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Building a weak shockwave from linear modes

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In shockwave theory, the density, velocity and pressure jumps are derived from the conservation equations. Here, we address the physics of a weak shock the other way around. We first show that the density profile of a weak shockwave in a fluid can be expressed as a sum of linear acoustic modes. The shock so built propagates at the speed of sound and matter is exactly conserved at the front crossing. Yet, momentum and energy are only conserved up to order 0 in powers of the shock amplitude. The density, velocity and pressure jumps are similar to those of a fluid shock, and an equivalent Mach number can be defined. A similar process is possible in magnetohydrodynamics. Yet, such a decomposition is found impossible for collisionless shocks due to the dispersive nature of ion acoustic waves. Weakly nonlinear corrections to their frequency do not solve the problem. Weak collisionless shocks could be inherently nonlinear, non-amenable to any linear superposition. Or they could be non-existent, as hinted by recent works.

Key words: astrophysical plasmas, plasma nonlinear phenomena, space plasma physics

1. Introduction

In a neutral fluid or in magnetohydrodynamics (MHD), the density, velocity or pressure jump of a shockwave can be straightforwardly determined from the conservation equations (Zel'dovich & Raizer 2002; Kulsrud 2005). In contrast, theoretical works dealing with shocks density profile or corrugations, are analytically more involved because they need to go beyond the conservation laws and deal with the proper dynamics of the fluid (Wouchuk 2001; Landau & Lifshitz 2013; Lemoine, Ramos & Gremillet 2016).

The goal of this article is to contribute to the second kind of study by proposing an analytical theory of weak shocks which could prove useful, for example, when studying the interaction of a shock with an obstacle (Chaudhuri *et al.* 2013). In the theory of shockwaves, the density, velocity and pressure jumps are derived from the conservation equations, while the shock profile is secondary. Here, we work the other way around. We start from an analytical expression for the density profile, then check the fulfilment of the conservations laws and finally derive the jumps.

The theory exposed in the following is limited to weak shocks because it relies on the following premise. In a homogenous fluid of density ρ_0, deviations from equilibrium

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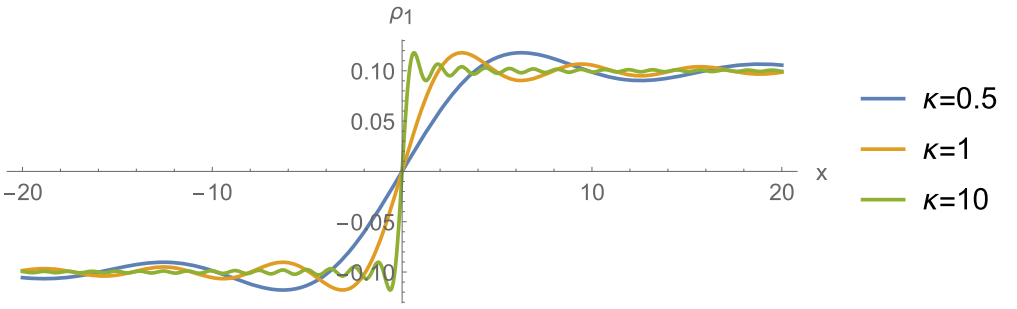


FIGURE 1. Reconstruction of the shock profile from (2.5) with $\delta\rho = 0.1$.

of amplitude $\delta\rho \ll \rho_0$ can be analysed using linear theory. It should then be possible to consider a weak shockwave of amplitude $\delta\rho$ as a weak perturbation of the medium, hence analysable in terms of linear modes. In § 2 of this article, we show that up to a good approximation, this program can be achieved in neutral fluid. In § 3, we find it is impossible to achieve in a collisionless ion/electron plasma. In the conclusion, we stress that what is achieved here for neutral fluids can also be achieved for MHD fluids. We also discuss the possible reasons why collisionless shocks escape the procedure.

2. Weak shocks in fluids as a sum of linear modes

Consider then a fluid at rest with homogenous density ρ_0. We study deviations from equilibrium of the form,

$$\rho(x,t) = \rho_0 + \rho_1(x,t), \tag{2.1}$$

with $\rho_1 \ll \rho_0$. Assume now that at time $t = 0$ the density profile of a weak shock has,

$$\rho_1(x,0) = \delta\rho\ \mathrm{sign}(x), \tag{2.2}$$

with $\delta\rho \ll \rho_0$. Can it be expressed as a superposition of linear acoustic modes?

The linear acoustic modes of a fluid are of the form,

$$\rho_1(x,t) = \varphi(k)\exp[\mathrm{i}k(x \pm c_s t)], \tag{2.3}$$

where c_s is the speed of sound in the medium. At $t = 0$, (2.3) reads $\rho(x,0) = \varphi(k)\,\mathrm{e}^{\mathrm{i}kx}$ so that, at this time, the possibility of expressing the profile (2.2) as a sum of linear modes simply stems for the possibility of Fourier expanding it. Fourier expansion at time $t = 0$ provides $\varphi(k) = -\mathrm{i}/k\pi$. We can therefore write,

$$\rho(x,0) = \rho_0 + \delta\rho \int_{-\infty}^{+\infty} \frac{-\mathrm{i}}{k\pi}\,\mathrm{e}^{\mathrm{i}kx}\,\mathrm{d}k. \tag{2.4}$$

It is interesting to observe how (2.4) reconstructs the shock density profile, as an increasing number of k values are accounted for. To this extent, we define,

$$\delta\rho^{\kappa}(x,0) = \delta\rho \int_{-\kappa}^{+\kappa} \varphi(k)\,\mathrm{e}^{\mathrm{i}kx}\,\mathrm{d}k. \tag{2.5}$$

Figure 1 plots this function for 3 values of κ. It takes large k values to reproduce the sharp front.

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2.1. *Propagating the wave packet*

It is straightforward to propagate the profile defined by (2.4), replacing $x \to x + c_s t$ to obtain a leftward propagation. The result reads,

$$\rho(x,t) = \rho_0 + \delta\rho \int_{-\infty}^{+\infty} \frac{-\mathrm{i}}{k\pi} \mathrm{e}^{\mathrm{i}k(x+c_s t)}\, \mathrm{d}k, \tag{2.6}$$

which is a superposition of normal modes of the form (2.3). In addition,

$$\rho_0 + \delta\rho \int_{-\infty}^{+\infty} \frac{-\mathrm{i}}{k\pi} \mathrm{e}^{\mathrm{i}k(x+c_s t)}\, \mathrm{d}k = \rho_0 + \delta\rho\ \mathrm{sign}(x + c_s t). \tag{2.7}$$

We thus find that choosing the same $\varphi(k)$, the superposition of linear modes (2.3) produces a propagating weak shock of amplitude $\delta\rho$.

2.2. *Velocity field*

To further study our weak shock we need to determine the velocity field. To this extent, consider the conservation of matter equation,

$$\frac{\partial(\rho_0 + \rho_1)}{\partial t} + \frac{\partial(\rho_0 + \rho_1)v}{\partial x} = 0, \tag{2.8}$$

we derive at first order[1] ,

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \frac{\partial v_1}{\partial x} = 0 \Rightarrow \frac{\partial v_1}{\partial x} = -\frac{1}{\rho_0}\frac{\partial \rho_1}{\partial t}. \tag{2.9}$$

From (2.6) we obtain the time derivative of ρ_1 as,

$$\frac{\partial \rho_1}{\partial t} = \delta\rho \int_{-\infty}^{+\infty} \frac{-\mathrm{i}}{k\pi} \mathrm{i}kc_s\, \mathrm{e}^{\mathrm{i}k(x+c_s t)}\, \mathrm{d}k, \tag{2.10}$$

so that,

$$\frac{\partial v_1}{\partial x} = -\frac{1}{\rho_0}\delta\rho \int_{-\infty}^{+\infty} \frac{-\mathrm{i}}{k\pi} \mathrm{i}kc_s\, \mathrm{e}^{\mathrm{i}k(x+c_s t)}\, \mathrm{d}k, \tag{2.11}$$

and,

$$\begin{aligned} v_1 &= -c_s \frac{\delta\rho}{\rho_0} \int_{-\infty}^{+\infty} \frac{-\mathrm{i}}{k\pi} \mathrm{e}^{\mathrm{i}k(x+c_s t)}\, \mathrm{d}k + \text{const.}, \\ &= -c_s \frac{\delta\rho}{\rho_0} \mathrm{sign}(x + c_s t) + \text{const.} \end{aligned} \tag{2.12}$$

Therefore, a velocity jump of amplitude $2c_s\delta\rho/\rho_0$ propagates at c_s, together with the density jump (2.6). The const. in this expression reflects the fact that it holds in any frame of reference. In the following we shall work in the frame of the shock front, setting const. $= c_s$.

[1] Note that v is first order since we work in the frame where the fluid is at rest.

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2.3. *Conserved quantities*

Shockwave theory usually begins with the conservation of matter, momentum and energy. Here, we built the shock from normal modes, without alluding to these laws. We now check them.

To this extent, we work in the shock frame. In this frame, we have from (2.7) and (2.12),

$$\rho = \begin{cases} \rho_0 - \delta\rho, & x < 0, \\ \rho_0 + \delta\rho, & x > 0, \end{cases} \quad v = c_s \times \begin{cases} 1 + \delta\rho/\rho_0, & x < 0, \\ 1 - \delta\rho/\rho_0, & x > 0. \end{cases} \tag{2.13}$$

For the conservation of matter, momentum and energy, we therefore check the conservation of the quantities ρv, $\rho v^2 + P$ and $\rho v^2/2 + \gamma P/(\gamma - 1)$, respectively ($\gamma$ is the adiabatic index of the fluid). That is,

(a) Conservation of matter

$$\rho v = \begin{cases} (\rho_0 - \delta\rho)c_s \left(1 + \dfrac{\delta\rho}{\rho_0}\right) = \rho_0 c_s \left(1 - \dfrac{\delta\rho^2}{\rho_0^2}\right), & x < 0, \\ (\rho_0 + \delta\rho)c_s \left(1 - \dfrac{\delta\rho}{\rho_0}\right) = \rho_0 c_s \left(1 - \dfrac{\delta\rho^2}{\rho_0^2}\right), & x > 0. \end{cases} \tag{2.14}$$

Matter is therefore exactly conserved through the front. With $P = \rho c_s^2/\gamma$ (see e.g. Landau & Lifshitz (2013), § 64) we check the conservation of momentum and energy.

(b) Conservation of momentum,

$$\rho v^2 + P = \begin{cases} (\rho_0 - \delta\rho)c_s^2 \left(1 + \dfrac{\delta\rho}{\rho_0}\right)^2 + (\rho_0 - \delta\rho)\dfrac{c_s^2}{\gamma} \\ \quad = \rho_0 c_s^2 \left(\dfrac{\gamma+1}{\gamma} + \dfrac{\gamma-1}{\gamma}\dfrac{\delta\rho}{\rho_0} - \left(\dfrac{\delta\rho}{\rho_0}\right)^2 - \left(\dfrac{\delta\rho}{\rho_0}\right)^3\right), & x < 0, \\ (\rho_0 + \delta\rho)c_s^2 \left(1 - \dfrac{\delta\rho}{\rho_0}\right)^2 + (\rho_0 + \delta\rho)\dfrac{c_s^2}{\gamma} \\ \quad = \rho_0 c_s^2 \left(\dfrac{\gamma+1}{\gamma} - \dfrac{\gamma-1}{\gamma}\dfrac{\delta\rho}{\rho_0} - \left(\dfrac{\delta\rho}{\rho_0}\right)^2 + \left(\dfrac{\delta\rho}{\rho_0}\right)^3\right), & x > 0. \end{cases} \tag{2.15}$$

Momentum is therefore conserved up to order 0 in $\delta\rho$, save for the specific case $\gamma = 1$ where it is conserved up to second order in $\delta\rho$. This would correspond to an isothermal gas which radiates the shock energy and maintains a constant temperature. In all other cases, momentum conservation is maintained only up to order 0 in $\delta\rho$.

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(c) Conservation of energy,

$$\frac{\rho v^2}{2}+\frac{\gamma}{\gamma-1}P=\begin{cases}\dfrac{1}{2}\rho_0 c_s^2\left(\dfrac{\gamma+1}{\gamma-1}+\dfrac{\gamma-3}{\gamma-1}\dfrac{\delta\rho}{\rho_0}-\left(\dfrac{\delta\rho}{\rho_0}\right)^2-\left(\dfrac{\delta\rho}{\rho_0}\right)^3\right), & x<0,\\ \dfrac{1}{2}\rho_0 c_s^2\left(\dfrac{\gamma+1}{\gamma-1}-\dfrac{\gamma-3}{\gamma-1}\dfrac{\delta\rho}{\rho_0}-\left(\dfrac{\delta\rho}{\rho_0}\right)^2+\left(\dfrac{\delta\rho}{\rho_0}\right)^3\right), & x>0.\end{cases} \tag{2.16}$$

Energy is therefore conserved up to order 0 in $\delta\rho$ for any γ.

In summary, matter is exactly conserved while momentum and energy are conserved to order 0 in $\delta\rho$. Such a discrepancy is due to the entropy jump, which vanishes in our case since it arises from the crossing of a structure eventually made up of linear Vlasov modes. However, since for a real weak shock, the entropy difference scales like $\delta\rho^3$ (Landau & Lifshitz 2013, § 86), considering it is 0 does yields incorrect results for the jump of the other quantities.

2.4. *Jumps and equivalent Mach number*

We can write the density jump of the shock as,

$$r=\frac{\rho_0+\delta\rho}{\rho_0-\delta\rho}=1+2\frac{\delta\rho}{\rho_0}+\mathcal{O}(\delta\rho)^2. \tag{2.17}$$

Conversely, the velocity jump reads,

$$r_v=\frac{c_s-c_s\dfrac{\delta\rho}{\rho_0}}{c_s+c_s\dfrac{\delta\rho}{\rho_0}}=\frac{\rho_0-\delta\rho}{\rho_0 v_0+\delta\rho}=\frac{1}{r}. \tag{2.18}$$

Finally, the pressure jump reads,

$$r_P=\frac{(\rho_0+\delta\rho)c_s^2}{(\rho_0-\delta\rho)c_s^2}=r. \tag{2.19}$$

Now, according to the shockwave theory (Thorne & Blandford 2017, p. 905),

$$\left.\begin{aligned} r&=\frac{(\gamma+1)\mathcal{M}^2}{(\gamma-1)\mathcal{M}^2+2}=1+\frac{4(\mathcal{M}-1)}{\gamma+1}+\mathcal{O}(\mathcal{M}-1)^2,\\ r_P&=\frac{2\gamma^2\mathcal{M}^2}{\gamma+1}-\frac{\gamma-1}{\gamma+1}=1+\frac{4(\mathcal{M}-1)}{\gamma+1}+\mathcal{O}(\mathcal{M}-1)^2,\end{aligned}\right\} \tag{2.20}$$

where $\mathcal{M}$ is the shock Mach number. It appears that, at first order in $(\mathcal{M}-1)$, the pressure and density jumps are equal. Noticing from (2.17) and (2.19) that our built-up shock also has the same density and pressure jumps, we can assign it an equivalent Mach number

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$\mathcal{M}_{\rm eq}$ by equating the Taylor expansions (2.17) and (2.20),

$$\text{define } \mathcal{M}_{\rm eq}/. \quad 2\frac{\delta\rho}{\rho_0} = \frac{4(\mathcal{M}_{\rm eq}-1)}{\gamma+1}, \tag{2.21}$$

giving,

$$\mathcal{M}_{\rm eq} = 1 + \frac{1+\gamma}{2\gamma}\frac{\delta\rho}{\rho_0}. \tag{2.22}$$

The shock that we just built from the superposition of linear modes mimics a weak fluid shocks with a Mach number given by (2.22).

3. Collisionless shocks

In a collisionless electron/proton plasma, ion acoustic waves (IAW) follow the dispersion,

$$\omega_k = \pm k\sqrt{\frac{k_B T_e/m_p}{1+k^2\lambda_D^2}}, \tag{3.1}$$

where T_e is the electronic temperature and λ_D the Debye length. These waves are Landau damped for $k > k_m = \lambda_D^{-1}\sqrt{T_e/T_p}$, T_p being the protonic temperature (Thorne & Blandford 2017, p. 1047). If the shock profile is to be given by an expression of the form,

$$\rho_1(x,t) = \int \varphi(k)\,\mathrm{e}^{\mathrm{i}kx-\mathrm{i}\omega_k t}\,\mathrm{d}k, \tag{3.2}$$

then again, setting $t=0$ implies $\varphi(k)$ is the Fourier transform of $\rho_1(x,0)$. Assuming for example a step function at $t=0$ as in § 2, (3.2) then propagates it for $t\neq 0$. After some algebra we get,

$$\rho_1(x,t) = \frac{\delta\rho}{\pi}\int_{-k_m}^{k_m}\sin\left(kx + kt\sqrt{\frac{k_B T_e/m_p}{1+k^2\lambda_D^2}}\right)\frac{\mathrm{d}k}{k}, \tag{3.3}$$

where the integration is over $[-k_m, k_m]$ instead of $[-\infty,\infty]$ to account for the Landau damping. Setting,

$$u = k\lambda_D, \quad X = x/\lambda_D, \quad \tau = t\frac{\sqrt{k_B T_e/m_p}}{\lambda_D}, \tag{3.4a--c}$$

gives

$$\rho_1(X,\tau) = \frac{\delta\rho}{\pi}\int_{-\sqrt{T_e/T_p}}^{\sqrt{T_e/T_p}}\sin\left[u\left(X+\frac{\tau}{\sqrt{1+u^2}}\right)\right]\frac{\mathrm{d}u}{u}. \tag{3.5}$$

Setting $X_s = X+\tau$ to follow the propagation, we get

$$\rho_1(X_s,\tau) = 2\frac{\delta\rho}{\pi}\int_0^{\sqrt{T_e/T_p}}\sin\left[uX_s\left(1+\frac{\tau f(u)}{X_s}\right)\right]\frac{\mathrm{d}u}{u}, \tag{3.6}$$

with,

$$f(u) = \frac{1-\sqrt{1+u^2}}{\sqrt{1+u^2}}. \tag{3.7}$$

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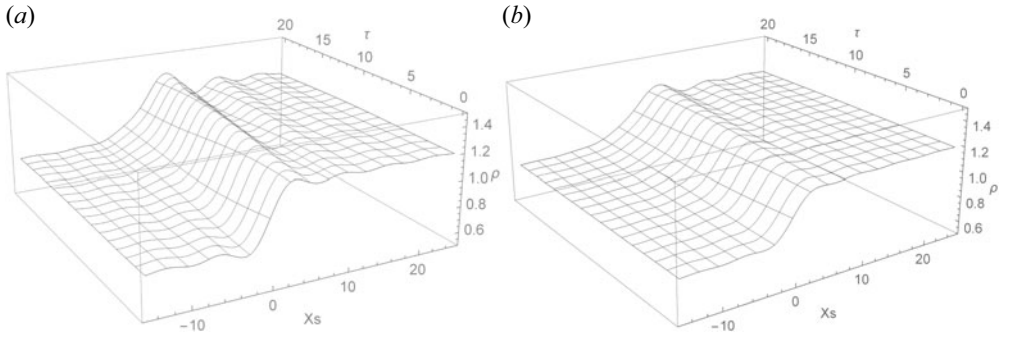


FIGURE 2. Density profile $\rho(X_s, \tau) = 1 + 0.2\rho_1(X_s, \tau)$, for $T_e/T_p = 1$. Here, $\delta\rho$ is set to 0.2 for more clarity. (*a*) Initial step function at $t = 0$. (*b*) Initial $\arctan(x/\lambda_D)$ function at $t = 0$.

Equation (3.6) is plotted in figure 2(*a*)[2]. It appears that the initial shape is not propagated without distortion. Figure 2(*b*) shows the same calculation, but for an initial profile given by $\arctan(x/\lambda_D)$. Again, propagation distorts the profile.

Considering the form of (3.1), the patterns match qualitatively the analysis of Krall (1997) (smaller wavelengths travel slower, hence trail behind the front). However, stationarity is not achieved in any frame.

Indeed we now prove that because of the form of the dispersion (3.1), it is impossible to build a IAW wave packet that does not distort when propagating.

Assume then a IAW wave packet of the form,

$$\rho(x, t) = \int \varphi(k)\, \mathrm{e}^{\mathrm{i}kx - \mathrm{i}\omega_k t}\, \mathrm{d}k. \tag{3.8}$$

To see if it can result in a stationary structure in a frame moving at speed V, we first operate $x \to X + Vt$ with $V \in \mathbb{R}$, and form the time derivative of the resulting expression,

$$\frac{\partial \rho(X, t)}{\partial t} = \int \mathrm{i}(kV - \omega_k)\varphi(k)\, \mathrm{e}^{\mathrm{i}kX + \mathrm{i}(kV - \omega_k)t}\, \mathrm{d}k. \tag{3.9}$$

If the packet (3.8) is to be stationary in some frame at velocity V then the expression above must vanish $\forall(X, t)$. In particular, setting $t = 0$ must give,

$$\int (kV - \omega_k)\varphi(k)\, \mathrm{e}^{\mathrm{i}kX}\, \mathrm{d}k = 0, \tag{3.10}$$

so that the Fourier transform of the function $F(u) = (uV - \omega_u)\varphi(u)$ must be 0. This implies $F(u) = 0$, that is, $\omega_u = uV$ or trivially $\varphi(u) = 0$.

Therefore, unless $\exists V \in R/\omega_u = uV$, it is impossible to build a IAW packet which is stationary is some frame. Conversely, if $\exists V \in \mathbb{R}/\omega_u = uV$, then any superposition of linear acoustic modes propagates without distortion since all the harmonics propagate at the same speed V.

Note that the problem does not come from the limited integration domain in (3.3) imposed by Landau damping, but from the dispersive form of ω_k in (3.1).

[2]The initial shape is not a step function because integration in (3.6) is not performed up to $+\infty$.

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4. Conclusion

We considered building up a weak shock from a superposition of linear modes. The attempt is successful for any medium where the dispersion relation is of the form $\omega_k = kV$. Conservation laws are met exactly for matter, and up to order 0, in powers of the shock amplitude, for the momentum and the energy.

As long as the linear modes of the fluid are of the form (2.3), that is, enjoy an acoustic dispersion like $\omega(k) = kc$, $c \in \mathbb{R}$, the reasoning above holds. In particular, the dispersion of the fast, intermediate and slow MHD modes read $\omega = kc_+$, $\omega = kc_A$ and $\omega = kc_-$, where c_A is the Alfvén speed and $c_\pm$ the velocity of the fast and slow modes (Kulsrud 2005, ch. 5). The reasoning above applies therefore to the 3 kinds of weak MHD shocks, substituting c_s by the corresponding velocity.

For collisionless plasma, the program cannot be realized, precisely because the dispersion relation is not of the suitable form. Nonlinear frequency shifts to (3.1) have been studied by Berger *et al.* (2013) and Affolter *et al.* (2019)[3]. Yet, they still do not result in a dispersionless expression for ω_k, which is a necessary condition to form a stationary profile in a moving frame, as proved in § 3.

The failure of our program for collisionless shocks can have two origins:

- Weak collisionless shocks are intrinsically nonlinear phenomena which cannot be captured by any superposition of linear modes. This would be in line with, for example, the theory of shocks in viscous fluids, where an analytical profile can be found (Landau & Lifshitz 2013, § 93) while the linear modes are all damped by viscosity (Zel'dovich & Raizer 2002, ch. 1, § 22). Similar considerations apply to solitons in a plasma (Tran 1979).

  Closely related is the long-known fact that collisionless shocks are possible because nonlinear wave steepening balances the spreading of the wave packet due to dispersion (see Tidman & Krall 1971, ch. 6, Galeev 1976; Kennel, Edmiston & Hada 1985; Balogh & Treumann 2013, § 2.1.4). Yet, introducing such effects, be it phenomenologically, would go against the main idea of our model which is precisely to reconstruct a shock from a superposition of linear modes.
- An extremely weak collisionless shock, with Mach number $1 + \varepsilon$, cannot exist. That would be coherent with the model of Bret & Narayan (2018) for collisionless shocks in pair plasmas, where no solutions could be found for Mach numbers lower than 1.34. Note that this latter model has been found correct for strong shocks (Haggerty, Bret & Caprioli 2021).

To our knowledge, no particle-in-cell simulations of collisionless shocks for Mach number $\mathcal{M} < 2$ have been achieved so far[4]. Future numerical works with $1 < \mathcal{M} < 2$ could help in resolving the present issue.

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[3] See for example (22) of Berger *et al.* (2013).
[4] See Guo, Sironi & Narayan (2018), Ha *et al.* (2018) and Kang, Ryu & Ha (2019) for $\mathcal{M} = 2$.

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Declaration of interest

The authors report no conflict of interest.

REFERENCES

AFFOLTER, M., ANDEREGG, F., DUBIN, D.H.E., VALENTINI, F. & DRISCOLL, C.F. 2019 Fluid and kinetic nonlinearities of near-acoustic plasma waves. *Phys. Plasmas* **26** (12), 122108.

BALOGH, A. & TREUMANN, R.A. 2013 *Physics of Collisionless Shocks: Space Plasma Shock Waves*. Springer.

BERGER, R.L., BRUNNER, S., CHAPMAN, T., DIVOL, L., STILL, C.H. & VALEO, E.J. 2013 Electron and ion kinetic effects on non-linearly driven electron plasma and ion acoustic waves. *Phys. Plasmas* **20** (3), 032107.

BRET, A. & NARAYAN, R. 2018 Density jump as a function of magnetic field strength for parallel collisionless shocks in pair plasmas. *J. Plasma Phys*. **84** (6), 905840604.

CHAUDHURI, A., HADJADJ, A., SADOT, O. & BEN-DOR, G. 2013 Numerical study of shock-wave mitigation through matrices of solid obstacles. *Shock Waves* **23** (1), 91–101.

GALEEV, A.A. 1976 Collisionless shocks. In *Physics of Solar Planetary Environement; Proceedings of the International Symposium on Solar-Terrestrial Physics, Boulder, CO, June 7–18, 1976* (ed. D.J. Williams), pp. 464–490. AGU.

GUO, X., SIRONI, L. & NARAYAN, R. 2018 Electron heating in low Mach number perpendicular shocks. II. Dependence on the pre-shock conditions. *Astrophys. J.* **858** (2), 95.

HA, J.-H., RYU, D., KANG, H. & VAN MARLE, A. J. 2018 Proton acceleration in weak quasi-parallel intracluster shocks: injection and early acceleration. *Astrophys. J.* **864** (2), 105.

HAGGERTY, C. C, BRET, A. & CAPRIOLI, D. 2021 Kinetic simulations of strongly magnetized parallel shocks: deviations from MHD jump conditions. *Mon. Not. R. Astron. Soc*. **509** (2), 2084–2090.

KANG, H., RYU, D. & HA, J.-H. 2019 Electron preacceleration in weak quasi-perpendicular shocks in high-beta intracluster medium. *Astrophys. J.* **876** (1), 79.

KENNEL, C.F., EDMISTON, J.P. & HADA, T. 1985 A quarter century of collisionless shock research. In *Collisionless Shocks in the Heliosphere: A Tutorial Review (A87-25326 09-92)*, pp. 1–36. American Geophysical Union.

KRALL, N.A. 1997 What do we really know about collisionless shocks? *Adv. Space Res*. **20** (4–5), 715–724.

KULSRUD, R. M 2005 *Plasma Physics for Astrophysics*. Princeton University Press.

LANDAU, L.D. & LIFSHITZ, E.M. 2013 *Fluid Mechanics*. Elsevier Science.

LEMOINE, M., RAMOS, O. & GREMILLET, L. 2016 Corrugation of relativistic magnetized shock waves. *Astrophys. J.* **827** (1), 44.

THORNE, K.S. & BLANDFORD, R.D. 2017 *Modern Classical Physics: Optics, Fluids, Plasmas, Elasticity, Relativity, and Statistical Physics*. Princeton University Press.

TIDMAN, D.A. & KRALL, N.A. 1971 *Shock Waves in Collisionless Plasmas*. Wiley-Interscience.

TRAN, M.Q. 1979 Ion acoustic solitons in a plasma: a review of their experimental properties and related theories. *Phys. Scr*. **20**, 317–327.

WOUCHUK, J.G. 2001 Growth rate of the linear Richtmyer-Meshkov instability when a shock is reflected. *Phys. Rev.* E **63** (5), 056303.

ZEL'DOVICH, Y. B & RAIZER, Y. P 2002 *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena*. Dover Publications.

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