J. Plasma Physics (2004), *vol.* 70, *part* 1, *pp.* 99–111. © 2004 Cambridge University Press
DOI: 10.1017/S0022377803002411 Printed in the United Kingdom

Self-consistent model of a dipole-like magnetosphere with an azimuthal solar wind flow

ANATOLY S. LEONOVICH[1,3], VITALY A. MAZUR[1,3] and JIN BIN CAO[2,3]

[1]Institute of Solar-Terrestrial Physics (ISTP), Russian Academy of Science, Siberian Branch, Irkutsk 33, P.O. Box 4026, 664 033, Russia

[2]Center for Space Science and Applied Research, Chinese Academy of Science, P.O. Box 8701, Beijing, China

[3]Russian–Chinese Joint Research Center on Space Weather

(Received 30 September 2002)

Abstract. We have constructed an analytical model for an axisymmetric magnetosphere with azimuthal plasma flow. A dipole-like model of the geomagnetic field was used. It is shown that within the framework of this model of the magnetosphere, plasma at each magnetic shell moves with a constant angular velocity. The possibility of formation of a twin plasmapause is demonstrated. Abrupt changes in plasma density are directed across magnetic shells separating regions with strongly differing plasma velocities, as well as on the cylindrical surface whose radius coincides with the equatorial radius of the plasmapause.

1. Introduction

The initiation of modern theoretical research on MHD oscillations of the magnetosphere is customarily associated with a paper of Dungey (1954). He demonstrated the possibility of existence of Alfven oscillations in the magnetosphere with a dipole magnetic field. Subsequently the longitudinal (along magnetic field lines) structure and spectrum of Alfven eigen-oscillations in such a model of the magnetosphere was investigated in detail (Radoski 1967; Radoski and Carovillano 1969). In inhomogeneous plasma Alfven waves interact with other branches of MHD oscillations: fast and slow magnetosonic waves. However, because of existing mathematical difficulties, it took a relatively long time until an adequate theoretical picture of such an interaction in the dipole model of the magnetosphere was constructed.

For that reason, early theoretical work where this investigation was treated analytically were based on using one-dimensionally inhomogeneous models of the magnetosphere with straight geomagnetic field lines (Southwood 1974; Chen and Hasegawa 1974). The theory of field-line resonance (FLR), developed in the cited references, describes the resonance excitation of the Alfven wave by a monochromatic fast magnetosonic wave. The source for magnetosonic waves can be provided by an instability of oscillations of the magnetospheric boundary, the magnetopause, with solar wind flow around it. The theory of such an instability, known as the Kelvin–Helmholtz instability (KHI), was constructed by McKenzie (1970) using

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

a plane stratified model of the medium with a rectilinear magnetic field. Similar models of the medium have been used to date in investigating the KHI at the magnetopause (Mills et al. 1999).

Later there appeared publications in which the FLR phenomenon was investigated in models of the magnetosphere with a dipole magnetic field (Leonovich and Mazur 1989; Chen and Cowley 1989). However, they and almost all subsequent work addressed only the field structure of the resonance Alfven wave excited at the resonance magnetic shell by monochromatic magnetosonic oscillations. The structure of the magnetosonic oscillations remained uncertain. On the other hand, a number of publications appeared, in which numerical methods were used to model the picture of a total field of MHD oscillations in the dipole magnetosphere under different boundary conditions on the magnetopause (Lee and Lysak 1989; 1991; 1994). Recently, there appeared papers in which a similar picture of the oscillations in a dipole-like magnetosphere was investigated analytically using the WKB approximation (Leonovich and Mazur 2000, 2001; Leonovich 2001).

However, in all the references cited above, it was assumed that plasma both inside the magnetosphere and in the solar wind region is at rest. When such a model is used, it is not possible to investigate the instability of the magnetospheric boundary which is, probably, one of the main sources of MHD oscillations of the magnetosphere. The central problem with such investigations is that currently there is no suitable self-consistent dipole (or dipole-like) model of the magnetosphere with traveling plasma. A relatively large number of publications have appeared to date, which calculate, usually numerically, the three-dimensional structure of the magnetosphere modified by the incoming solar wind stream (Chen and Wolf 1972; Tsyganenko 1987; Ogino et al. 1994). However, because of the cumbersome nature of these models and the inconsistency of the magnetic field geometry with the dipole geometry used in modern theoretical work, they are not yet used in analytical (and numerical) calculations of MHD oscillations.

The objective of this paper is to construct a self-consistent model of the magnetosphere with a dipole-like magnetic field in the presence of solar wind plasma flow around it. Furthermore, the main requirements of such a model include a maximum possible simplicity such that it could be used in analytical (and numerical) calculations, as well as the unconditional self-consistency of model characteristics from the mathematical standpoint. The latter requirement signifies that the calculated model parameters must satisfy a complete system of equations of ideal magnetohydrodynamics. A choice of the free parameters must ensure a maximum possible approximation of the calculated characteristics of the medium to those observed in the real magnetosphere.

Of course, such a model cannot totally adequately describe all properties of the real magnetosphere. However, it is quite sufficient for the problem for which it is designed: the problem of modeling the instability of magnetospheric boundary oscillations with a dipole-like geometry of the magnetic field. It seems plausible that the resulting plasma-distribution patterns in the magnetospheric cavity can be extended—with some correction—to the real magnetosphere.

2. Basic equations and coordinate system

A closed system of equations of ideal MHD describing the plasma dynamics in a magnetic field has the form:

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

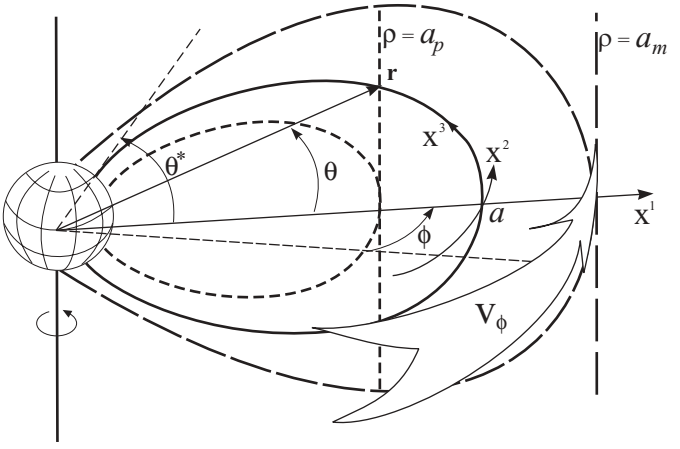


Figure 1. Model of the axisymmetric magnetosphere and coordinate systems: (x^1, x^2, x^3)—curvilinear orthogonal coordinate system tied with magnetic field lines, (a, φ, θ)—coordinate system as used in calculations. Curvilinear dashed lines correspond to the plasmapause ($\rho = a_\mathrm{p}$) and magnetopause ($\rho = a_\mathrm{m}$).

$$\rho_0 \frac{d\mathbf{v}_0}{dt} = -\nabla P_0 + \frac{1}{4\pi}[\mathrm{curl}\mathbf{B}_0 \times \mathbf{B}_0], \tag{2.1a}$$

$$\frac{\partial \mathbf{B}_0}{\partial t} = \mathrm{curl}[\mathbf{v}_0 \times \mathbf{B}_0], \tag{2.1b}$$

$$\frac{\partial \rho_0}{\partial t} + \mathrm{div}(\rho_0 \mathbf{v}_0) = 0, \tag{2.1c}$$

$$\frac{d}{dt}\frac{P_0}{\rho_0^\gamma} = 0, \tag{2.1d}$$

where $\mathbf{B}_0$ is the vector of magnetic field strength, $\mathbf{v}_0$ is the velocity vector, ρ_0 is plasma density, P_0 is plasma pressure, and γ is the adiabatic index. For determining the plasma temperature, this system of equations should be complemented by the state equation $P_0(\rho_0, T)$. In the simplest case it is possible to make use of the state equation of an ideal gas $P_0 = n_0 k T_0$, where n_0 is plasma concentration and k is the Boltzmann constant. In (2.1a) and (2.1d), d/dt is a total Lagrangian derivative taking into account both a local change of plasma characteristics and a change associated with the plasma transport with the velocity $\mathbf{v}_0$.

We now choose a model of the geomagnetic field, an equation of field lines of which in the meridional plane has the form

$$r = a \cos^\nu \theta, \tag{2.2}$$

where r is the radius of a point on the field line in the spherical coordinate system (see Fig. 1), a is the equatorial radius of the field line, θ is the latitude measured from the equatorial surface, and the field strength is given by the expression

$$\mathbf{B}_0(a, \theta) = \bar{B}\left(\frac{\bar{a}}{a}\right)^{\mu+1} \frac{\sqrt{1 + (\nu^2 - 1)\sin^2\theta}}{(\cos\theta)^{2(\nu+1)}}. \tag{2.3}$$

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

If $\bar{a} = R_E$ (where R_E is the Earth's radius), then $\bar{B} = 0.32\,\mathrm{G}$ is the equatorial magnetic field strength on the ground. When $\nu = \mu = 2$, (2.2) and (2.3) describe a dipole magnetic field. When $\nu > 0$, (2.2) and (2.3) make it possible to model a sufficiently broad class of magnetic fields with a dipole-like configuration of field lines.

The MHD equations have the simplest form in the coordinate system tied with magnetic field lines. In this paper we are using an orthogonal curvilinear coordinate system (x^1, x^2, x^3) in which the coordinate x^1 numbers the magnetic shells, x^2 specifies the field line on the selected shell, and x^3 specifies the point of a given field line (see Fig. 1). The technique of a constructive description of such a coordinate system was described by Klimushkin et al. (1995). As was pointed out by Salat and Tataronis (2000), this can only be done in the absence of magnetic field shear. For the sake of simplicity, in this paper it will be assumed that the azimuthal component of the magnetic field is absent ($B_2 = 0$).

The coordinates x^1, x^2, x^3 can also be represented by arbitrary functions uniquely related to each of them separately. For performing calculations, it is convenient to use, as the coordinate x^1, the equatorial radius of a field line a, as the coordinate x^2 the azimuthal angle φ, and as the longitudinal coordinate x^3 the latitude θ (or the field-line length ℓ measured from the equatorial surface). Note that the coordinates (a, φ, θ) are, of course, not orthogonal. In this case the metric tensor components that determine the length elements along the coordinate lines have the form:

$$g_1 = \frac{(\cos\theta)^{2(\nu+1)}}{1 + (\nu^2 - 1)\sin^2\theta}, \quad g_2 = a^2(\cos\theta)^{2(\nu+1)}. \tag{2.4}$$

The third metric tensor component can be determined through the ratio of segments of two field lines with the equatorial radii a and a_0 enclosed between two infinitely near-lying coordinate surfaces x^3 and $x^3 + dx^3$. If the field line with the equatorial radius a is intersected by the coordinate surface $x^3 = const$ at the latitude θ and the field line with the equatorial radius a_0 is intersected at the latitude θ_0, then an expression for g_3 has the form

$$\frac{g_3(a,\theta)}{g_3(a_0,\theta_0)} = \left(\frac{a}{a_0}\right)^{2(\mu+1)} \left(\frac{\cos\theta}{\cos\theta_0}\right)^{2\nu(\nu+1)} \frac{1 + (\nu^2 - 1)\sin^2\theta_0}{1 + (\nu^2 - 1)\sin^2\theta}. \tag{2.5}$$

It is easy to check that the equation $\mathrm{div}\mathbf{B}_0 = 0$, which in the selected coordinate system reduces to

$$\frac{\partial}{\partial x^3}\frac{\sqrt{g}}{g_3}B_3 = \frac{\partial}{\partial x^3}\sqrt{g_1 g_2}B_0 = 0,$$

holds for the magnetic field of (2.2), (2.3). When $\nu = \mu$ the condition $\mathrm{curl}\mathbf{B}_0 = 0$ is also satisfied, which implies the absence of currents in the plasma.

In the next sections we construct a stationary ($\partial/\partial t = 0$) solution to the system (1) to define the spatial distribution of plasma characteristics.

3. Plasma velocity ($\mathbf{v_0}$)

For the sake of simplicity, it will be assumed that in the model under consideration the plasma moves only in the azimuthal direction $\mathbf{v}_0 = (0, v_\varphi = v_2/\sqrt{g_2}, 0)$ (see Fig. 1). At equilibrium the centrifugal force is compensated by the plasma pressure and magnetic field gradient. The azimuthal symmetry not only of the magnetic field but also of the plasma will also be assumed. Furthermore, (2.1c) and (2.1d) hold

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

identically since each term of them is zero. The first and third components of the vector equation (2.1b) are also identically equal to zero, and the second component gives

$$\frac{B_3}{g_3}\nabla_3\frac{v_2}{g_2}=0. \tag{3.1}$$

In any axisymmetric coordinate system in which the azimuthal angle φ is used as the coordinate x^2, the g_2 component of the metric tensor may be represented as $g_2=\rho^2$, where ρ is the radius measured from the symmetry axis. Taking into consideration that $\Omega=v_\varphi/\rho=v_2/g_2$ is the angular velocity of the plasma and $\mathbf{B}_0=(0,0,B_3/\sqrt{g_3})$, (3.1) may be rewritten as

$$\mathbf{B_0}\nabla\mathbf{\Omega}=\mathbf{0}. \tag{3.2}$$

This equation implies that in the model under consideration the angular velocity of the plasma is constant on each magnetic shell and depends only on the coordinate x^1: $\Omega\equiv\Omega(x^1)$. Thus azimuthal rotation of the plasma on each magnetic shell has a constant angular velocity and does not alter the geometry of the magnetic field line.

This means in particular that the *arbitrary boundaries separating regions with sharply differing plasma velocities (such as the plasmapause and magnetopause) must coincide with one of the magnetic shells*, as shown in Fig. 1.

Subsequent calculations will use the following model of $\Omega(x^1)$:

$$\Omega(a)=\frac{d(a)}{2}\left[(\Omega_\mathrm{m}+\Omega_\mathrm{sw})-(\Omega_\mathrm{m}-\Omega_\mathrm{sw})\tanh\frac{a-a_\mathrm{m}}{\Delta_\mathrm{mp}}\right], \tag{3.3}$$

where Ω_m and Ω_sw are the typical values of the angular velocity of plasma in the neighborhood of the magnetopause in the magnetosphere and in the solar wind, respectively, a_m is the equatorial radius of the magnetopause, and Δ_mp is the characteristic thickness of the plasmasheath. The coefficient $d(a)$ makes it possible to model the character of plasma motion inside the magnetosphere and in the solar wind.

4. Plasma density (ρ_0) and Alfven velocity (A)

With the selected models of the magnetic field and plasma velocity, the second component of the vector equation (2.1a) goes to zero identically, and the other two equations may be written as

$$\frac{\rho_0\Omega^2}{2}\nabla_1 g_2=\nabla_1 P_0+\frac{1}{g_3}\nabla_1\frac{B_3^2}{8\pi}, \tag{4.1}$$

$$\frac{\rho_0\Omega^2}{2}\nabla_3 g_2=\nabla_3 P_0. \tag{4.2}$$

It is evident that the equilibrium of rotating plasma along a field line is sustained by the gas kinetic pressure gradient, and across magnetic shells by a total magnetic and gas kinetic pressure gradient.

This suggests an important conclusion. We now pass to a cylindrical coordinate system (ρ,φ,z) in which the metric tensor components $g_1=g_3=1$, $g_2=\rho^2$. If the magnetic field is a force-free one, then from (4.2) it follows that plasma pressure depends on the radius $P_0\equiv P_0(\rho)$ only. From (4.1) it follows in this case that the

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

product $\rho_0\Omega^2$ is also only a function of the radius ρ, i.e. its value is constant on cylindrical surfaces $\rho = const$.

In order to solve the system of equations (4.1), (4.2) in a general form, we differentiate (4.1) and (4.2) with respect to x^3 and x^1, respectively, and subtract one from the other. As a result, we obtain

$$\nabla_3(\rho_0\Omega^2) - \frac{\nabla_3 g_2}{\nabla_1 g_2}\nabla_1(\rho_0\Omega^2) = \frac{\nabla_3 g_3^{-1}\nabla_1 B_3^2/8\pi}{\nabla_1 g_2}. \tag{4.3}$$

We seek the solution for this equation by the method of characteristics. The characteristics of the operator appearing on the left-hand side of (4.3) are determined by the equation

$$\frac{dx^1}{dx^3} = -\frac{\nabla_3 g_2}{\nabla_1 g_2},$$

which reduces to a full differential $dg_2 = 0$, i.e. the metric tensor component g_2 does not vary along the characteristic. In an axisymmetric coordinate system we have $g_2 = \rho^2$ and, hence, the characteristics in the plane $x^2 = const$ represent straight lines parallel to the symmetry axis. Two of them in Fig. 1 are shown by vertical dashed lines.

If we introduce the coordinate s that varied along the characteristic, then by parameterizing the coordinates x^1 and x^3:

$$\frac{dx^1}{ds} = 1, \quad \frac{dx^3}{ds} = -\frac{\nabla_3 g_2}{\nabla_1 g_2},$$

we bring (4.3) to the form

$$2\frac{\partial \rho_0\Omega^2}{\partial s} = \frac{1}{4\pi}\left(\frac{\partial g_2}{\partial \rho}\right)^{-1}\frac{\partial}{\partial s}\frac{1}{g_3}\frac{\partial B_3^2}{\partial \rho}, \tag{4.4}$$

where ρ, the cylindrical radius, is used as the coordinate numbering the characteristics, and the origin $s = 0$ is taken on the equatorial surface.

Since $\partial g_2/\partial\rho = 2\rho$ does not depend on s, (4.4) is readily integrated and, returning again to the dipole coordinates (a, φ, θ), we obtain

$$\rho_0(a,\theta) = \rho_0(\rho,0)\frac{\Omega^2(\rho)}{\Omega^2(a)} + \frac{\nu-\mu}{8\pi\rho\Omega^2(a)}\left[\frac{B_0^2(a,\theta)}{a} - \frac{B_0^2(\rho,0)}{\rho}\right]. \tag{4.5}$$

Here the explicit form of the functions g_2 and B_3 is used. As would be expected, in a force-free field ($\nu = \mu$) the product $\rho_0(a,\theta)\Omega^2(a) = \rho_0(\rho,0)\Omega^2(\rho)$ does not depend on θ, i.e. is conserved along the characteristic $\rho = const$.

One of the most important parameters defining the character of plasma oscillations is the value of the Alfven velocity $A = B_0/\sqrt{4\pi\rho_0}$. Using for B_0 the expression (2.3) and for ρ_0 (4.5), we obtain a general expression for a two-dimensional (in the plane of the magnetic meridian) distribution

$$A(a,\theta) = \left[f(a)(\cos\theta)^{\nu+1} + \left(\frac{\Omega^2(\rho)}{\Omega^2(a)}\frac{1}{A^2(\rho,0)} - f(a)\right)\beta(\theta)\right]^{-1/2}, \tag{4.6}$$

where

$$f(a) = \frac{(\nu-\mu)}{2\rho^2\Omega^2(a)}, \quad \beta(\theta) = \frac{(\cos\theta)^{2(\nu+1)(1-\mu)}}{1+(\nu^2-1)\sin^2\theta}.$$

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

In a force-free magnetic field ($\nu = \mu$) the expression for A is simplified

$$A(a,\theta) = A(\rho,0)\frac{\Omega(a)}{\Omega(\rho)\sqrt{\beta(\theta)}}. \tag{4.7}$$

5. Plasma pressure (P_0)

Equilibrium plasma pressure with a given distribution of the magnetic field $B_0(a,\theta)$ and density $\rho_0(a,\theta)$ is defined by the pair of equations (4.1) and (4.2). Integrating (4.2) along a field line we obtain

$$\begin{aligned} P_0(a,\theta) &= P_0(a,0) + \frac{\Omega^2(a)}{2}\int_0^\theta \rho_0(a,\theta')\frac{\partial g_2(a,\theta')}{\partial\theta'}\,d\theta' \\ &= P_0(a,0) - (\nu+1)a^2\Omega^2(a)\int_0^\theta \rho_0(a,\theta')\sin\theta'(\cos\theta')^{2\nu+1}\,d\theta', \end{aligned} \tag{5.1}$$

where $P_0(a,0)$ is the boundary value on the equatorial plane, for determining which we integrate (4.1), assuming that $\theta = 0$:

$$P_0(a,0) = P_0(\bar{a},0) + \int_{\bar{a}}^a \rho_0(a',0)\Omega^2(a')a'\,da' - \frac{\nu-\mu}{4\pi}\int_{\bar{a}}^a \frac{B_0^2(a',0)}{a'}\,da', \tag{5.2}$$

where $\bar{a}$ is the equatorial radius of the boundary magnetic shell, which can be taken to be equal to the radius of the ionosphere.

6. Results of numerical calculations and discussion

It will be assumed that the plasma velocity decreases rapidly inward in the magnetosphere, and in the solar wind it remains virtually constant. In this case, for the function $d(a)$ used in (3.3) we can choose the following model

$$d(a) = \begin{cases} \exp\left(-\alpha(a)(a-a_\mathrm{m})^2/\Delta_m^2\right), & a \le a_\mathrm{m} \\ a_\mathrm{m}/a, & a > a_\mathrm{m} \end{cases}$$

where Δ_m is the characteristic scale of plasma decrease in the magnetosphere, and $\alpha(a)$ is a coefficient of order unity. We use the following numerical values of the problem parameters: $a_\mathrm{m} = 10\,R_\mathrm{E}$ and $\Delta_\mathrm{mp} = 0.5\,R_\mathrm{E}$, where R_E is the Earth's radius, $\Omega_\mathrm{m} = v_{\varphi\mathrm{m}}/a_\mathrm{m}$, $\Omega_\mathrm{sw} = v_{\varphi\mathrm{sw}}/a_\mathrm{m}$, where $v_{\varphi\mathrm{m}} = 50\,\mathrm{km\,s^{-1}}$, and $v_{\varphi\mathrm{sw}} = 400\,\mathrm{km\,s^{-1}}$ are the characteristic azimuthal velocities of plasma in the neighborhood of the magnetopause in the magnetosphere and in the solar wind, and $\Delta_\mathrm{m} = 10\,R_\mathrm{E}$.

Curves 1 and 2 in Fig. 2 show the equatorial distribution of the plasma velocity for two different distributions of $\Omega(a)$ specified by the function $\alpha(a)$. Curve 1 corresponds to the value of $\alpha = 1$ inside the entire magnetosphere, and curve 2 is obtained for $\alpha(a) = (3 - \tanh(a-a_\mathrm{p})/\Delta_\mathrm{p})/4$, where $a_\mathrm{p} = 4\,R_\mathrm{E}$ is the equatorial radius, and $\Delta_\mathrm{p} = 0.5\,R_\mathrm{E}$ is the characteristic thickness of the plasmapause. Using such a function $\alpha(a)$ makes it possible to model a sharp change of v_φ on the plasmapause. As will be evident from subsequent calculations, such a distribution of v_φ will lead to an abrupt jump of plasma density at the passage through the plasmapause.

We now consider what will be brought about by a sharp change of the equatorial density $\rho_0(a,0)$ and the velocity $\Omega(a)$ on the plasmapause and magnetopause. Note

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

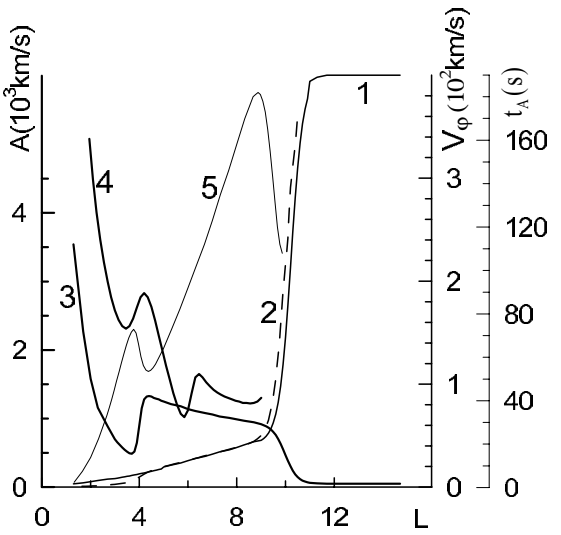


Figure 2. Distribution across magnetic shells of the azimuthal plasma velocity v_φ (curves 1 and 2), the Alfven velocity $A(a,\theta)$ (curves 2—in the equatorial plane $\theta=0$, curve 4—along the radius **r** at the angle $\theta=30^\circ$ to the equator), and of the fundamental harmonic period of Alfven eigen-oscillations of the magnetosphere t_A (curve 5). L—MacIlwain parameter.

that the model under consideration assumes a smooth variation in the magnetic field distribution, even at the passage through the magnetopause. We model the sharp changes in the Alfven velocity distribution at the passage through the plasmapause and magnetopause by the variation of the plasma density $\rho_0(a,\theta)$. Let us specify the equatorial distribution of the Alfven velocity $A(a,0)$, in accordance with the model of Carpenter and Anderson (1992), such as shown in Fig. 2 (curve 3). The analytical model of such a distribution was described in detail by Leonovich and Mazur (2000).

Figure 3 plots the distributions $A(a,\theta)$ along field lines obtained from (4.6) and (4.7), for different magnetic shells. For field lines on magnetic shells $L=2$ (curve 1) and $L=6$ (curve 3), the distributions $A(a,\theta)$ are presented for three different values of the parameters $\nu=\mu=1.75;2;2.5$ determining the geometry of a force-free magnetic field. As follows from (4.7), the value of $A(a,\theta)$ at the ionosphere, with a given boundary value of $A(a,0)$ at the equator, is determined by the distribution $\Omega(a)$. In calculations of the plots presented in Fig. 3, the values of the parameter Δ_m in the function $d(a)$ (when $\alpha(a)=1$) were chosen such that on all field lines inside the magnetosphere the values of A at the ionosphere are approximately identical.

The distributions A, shown in Fig. 3, have the following feature. The variation of A along field lines lying under the plasmapause in the plasmasphere proceeds monotonically, and along field lines outside the plasmasphere it shows a sharp jump. The origin of this jump is readily understood from the expression (4.7). The jump is caused by a sharp change of $A(\rho,0)$ when the field line crosses the characteristic $\rho=a_p$ tangent to the plasmapause at the equator. Thus in the movement along a field line with the equatorial radius $a>a_p$, as soon as the traveling point crosses the cylindrical surface $\rho=a_p$, there occurs a sharp change of $A(a,\theta)$.

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

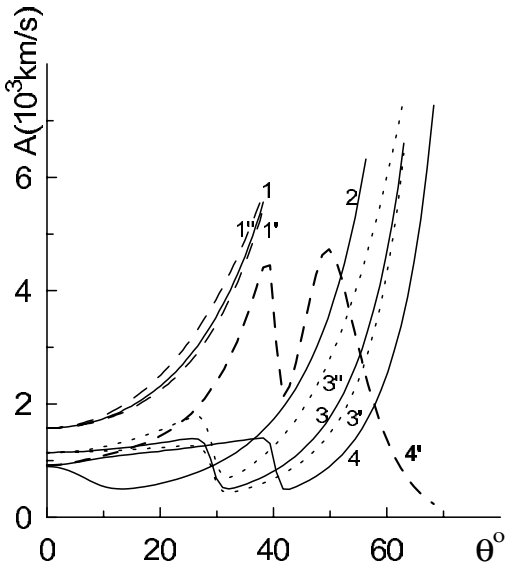


Figure 3. Distribution along field lines of the value of the Alfven velocity $A(a,\theta)$ at different magnetic shells: 1 $L=2$, 2 $L=4$, 3 $L=6$, 4 $L=9$. Solid lines show the dependences $A(a,\theta)$ corresponding to a dipole magnetic field ($\nu=\mu=2$ in (2.3)). Dashed lines $1'$ and $3'$ correspond to a force-free field $\nu=\mu=1.75$, $1''$ and $3''$ correspond to a force-free field $\nu=\mu=2.5$, and dashed line $4'$ corresponds to a force field $\nu=2, \mu=\nu+10^{-6}$.

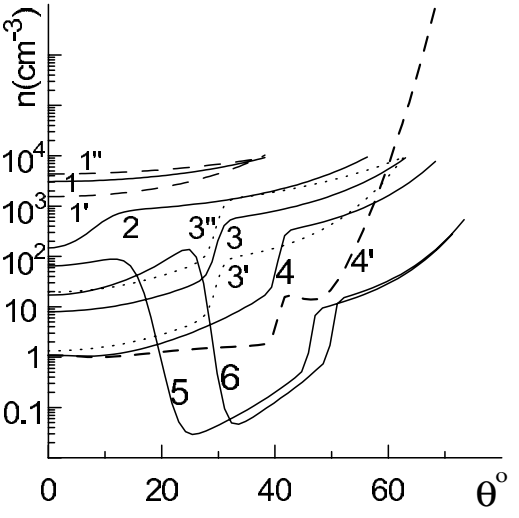


Figure 4. Distribution along field lines of the plasma density $n_0(a,\theta)$ at magnetic shells: 1 $L=2$, 2 $L=4$, 3 $L=6$, 4 $L=9$, 5 $L=12$, 6 $L=15$. The designations used here are the same as in Fig. 3.

Figure 4 presents the corresponding distributions of plasma density along a field line. It is evident that the crossing of this surface is accompanied by a sharp change in concentration. However, in accordance with generally accepted criteria, such a surface should also be defined as the plasmapause. This surface does not

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

coincide with any one of the magnetic shells and does not have any projection onto the ionosphere. Let us define such a surface as the plasmapause of the second type.

It is obvious that a sharp change of $A(a,\theta)$ and $\rho_0(a,\theta)$ will also occur for the crossing of the magnetic shell on which $\Omega(a)$ changes dramatically. Curve 2 in Fig. 2 describes just such a dependence of $\Omega(a)$ in the region $a=a_p$. Let us designate this magnetic shell as the plasmapause of the first type. This surface is mapped along field lines into the ionosphere, into the region of the main ionospheric trough.

Hence, in the model of the magnetosphere under consideration *the co-existence of the plasmapause of two different types is possible*. If a satellite orbiting in the magnetosphere crosses both of these surfaces, it must detect a double plasmapause. Curve 4 in Fig. 2 represents the distribution of the Alfven velocity which can be measured by the satellite traveling along the radius $\mathbf{r}$ (see Fig. 1) at the angle $\theta=30^\circ$ to the equatorial plane. A double plasmapause is seen. The characteristic 'knee' on the magnetic shell $L=6$ represents the plasmapause of the second type.

While the presence of the plasmapause of the first type associated with a sharp change of $\Omega(a)$ does not arouse any special questions, the formation of the plasmapause of the second type is not quite an obvious phenomenon at present. Its unusual shape, a cylindrical surface of radius $\rho=a_p$, is of course associated with quite a definite choice of the plasma motion in the model of the medium under consideration, the azimuthal rotation. Furthermore, plasma equilibrium is ensured by the equality of the centrifugal force and radial pressure gradient. In the real magnetosphere plasma motion has a still more complicated character. For that reason, the shape of the plasmapause of the second type can differ greatly from that calculated in this paper. In any case, the presence of such a plasmapause and its possible shape need further investigation using satellites.

One further peculiarity of the Alfven velocity and plasma density distribution along the field line at the magnetic shell $L=9$ is represented in Figs. 2 and 3 by curves 4 and 4′. Curve 4 is calculated for a dipole magnetic field ($\nu=\mu=2$). Curve 4′ is calculated for the magnetic field whose field lines are similar to dipole field lines ($\nu=2$), but with a slight difference of the index $\mu=\nu+10^{-6}$, which makes the field a force field (i.e. having a non-zero magnetic pressure gradient). It is evident that near the equatorial surface both distributions are nearly identical. However, in the case of the motion along a field line the distributions differ greatly as one approaches the ionosphere. In a force-free magnetic field the distributions $A(a,\theta)$ and $n_0(a,\theta)$ are quite adequate to describe those actually observed in the magnetosphere. In a force field, as one approaches the ionosphere, they differ from actual values by several orders of magnitude. Of course, this does not imply that in the real magnetosphere the magnetic field can be only a force-free one. In near-equatorial regions of the outer magnetosphere the geomagnetic field is most likely to be a force field. However, as one approaches the ionosphere, it becomes a force-free field, which ensures the actually observed values of plasma density. Unfortunately, in terms of a simple model of the magnetic field (2.2), (2.3) used in this paper it is not possible to take into account such a change of the character of field behavior along field lines.

Curves 5 and 6 in Fig. 4 correspond to the plasma-density distribution outside the magnetosphere on the magnetic shells $L=12$ and $L=15$. A sharp change of $n_0(a,\theta)$ at $\theta=50^\circ$ and $\theta=20$–30° is caused by the field-line crossing of the 'plasmapause

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

of the second type' and of the analogous cylindrical surface associated with a sharp change of $\rho_0(a,0)$ at the magnetopause. However, for regions outside the magnetosphere, using the magnetic field model of (2.2), (2.3) is not justified because the magnetic field geometry in the solar wind is entirely dissimilar to that existing in the magnetosphere. The fact that this model of the magnetic field outside the magnetosphere is employed in this study was dictated solely by considerations of conservation of the simplicity of the self-consistent model of the medium as well as by the specific character of its further employment. This model is intended for theoretical investigations of the MHD oscillations of the magnetosphere. The main objective within the framework of this model must be to investigate the MHD instability of the magnetospheric boundary in the case of solar wind flow around it. As shown by Leonovich and Mazur (2000), most of these oscillations, magnetosonic waves, are localized near the equator. The model of the medium that is addressed in this paper models sufficiently well the Alfven velocity distribution $A(a,\theta)$ in the near-equatorial region, both in the magnetosphere and in the solar wind. Curve 3 in Fig. 2 represents the equatorial distribution $A(a,0)$, and curve 5 represents the value of the main period of Alfven eigen-oscillations of the magnetosphere

$$t_A = \oint \frac{d\ell}{A(a,\theta)},$$

where $d\ell = a(\cos\theta)^{\nu-1}\sqrt{1+(\nu^2-1)\sin^2\theta}\,d\theta$ is the length element along a field line, and the integral along the closed contour implies integration along the field line between magnetoconjugate ionospheres 'back and forth'.

Figure 5 presents the distribution of level lines of the parameter $\beta = 8\pi P_0(a,\theta)/B_0^2(a,\theta)$ (in the meridional plane), the gas kinetic to magnetic plasma pressure ratio. For evaluating the integrals of (14), (15), as ultimate pressure we chose to use the pressure $P_0(\bar{a},0) = 0.2\,\text{nPa}$ at the ionosphere (at $h = 1500\,\text{km}$ over the ground), which, with the plasma density $n_0 = 10^4\,\text{cm}^{-3}$, corresponds to the temperature $T_0 = 1500\,\text{K}$. It is customary to use as the magnetospheric boundary such a surface on which total kinetic and gas kinetic solar wind plasma pressure compares with geomagnetic field pressure ($\beta = 1$). It is evident that in the magnetospheric model under consideration this surface differs greatly from the magnetopause that was defined above as the boundary separating regions of rapidly moving solar wind plasma and slow convection of magnetospheric plasma. These surfaces coincide sufficiently closely in the near-equatorial region only.

7. Main results of this study

The main results obtained in this study may be summarized as follows.

1. We have constructed a self-consistent analytical model for the axisymmetric magnetosphere with solar wind flow around it. The solar wind was modeled by a plasma flow rotating in the azimuthal direction.
2. It has been shown that within the framework of this model plasma rotation at each magnetic shell proceeds with a constant angular velocity.
3. We have obtained two-dimensional (in the plane of the magnetic meridian) density, pressure, and Alfven velocity distribution of the plasma.

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

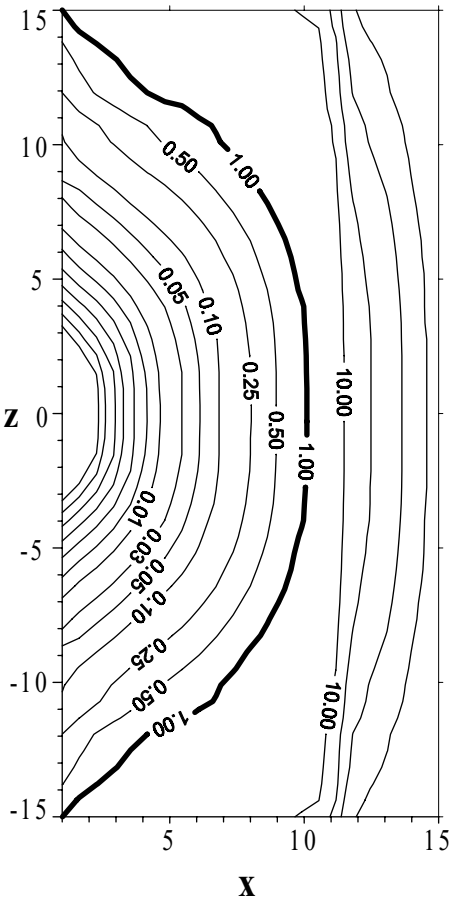


Figure 5. Map of level lines of the parameter $\beta = 8\pi P_0(a,\theta)/B_0^2(a,\theta)$ in the meridional plane. The coordinates $x = r\cos^{\nu+1}\theta$ and $z = r\cos^{\nu}\theta\sin\theta$ are expressed in units of the Earth's radius.

4. It has been shown that in the magnetospheric model under consideration the existence of the plasmapause of two different types is possible. The plasmapause of the first type coincides with the magnetic shell, across which a sharp change of the plasma velocity occurs. In the magnetospheric model under consideration the plasmapause of the second type represents a cylindrical surface, the radius of the generatrix of which coincides with the equatorial radius of the plasmapause of the first type. When a satellite moves at an angle to the equatorial surface, a double (or of larger multiplicity) plasmapause must be observed.

Acknowledgements

We are grateful to V. G. Mikhalkovsky for his assistance in preparing the English version of the manuscript.

This work is partially supported by the grants of Russian Foundation for Basic Research (RFBR No. 02-05-39020) and National Scientific Foundation of China (NSFC No. 40025413).

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press

References

Carpenter, D. L. and Anderson, R. R. 1992 An ISEE/whistler model of equatorial electron density in the magnetosphere. *J. Geophys. Res.* **97**, 1097–1108.

Chen, L. and Cowley, S. C. 1989 On field line resonances of hydromagnetic Alfven waves in a dipole magnetic field. *Geophys. Res. Lett.* **16**, 895–897.

Chen, L. and Hasegawa, A. 1974 A theory of long period magnetic pulsation. 1. Steady state excitation of field line resonances. *J. Geophys. Res.* **79**, 1024–1032.

Chen, A. J. and Wolf, R. A. 1972 Effect on the plasmasphere of time-varying convection electric field. *Planet. Space Sci.* **20**, 483–509.

Dungey, J. W. 1954 Electrodynamics of the outer atmospheres. Ionospheric Science Report no. 69, Ionos. Res. Lab., Pennsylvania State.

Klimushkin, D. Yu., Leonovich, A. S. and Mazur, V. A. 1995 On the propagation of transversally small scale standing Alfven waves in a three dimensionally inhomogeneous magnetosphere. *J. Geophys. Res.* **100**, 9527–9536.

Lee, D.-H. and Lysak, R. L. 1989 Monochromatic ULF wave coupling in the dipole model: the impulsive excitation. *J. Geophys. Res.* **94**, 17 097–17 109.

Lee, D.-H. and Lysak, R. L. 1991 Monochromatic ULF wave excitation coupling in the dipole magnetosphere. *J. Geophys. Res.* **96**, 5811–5823.

Lee, D.-H. and Lysak, R. L. 1994 Numerical studies on ULF wave structures in the dipole model. In *Solar Wind Sources of Magnetospheric Ultra-Low-Frequency Waves* (*Geophys. Monogr. Ser. 81*) (ed. M. J. Engebretson et al.). Washington: AGU, pp. 293–297.

Leonovich, A. S. 2001 A theory of field line resonance in a dipole-like axisymmetric magnetosphere. *J. Geophys. Res.* **106**, 25 803–25 812.

Leonovich, A. S. and Mazur, V. A. 1989 Resonance excitation of standing Alfven waves in an axisymmetric magnetosphere (monochromatic oscillations). *Planet. Space Sci.* **37**, 1095–1108.

Leonovich, A. S. and Mazur, V. A. 2000 Structure of magnetosonic eigenoscillations of an axisymmetric magnetosphere. *J. Geophys. Res.* **105**, 27 707–27 716.

Leonovich, A. S. and Mazur, V. A. 2001 On the spectrum of magnetosonic eigenoscillations of an axisymmetric magnetosphere. *J. Geophys. Res.* **106**, 3919–3928.

McKenzie, J. F. 1970 Hydromagnetic wave interaction with the magnetopause and the bow shock. *Planet. Space Sci.* **18**, 1–23.

Mills, K. J., Wright, A. N. and Mann, I. R. 1999 Kelvin – Helmholtze driven modes of the magnetosphere. *Physics of Plasmas* **6**, 4070–4087.

Ogino, T. R., Walker, R. J. and Ashour-Abdalla, M. 1994 A global magnetohydrodynamic simulation of the response of the magnetosphere to a northward turning of the interplanetary magnetic field. *J. Geophys. Res.* **99**, 11 027–11 042.

Pilipenko, V., Fedorov, E., Mazur, N., Engebretson, M. J. and Hughes, W. J. 1999 Magnetohydrodynamic waveguide/resonator for Pc3 pulsations at cusp latitudes. *Earth Planet. Space* **51**, 441–448.

Radoski, H. R. 1967 Highly asymmetric MHD resonances. The guided poloidal mode. *J. Geophys. Res.* **72**, 4026–4033.

Radoski, H. R. and Carovillano, R. R. 1969 Axisymmetric plasmaspheric resonances. Toroidal mode. *Phys. Fluids* **9**, 285–291.

Salat, A. and Tataronis, J. A. 2000 Condition for existence of orthogonal coordinate systems oriented by magnetic field lines. *J. Geophys. Res.* **105**, 13 055–13 064.

Southwood, D. J. 1974 Some features of field line resonances in the magnetosphere. *Planet. Space Sci.* **22**, 483–492.

Tsyganenko, N. 1987 Global quantitative models of geomagnetic field in the cislunar magnetosphere for different disturbance levels. *Planet. Space Sci.* **35**, 1347–1358.

https://doi.org/10.1017/S0022377803002411 Published online by Cambridge University Press