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Instability of anomalous viscosity disks around young stellar objects

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Abstract. An accretion disk with vertical structure around a young stellar object is investigated, considering anomalous magnetic viscosity. It was found that the I-mode was unstable in the outer disk, and sometimes turned stable in the inner disk, for the standard α model. However, we find that the I-mode is always unstable in the inner disk, owing to the anomalous magnetic viscosity.

1. Introduction

As a powerful and attractive theoretical model, accretion disks (ADs) are widely believed to be sources of high-energy radiation for X-ray binaries and active galactic nuclei (AGNs). Since the standard thin-disk model was constructed in the early 1970s (Shakura and Sunyaev 1973), the stability of the disk has become an important area in the theory of ADs. This is because many quasi-periodic variations in X-ray binaries and AGNs are believed to be related to the instabilities of the disk. Shortly after the publication of Shakura and Sunyaev's work, the inner region of the standard thin disk was found to be secularly and thermally unstable (Lightman and Eardley 1974; Shakura and Sunyaev 1976). The rapid growth of these unstable modes may result in the breakdown of the thin accretion disk configuration. Subsequently, much research was done concerning disk instabilities. Except the thermal mode and the viscous mode, there are two acoustic modes in ADs (Wallinder 1991; Wu and Yang 1994; Khoperskov and Khrapov 1995; Wu et al. 1995), the O-mode (propagating outward) and the I-mode (propagating inward). The O-mode is unstable throughout the disk, while the I-mode is unstable in the outer disk but sometimes stable in the inner disk (Wu and Yang 1994).

In the standard thin-disk model it is usually assumed that the disk is geometrically thin, $H(r) \ll r$ (H is the thickness of the disk and r is the radius). It involves averaging over the z-coordinate of the three-dimensional hydrodynamic equations, assuming a number of additional conditions to be fulfilled (Shakura and Sunyaev 1973; Khoperskov and Khrapov 1995). However, Khoperskov and Khrapov (1999) pointed out that it was necessary to consider the vertical structure for perturbation wavelengths $\lambda \leqslant h$ (h is the half-thickness of the disk).

ADs are hydrodynamic disks with anomalous turbulent viscosity. Unfortunately, the turbulent viscosity is a rather difficult problem and has disturbed astrophysicists for a long time. We know little about it. So we usually use the standard α model: $\nu = \alpha C_s H$ (Shakura and Sunyaev 1973) instead. In this paper, we use the

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anomalous magnetic viscosity found by Li and Zhang (2002) to investigate the acoustic perturbations of ADs with vertical structure around young stellar objects (YSOs). We find that the I-mode is always unstable in the inner disk. It is different from Wu and Yang's results mentioned above.

This paper is organized as follows. In Sec. 2 we simply introduce the anomalous magnetic viscosity in ADs. In Sec. 3 we define the ADs model and in Sec. 4 we consider the dynamics of linear acoustic perturbations. Finally, we present numerical calculations in Sec. 5 and give conclusions in Sec. 6.

2. Anomalous magnetic viscosity

To investigate the structure of the disk, we must know the anomalous viscosity $\eta_{\rm m}$. Recently, a solution was suggested by Li and Zhang (2002).

Here, we briefly introduce the calculation processes of the magnetic kinematic viscosity. The magnetic viscous stress tensor by the self-generated spatially intermittent flux is

$$t_{ij}^{\rm m} = \frac{\langle \delta B_i \delta B_j - \frac{1}{2}\delta_{ij}(\delta \mathbf{B})^2 \rangle}{4\pi}. \tag{1}$$

The work done on the volume $d\mathbf{r}$, per unit time, by the stress is $(-(\partial t_{ij}^{\rm m}/\partial x_j)v_i)d\mathbf{r}$, which contributes to viscous dissipation, resulting in the change of entropy within the volume

$$\dot{S} = \int \frac{1}{T}\left(-\frac{\partial t_{ij}^{\rm m}}{\partial x_j} v_i\right) d\mathbf{r} = \int t_{ij}^{\rm m} \frac{V_{ij}}{T}\, d\mathbf{r}, \tag{2}$$

where

$$V_{ij} \equiv \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right). \tag{3}$$

This corresponds to the linear relation of the 'flow' $t_{ij}^{\rm m}$ and the 'force' $-V_{ij}/T$ (Lifshitz and Pitaevskii 1981):

$$t_{ij}^{\rm m} = \gamma_{ij;lk}\frac{V_{lk}}{T} = \eta_{ij;lk}V_{lk}, \tag{4}$$

or

$$t_{ij}^{\rm m} = \eta_{\rm m}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial v_k}{\partial x_k}\right),$$

where

$$\eta_{ij;lk} = \eta_{\rm m}(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl} - \tfrac{2}{3}\delta_{ij}\delta_{lk}),$$

and $\eta_{\rm m}$ is the magnetic viscosity of the intermittent flux defined through the kinetic coefficient as $\eta_{\rm m} = \eta_{lk;lk}$ ($l \neq k$, no summation over repeated lk). In ADs, the shear stress tensor V_{lk} has dominant $r\varphi$-component and then (4) becomes

$$t_{ij}^{\rm m} = \eta_{ij;r\varphi}\frac{1}{2}\left(\frac{\partial v_\varphi}{\partial r} - \frac{v_\varphi}{r}\right) = \eta_{ij;r\varphi}\frac{1}{2}r\frac{\partial \Omega(r)}{\partial r}, \tag{5}$$

with $\Omega = v_\varphi/r$. Furthermore, from (1) and (5), we have that

$$\frac{|\langle \delta B_r \delta B_\varphi \rangle|}{4\pi} = \eta_{\rm m}\frac{1}{2}r\left|\frac{\partial \Omega(r)}{\partial r}\right|. \tag{6}$$

It is assumed that self-generated magnetic fields by the transverse modes are

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statistically isotropic on the scales of interest:

$$\tfrac{1}{3}\langle(\delta\mathbf{B})^2\rangle \approx \langle\delta B_r \delta B_\varphi\rangle. \tag{7}$$

In this case, (5) and (7), combined with a self-similar solution for the collapse magnetic flux (Li and Zhang 2002)

$$\frac{\delta\mathbf{B}^2_{\max}}{8\pi} = \frac{16}{9}\mu c^2 \rho \frac{(|\mathbf{E}_0|^2)^{4/3}}{\alpha^{2/3}},$$

give that the magnetic kinematic viscosity $\nu_{\rm m}$ can be written as (Li and Zhang 2002)

$$\nu_{\rm m} = 7\times 10^{-12}\frac{c^2}{r|\partial\Omega(r)/\partial r|}T_0^{2/3}\left(\frac{T_{\rm e}}{T_0}\right)^{2/3}\left(\frac{\bar{W}_0^P}{10^{-5}}\right)^{4/3}\ ({\rm cm^2\ s^{-1}}). \tag{8}$$

3. Model and basic equations

Now we consider an axisymmetric differentially rotating gas disk with vertical structure around an YSO in the gravitational field of a mass M. Without including self-gravity and relativistic effects, and adopting cylindrical coordinates, we have

$$\Psi_{(r,z)} = -\frac{GM}{(r^2+z^2)^{1/2}} \approx -\frac{GM}{r} + \frac{1}{2}\Omega_k^2 z^2, \tag{9}$$

where G is the gravitational constant and $\Omega_k = (GM/r^3)^{1/2}$ is the Keplerian angular velocity. If the disk is Keplerian, we can obtain

$$f_\varphi = -\frac{3}{2r^2}\frac{\partial}{\partial r}(r^2\rho\nu_{\rm m}\Omega_k), \tag{10}$$

where ρ is the volume density of matter in the disk. Now, taking the new anomalous viscosity into consideration, we have from (8) that

$$\nu_{\rm m} = \nu_{{\rm m}i}\left(\frac{r}{r_i}\right)^{3/2}\left(\frac{T}{T_i}\right)^{2/3}, \tag{11}$$

where the subscript 'i' signifies the values taken at the inner edge of the disk.

We use the axisymmetric hydrodynamic equations with cylindrical coordinates. The equations of motion and continuity and the thermal equation have the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{\partial\Psi}{\partial r}, \tag{12}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \frac{1}{\rho}f_\varphi, \tag{13}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{\partial\Psi}{\partial z}, \tag{14}$$

$$\frac{\partial\rho}{\partial t} + \frac{\partial(r\rho u)}{r\partial r} + \frac{\partial(\rho w)}{\partial z} = 0, \tag{15}$$

$$\frac{dS}{dt} = \frac{Q}{T}, \tag{16}$$

where P is the pressure, $\mathbf{V} = (u, v, w)$ is the velocity, S is the entropy, and Q is the rate of heat. When we investigate the equilibrium of the disk, it is assumed that the equilibrium velocity in the thin disk has only r and φ components: $\mathbf{V}_0 = (U_0, V_0, 0)$

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and usually we have $|U_0/V_0| \approx \alpha(h/r)^2$. The pressure and density have the form (Khoperskov and Khrapov 1999)

$$P_0(r,z) = P_0(r)F(z)^a, \quad \rho_0(r,z) = \rho_0(r)F(z)^b, \tag{17}$$

where

$$F(z) = 1 - \frac{z^2}{h^2}, \quad a = \frac{n}{n-1}, \quad b = \frac{1}{n-1},$$

and n is the polytropic index.

Observation has shown $T \propto r^{-1/2}$ in ADs around YSOs (Beckwith 1994). We can assume that $P_0(r) \propto r^{-3/2}$ and $\rho_0(r) \propto r^{-1}$, considering the ideal-gas equation of state. By use of (11) we can obtain from (10) that

$$f_\varphi = -\frac{3\rho}{2r}\Omega_k\nu_\mathrm{m}\left(\frac{5}{3} + \frac{r}{\rho}\frac{\partial\rho}{\partial r}\right). \tag{18}$$

With these conditions and assumptions the equilibrium equations are

$$\frac{V_0^2}{r} = \frac{1}{\rho_0}\frac{\partial P_0}{\partial r} + \frac{\partial\Psi}{\partial r}, \tag{19}$$

$$U_0\frac{\partial rV_0}{r\partial r} = -\frac{\Omega_k\nu_\mathrm{m}}{r}, \tag{20}$$

$$\frac{\partial P_0}{\partial z} = -\rho_0\frac{\partial\Psi}{\partial z}. \tag{21}$$

Using (17) and (21), the relation

$$C_\mathrm{s}^2(r,z) = \frac{\gamma P_0}{\rho_0} = \frac{\gamma P_0(r)}{\rho_0(r)}F(z) = \frac{\gamma\Omega_k^2h^2}{2a}F(z) \tag{22}$$

defines the adiabatic sound speed (γ is the adiabatic index). From now on, we choose $n = \gamma = 5/3$, so $a = 5/2, b = 3/2$. Then we obtain the equilibrium velocities

$$V_0(r,z) = r\Omega(r,z) = \sqrt{r^2\Omega_k^2 - \frac{3}{2}\frac{C_s^2}{\gamma}} \approx r\Omega_k, \tag{23}$$

$$U_0(r,z) = -2\frac{\Omega_k\nu_\mathrm{m}}{V_0} \approx -2\frac{\nu_\mathrm{m}}{r}. \tag{24}$$

4. Linear analysis

In the framework of the standard linear analysis, $y(u,v,w,P,\rho) = y_0 + \tilde{y}$. In the linear approximation ($|\tilde{y}| \ll |y_0|$) and after neglecting some higher-order perturbed terms, we obtain linear perturbed equations as follows:

$$\frac{\partial\tilde{u}}{\partial t} + U_0\frac{\partial\tilde{u}}{\partial r} + \tilde{u}\frac{\partial U_0}{\partial r} - \frac{2V_0}{r}\tilde{v} = -\frac{1}{\rho_0}\frac{\partial\tilde{P}}{\partial r} + \frac{\tilde{\rho}}{\rho_0^2}\frac{\partial P_0}{\partial r}, \tag{25}$$

$$\frac{\partial\tilde{v}}{\partial t} + U_0\frac{\partial(r\tilde{v})}{r\partial r} + \tilde{u}\frac{\partial(rV_0)}{r\partial r} = \frac{3}{2}\Omega_k\nu_\mathrm{m}\left(-\frac{1}{\rho_0}\frac{\partial\tilde{\rho}}{\partial r} + \frac{\tilde{\rho}}{\rho_0^2}\frac{\partial\rho_0}{\partial r}\right), \tag{26}$$

$$\frac{\partial\tilde{w}}{\partial t} + U_0\frac{\partial\tilde{w}}{\partial r} = -\frac{1}{\rho_0}\frac{\partial\tilde{P}}{\partial z} - g\frac{\tilde{\rho}}{\rho_0}, \tag{27}$$

$$\frac{\partial\tilde{\rho}}{\partial t} + \frac{\partial[r(U_0\tilde{\rho} + \rho_0\tilde{u})]}{r\partial r} + \frac{\partial(\rho_0\tilde{w})}{\partial z} = 0, \tag{28}$$

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where

$$g \equiv \frac{\partial \Psi}{\partial z} = -\frac{1}{\rho_0}\frac{\partial P_0}{\partial z}.$$

As we study only the dynamics of acoustic instability, we set $Q = 0$ (Khoperskov and Khrapov 1999). The thermal equation becomes

$$\frac{\partial \tilde{S}}{\partial t} + U_0\frac{\partial \tilde{S}}{\partial r} + \tilde{u}\frac{\partial S_0}{\partial r} + \tilde{w}\frac{\partial S_0}{\partial z} = 0, \tag{29}$$

where $S_0 = C_V \ln(P_0/\rho_0^{\gamma})$ is the entropy of the gas at equilibrium. On the other hand,

$$\tilde{S} = \tilde{S}(\tilde{P}, \tilde{\rho}) = C_V\frac{\tilde{P}}{P_0} - C_P\frac{\tilde{\rho}}{\rho_0}, \tag{30}$$

where C_V and C_P are the specific heats at constant density and pressure. Using $C_P/C_V = \gamma$, one obtains from (29) that

$$\frac{1}{P_0}\frac{\partial \tilde{P}}{\partial t} - \frac{\gamma}{\rho_0}\frac{\partial \tilde{\rho}}{\partial t} + \frac{U_0}{P_0}\left(\frac{\partial \tilde{P}}{\partial r} - \frac{\tilde{P}}{P_0}\frac{\partial P_0}{\partial r}\right) - \frac{\gamma U_0}{\rho_0}\left(\frac{\partial \tilde{\rho}}{\partial r} - \frac{\tilde{\rho}}{\rho_0}\frac{\partial \rho_0}{\partial r}\right) + \frac{\tilde{u}}{6r} = 0. \tag{31}$$

The short-wave approximation in the radial direction ($kr \gg 1$, k is the radial wavenumber) allows the solution to be written as

$$\tilde{y}(r, z, t) \propto \exp(ikr - i\omega t), \tag{32}$$

where ω is the perturbation frequency. Taking into account (32), the perturbed equations become

$$\left(\sigma - \frac{U'}{6}\right)\tilde{u} + 2\Omega_k\tilde{v} - \frac{ik}{\rho_0}\tilde{P} - \frac{3C_s^2}{2\gamma\rho_0 r}\tilde{\rho} = 0, \tag{33}$$

$$\frac{1}{2}\Omega_k\tilde{u} - (\sigma - U')\tilde{v} + \frac{3\Omega_k\nu_m}{2\rho_0 r}(1 + ikr)\tilde{\rho} = 0, \tag{34}$$

$$\sigma\tilde{w} - \frac{g}{\rho_0}\tilde{\rho} = 0, \tag{35}$$

$$ik\rho_0\tilde{u} + \frac{\partial \rho_0}{\partial z}\tilde{w} - \left(\sigma - \frac{7U'}{6}\right)\tilde{\rho} = 0, \tag{36}$$

$$\frac{1}{6r}\tilde{u} - \frac{\sigma - (3U'/2)}{\rho_0 C_s^2/\gamma}\tilde{P} + \frac{\sigma - U'}{\rho_0/\gamma}\tilde{\rho} = 0, \tag{37}$$

where $\sigma = i(\omega - kU_0)$ and $U' = U_0/r = -2\nu_m/r^2$.

From (33)–(37), we get the dispersion equation

$$\sigma^5 - \frac{23}{6}U'\sigma^4 + \left\{\left[\frac{181}{36}U'^2 + \Omega_k^2 + \frac{3\Omega_k^2 z^2}{F(z)h^2} + \frac{1}{3}\Omega_k^2 F(z)h^2k^2\right] - \left[\frac{k}{3r}\Omega_k^2 F(z)h^2\right]i\right\}\sigma^3$$
$$+ \left\{\left[-\frac{8}{3}U'\Omega_k^2 - 3\Omega_k^2\nu_m k^2 - \frac{2}{3}U'\Omega_k^2 F(z)h^2k^2 - \frac{8U'\Omega_k^2 z^2}{F(z)h^2} - \frac{179}{72}U'^3\right]\right.$$
$$\left. + \left[\frac{3k}{r}\Omega_k^2\nu_m - \frac{61k}{90r}U'\Omega_k^2 F(z)h^2\right]i\right\}\sigma^2$$

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$$+\left\{\left[\frac{3\Omega_k^4 z^2}{F(z)h^2}+\frac{9}{2}U'\Omega_k^2\nu_{\mathrm{m}}k^2+\frac{1}{3}U'^2\Omega_k^2F(z)h^2k^2+\frac{7}{24}U'^4+\frac{7}{4}U'^2\Omega_k^2\right.\right.$$
$$\left.\left.+\frac{23}{4}\frac{U'^2\Omega_k^2z^2}{F(z)h^2}\right]+\left[-\frac{k}{10r}\Omega_k^4z^2-\frac{2k}{15r}U'^2\Omega_k^2F(z)h^2-\frac{9k}{2r}U'\Omega_k^2\nu_{\mathrm{m}}\right]i\right\}\sigma$$
$$+\left\{\left[-\frac{3U'^3\Omega_k^2z^2}{F(z)h^2}-\frac{9}{2}\frac{U'\Omega_k^4z^2}{F(z)h^2}\right]+\left(\frac{k}{10r}U'\Omega_k^4z^2\right)i\right\}=0.$$

5. Numerical result

If we set $W=\omega/\Omega_k$ (W is the dimensionless frequency), $K=hk$ (K is the dimensionless wavenumber), $R=r/r_i$, $Z=z/h$, $D=h/r=0.05$, and

$$N=\frac{\nu_{\mathrm{m}}}{r^2\Omega_k}=\frac{\nu_{\mathrm{m}i}}{r_i^2\Omega_{ki}}R^{2/3}=\frac{1}{2}\left|\frac{U_{0i}}{V_{0i}}\right|R^{2/3}\approx\frac{1}{2}\alpha D^2R^{2/3},$$

the dimensionless dispersion equation is

$$\Delta^5-\frac{23}{6}(-2N)\Delta^4$$
$$+\left\{\left[\frac{181}{36}(-2N)^2+1+\frac{3Z^2}{F(Z)}+\frac{1}{3}K^2F(Z)\right]-\left[\frac{1}{3}KDF(Z)\right]i\right\}\Delta^3$$
$$+\left\{\left[-3N\frac{K^2}{D^2}-\frac{8}{3}(-2N)-\frac{2}{3}K^2F(Z)(-2N)-8\frac{Z^2(-2N)}{F(Z)}-\frac{179}{72}(-2N)^3\right]\right.$$
$$\left.+\left[3\frac{K}{D}N-\frac{61}{90}KDF(Z)(-2N)\right]i\right\}\Delta^2$$
$$+\left\{\left[3\frac{Z^2}{F(Z)}+\frac{7}{4}(-2N)^2+\frac{9K^2}{2D^2}(-2N^2)+\frac{1}{3}K^2F(Z)(-2N)^2\right.\right.$$
$$\left.+\frac{23}{4}\frac{Z^2}{F(Z)}(-2N)^2+\frac{7}{24}(-2N)^4\right]$$
$$\left.+\left[-\frac{1}{10}KDZ^2-\frac{2}{15}KDF(Z)(-2N)^2-\frac{9K}{2D}(-2N^2)\right]i\right\}\Delta$$
$$+\left\{\left[3\frac{Z^2}{F(Z)}(-2N)^3-\frac{9}{2}\frac{Z^2}{F(Z)}(-2N)\right]+\left[\frac{1}{10}KDZ^2(-2N)\right]i\right\}=0,$$

where

$$\Delta=\frac{\sigma}{\Omega_k}=i\left(\frac{\omega}{\Omega_k}-\frac{kU_0}{\Omega_k}\right)=i\left(W+2\frac{K}{D}N\right),\quad F(Z)=1-Z^2.$$

Because $\alpha<1$, we take $N=0.5\times10^{-4}R^{2/3}$. To investigate the instability of the inner disk with z-structure, we choose $R=2, Z=0, 0.5, 0.9$. A positive imaginary part of the dimensionless frequency W means that the mode is unstable. The O-mode (propagating outward) has a negative real part of W, while the I-mode (propagating inward) has a positive one.

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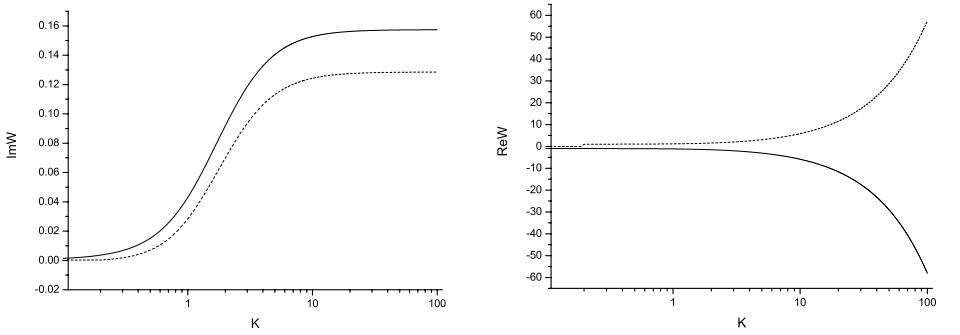


Figure 1. The dependence of dimensionless frequency W on the dimensionless wavenumber K in the inner disk, where $R = 2$, $Z = 0$. The solid curve corresponds to the O-mode and the dashed curve to the I-mode. They are all unstable.

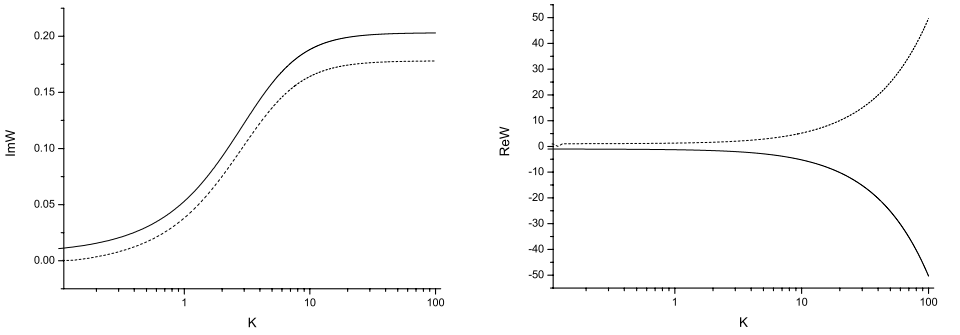


Figure 2. The dependence of dimensionless frequency W on the dimensionless wavenumber K in the inner disk, where $R = 2$, $Z = 0.5$. The solid curve corresponds to the O-mode and the dashed curve to the I-mode.

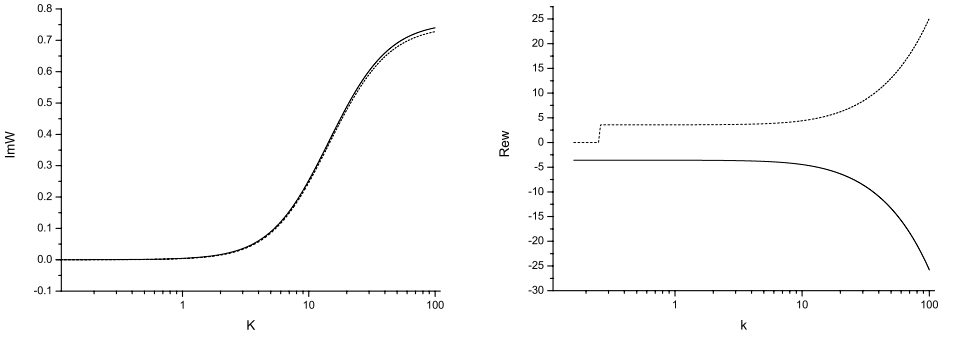


Figure 3. The dependence of dimensionless frequency W on the dimensionless wavenumber K in the inner disk, where $R = 2$, $Z = 0.9$. The solid curve corresponds to the O-mode and the dashed curve to the I-mode.

At each point we choose, the O-mode and I-mode are all unstable (Figs 1–3). Wu and Yang (1994) found that the I-mode was unstable in the outer disk, but sometimes turned stable in the inner disk. So we change R to 1. Here we just show

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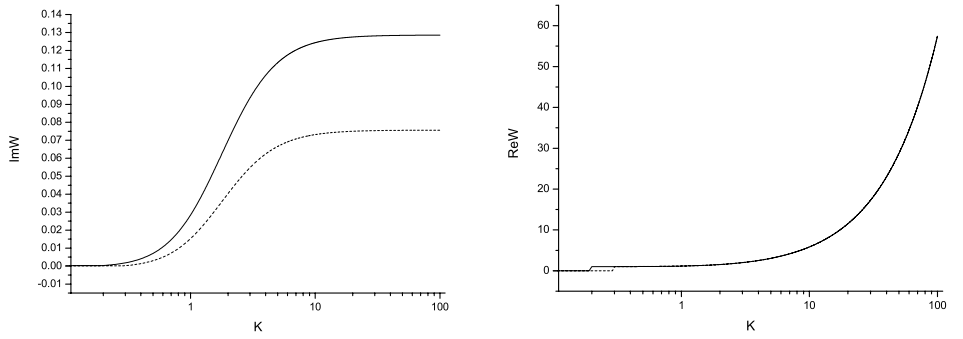


Figure 4. The dependence of the dimensionless frequency W on the dimensionless wavenumber K for the I-mode for different R when $Z = 0$. The solid curve corresponds to $R = 2$, and the dashed curve to $R = 1$. The I-mode is always unstable.

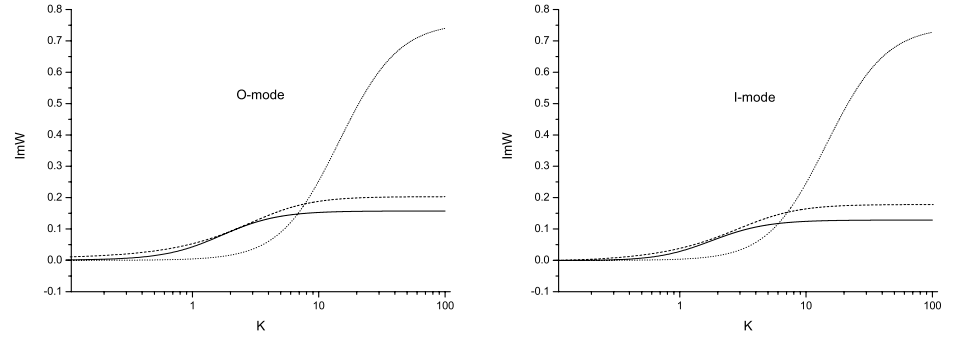


Figure 5. The unstable mode with different Z ($R = 2$). The solid, dashed and dotted curves represent $Z = 0, 0.5$ and 0.9, respectively. The left figure corresponds to the O-mode and the right figure corresponds to the I-mode.

the situation when $Z = 0$ (Fig. 4). It is clear that the I-mode is always unstable in the inner disk, although the growth rate changes. Wu and Yang (1994) used the α model, but we consider the anomalous magnetic viscosity (Li and Zhang 2002). So we think the anomalous viscosity causes the I-mode to always be unstable in the inner disk.

From Figs 1–3 we can also find that the growth rate changes with different Z. We are in agreement with Khoperskov and Khrapov (1999). The vertical structure should not be ignored when we discuss the short-wavelength perturbations ($K > 3$). However, when $K < 1$, the averaging over the z-coordinate is still reasonable (Fig. 5).

6. Conclusions

In summary, we investigate an accretion disk with vertical structure around a young stellar object, considering the anomalous viscosity. The two kinds of acoustic modes: O-mode and I-mode are all unstable in the inner disk. Different from Wu and Yang (1994), we find that the I-mode is always unstable because of the anomalous viscosity. Moreover, we should consider the vertical structure when the short-wavelength perturbations are investigated.

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Erratum

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