corporate risk management and managerial career concerns. We show that managers with low (high) initial reputation have high (low) career concerns about keeping their jobs and receiving all future income. These managers are more likely to speculate (hedge) early in their careers. In the later stage of their careers when managers have less career concerns, there is no speculative motive for self interested managers. On the other hand, highly reputable managers have minimal career concerns and they engage in neither hedging nor speculation early in their careers, but they may choose to hedge after poor early performance.

SHARPE, I. G. & STADNIK, A. *APRA's Expert Judgment Ratings and Solvency Cover of Australian General Insurers*. 593-616. The Australian Prudential Regulation Authority's (APRA's) supervisors use expert judgment to rate the risk of failure of general insurers (GIs). Using statistical data, we model the determinants of GI ratings and solvency cover and find: (1) sufficient predictive power in statistical data to identify GIs for earlier review and assist in quality assurance of APRA's ratings, and (2) that profitability, solvency cover, investment, and underwriting risk play different roles in rating foreign branch and Australian-incorporated GIs. We conclude that supervisors generally correctly incorporate our a priori expectations of the effects of risk indicators on GI risk into their ratings.

WANG, J. L., CHUNG, C.-F. & TZENG, L. Y. *An Empirical Analysis of the Effects of Increasing Deductibles on Moral Hazard*. 551-566. Using information on timing and number of claims in

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a unique data set pertaining to comprehensive automobile insurance with the increasing deductible provision in Taiwan, the authors provide new evidence for moral hazard. Time-varying correlations between the choice of the insurance coverage and claim occurrence are significantly positive and exhibit a smirk pattern across policy months. This empirical finding supports the existence of asymmetric information. A subsample estimation depicts insured drivers' significant responses to increasing deductibles, which implies the existence of moral hazard. According to the probit regression results, the increasing deductible makes policyholders who have ever filed claims less likely to file additional claims later in the policy year. The empirical findings strongly support the notion that the increasing deductible provision helps control moral hazard.

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ARENA, M. *Does insurance market activity promote economic growth?: A cross-country study for industrialized and developing countries.* 921-946. Insurance market activity may contribute to economic growth, both as financial intermediary and provider of risk transfer and indemnification, by allowing different risks to be managed more efficiently and by mobilizing domestic savings. During the last decade, there has been faster growth in insurance market activity, particularly in emerging markets, given the process of financial liberalization and integration, which raises questions about the overall impact on economic growth. This article tests whether there is a causal relationship between insurance market activity (life and nonlife insurance) and economic growth. Using the generalized method of moments (GMM) for dynamic models of panel data for 55 countries between 1976 and 2004, I find robust evidence for this relationship. Both life and nonlife insurance have a positive and significant causal effect on economic growth. For life insurance, high-income countries drive the results, and for nonlife insurance, both high-income and developing countries drive the results.

BAJTELSMIT, V. & THISTLE, P. D. *The reasonable person negligence standard and liability insurance.* 815-823. The article shows that, under the reasonable person negligence rule, heterogeneity of potential injurers can be sufficient to create a demand for liability insurance. Potential injurers with a low probability of accidents or a high cost of exercising care have optimal levels of care that are below the negligence standard. For these groups, it may be less costly to be negligent and purchase insurance than to comply with the negligence standard. We show that the availability of insurance is socially desirable.

BLAU, B. M. & VAN NESS, R. A. *Capitalizing on catastrophe: short selling insurance stocks around hurricanes Katrina and Rita.* 967-996. We develop several hypotheses regarding short-selling activity around Hurricanes Katrina and Rita. We find that abnormal short selling does not increase until 2 trading days after the landfall of Katrina and that short-selling activity is much more significant around Rita. We find a substantial increase in short-selling activity in the trading days prior to the landfall of Rita and relatively less short-selling activity in the trading days after landfall. There is little evidence that suggests that traders short insurance stocks with more potential exposure in the Gulf region than other insurance stocks in the days before landfall.

CARSON, J. M., ELYASIANI, E. & MANSUR, I. *Market risk, interest rate risk, and interdependencies in insurer stock returns: a system-GARCH model.* 873-891. We examine market risk, interest rate risk, and interdependencies in returns and return volatilities across three insurer segments within a System-GARCH framework. Three main results are obtained: market risk is greatest for accident and health (A&H) insurers, followed by life (Life) and property and casualty (P&C) insurers; interest rate sensitivity is negative and greatest for Life insurers; and

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interdependencies in returns are significant with the magnitude being strongest between P&C and A&H insurers. The implication is that greatest diversification benefits arise between Life and the other segments of the insurance industry. Market risk and interest rate risk for diversified firms are smaller than those for nondiversified firms for both product and geographic diversification.

GONG, G. & WEBB, A. *Mortality heterogeneity and the distributional consequences of mandatory annuitization.* 1055-1079. This article investigates the distributional consequences of mandatory annuitization. Using Health and Retirement Study (HRS) data and accounting for longevity risk pooling within marriage and preannuitized wealth, we find substantial redistribution away from disadvantaged groups in expected utility terms. Using HRS data on subjective survival probabilities, we construct a subjective life table for each individual in the HRS. We calculate the value each household would place on annuitization, based on the husband and wife's subjective life tables, and the household's degree of risk aversion and proportion of preannuitized wealth. A significant minority would perceive themselves as suffering a loss from mandatory annuitization.

HORNEFF, W. J., MAURER, R. H. & STAMOS, M. Z. *Optimal gradual annuitization: quantifying the costs of switching to annuities.* 1019-1038. We compute the optimal dynamic annuitization and asset allocation policy for a retiree with Epstein–Zin preferences, uncertain investment horizon, potential bequest motives, and pre-existing pension income. In our setting the retiree can decide each year how much he consumes and how much he invests in stocks, bonds, and life annuities, while the prior literature mostly considered restricted so-called deterministic or stochastic switching strategies. We show that postponing the annuity purchase is no longer optimal in the gradual annuitization (GA) case since investors are able to attain the optimal mix between liquid assets (stocks and bonds) and illiquid life annuities each year. In order to assess potential utility losses, we benchmark various restricted annuitization strategies against the unrestricted GA strategy.

HUANG, H., MILEVSKI, M. A. & WANG, J. *Portfolio choice and life insurance: the CRRA case.* 847-872. We solve a portfolio choice problem that includes life insurance and labor income under constant relative risk aversion (CRRA) preferences. We focus on the correlation between the dynamics of human capital and financial capital and model the utility of the family as opposed to separating consumption and bequest. We simplify the underlying Hamilton–Jacobi–Bellman equation using a similarity reduction technique that leads to an efficient numerical solution. Households for whom shocks to human capital are negatively correlated with shocks to financial capital should own more life insurance with greater equity/stock exposure. Life insurance hedges human capital and is insensitive to the family's risk aversion, consistent with practitioner guidance.

HUANG, R. J. & TZENG, L. Y. *Consumption externality and equilibrium underinsurance.* 1039-1054. Relative consumption has been found to be crucial in many areas, such as asset pricing, the design of taxation, and economic growth. This article extends this line of research to the individual's insurance decision. We first define "keeping up with the Joneses" in the purchase of insurance and find that jealousy does not necessarily give rise to "keeping up with the Joneses". We also identify several sufficient conditions that cause the optimal coverage in the private market to be less than the social optimum (equilibrium underinsurance). Jealousy is found to be neither a sufficient nor a necessary condition for equilibrium underinsurance. We further show that a social welfare maximizing government could adopt a tax system to correct for the consumption externality and make individuals better off.

LIEBENBERG, A. P. & SOMMER, D. W. *Effects of corporate diversification: evidence from the property–liability insurance industry.* 893-919. Using a sample of property–liability insurers

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over the period 1995-2004, we develop and test a model that explains performance as a function of line-of-business diversification and other correlates. Our results indicate that undiversified insurers consistently outperform diversified insurers. In terms of accounting performance, we find a diversification penalty of at least 1 percent of return on assets or 2 percent of return on equity. These findings are robust to corrections for potential endogeneity bias, alternative risk measures, alternative diversification measures, and an alternative estimation technique. Using a market-based performance measure (Tobin's Q) we find that the market applies a significant discount to diversified insurers. The existence of a diversification penalty (and diversification discount) provides strong support for the strategic focus hypothesis. We also find that insurance groups underperform unaffiliated insurers and that stock insurers outperform mutuals.

LIGON, J. A. & THISTLE, P. D. *Adverse selection with frequency and severity risk: alternative risk-sharing provisions.* 825-846. The analysis considers an insurance market with adverse selection where individuals' loss distributions may differ with respect to both the frequency and severity of loss. We show that the combination of deductibles and coinsurance can be used to sort rationed policyholders. Because of their screening properties, coinsurance and deductibles may both be equilibrium forms of risk sharing for a particular insurer facing asymmetric information, with different rationed consumers choosing different risk-sharing provisions.

POPE, N. & MA, Y.-L. *The market structure — performance relationship in the international insurance sector.* 947-966. This article tests the applicability of the structure–conduct–performance (SCP) hypothesis in the international nonlife insurance marketplace. We employ a panel data regression methodology that includes 23 nations (developed and developing countries) over the time period of 1996 to 2003. The results reveal that the interaction of market liberalization and market concentration shares a complex relationship with market profitability. Our results show that the expectations associated with the SCP hypothesis are supported when the levels of liberalization are low. However, for markets that are highly liberalized the presence of foreign insurers significantly alters the dynamics of nonlife insurance markets.

THOMAS, R. G. *Loss coverage as a public policy objective for risk classification schemes.* 997-1018. This article suggests that from a public policy perspective, some degree of adverse selection may be desirable in some insurance markets. The article suggests that a public policymaker should consider the criterion of "loss coverage", and that in some markets a policymaker may wish to regulate risk classification with a view to increasing loss coverage. Either too much or too little risk classification may reduce loss coverage. The concept is explored by means of examples and formulaic and graphical interpretations. An application to the UK life insurance market is considered.

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BADESCU, A. & LANDRIAULT, D. *Recursive calculation of the dividend moments in a multi–threshold risk model.* 74-88. In this article, we consider the class of risk models with Markovian claim arrivals studied by Badescu *et al*. (2005) and Ramaswami (2006), among others. Under

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a multi-threshold dividend structure, we develop a recursive algorithm for the calculation of the moments of the discounted dividend payments before ruin. Capitalizing on the connection between an insurer's surplus process and its corresponding fluid flow process, our approach generalizes results obtained by Albrecher and Hartinger (2007) and Zhou (2006) in the framework of the classical compound Poisson risk model (with phase-type claim sizes). Contrary to the traditional analysis of the discounted dividend payments in risk theory, we develop a sample-path-analysis procedure that allows the determination of these moments with or without ruin occurrence (separately). Numerical examples are then considered to illustrate our main results and show the contribution of each component to the moments of the discounted dividend payments.

Cowell, M. J. *Discussion of 'Trajectories of morbidity, disability, and mortality among the U.S. elderly population: evidence from the 1984-1999 NLTCS', Stallard, Eric, July 2007.* 95-98.

Gerber, H. U. & Shiu, E. S. W. *Discussion of: 'Asset allocation with hedge funds on the menu', Boyle, Phelim and Liew, Sun Siang, October 2007.* 89-90.

Li, J. S. H. & Ng, A. C. Y. *Discussion of 'Markov aging process and phase-type law of mortality', Lin, X Sheldon and Liu, Xiaoming, October 2007.* 90-94.

Li, Z., Tan, K. S. & Yang, H. *Multiperiod optimal investment consumption strategies with mortality risk and environment uncertainty.* 47-64. In this article we investigate three related investment-consumption problems for a risk-averse investor: (1) an investment-only problem that involves utility from only terminal wealth, (2) an investment-consumption problem that involves utility from only consumption, and (3) an extended investment-consumption problem that involves utility from both consumption and terminal wealth. Although these problems have been studied quite extensively in continuous-time frameworks, we focus on discrete time. Our contributions are (1) to model these investment-consumption problems using a discrete model that incorporates the environment risk and mortality risk, in addition to the market risk that is typically considered, and (2) to derive explicit expressions of the optimal investment-consumption strategies to these modeled problems. Furthermore, economic implications of our results are presented. It is reassuring that many of our findings are consistent with the well-known results from the continuous-time models, even though our models have the additional features of modeling the environment uncertainty and the uncertain exit time.

Rosenberg, M. A. & Farrell, P. M. *Predictive modeling of costs for a chronic disease with acute high–cost episodes.* 1-18. Chronic diseases account for 75% of U.S. national health care expenditures as estimated by the Centers for Disease Control. Many chronic diseases are punctuated by acute episodes of illnesses that occur randomly and create cost spikes in utilization from one year to the next. Modeling to account for these random events provides better estimates of (1) future costs, and (2) their variability. A Bayesian statistical model is used to predict the incidence and cost of hospitalizations for one chronic disease. A two-part statistical model is described that separately models the utilization and cost of hospitalization. Individual demographic characteristics are included as well as a simple biological classification system to adjust for the severity of disease among individuals. Results by child, as well as by calendar year, are presented. Using a simple approach to incorporate severity, the model produces reasonable estimates of the number of hospitalizations and cost of hospitalization for the group in total, as well as for a separate group of High Utilizers. The study reflects real-world experiences of persons entering and leaving a group. Modeling at an individual level provides a way to adjust for individual-level severity. The ability to model uneven and unpredictable occurrence of utilization, and potential cost, would be beneficial in the design of insurance programs or for disease management programs.

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Siu, T. K., Erlwein, C. & Mamon, R. S. *The pricing of credit default swaps under a markov–modulated Merton's structural model.* 19-46. We consider the valuation of credit default swaps (CDSs) under an extended version of Merton's structural model for a firm's corporate liabilities. In particular, the interest rate process of a money market account, the appreciation rate, and the volatility of the firm's value have switching dynamics governed by a finite-state Markov chain in continuous time. The states of the Markov chain are deemed to represent the states of an economy. The shift from one economic state to another may be attributed to certain factors that affect the profits or earnings of a firm; examples of such factors include changes in business conditions, corporate decisions, company operations, management strategies, macroeconomic conditions, and business cycles. In this article, the Esscher transform, which is a well-known tool in actuarial science, is employed to determine an equivalent martingale measure for the valuation problem in the incomplete market setting. Systems of coupled partial differential equations (PDEs) satisfied by the real-world and risk-neutral default probabilities are derived. The consequences for the swap rate of a CDS brought about by the regime switching effect of the firm's value are investigated via a numerical example for the case of a two-state Markov chain. We perform sensitivity analyses for the real-world default probability and the swap rate when different model parameters vary. We also investigate the accuracy and efficiency of the PDE approach by comparing the numerical results from the PDE approach to those from the Monte Carlo simulation.

Zaks, Y., Frostig, E. & Levikson, B. *Pricing a heterogeneous portfolio based on a demand function.* 65-73. Consider a portfolio containing a number of risk classes. Each class has its own demand function, which determines the number of insureds in this class as a function of the premium. The insurer determines the premiums based on the number of insureds in each class. The "market" reacts by updating the number of the policyholders, then the insurer updates the premium, and so on. We show that this process has an equilibrium point, and then we characterize this point.

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North American Actuarial Journal

Volume **12** (2), 2008

Balasooriya, U. & Low, C. K. *Modeling insurance claims with extreme observations: transformed Kernel density and generalized Lambda distribution.* 129-142. In modeling insurance claims, when there are extreme observations in the data, the commonly used loss distributions often are able to fit the bulk of the data well but fail to do so at the tail. One approach to overcome this problem is to focus on the extreme observations only and model them with the generalized Pareto distribution, as supported by extreme value theory. However, this approach discards useful information about the small and medium-sized claims, which is important for many actuarial purposes. In this article we consider modeling large skewed data using a highly flexible distribution, the generalized lambda distribution, and the recently proposed semiparametric transformed kernel density estimation. Our results suggest that both these approaches are credible options for the investigator when modeling insurance claims data that typically contain large extreme observations. In addition, even at the extreme tails they perform well when compared with the generalized Pareto distribution.

Gerber, H. U. & Shiu, E. S. W. *Authors reply to discussion of: 'On optimal dividend strategies in the compound Poisson model', Ko, Bangwon, July 2006.* 216-219.

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LI, J. S. H., HARDY, M. R. & TAN, K. S. *Threshold life tables and their applications.* 99-115. The rapid emergence of centenarians has highlighted the importance of survival probabilities at extreme ages and has motivated actuaries to look for alternative ways to close of life tables in place of assigning a death probability of 1 at an arbitrarily chosen age. Using the asymptotic results of modern extreme value theory, we propose a model, which we call the threshold life table, to extrapolate survival distributions to extreme ages and to determine the appropriate end point of a life table. By combining the threshold life table with the Lee–Carter model for stochastic mortality forecasting, we consider applications to the valuation of a life annuity portfolio and to the prediction of the highest attained age. We illustrate the theoretical results using U.S., Canadian, and Japanese mortality data.

LI, S. & BADESCU, A. L. *Discussion of: 'The discounted joint distribution of the surplus prior to ruin and the deficit at ruin in a Sparre Andersen model', Ren, Jiandong, July 2007.* 208-212.

LUO, S. *Ruin minimization for insurers with borrowing constraints.* 143-174. We consider an optimal dynamic control problem for an insurance company with opportunities of proportional reinsurance and investment. The company can purchase proportional reinsurance to reduce its risk level and invest its surplus in a financial market that has a Black–Scholes risky asset and a risk-free asset. When investing in the risk-free asset, three practical borrowing constraints are studied individually: (B1) the borrowing rate is higher than lending (saving) rate, (B2) the dollar amount borrowed is no more than $K > 0$, and (B3) the proportion of the borrowed amount to the surplus level is no more than $k > 0$. Under each of the constraints, the objective is to minimize the probability of ruin. Classical stochastic control theory is applied to solve the problem. Specifically, the minimal ruin probability functions are obtained in closed form by solving Hamilton-Jacobi-Bellman (HJB) equations, and their associated optimal reinsurance-investment policies are found by verification techniques.

MERZ, M. & WÜTHRICH, M. V. *Prediction error of the multivariate chain ladder reserving method.* 175-197. In this paper we consider the claims reserving problem in a multivariate context: that is, we study the multivariate chain-ladder (CL) method for a portfolio of N correlated runoff triangles based on multivariate age-to-age factors. This method allows for a simultaneous study of individual runoff subportfolios and facilitates the derivation of an estimator for the mean square error of prediction (MSEP) for the CL predictor of the ultimate claim of the total portfolio. However, unlike the already existing approaches we replace the univariate CL predictors with multivariate ones. These multivariate CL predictors reflect the correlation structure between the subportfolios and are optimal in terms of a classical optimality criterion, which leads to an improvement of the estimator for the MSEP. Moreover, all formulas are easy to implement on a spreadsheet because they are in matrix notation. We illustrate the results by means of an example.

PENG, L. *Estimating the probability of a rare event via elliptical copulas.* 116-128. A rare event happens with an extremely small probability but may cost billions of dollars. How to model and estimate the small probability of such an event is of importance to the insurance industry. Based on multivariate extreme value theory, methods have been proposed to extrapolate data into a far tail region. However, questions still remain open, such as the direction of extrapolation for a multivariate distribution and threshold selection for both marginals and the tail dependence function. In this paper we provide a way to estimate the probability of a rare event via modeling marginals and dependence by heavy tailed distributions and elliptical copulas, respectively. Hence, the direction of extrapolation becomes irrelevant. Moreover we employ recent threshold selection procedures to choose tuning parameters automatically.

REN, J. *On the laplace transform of the aggregate discounted claims with Markovian arrivals.* 198-207. We present an explicit formula for the Laplace transform of the distribution of the

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aggregate discounted claims when interclaim times follow a Markovian arrival process. In addition, we derive explicit formulas for the first two moments and then show that the higher moments may be obtained by numerically solving a system of ordinary differential equations.

SIU, T. K. *Discussion of: 'asset allocation with hedge funds on the menu', Boyle, Phelim and Liew, Sun Siang, October 2007.* 213-216.

NORTH AMERICAN ACTUARIAL JOURNAL

Volume **12** (3), 2008

BERNSTEIN, D. *Intergenerational transfers and insurance policy design.* 292-298. Group health insurance policies offering an identical benefit package to every member of the group result in lower expected health benefits for younger cohorts than older cohorts. The dispersion in insurance benefits across age groups differs among insurance policies. Simulation results presented in this paper demonstrate that a shift from comprehensive health insurance to highdeductible health insurance decreases the share of expected benefits going to younger cohorts. An estimated 81.5% of the 23-to-32-year-old cohort is expected to receive less than $500 in health benefits during a year for one prototypical high-deductible health plan. Low expected benefits for younger relatively healthy cohorts could increase the number of younger individuals who eschew health coverage. Age-rated premiums are probably the most straightforward way to stimulate demand for high-deductible health plans among younger healthier individuals.

CHEUNG, E. C. K. *Discussion of, 'Recursive calculation of the dividend moments in a multi-threshold risk model', Badescu, Andrei and Landriault, David, January 2008.* 336-340.

CHEUNG, E. C. K., DICKSON, D. C. M. & DREKIC, S. *Moments of discounted dividends for a threshold strategy in the compound Poisson risk model.* 299-318. We consider a compound Poisson risk model in which part of the premium is paid to the shareholders as dividends when the surplus exceeds a specified threshold level. In this model we are interested in computing the moments of the total discounted dividends paid until ruin occurs. However, instead of employing the traditional argument, which involves conditioning on the time and amount of the first claim, we provide an alternative probabilistic approach that makes use of the (defective) joint probability density function of the time of ruin and the deficit at ruin in a classical model without a threshold. We arrive at a general formula that allows us to evaluate the moments of the total discounted dividends recursively in terms of the lower-order moments. Assuming the claim size distribution is exponential or, more generally, a finite shape and scale mixture of Erlangs, we are able to solve for all necessary components in the general recursive formula. In addition to determining the optimal threshold level to maximize the expected value of discounted dividends, we also consider finding the optimal threshold level that minimizes the coefficient of variation of discounted dividends. We present several numerical examples that illustrate the effects of the choice of optimality criterion on quantities such as the ruin probability.

FREEDMAN, B. *Efficient post-retirement asset allocation.* 228-241. To examine post-retirement asset allocation, an extension to the classic Markowitz risk-return framework is suggested. Assuming that retirees make constant (real dollar) annual withdrawals from their portfolios, reward and risk measures are defined to be the mean and standard deviation of wealth remaining at end of life. Asset returns and time of death are both treated as random variables. Assuming constant lifetime asset allocation, the risk and reward measures can be evaluated analytically, and an efficient frontier can be determined. Life annuities can be used to extend

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the left-hand (low-risk) side of the efficient frontier. The desired level of wealth at end of life can be used to choose a desirable portfolio on the efficient frontier. The desirable portfolio strongly depends on the withdrawal rate. It is suggested (although not proven) that asset allocations strategies that vary with age do not add efficiency in this model, and asset allocation strategies that vary with wealth can add efficiency.

HUH, J. & KOLKIEWICZ, A. *Computation of multivariate barrier crossing probability and its applications in credit risk models.* 263-291. In this paper we consider computational methods of finding exit probabilities for a class of multivariate diffusion processes. Although there is an abundance of results for one-dimensional diffusion processes, for multivariate processes one has to rely on approximations or simulation methods. We adopt a Large Deviations approach to approximate barrier crossing probabilities of a multivariate Brownian Bridge. We use this approach in conjunction with simulation methods to develop an efficient method of obtaining barrier crossing probabilities of a multivariate Brownian motion. Using numerical examples, we demonstrate that our method works better than other existing methods. We mainly focus on a three-dimensional process, but our framework can be extended to higher dimensions. We present two applications of the proposed method in credit risk modeling. First, we show that we can efficiently estimate the default probabilities of several correlated credit risky entities. Second, we use this method to efficiently price a credit default swap (CDS) with several correlated reference entities. In a conventional approach one normally adopts an arbitrary copula to capture dependency among counterparties. The method we propose allows us to incorporate the instantaneous variance-covariance structure of the underlying process into the CDS prices.

MUERMANN, A. *Market price of insurance risk implied by catastrophe derivatives.* 221-227. Insurance derivatives facilitate the trading of insurance risks on capital markets, such as catastrophe derivatives that were traded on the Chicago Board of Trade. Simultaneously, insurance risks are traded through reinsurance portfolios. In this paper we make inferences about the market price of risk implied by the information embedded in the prices of these two assets.

SIU, T. K. *Discussion of, 'The Discounted Joint Distribution of the Surplus Prior to Ruin and the Deficit at Ruin in a Sparre Andersen Model', Ren, Jiandong, July 2007.* 341-342.

TAYLOR, G. *A simple model of insurance market dynamics.* 242-262. This paper constructs and studies a simple but realistic model of an insurance market. The model has a minimalist construction in the sense that the number of parameters defining it is strictly limited and the elimination of any one of them would destroy its realism. There are 11 essential parameters. Each of the parameters has a physical interpretation. Some determine competitive effects within the market, some barriers to entry, and so on. The effect of each on various aspects of the market is examined in the presence of simulated loss experience. The aspects of the market considered include stability of premium rates, profitability, and market concentration. Some of the parameters are capable of use as regulatory controls. Two parameters, in addition to the original 11, are explicit price controls. Despite its simplicity, the model displays considerably complex behavior. Some results are intuitive, but some are not. For this reason, regulatory controls need to be applied with great caution lest they induce perverse effects, possibly even the reverse of those intended. The effect of the parameters on market behavior is first studied in the absence of catastrophic events from the loss experience. Subsequently, the effect of a single such event is studied.

TSAI, C. C.-L. *Ordering ruin probabilities resulting from layer-based claim amounts for surplus process perturbed by diffusion.* 319-335. In this paper we study orders of pairs of ruin probabilities resulting from two claim severity random variables X and Y for a continuous

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time surplus process perturbed by diffusion, each of which is the underlying risk Z with or without a deductible and/or a policy limit imposed, called a layer of Z. The deductibles and policy limits for X and Y could be the same or different. Under some condition regarding the relative security loadings, we find that the layer with a policy limit and the layer with a deductible yield the lowest and highest ruin probabilities, respectively, provided that Z has a decreasing failure rate, and the layer with and the layer without both a deductible and a policy limit produce the smallest and largest ruin probabilities, respectively, provided that Z has an increasing failure rate. Numerical examples are also given to illustrate the results of the proposed theorems for ordering ruin probabilities resulting from layers of two random variables distributed as a single exponential and a mixture of two exponentials.

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Scandinavian Actuarial Journal

Volume **1**, 2008

Landriault, D. *Randomized dividends in the compound binomial model with a general premium rate.* 1-15. In this paper, we consider the compound binomial model with a multi-threshold dividend structure and randomized dividend payments. Using the roots of a generalization of Lundberg's fundamental equation and the general theory on difference equations, we derive an explicit expression for the Gerber-Shiu discounted penalty function with any initial surplus u (u). This result generalizes the main result of Tan & Yang (2006) regarding the recursive calculation of some Gerber-Shiu functions in a special class of risk models, namely the compound binomial model with a unit premium and a single threshold dividend structure. Finally, an explicit expression is also derived for the expected discounted dividend payments before ruin.

Lefevre, C. & Loisel, S. *On finite-time ruin probabilities for classical risk models.* 41-60. This paper examines the problem of ruin in the classical compound binomial and compound Poisson risk models. Our primary purpose is to extend to those models an exact formula derived by Picard & Lefèvre (1997) for the probability of (non-)ruin within finite time. First, a standard method based on the ballot theorem and an argument of Seal-type provides an initial (known) formula for that probability. Then, a concept of pseudo-distributions for the cumulated claim amounts, combined with some simple implications of the ballot theorem, leads to the desired formula. Two expressions for the (non-)ruin probability over an infinite horizon are also deduced as corollaries. Finally, an illustration within the framework of Solvency II is briefly presented.

Pfeifer, D. & Strassburger, D. *Solvency II: stability problems with the SCR aggregation formula.* 61-77. One of the central issues in the Solvency II process will be an appropriate calculation of the Solvency Capital Requirement (SCR). This is the economic capital that an insurance company must hold in order to guarantee a one-year ruin probability of at most 0.5%. In the so-called standard formula, the overall SCR is calculated from individual SCRs in a particular way that imitates the calculation of the standard deviation for a sum of normally distributed risks (SCR aggregation formula). However, in order to cope with skewness in the individual risk distributions, this formula must be calibrated accordingly in

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order to maintain the prescribed level of confidence. In this paper, we want to show that the methods proposed and discussed thus far still show stability problems within the general setup.

ZAKS, Y. *The optimal claiming strategies in a Bonus-Malus System and the monotony property.* 34-40. In the classical Bonus-Malus System (BMS) there are several premium levels, e.g., M. A premium level i consists of a premium (i) and a deductible (di). In this paper, we consider the expected cost for a horizon of n periods of a policyholder in level i. This expectation is denoted by $En(i)$. When damage of size x occurs, the policyholder should decide whether or not to claim. The minimum damage for which the policyholder will claim should be the solution of. We will show that $En - 1(i-1) - En - 1(i+1)$, hence $x = di + En - 1(i+1)En - 1(i-1)$, is a valid solution. This paper is dedicated to the memory of Professor Benjamin (Benny) Zeev Levikson.

ZIMBIDIS, A. A. *Premium and reinsurance control of an ordinary insurance system with liabilities driven by a fractional Brownian motion.* 16-33. This paper investigates the problem of premium and reinsurance control of an ordinary insurance system when liabilities are driven by a fractional Brownian motion. The reserve equation is considered using two alternative routes: the first with no reinsurance option, and the second with some controllable proportional reinsurance coverage. Recent results from the theory of fractional linear-quadratic control (fractional calculus) are discussed, partially extended and utilized to derive compact analytical formulae for the optimal functionals of the safety loading (consequently for the respective premium rate), and the volume of the retained risk (or equivalently, for the proportion of the reinsurance coverage).

SCANDINAVIAN ACTUARIAL JOURNAL

Volume **2-3**, 2008

CAIRNS, A. J. G., BLAKE, D. & DOWD, K. *Modelling and management of mortality risk: a review.* 79-113. In the first part of the paper, we consider the wide range of extrapolative stochastic mortality models that have been proposed over the last 15-20 years. A number of models that we consider are framed in discrete time and place emphasis on the statistical aspects of modelling and forecasting. We discuss how these models can be evaluated, compared and contrasted. We also discuss a discrete-time market model that facilitates valuation of mortality-linked contracts with embedded options. We then review several approaches to modelling mortality in continuous time. These models tend to be simpler in nature, but make it possible to examine the potential for dynamic hedging of mortality risk. Finally, we review a range of financial instruments (traded and over-the-counter) that could be used to hedge mortality risk. Some of these, such as mortality swaps, already exist, while others anticipate future developments in the market.

DAHL, M., MELCHIOR, M. & MOELLER, T. *On systematic mortality risk and risk-minimization with survivor swaps.* 114-146. A new market for so-called mortality derivatives is now appearing with survivor swaps (also called mortality swaps), longevity bonds and other specialized solutions. The development of these new financial instruments is triggered by the increased focus on the systematic mortality risk inherent in life insurance contracts, and their main focus is thus to allow the life insurance companies to hedge their systematic mortality risk. At the same time, this new class of financial contract is interesting from an investor's point of view, since it increases the possibility for an investor to diversify the investment portfolio. The systematic mortality risk stems from the uncertainty related to the future development of the mortality intensities via stochastic processes. We consider two different

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portfolios of insured lives, where the underlying mortality intensities are correlated, and study the combined financial and mortality risk inherent in a portfolio of general life insurance contracts. In order to hedge this risk, we allow for investments in survivor swap and derive risk-minimizing strategies in markets where such contracts are available. The strategies are evaluated numerically.

JARNER, S. F., KRYGER, E. M. & DENGSOE, C. *The evolution of death rates and life expectancy in Denmark*. 147-173. From 1835 to date Denmark has experienced an increase in life expectancy at birth of about 40 years for both sexes. Over the course of the last 170 years, life expectancy at birth has increased from 40 to 80 years for women and from 36 to 76 years for men, and it continues to rise. Using a new methodology, we show that about half of the total historic increase can be attributed to the sharp decline in infant and young age death rates up to 1950. However, life expectancy gains fro 1950 to date can be primarily attributed to improvements in the age-specific death rates for the age group from 50 to 80, although there is also a noticeable contribution from the further decline in infant mortality over this period. With age-specific death rates up to age 60 now at a very low absolute level, substantial future life expectancy improvements must necessarily arise from improvements in age-specific death rates for ages 60 and above. Using the developed methodology, we quantify the impact of further reductions in age-specific mortality. Despite being one of countries with the highest life expectancy at the beginning of the 20th century, and despite the spectacular historic increase in life expectancy since then, Denmark is, in fact, lagging behind compared to many other countries, notably the other Nordic countries. The main reason is alarming excess mortality for cause-specific death rates related to ischaemic heart diseases and in particular a number of cancer diseases. Age-specific death rates continue to improve in most countries, and a likely scenario is that in the future Denmark will experience improvement rates at the international level or perhaps even higher as a result of a catch-up effect.

MAKINEN, M. *Reference mortality K2004 of personal life insurance policies in Finland.* 174-183. This article presents the reference mortality model K2004 approved by the Actuarial Society of Finland and the technique that was implemented in developing it. Initially, I will present the historical development of individual mortality rates in Finland. Then, the requirements posed for a modern mortality modelling will be presented. Reference mortality model K2004 is based on total population mortality rates, which were adjusted to correspond with that portion of the population that has a life insurance policy. First, the model presents a margin of the observed life insurance mortality rate in the total population with a Lee–Carter method together with a forecast, where the downward trend in mortality rates is expected to continue at the rate illustrated since the 1960s. Then, the mortality rate has been adjusted into life insurance mortality per age so that it corresponds to the differences observed between total population and the portion of population that has a life insurance during 1991-2001. Finally, a cohort and gender-specific functional margin will be presented to obtained data.

SAMUELSSON, E. *Mortality among Swedish insured.* 184-199. In this paper, the mortality among the Swedish voluntarily insured is described. It is based on calculations of the mortality among the Swedish insured from 2001-2005, and in the total Swedish population. The total population data has been used to compute the mortality trend with the Lee–Carter model.

SCANDINAVIAN ACTUARIAL JOURNAL

Volume **4,** 2008

LAU, J. W. & SIU, T. K. *Modelling long-term investment returns via Bayesian infinite mixture time series models.* 243-282. This paper introduces the class of Bayesian infinite mixture time

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series models first proposed in Lau & So (2004) for modelling long-term investment returns. It is a flexible class of time series models and provides a flexible way to incorporate full information contained in all autoregressive components with various orders by utilizing the idea of Bayesian averaging or mixing. We adopt a Bayesian sampling scheme based on a weighted Chinese restaurant process for generating partitions of investment returns to estimate the Bayesian infinite mixture time series models. Instead of using the point estimates, as in the classical or non-Bayesian approach, the estimation in this paper is performed by the full Bayesian approach, utilizing the idea of Bayesian averaging to incorporate all information contained in the posterior distributions of the random parameters. This provides a natural way to incorporate model risk or uncertainty. The proposed models can also be used to perform clustering of investment returns and detect outliers of returns. We employ the monthly data from the Toronto Stock Exchange 300 (TSE 300) indices to illustrate the implementation of our models and compare the simulated results from the estimated models with the empirical characteristics of the TSE 300 data. We apply the Bayesian predictive distribution of the logarithmic returns obtained by the Bayesian averaging or mixing to evaluate the quantile-based and conditional tail expectation risk measures for segregated fund contracts via stochastic simulation. We compare the risk measures evaluated from our models with those from some well-known and important models in the literature, and highlight some features that can be obtained from our models.

Ohlsson, E. *Combining generalized linear models and credibility models in practice.* 301-314. Rating of non-life insurance contracts commonly employs multiplicative models, which are estimated by generalized linear models (GLMs); another useful tool for rate making are credibility models. The object of this paper is to demonstrate how these can be combined in practice, to solve the problem with multi-level factors — rating factors with too many levels for GLM estimation. In particular, we consider car model classification in motor insurance, using data from a Swedish insurance company.

Taylor, G. *Second-order Bayesian revision of a generalised linear model.* 202-242. It is well known that the exponential dispersion family (EDF) of univariate distributions is closed under Bayesian revision in the presence of natural conjugate priors. However, this is not the case for the general multivariate EDF. This paper derives a second-order approximation to the posterior likelihood of a naturally conjugated generalised linear model (GLM), i.e., multivariate EDF subject to a link function (Section 5.5). It is not the same as a normal approximation. It does, however, lead to second-order Bayes estimators of parameters of the posterior. The family of second-order approximations is found to be closed under Bayesian revision. This generates a recursion for repeated Bayesian revision of the GLM with the acquisition of additional data. The recursion simplifies greatly for a canonical link. The resulting structure is easily extended to a filter for estimation of the parameters of a dynamic generalised linear model (DGLM) (Section 6.2). The Kalman filter emerges as a special case. A second type of link function, related to the canonical link, and with similar properties, is identified. This is called here the companion canonical link. For a given GLM with canonical link, the companion to that link generates a companion GLM (Section 4). The recursive form of the Bayesian revision of this GLM is also obtained (Section 5.5.3). There is a perfect parallel between the development of the GLM recursion and its companion. A dictionary for translation between the two is given so that one is readily derived from the other (Table 5.1). The companion canonical link also generates a companion DGLM. A filter for this is obtained (Section 6.3). Section 1.2 provides an indication of how the theory developed here might be applied to loss reserving. A sequel paper, providing numerical illustrations of this, is planned.

Wüthrich, M. V., Merz, M. & Buhlmann, H. *Bounds on the estimation error in the chain ladder method.* 283-300. Buchwalder *et al.* (2006) have illustrated that there are different

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approaches for the derivation of an estimate for the parameter estimation error in the distribution-free chain ladder reserving method. In this paper, we demonstrate that these approaches provide estimates that are close to each other for typical parameters. This is carried out by proving upper and lower bounds.

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SOUTH AFRICAN ACTUARIAL JOURNAL

Volume **8**, 2008

CLOVER, R. J. *Taxation of life insurance in South Africa revisited.* 1-34. This paper describes the theoretical framework underlying the four-funds basis for taxing life insurance in South Africa and records the historical steps in the development of the basis from inception to its current form. It goes on to evaluate the basis in its current form and to propose possible changes that should be considered.

DA SILVA, R. & WAYBURNE, L. *The effects of HIV/AIDS on medical schemes in South Africa.* 35-91. With the high level of the HIV epidemic in South Africa, medical schemes continue to be at risk. Risk-management strategies need to take into account that the disease is not notifiable and that there is legislated open enrolment and community rating of contributions, as well as prescribed minimum benefits for HIV/AIDS. As a result, many schemes have introduced HIV disease management and awareness programmes that are aimed at improving the health of HIV-positive beneficiaries and preventing new infections. This paper provides an analysis of current developments in the medical-scheme industry with respect to HIV/AIDS. For a sample of the medical-scheme membership, HIV prevalence estimates are presented with the associated cost effects. This sample is however not necessarily representative of the medical-scheme population. The intention is therefore to develop a methodology for producing indicative results to inform management decisions. The opportunity to lower costs related to HIV/AIDS through proactive risk management is investigated.

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