Bull. Aust. Math. Soc. **113** (2026), 194–202
doi:10.1017/S0004972725100476

RAMSEY NUMBERS OF CYCLES VERSUS MULTIPLE ODD WHEELS

ZHAOFA WANG and YANBO ZHANG

(Received 30 May 2025; accepted 22 July 2025; first published online 1 September 2025)

Abstract

Given two graphs G and H, the Ramsey number $R(G, H)$ is the smallest positive integer N such that every graph of order N contains G or its complement contains H as a subgraph. Let C_n denote the cycle on n vertices and let tW_{2m+1} denote the disjoint union of t copies of the $(2m + 2)$-vertex wheel W_{2m+1}. We show that for integers $m \geq 1$, $t \geq 2$ and $n \geq (6m + 3)t - 6m + 999$,

$$R(C_n, tW_{2m+1}) = 3n + t - 3.$$

This result extends several previous results and settles a conjecture posed by Sudarsana ['A note on the Ramsey number for cycle with respect to multiple copies of wheels', *Electron. J. Graph Theory Appl.* **9**(2) (2021), 561–566].

2020 *Mathematics subject classification*: primary 05D10; secondary 05C55.

Keywords and phrases: Ramsey number, cycle, wheel.

1. Introduction

Ramsey theory is concerned with the emergence of order in large and complex structures. One of its central concepts is the Ramsey number, which captures the threshold at which a certain structure must appear.

Given two graphs G and H, the *Ramsey number* $R(G, H)$ is defined as the smallest positive integer N such that every graph Γ on N vertices either contains G as a subgraph or its complement $\overline{\Gamma}$ contains H as a subgraph. Equivalently, $R(G, H)$ is the smallest integer N such that in every red–blue edge-colouring of the complete graph K_N, we can always find either a red copy of G or a blue copy of H.

We use C_n to denote the cycle on n vertices, and W_n to denote the graph obtained by adding an extra vertex to C_n and connecting it to every vertex on the cycle. We often refer to such a wheel by the parity of the order of its outer cycle. That is, an even wheel is a wheel of odd order and an odd wheel is a wheel of even order. As cycles and wheels

The second author was partially supported by the National Natural Science Foundation of China (NSFC) under Grant No. 11601527 and by the Natural Science Foundation of Hebei Province under Grant No. A2023205045.

© The Author(s), 2025. Published by Cambridge University Press on behalf of Australian Mathematical Publishing Association Inc.

https://doi.org/10.1017/S0004972725100476 Published online by Cambridge University Press

are basic graph classes in graph theory, the Ramsey number $R(C_n, W_\ell)$ has attracted considerable attention since the last century. In 1983, Burr and Erdős [5] initiated the study of cycle–wheel Ramsey numbers and proved that $R(C_3, W_\ell) = 2\ell + 1$ for $\ell \geq 5$. However, when C_3 is replaced by C_4, the problem becomes significantly more complex. In 2014, Zhang *et al.* [19] showed that $R(C_4, W_\ell) = R(C_4, K_{1,\ell})$ for $\ell \geq 6$. The exact value of the latter is only known in certain special cases, and the general upper bound problem encompasses a \$100 conjecture of Erdős [6]. In addition to the aforementioned studies focused on small cycles (C_3 and C_4), substantial progress has been made on Ramsey numbers for more general cycle–wheel combinations. For Ramsey numbers involving odd cycles and large wheels, a series of results were obtained by Zhang *et al.* [21], Sanhueza-Matamala [13] and Alweiss [1]. For even cycles and odd wheels, when $n < \ell < 3n/2$, Zhang *et al.* [20] also determined the exact value of $R(C_n, W_\ell)$.

When the order of the cycle is large relative to that of the wheel, the parity of the wheel plays a crucial role in determining the Ramsey number. For clarity, we consider odd wheels W_{2m+1} and even wheels W_{2m} separately. In this context, Surahmat *et al.* [16, 17] proposed the following two conjectures.

CONJECTURE 1.1. If $n \geq 2m + 1 \geq 3$ and $(n, m) \neq (3, 1)$, then $R(C_n, W_{2m+1}) = 3n - 2$.

CONJECTURE 1.2. If $n \geq 2m \geq 4$ and $(n, m) \neq (4, 2)$, then $R(C_n, W_{2m}) = 2n - 1$.

Subsequent research has delved deeply into these conjectures in an effort to verify them.

Regarding the odd wheel W_{2m+1}, when does the equality $R(C_n, W_{2m+1}) = 3n - 2$ hold? In 2008, Surahmat *et al.* [17] proved that this formula holds when $m \geq 2$ and $n > 5m - 2$. In 2010, Shi [14] improved the bound to $n > 70$ or $n \geq 3m + 3$. Later that same year, Zhang *et al.* [18] further reduced the requirement to $n \geq 20$. Ultimately, in 2012, Chen *et al.* [8] completely resolved Conjecture 1.1.

As for the even wheel W_{2m}, when does the equality $R(C_n, W_{2m}) = 2n - 1$ hold? In 2006, Surahmat *et al.* [16] proved that the formula holds when $n \geq 5m - 1$. In 2009, Chen *et al.* [7] improved the bound to $n \geq 3m + 1$. Later, in 2015, Zhang *et al.* [20] further lowered the requirement to $n \geq 2m + 502$. Combining all known results, it follows that the statement $R(C_n, W_{2m}) = 2n - 1$ holds whenever $m \geq 2$ and $n \geq 2m + \min\{m + 1, 502\}$, thereby almost resolving Conjecture 1.2.

Extending these results, Sudarsana [15] studied Ramsey numbers of large cycles versus disjoint unions of multiple wheels. We use tW_ℓ to denote the disjoint union of t copies of the wheel W_ℓ. Sudarsana [15] proposed the following conjecture.

CONJECTURE 1.3. For $t \geq 1$ and $m \geq 2$, there exists an integer n_0 such that

$$R(C_n, tW_{2m}) = 2n + t - 2 \quad \text{and} \quad R(C_n, tW_{2m+1}) = 3n + t - 3 \quad \text{for all } n \geq n_0.$$

In our paper 'Ramsey numbers of cycles versus multiple wheels' (submitted), we proved that Sudarsana's conjecture holds in the case of even wheels.

THEOREM 1.4. *For $t \geq 1$ and $m \geq 2$, if $n \geq 4tm$, then $R(C_n, tW_{2m}) = 2n + t - 2$.*

https://doi.org/10.1017/S0004972725100476 Published online by Cambridge University Press

The main result of this paper confirms that Sudarsana's conjecture also holds for odd wheels.

THEOREM 1.5. *For $t \geq 2$ and $m \geq 1$, if $n \geq (6m+3)t - 6m + 999$, then*

$$R(C_n, tW_{2m+1}) = 3n + t - 3.$$

For $t = 1$, $n \geq 2m+1$ and $(n, m) \neq (3, 1)$, Chen *et al.* [8] had already established that $R(C_n, W_{2m+1}) = 3n - 2$. Therefore, Conjecture 1.3 proposed by Sudarsana is now fully confirmed.

The structure of this paper is as follows: we present six useful lemmas in Section 2 and give a complete proof of Theorem 1.5 in Section 3.

To complete this section, we introduce additional notation. We use $[n]$ to denote the set $\{1, 2, \ldots, n\}$. The symbols $|G|$, $\delta(G)$, $\Delta(G)$, $\omega(G)$ and $\chi(G)$ denote respectively the number of vertices, the minimum degree, the maximum degree, the number of connected components and the chromatic number of a graph G. The *circumference* of a graph G, denoted by $c(G)$, is the length of the longest cycle in G, while the *girth* of G, denoted by $g(G)$, is the length of the shortest cycle in G. The *connectivity* $\kappa(G)$ of a graph G refers to the minimum number of vertices whose removal results in a disconnected graph. The *closure* $\mathrm{cl}(G)$ of a graph $G = (V, E)$ is the graph obtained by recursively adding edges between nonadjacent vertex pairs whose degree sum is at least $|V|$, until no such pair remains. This process results in a unique graph, regardless of the order in which the edges are added. The *chromatic surplus* of G refers to the number of vertices in the smallest colour class among all proper colourings of G using $\chi(G)$ colours. Given a subset $U \subseteq V(G)$, we write $G[U]$ for the subgraph of G induced by U. The graph obtained by removing the vertex set U and all incident edges from G is denoted by $G - U$; if $U = \{u\}$, we simply write $G - u$. For disjoint subsets $U_1, U_2 \subseteq V(G)$, the notation $G[U_1, U_2]$ denotes the subgraph induced by all edges with one endpoint in U_1 and the other in U_2. The disjoint union of graphs G and H is denoted by $G \cup H$. For a subgraph H and a vertex u in H, let $d_H(u)$ denote the number of neighbours of u in H. Notation and terminology not explicitly defined follow those in the textbook by Bondy and Murty [3].

2. Useful lemmas

LEMMA 2.1 (Dirac [9]). *Let G be a 2-connected graph of order $n \geq 3$ with minimum degree $\delta = \delta(G)$. Then, $c(G) \geq \min\{2\delta, n\}$.*

LEMMA 2.2 (Rosta [12], Faudree and Schelp [10]). *We have $R(C_n, C_{2m+1}) = 2n - 1$ for $n \geq 2m + 1 \geq 3$ and $(n, m) \neq (3, 1)$.*

LEMMA 2.3 (Burr and Erdős [5], Greenwood and Gleason [11]). *We have*

$$R(C_3, W_{2m+1}) = \begin{cases} 9 & \text{if } m = 1, \\ 4m + 3 & \text{if } m \geq 2. \end{cases}$$

https://doi.org/10.1017/S0004972725100476 Published online by Cambridge University Press

LEMMA 2.4 (Bondy and Chvátal [2]). *A graph is Hamiltonian if and only if its closure is Hamiltonian.*

LEMMA 2.5 (Brandt *et al.* [4]). *Let G be a nonbipartite graph of order n with $\delta(G) \geq (n+2)/3$. Then, G is weakly pancyclic with girth 3 or 4.*

LEMMA 2.6 (Brandt *et al.* [4]). *Let G be a 2-connected nonbipartite graph of order n with $\delta(G) \geq n/4 + 250$. Then, G is weakly pancyclic, unless its odd girth is 7. In that case, G contains a cycle of every length from 4 up to its circumference, except possibly a cycle of length 5.*

3. Proof of Theorem 1.5

For the lower bound, consider the graph $K_{n-1} \cup K_{n-1} \cup K_{n-1} \cup K_{t-1}$. Clearly, it does not contain C_n as a subgraph. Its complement is the complete 4-partite graph $K_{n-1,n-1,n-1,t-1}$, which has chromatic surplus $t-1$. However, the graph tW_{2m+1} has chromatic number 4 and chromatic surplus t, and therefore cannot be contained as a subgraph in $K_{n-1,n-1,n-1,t-1}$. It follows that

$$R(C_n, tW_{2m+1}) \geq 3n + t - 3.$$

Let $t \geq 2$, $m \geq 1$ and $n \geq (6m+3)t - 6m + 999$. Let G be a graph of order $3n + t - 3$. To establish the upper bound, assume for contradiction that G does not contain C_n as a subgraph and that its complement $\overline{G}$ does not contain tW_{2m+1} as a subgraph. We shall derive a contradiction.

First, select as many vertex-disjoint copies of W_{2m+1} in the graph $\overline{G}$ as possible and suppose that a total of x such copies are selected. By the assumption, we have $x \leq t-1$. Next, remove all the vertices of these x disjoint copies of W_{2m+1} from G and denote the resulting graph by G'. By the maximality of x, the complement $\overline{G'}$ contains no copy of W_{2m+1}.

We now proceed with the following sequence of claims.

CLAIM 1. *We have $|G'| \geq 3n - (2m+1)t + 2m - 1$. Consequently, $|G'| \geq 8n/3 + 332$ and $|G'| \geq 16m + 3012$.*

PROOF. All three lower bounds follow from straightforward calculations:

$$\begin{aligned} |G'| &= |G| - x(2m+2) \\ &\geq |G| - (t-1)(2m+2) \\ &= (3n + t - 3) - (t-1)(2m+2) \\ &= 3n - (2m+1)t + 2m - 1. \end{aligned}$$

Since $t \geq 2$ and $n \geq (6m+3)t - 6m + 999$, it follows that

$$3|G'| \geq 9n - (6m+3)t + 6m - 3 \geq 8n + 996.$$

Therefore, $|G'| \geq 8n/3 + 332$. Substituting $n \geq (6m+3)t - 6m + 999 \geq 6m + 1005$ into the inequality above yields $|G'| \geq 16m + 3012$. □

https://doi.org/10.1017/S0004972725100476 Published online by Cambridge University Press

CLAIM 2. *We have* $\delta(G') \geq |G'| - 2n + 1$, $\delta(G') \geq |G'|/4 + 250$ *and* $\delta(G') \geq n/2 + 1$.

PROOF. We first prove that $\delta(G') \geq |G'| - 2n + 1$. Suppose otherwise, that is, $\delta(G') \leq |G'| - 2n$. Then, $\Delta(\overline{G'}) = |G'| - 1 - \delta(G') \geq 2n - 1$. Let u_0 be a vertex of minimum degree in G' and denote by G_0 the subgraph induced by the set of nonneighbours of u_0 in G'. Then, $|G_0| \geq 2n - 1$. By Lemma 2.2, either G_0 contains a copy of C_n as a subgraph or $\overline{G_0}$ contains a copy of C_{2m+1} as a subgraph, which together with u_0 would form a copy of W_{2m+1} in $\overline{G'}$. Both cases lead to a contradiction. Hence, $\delta(G') \geq |G'| - 2n + 1$.

To prove $\delta(G') \geq |G'|/4 + 250$, it suffices to show that $|G'| - 2n + 1 \geq |G'|/4 + 250$, which is equivalent to proving $3|G'| \geq 8n + 996$. Therefore, by Claim 1, we conclude that $\delta(G') \geq |G'|/4 + 250$. Since $\delta(G') \geq |G'| - 2n + 1$ and $|G'| \geq 5n/2$, it follows that $\delta(G') \geq n/2 + 1$. □

CLAIM 3. G' *is nonbipartite.*

PROOF. Suppose, in contrast, that G' is bipartite. Then, its independence number satisfies $\alpha(G') \geq |G'|/2$. It follows from Claim 1 that $\alpha(G') \geq 2m + 2$. Hence, $\overline{G'}$ contains K_{2m+2} as a subgraph and thus also contains W_{2m+1} as a subgraph, which contradicts our assumption. Therefore, G' must be nonbipartite. □

CLAIM 4. $\kappa(G') \leq 1$.

PROOF. Since $|G'| \geq 4m + 5$, by Lemma 2.3, either G' contains C_3 as a subgraph or $\overline{G'}$ contains W_{2m+1} as a subgraph. The latter contradicts our assumption. Thus, $g(G') = 3$.

Suppose $\kappa(G') \geq 2$. Then, by Lemma 2.1 and Claims 1 and 2, we have $c(G') \geq \min\{2\delta(G'), |G'|\} \geq n$.

Moreover, if $\kappa(G') \geq 2$, then by Lemma 2.6 and Claims 2 and 3, the graph G' is weakly pancyclic. Hence, G' contains C_n as a subgraph, which is again a contradiction. Therefore, we conclude that $\kappa(G') \leq 1$. □

CLAIM 5. *There exist two vertices* u_1 *and* u_2 *in* G' *such that* $G' - \{u_1, u_2\}$ *has at least three connected components.*

PROOF. By Claim 2, $\delta(G') \geq |G'|/4 + 250 \geq 250$. Hence, every connected component of G' contains at least 251 vertices. From Claim 4, we have $\kappa(G') \leq 1$. If $\kappa(G') = 0$, then for any vertex u_1 in G', the graph $G' - u_1$ has at least two connected components. If $\kappa(G') = 1$, let u_1 be a cut vertex of G'. Again, $G' - u_1$ has at least two connected components.

If $G' - u_1$ has at least three connected components, then for any vertex u_2 in $G' - u_1$, the graph $G' - \{u_1, u_2\}$ also has at least three connected components.

If $G' - u_1$ has exactly two connected components and one of them is not 2-connected, let u_2 be a cut vertex of that component. Then, $G' - \{u_1, u_2\}$ has at least three connected components.

We now consider the case where $G' - u_1$ has exactly two connected components and each component is 2-connected. Without loss of generality, let G_1 and G_2 be the

https://doi.org/10.1017/S0004972725100476 Published online by Cambridge University Press

subgraphs induced by these two components, with $|G_1| \geq |G_2|$. It then follows that $|G_1| \geq n$ and $\kappa(G_1) \geq 2$.

Since $\kappa(G_1) \geq 2$, by Lemma 2.1 and Claims 1 and 2, the circumference of G_1 satisfies

$$c(G_1) \geq \min\{2\delta(G_1), |G_1|\} \geq \min\{2(\delta(G') - 1), |G_1|\} \geq n.$$

If G_1 is bipartite, then $\alpha(G_1) \geq (|G'| - 1)/4 \geq 2m + 2$. Thus, $\overline{G_1}$ contains K_{2m+2} as a subgraph, which implies that $\overline{G_1}$ contains W_{2m+1} as a subgraph, which is a contradiction. Hence, G_1 is not bipartite.

Since $|G_2| \geq \delta(G' - u_1) + 1 \geq \delta(G')$, we have

$$|G_1| = |G'| - 1 - |G_2| \leq |G'| - \delta(G') - 1.$$

From Claim 2, it follows that $\delta(G') - 1 \geq (|G'| - \delta(G') + 1)/3$ and, thus, $\delta(G_1) \geq \delta(G') - 1 \geq (|G_1| + 2)/3$. By Lemma 2.5, G_1 is weakly pancyclic and has girth 3 or 4.

Combining the weak pancyclicity of G_1 with $c(G_1) \geq n$, we conclude that G_1 contains C_n as a subgraph, leading to a contradiction. This completes the proof. □

CLAIM 6. *There exist two vertices u_1 and u_2 in G' such that $G' - \{u_1, u_2\}$ has exactly three connected components.*

PROOF. We proceed by contradiction. Suppose not. Then, by Claim 5, there exist two vertices u_1 and u_2 in G' such that $G' - \{u_1, u_2\}$ has at least four connected components. Each connected component has order at least

$$\delta(G' - \{u_1, u_2\}) + 1 \geq \delta(G') - 1 \geq m.$$

Therefore, the complement of $G' - \{u_1, u_2\}$ contains the complete 4-partite graph $K_{m,m,m,m}$ as a subgraph and, thus, contains W_{2m+1} as a subgraph. This contradicts the assumption that $\overline{G'}$ does not contain W_{2m+1} as a subgraph. □

Let U_1, U_2 and U_3 denote the three connected components of $G' - \{u_1, u_2\}$. We now state the next claim.

CLAIM 7. *For each $j \in [3]$, the subgraph $G[U_j]$ is a complete graph and*

$$n/2 + m \leq |U_j| \leq n - 1.$$

PROOF. For each $j \in [3]$, we have

$$|U_j| \geq \delta(G[U_j]) + 1 \geq \delta(G') - 1 \geq |G'| - 2n \geq n - (2m + 1)t + 2m - 1 \geq n/2 + m.$$

The last three inequalities follow from Claims 2, 1 and the lower bound on n.

Suppose, in contrast, that $G[U_1]$ is not a complete graph. Then, there exist vertices $u, v \in U_1$ such that $uv \in E(\overline{G}[U_1])$. Let $U_2' \subseteq U_2$ and $U_3' \subseteq U_3$ be arbitrary subsets of size m. Then, the induced subgraph $\overline{G}[\{u, v\} \cup U_2' \cup U_3']$ contains a copy of W_{2m+1} centred at either u or v, which yields a contradiction. Therefore, $G[U_1]$ must be a complete graph. By the same argument, $G[U_2]$ and $G[U_3]$ are also complete graphs.

https://doi.org/10.1017/S0004972725100476 Published online by Cambridge University Press

Finally, we show that $|U_j| \le n-1$. Suppose otherwise that $|U_j| \ge n$. Since $G[U_j]$ is a complete graph, it must contain a cycle C_n as a subgraph, which contradicts our assumption. Hence, $|U_j| \le n-1$. □

CLAIM 8. *For each $i \in [2]$, there exists some $j \in [3]$ such that the vertex u_i has at least $|U_j| - m + 1$ neighbours in U_j.*

PROOF. Suppose the claim is false. The vertex u_i has at least m nonneighbours in each of U_1, U_2 and U_3. Select m such nonneighbours from each of U_1, U_2 and U_3. Together with u_i, these vertices induce a copy of the complete 4-partite graph $K_{1,m,m,m}$ in $\overline{G'}$, and it contains W_{2m+1} as a subgraph. This contradiction completes the proof. □

Recall that the graph $\overline{G} - V(G')$ contains x vertex-disjoint copies of W_{2m+1}. For each such copy of W_{2m+1}, we have the following claim.

CLAIM 9. *In each W_{2m+1}, except for at most one vertex, every other vertex has at least $|U_j| - 2m + 1$ neighbours in some U_j with $j \in [3]$.*

PROOF. We proceed by contradiction. Suppose that in some W_{2m+1}, say F, there exist two vertices v_1 and v_2 such that for every $j \in [3]$, each of them has at most $|U_1| - 2m$ neighbours in U_j.

Under this assumption, we can choose m nonneighbours of v_1 in U_1 (denoted by X_1) and m nonneighbours of v_2 in U_1 (denoted by Y_1) such that $X_1 \cap Y_1 = \emptyset$. Similarly, choose m nonneighbours of v_1 in U_2 (denoted by X_2) and m nonneighbours of v_2 in U_2 (denoted by Y_2) with $X_2 \cap Y_2 = \emptyset$. Likewise, choose m nonneighbours of v_1 in U_3 (denoted by X_3) and m nonneighbours of v_2 in U_3 (denoted by Y_3) with $X_3 \cap Y_3 = \emptyset$.

Since U_1, U_2 and U_3 are the three connected components of $G' - \{v_1, v_2\}$, the subgraph $\overline{G}[U_1, U_2, U_3]$ forms a complete tripartite graph. Consequently, in $\overline{G}$, the induced subgraph $\overline{G}[\{v_1\} \cup X_1 \cup X_2 \cup X_3]$ contains a copy of W_{2m+1} centred at v_1, and $\overline{G}[\{v_2\} \cup Y_1 \cup Y_2 \cup Y_3]$ contains a copy of W_{2m+1} centred at v_2.

By removing F from the original x vertex-disjoint copies of W_{2m+1} and adding the two newly constructed ones, we obtain $x+1$ vertex-disjoint copies of W_{2m+1} in $\overline{G}$, which contradicts the maximality of x. This completes the proof of the claim. □

Next, we classify the vertices outside $\bigcup_{j=1}^{3} U_j$ as follows: if a vertex has at least $|U_j| - 2m + 1$ neighbours in U_j, then we assign it to V_j for $j \in [3]$. According to Claims 8 and 9, there are at most x vertices not contained in $\bigcup_{j=1}^{3}(U_j \cup V_j)$. In other words, the number of vertices in $\bigcup_{j=1}^{3}(U_j \cup V_j)$ is at least $3n + t - 3 - x \ge 3n - 2$. By the pigeonhole principle, there exists some $j \in [3]$ such that $|U_j \cup V_j| \ge n$. Without loss of generality, assume that $|U_1 \cup V_1| \ge n$. We then select $n - |U_1|$ vertices from V_1 and still denote this subset by V_1.

Let $H := G[U_1 \cup V_1]$ be the induced subgraph and denote its closure by $\mathrm{cl}(H)$. We now show that $\mathrm{cl}(H)$ is the complete graph K_n. By Claims 7, 8 and 9, for every pair of nonadjacent vertices $\{u, v\}$ in H, where $u \in U_1$ and $v \in V_1$,

$$d_H(u) + d_H(v) \ge (|U_1| - 1) + (|U_1| - 2m + 1) = 2|U_1| - 2m \ge n.$$

https://doi.org/10.1017/S0004972725100476 Published online by Cambridge University Press

Hence, when constructing $\mathrm{cl}(H)$, we add edges uv to H such that each vertex in V_1 eventually becomes adjacent to all vertices in U_1. Therefore, every vertex in V_1 has degree at least $|U_1|$ in $\mathrm{cl}(H)$. Furthermore, for every pair of nonadjacent vertices $\{u', v'\}$ in V_1, we have $d_{\mathrm{cl}(H)}(u') + d_{\mathrm{cl}(H)}(v') \geq 2|U_1| \geq n$, so we continue adding the edge $u'v'$. Following this procedure, the resulting graph $\mathrm{cl}(H)$ becomes a complete graph of order n.

Since complete graphs are Hamiltonian, $\mathrm{cl}(H)$ is Hamiltonian. By Lemma 2.4, H is also Hamiltonian and thus contains a C_n as a subgraph, which is a final contradiction.

References

[1] R. Alweiss, 'Ramsey numbers of odd cycles versus larger even wheels', *Discrete Math.* **341**(4) (2018), 981–989.

[2] J. A. Bondy and V. Chvátal, 'A method in graph theory', *Discrete Math.* **15**(2) (1976), 111–135.

[3] J. A. Bondy and U. S. R. Murty, *Graph Theory*, Graduate Texts in Mathematics, 244 (Springer, London, 2008).

[4] S. Brandt, R. J. Faudree and W. Goddard, 'Weakly pancyclic graphs', *J. Graph Theory* **27**(3) (1998), 141–176.

[5] S. A. Burr and P. Erdős, 'Generalizations of a Ramsey-theoretic result of Chvátal', *J. Graph Theory* **7**(1) (1983), 39–51.

[6] S. A. Burr, P. Erdős, R. J. Faudree, C. C. Rousseau and R. H. Schelp, 'Some complete bipartite graph-tree Ramsey numbers', in: *Graph Theory in Memory of G. A. Dirac*, Annals of Discrete Mathematics, 41 (eds. L. D. Andersen, I. T. Jakobsen, C. Thomassen, B. Toft and P. D. Vestegaard) (North-Holland, Amsterdam, 1989), 79–89.

[7] Y. Chen, T. C. E. Cheng, Z. Miao and C. T. Ng, 'The Ramsey numbers for cycles versus wheels of odd order', *Appl. Math. Lett.* **22**(12) (2009), 1875–1876.

[8] Y. Chen, T. C. E. Cheng, C. T. Ng and Y. Zhang, 'A theorem on cycle-wheel Ramsey number', *Discrete Math.* **312**(5) (2012), 1059–1061.

[9] G. A. Dirac, 'Some theorems on abstract graphs', *Proc. Lond. Math. Soc. (3)* **2** (1952), 69–81.

[10] R. J. Faudree and R. H. Schelp, 'All Ramsey numbers for cycles in graphs', *Discrete Math.* **8** (1974), 313–329.

[11] R. E. Greenwood and A. M. Gleason, 'Combinatorial relations and chromatic graphs', *Canad. J. Math.* **7** (1955), 1–7.

[12] V. Rosta, 'On a Ramsey type problem of J. A. Bondy and P. Erdős, I, II', *J. Combin. Theory Ser. B* **15** (1973), 94–120.

[13] N. Sanhueza-Matamala, 'Stability and Ramsey numbers for cycles and wheels', *Discrete Math.* **339**(5) (2016), 1557–1565.

[14] L. Shi, 'Ramsey numbers of long cycles versus books or wheels', *European J. Combin.* **31**(3) (2010), 828–838.

[15] I. W. Sudarsana, 'A note on the Ramsey number for cycle with respect to multiple copies of wheels', *Electron. J. Graph Theory Appl. (EJGTA)* **9**(2) (2021), 561–566.

[16] Surahmat, E. T. Baskoro and I. Tomescu, 'The Ramsey numbers of large cycles versus wheels', *Discrete Math.* **306**(24) (2006), 3334–3337.

[17] Surahmat, E. T. Baskoro and I. Tomescu, 'The Ramsey numbers of large cycles versus odd wheels', *Graphs Combin.* **24**(1) (2008), 53–58.

[18] L. Zhang, Y. Chen and T. C. E. Cheng, 'The Ramsey numbers for cycles versus wheels of even order,' *European J. Combin.* **31**(1) (2010), 254–259.

[19] Y. Zhang, H. Broersma and Y. Chen, 'A remark on star-C_4 and wheel-C_4 Ramsey numbers', *Electron. J. Graph Theory Appl. (EJGTA)* **2**(2) (2014), 110–114.

https://doi.org/10.1017/S0004972725100476 Published online by Cambridge University Press

[20] Y. Zhang, H. Broersma and Y. Chen, 'Three results on cycle-wheel Ramsey numbers', *Graphs Combin.* **31**(6) (2015), 2467–2479.
[21] Y. Zhang, Y. Zhang and Y. Chen, 'The Ramsey numbers of wheels versus odd cycles', *Discrete Math.* **323** (2014), 76–80.

ZHAOFA WANG, School of Mathematical Sciences,
Hebei Normal University, Shijiazhuang 050024, PR China
and
Hebei Research Center of the Basic Discipline Pure Mathematics,
Shijiazhuang 050024, PR China
e-mail: zfwang.edu@outlook.com

YANBO ZHANG, School of Mathematical Sciences,
Hebei Normal University, Shijiazhuang 050024, PR China
and
Hebei Research Center of the Basic Discipline Pure Mathematics,
Shijiazhuang 050024, PR China
e-mail: ybzhang@hebtu.edu.cn

https://doi.org/10.1017/S0004972725100476 Published online by Cambridge University Press