J. Plasma Physics (2012), *vol.* 78, *part* 3, *pp.* 265–278. © Cambridge University Press 2012
doi:10.1017/S0022377812000013

Ion-acoustic envelope excitations in multispecies plasma with non-thermally distributed electrons

PARVEEN BALA[1], TARSEM SINGH GILL[2] and HARVINDER KAUR[3]

[1]Deparment of Math. Stat. & Physics, Punjab Agricultural University, Ludhiana-141004, India (pravi2506@gmail.com)
[2]Deparment of Physics, Guru Nanak Dev University, Amritsar-143005, India
[3]Deparment of Physics, Khalsa College, Amritsar-143002, India

(Received 12 December 2011; accepted 3 January 2012; first published online 6 February 2012)

Abstract. By using the standard reductive perturbation technique, a nonlinear Schrödinger equation is derived to study the stability of finite amplitude ion-acoustic waves in an unmagnetized plasma consisting of warm positive and negative ions and non-thermal electrons. The effect of relative temperature of positive and negative ions, their charge and mass ratios, density ratios and non-thermally distributed electrons on modulational instability of fast and slow ion-acoustic mode is investigated. It is found that these parameters significantly change the domain of modulation instability. Both, envelope and dark solitons appear in different regions of parameter space.

1. Introduction

Nonlinear behaviour of natural phenomena is one of the most important aspects of modern plasma physics research. In the nonlinear wave studies, the propagation of solitary waves is an important aspect as it captures the basic physics issues as the characteristics of interaction between waves and plasma. Modulational instability (MI) is a ubiquitous phenomenon, first studied some four decades back in context of water waves (Benjamin and Feir 1967; Whitam 1967) and is exhibited in many nonlinear waves. Since then, there has been growing interest in the study of MI in various physical systems, e.g. laser plasma related parametric coupling (Shivamogi 1988; El-Dib 2000; Kourakis et al. 2005a), multi-component plasmas (Chhabra and Sharma 1986; Mishra et al. 1993; Mishra et al. 1994; Mishra et al. 1995; Hertzberg et al. 2004; Kourakis et al. 2006), space and astrophysical plasmas (Tstovich and Wharton 1978; Bharuthram 1992; Verheest et al. 1995; Gill et al. 2007; Kourakis et al. 2007), dusty and complex plasma (Amin et al. 1998; Tang and Jukui 2003; Xue and He 2003; Kourakis and Shukla 2004a; Kourakis and Shukla 2004b; Xue 2004, 2004a; Duan 2006; Mishra and Chowdhury 2006a), Bose–Einstein condensates (Shukla et al. 2001; Theocharis et al. 2003; Kourakis et al. 2005b; Xue 2005; Abdullaev et al. 2007; Law et al. 2008) etc, solid state physics (Davydov 1985; Rapti et al. 2004; Griffiths et al. 2006; Johansson 2006), signal transmission line (Remoissenet 1994; Bilbault et al. 1995; Yemele and Marquie 2003), fibre communication (Tan and Boyd 2001; Ganapathy et al. 2006; Aggrawal 2007; Ballav and Chowdhury 2007; Ndzana et al. 2007; Zhu 2007; Nohara et al. 2008), charge transport in molecular system (Dauxois and Peyrard 2006) *etc*. From the theoretical point of view nonlinear phenomena related localized structures, instabilities, wave-interaction, wave-particle interaction and methodology related to plasma modes have been reported in standard text (Davidson 1972). The non-envelope related structures governed by Korteweg-de-Vries (KdV) or modified Korteweg-de-Vries ($mKdV$) derived using reductive perturbation technique, had received extensive attention as witnessed by increasing number of publications during the last four decades.

By incorporating the effects of harmonic generation and ponderomotive nonlinearities, many authors have derived the nonlinear Schrödinger equation ($NLSE$) which governs the dynamics of nonlinear ion-acoustic envelope solitons in plasma. For large amplitude waves interacting with background media, nonlinear harmonic generation due to carrier wave self-interaction comes into picture. Self-modulation involving second harmonic generation is responsible for modulational instabilities and energy localization via the formulation of envelope excitations. A comprehensive review of many original and useful publications in this field is also given by Infeld and Rowland (1990) and Vladimirov et al. (1995). Experimental observations of the MI of monochromatic ion-acoustic waves have been reported by Watanabe (1977).

Inspired by the earlier work, some research work on MI of different wave modes in multi-component plasmas relevant to space, astrophysical and laboratory have been reported (Salahuddin 2002; Kourakis and Shukla 2005c). Space plasmas contain several ions having different charge states, are usually of multi-component type. *In situ* measurements by various satellite missions

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

provide abundant information about stationary nonlinear ES/EM structures occurring in space plasma. Localized modulated wave packets are encountered in abundance, *e.g.* in the Earth's magnetosphere, where they are associated with localized field and/or density variation simultaneously observed (Pottelette et al. 1999). Negative ions are found in D-region of Earth's ionosphere (Swider 1988). As an approximation, plasma can be treated collissionless in the upper part of the D-region of Earth's ionosphere where collision frequency is expected to be small and neglected (Mishra et al. 1995). In a series of experiments, it has been established that ion-acoustic waves in a plasma consisting of negative ions to be unstable with respect to parallel modulation (Tsukabayashi and Nakamura 1990; Bailung and Nakamura 1993; Bailung et al. 1996). McFadden et al. (2003) have further suggested that localized structures observed via satellite in space missions are not of unmodulated KdV solitons but have resemblance to modulated wave packets.

Both linear and nonlinear properties of the plasmas are influenced by the velocity distributions of the particle constituents of the plasma. Some recent observations show that the particles may not follow Maxwellian distribution and based on the data, particle distributions are better modelled by velocity distribution having flat top with high energy tails. These distributions namely, non-thermal particle distributions have abundance of superthermal particles. Another type of non-Maxwellian distribution is non-isothermal velocity distribution, resulting from the formation of space holes. This vortex distribution involving particle trapping is not only observed in space plasmas but also in laboratory (Saeki et al. 1979; Guio et al. 2003). Use of non-thermal distributions accounted for observed nonlinear structures of Freja (Cairns et al. 1995). For the last decade, there have been prolific literature on envelope and non-envelope solitons using non-Maxwellian particle distributions in different plasma systems (El-labany et al. 2003; Jukui 2003; Mishra and Chowdhury 2003; Singh and Lakhina 2004; Tang and Xue 2004; Gill et al. 2006; Mishra and Chowdhury 2006; Sahu and Roychoudhury 2006; El-Shewy 2007; Gill et al. 2007). This renewed our interest to study nonlinear wave structures in such multicomponent plasmas.

2. Governing equations

We consider a collisionless unmagnetized plasma consisting of non-thermal electron distribution, warm positive and negative ion species having temperatures T_p and T_n which are divided into two distinct groups. We assume that low frequency electrostatic waves propagate in the plasma.

To explain observation made from the Freja satellite, Cairns et al. (1995) assumed a distribution of electrons which is non-thermal with an excess of energetic particles. The number density of the electron fluid, with non-thermal electrons is given by:

$$n_e = (1 - \beta\phi + \beta\phi^2)e^{\phi}, \tag{1}$$

where

$$\beta = \frac{4\gamma'}{1 + 3\gamma'}.$$

Here γ' is a parameter determining the number of non-thermal electrons present in our non-thermal plasma model (Cairns et al. 1995). This is non-Maxwellian distribution function which contains high energy electrons component. Such distributions are very common in auroral zone of ionosphere (Singh and Lakhina 2004; Gill et al. 2007).

The nonlinear behaviour of the ion-acoustic waves may be described by the following set of normalized fluid equations:

$$\frac{\partial n_p}{\partial t} + \frac{\partial (n_p v_p)}{\partial x} = 0, \tag{2}$$

$$\frac{\partial v_p}{\partial t} + v_p \frac{\partial v_p}{\partial x} = -\frac{1}{\delta}\frac{\partial \phi}{\partial x} - \frac{\sigma_p}{\delta Z_p}\frac{1}{n_p}\frac{\partial n_p}{\partial x}, \tag{3}$$

$$\frac{\partial n_n}{\partial t} + \frac{\partial (n_n v_n)}{\partial x} = 0, \tag{4}$$

$$\frac{\partial v_n}{\partial t} + v_n \frac{\partial v_n}{\partial x} = \frac{\epsilon_z}{\delta\eta}\frac{\partial \phi}{\partial x} - \frac{\sigma_n}{\delta\eta Z_p}\frac{1}{n_n}\frac{\partial n_n}{\partial x}, \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - \frac{n_p}{1 - \alpha\epsilon_z} + \frac{\alpha\epsilon_z}{1 - \alpha\epsilon_z} n_n, \tag{6}$$

where

$$\delta = \frac{\eta + \alpha\epsilon_z^2}{\eta(1 - \alpha\epsilon_z)}, \alpha = \frac{n_n^{(0)}}{n_p^{(0)}}, \epsilon_z = \frac{Z_n}{Z_p},$$
$$\eta = \frac{m_n}{m_p}, \sigma_p = \frac{T_p}{T_e}, \sigma_n = \frac{T_n}{T_e}. \tag{7}$$

In the above equations n_p, v_p and n_n, v_n are the densities and fluid velocities of positive and negative ion species, respectively. $n_p^{(0)}$, $n_n^{(0)}$ are the equilibrium densities of two ion components, respectively. Further, ϕ is the electrostatic potential. η is the mass ratio of the negative ion species to the positive ion species, α is the equilibrium density ratio of the negative ion to positive ion species and ϵ_z is the charge multiplicity ratio of the negative ion to positive ion species. In (1)–(6), velocities (v_p, v_n), potential (ϕ), time (t) and space coordinate (x) have been normalized with respect to the effective ion-acoustic speed in the mixture, $C_s = [T_e\delta Z_p/m_p]^{1/2}$, thermal potential T_e/e, inverse of ion plasma frequency in the mixture $\omega_{pi}^{-1} = [m_p/4\pi n_{eo}e^2 Z_p\delta]^{1/2}$ and Debye length $\lambda_D = [T_e/4\pi n_{eo}e^2]^{1/2}$, respectively. Ion densities n_p, n_n and n_e are normalized with their corresponding equilibrium values.

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

3. Derivation of nonlinear Schrödinger equation

Modulational instability of ion-acoustic wave is studied using reductive perturbation technique (Washimi and Taniuti 1966). We derive here nonlinear Schrödinger equation which governs the slow modulation of wave amplitude. Reductive perturbation technique in which dependent variables are expanded, also make use stretching of space and time coordinates. The following set of stretched (slow) space and time variables are appropriate for our problem:

$$\begin{aligned}\xi &= \epsilon(x-\lambda_g t),\\ \tau &= \epsilon^2 t,\end{aligned} \tag{8}$$

where λ_g is the group velocity to be determined by the compatibility requirement. Here ϵ is a small formal expansion parameter and is the measure of the perturbation. The condition $\epsilon \ll 1$ implies that the plasma dimension must be much larger than the Debye length, which is satisfied in the most cases of interest. We will assume that all perturbed quantities depend on the fast scale via the phase $\theta = kx-\omega t$ only, while the slow scales enter the argument of the lth harmonic amplitude, say for density as $n_{p,n}^{(n')}(l)$. Following this prescription, the dependent variables are expanded as

$$\begin{aligned}n_{p,n} &= 1+\sum_{n'=1}^{\infty}\sum_{l=-\infty}^{\infty}\epsilon^{n'} n_{p,n}^{n'}(l)(\xi,\tau)e^{il(kx-\omega t)},\\ v_{p,n} &= \sum_{n'=1}^{\infty}\sum_{l=-\infty}^{\infty}\epsilon^{n'} v_{p,n}^{n'}(l)(\xi,\tau)e^{il(kx-\omega t)},\\ \phi &= \sum_{n'=1}^{\infty}\sum_{l=-\infty}^{\infty}\epsilon^{n'}\phi_l^{n'}(\xi,\tau)e^{il(kx-\omega t)},\end{aligned} \tag{9}$$

where the subscripts p,n refer to positive and negative ions, respectively. Also $n_{p,n}$, $v_{p,n}$ and ϕ satisfy the reality condition $A_{-l}^{(n')} = A_l^{(n')*}$ and asterisk denotes the complex conjugate. Using (1) through (9) in (2) to (6) and collecting the terms of different powers of ϵ, we get the reduced equations. For the first order ($n'=1$), we get

$$-i\omega n_p^{(1)}(1)+ikv_p^{(1)}(1)=0, \tag{10}$$

$$-i\omega v_p^{(1)}(1)+ik\frac{\sigma_p}{\delta Z_p}n_p^{(1)}(1)+ik\frac{1}{\delta}\phi_1^{(1)}=0, \tag{11}$$

$$-i\omega n_n^{(1)}(1)+ikv_n^{(1)}(1)=0, \tag{12}$$

$$-i\omega v_n^{(1)}(1)+ik\frac{\sigma_n}{\eta\delta Z_p}n_n^{(1)}(1)-ik\frac{\epsilon_z}{\eta\delta}\phi_1^{(1)}=0, \tag{13}$$

$$(1-\beta+k^2)\phi_1^{(1)}-\frac{1}{(1-\alpha\epsilon_z)}n_p^{(1)}(1)+\frac{\alpha\epsilon_z}{(1-\alpha\epsilon_z)}n_n^{(1)}(1)=0. \tag{14}$$

Algebraic manipulations of these equations lead to the following dispersion relation:

$$\begin{aligned}\frac{\omega^2}{k^2} &= \frac{1}{2}\left(\frac{1}{1-\beta+k^2}+\frac{\eta\sigma_p+\sigma_n}{\eta\delta Z_p}\right)\\ &\pm\frac{1}{2}\left[\left(\frac{1}{1-\beta+k^2}+\frac{\eta\sigma_p+\sigma_n}{\eta\delta Z_p}\right)^2\right.\\ &\left.-\frac{4}{\eta\delta^2 Z_p}\left(\frac{\sigma_p\sigma_n}{Z_p}+\frac{1}{(1-\alpha\epsilon_z)}\frac{\alpha\epsilon_z^2\sigma_p+\sigma_n}{(1-\beta+k^2)}\right)\right]^{1/2}.\end{aligned} \tag{15}$$

It may be mentioned that for cold ions, i.e. for $\sigma_p = \sigma_n = 0$, and for Maxwelllian distribution $\beta = 0$, the above dispersion relation reduces to that of Mishra et al. (1993). From (10) to (13), we can express the first order quantities in terms of $\phi_1^{(1)}$ as

$$\begin{aligned}n_p^{(1)}(1) &= a\phi_1^{(1)}(1), \qquad n_n^{(1)}(1) = b\phi_1^{(1)}(1),\\ v_p^{(1)}(1) &= \frac{\omega}{k}a\phi_1^{(1)}(1), \qquad v_n^{(1)}(1) = \frac{\omega}{k}b\phi_1^{(1)}(1),\end{aligned} \tag{16}$$

where,

$$a=\frac{1}{\delta\left(\frac{\omega^2}{k^2}-\frac{\sigma_p}{\delta Z_p}\right)}, \qquad b=-\frac{\epsilon_z}{\eta\delta\left(\frac{\omega^2}{k^2}-\frac{\sigma_n}{\eta\delta Z_p}\right)}. \tag{17}$$

For the second order ($n'=2$), reduced equation with $l=1$, we get

$$-i\omega n_p^{(2)}(1)+ikv_p^{(2)}(1)=\lambda_g\frac{\partial n_p^{(1)}(1)}{\partial\xi}-\frac{\partial v_p^{(1)}(1)}{\partial\xi}, \tag{18}$$

$$\begin{aligned}&-i\omega v_p^{(2)}(1)+ik\frac{\sigma_p}{\delta Z_p}n_p^{(2)}(1)+ik\frac{1}{\delta}\phi_1^{(2)}\\ &=\lambda_g\frac{\partial v_p^{(1)}(1)}{\partial\xi}-\frac{\sigma_p}{\delta Z_p}\frac{\partial n_p^{(1)}(1)}{\partial\xi}-\frac{1}{\delta}\frac{\partial\phi_1^{(1)}}{\partial\xi},\end{aligned} \tag{19}$$

$$-i\omega n_n^{(2)}(1)+ikv_n^{(2)}(1)=\lambda_g\frac{\partial n_n^{(1)}(1)}{\partial\xi}-\frac{\partial v_n^{(1)}(1)}{\partial\xi}, \tag{20}$$

$$\begin{aligned}&-i\omega v_n^{(2)}(1)+ik\frac{\sigma_n}{\eta\delta Z_p}n_n^{(2)}(1)-ik\frac{\epsilon_z}{\eta\delta}\phi_1^{(2)}\\ &=\lambda_g\frac{\partial v_n^{(1)}(1)}{\partial\xi}-\frac{\sigma_n}{\eta\delta Z_p}\frac{\partial n_n^{(1)}(1)}{\partial\xi}+\frac{\epsilon_z}{\eta\delta}\frac{\partial\phi_1^{(1)}}{\partial\xi},\end{aligned} \tag{21}$$

$$\begin{aligned}&(1-\beta+k^2)\phi_1^{(2)}-\frac{1}{(1-\alpha\epsilon_z)}n_p^{(2)}(1)+\frac{\alpha\epsilon_z}{(1-\alpha\epsilon_z)}n_n^{(2)}(1)\\ &=2ik\frac{\partial\phi_1^{(1)}}{\partial\xi}.\end{aligned} \tag{22}$$

Using (16) to (22), we can put the second order quantities $n_{p,n}^{(2)}$, $v_{p,n}^{(2)}$ in terms of $\phi_1^{(2)}$ and $\frac{\partial\phi_1^{(1)}}{\partial\xi}$. Then, these are solved algebraically to obtain the following compatibility condition:

$$\lambda_g=\frac{k}{\omega}\left[\frac{\omega^2}{k^2}-\frac{k^2}{\delta}\frac{(1-\alpha\epsilon_z)}{(a^2+\alpha\eta b^2)}\right]. \tag{23}$$

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

The second harmonic mode of the carrier, which comes from nonlinear self-interaction, is also obtained in terms of $[\phi_1^{(1)}]^2$. The component $l = 2$ for the second order, $n' = 2$, reduced equations determine the second order quantities. They turn out to be

$$n_p^{(2)}(2) = A_{n_p}\left[\phi_1^{(1)}\right]^2, \tag{24}$$

$$v_p^{(2)}(2) = A_{v_p}\left[\phi_1^{(1)}\right]^2, \tag{25}$$

$$n_n^{(2)}(2) = A_{n_n}\left[\phi_1^{(1)}\right]^2, \tag{26}$$

$$v_n^{(2)}(2) = A_{v_n}\left[\phi_1^{(1)}\right]^2, \tag{27}$$

$$\phi_2^{(2)} = A_\phi\left[\phi_1^{(1)}\right]^2, \tag{28}$$

where

$$\begin{aligned}
A_\phi &= \frac{a^3\delta\left(\frac{3\omega^2}{k^2} - \frac{\sigma_p}{\delta Z_p}\right) + \alpha\eta\delta b^3\left(\frac{3\omega^2}{k^2} - \frac{\sigma_n}{\eta\delta Z_p}\right)}{6k^2(1-\alpha\epsilon_z)} - \frac{1}{6k^2},\\
A_{n_p} &= aA_\phi + \frac{a^3\delta}{2}\left(\frac{3\omega^2}{k^2} - \frac{\sigma_p}{\delta Z_p}\right),\\
A_{n_n} &= bA_\phi - \frac{b^3}{2}\frac{\eta\delta}{\epsilon_z}\left(\frac{3\omega^2}{k^2} - \frac{\sigma_n}{\eta\delta Z_p}\right),\\
A_{v_p} &= \frac{\omega}{k}A_{n_p} - a^2\frac{\omega}{k},\\
A_{v_n} &= \frac{\omega}{k}A_{n_n} - b^2\frac{\omega}{k}.
\end{aligned} \tag{29}$$

The nonlinear self-interaction of the carrier wave also leads to the creation of a zeroth order harmonic. Its strength is analytically determined by taking $l = 0$ component of the third-order reduced equations, i.e. for $n' = 2$, $l = 0$. The result is expressed in terms of the square of modulus of $n' = 1$, $l = 1$ quantities, i.e. $|\phi_1^{(1)}|^2 = \phi_1^{(1)}\phi_1^{(1)*}$

$$n_{0p}^{(2)}(2) = B_{n_p}\left|\phi_1^{(1)}\right|^2, \tag{30}$$

$$v_{0p}^{(2)}(2) = B_{v_p}\left|\phi_1^{(1)}\right|^2, \tag{31}$$

$$n_{0n}^{(2)}(2) = B_{n_n}\left|\phi_1^{(1)}\right|^2, \tag{32}$$

$$v_{0n}^{(2)}(2) = B_{v_n}\left|\phi_1^{(1)}\right|^2, \tag{33}$$

$$\phi_0^{(2)} = B_\phi\left|\phi_1^{(1)}\right|^2, \tag{34}$$

where,

$$B_\phi = \frac{\frac{a^2}{\left(\lambda_g^2 - \frac{\sigma_p}{\delta Z_p}\right)}\left(2\lambda_g\omega/k + \frac{1}{\delta a}\right) - \alpha\epsilon_z\frac{b^2}{\left(\lambda_g^2 - \frac{\sigma_n}{\eta\delta Z_p}\right)}\left(2\lambda_g\omega/k - \frac{\epsilon_z}{\eta\delta b}\right) - (1-\alpha\epsilon_z)}{(1-\beta)(1-\alpha\epsilon_z) - \left(\frac{1}{\delta\left(\lambda_g^2 - \frac{\sigma_p}{\delta Z_p}\right)} + \frac{\alpha\epsilon_z^2}{\eta\delta\left(\lambda_g^2 - \frac{\sigma_n}{\eta\delta Z_p}\right)}\right)},$$

$$B_{n_p} = \frac{1}{\delta}\frac{B_\phi}{\left(\lambda_g^2 - \frac{\sigma_p}{\delta Z_p}\right)} + \frac{a^2}{\left(\lambda_g^2 - \frac{\sigma_p}{\delta Z_p}\right)}\left(2\frac{\omega}{k}\lambda_g + \frac{1}{\delta a}\right),$$

$$B_{n_n} = -\frac{\epsilon_z}{\eta\delta}\frac{B_\phi}{\left(\lambda_g^2 - \frac{\sigma_n}{\eta\delta Z_p}\right)} + \frac{b^2}{\left(\lambda_g^2 - \frac{\sigma_n}{\eta\delta Z_p}\right)}\left(2\frac{\omega}{k}\lambda_g - \frac{\epsilon_z}{\eta\delta b}\right),$$

$$B_{v_p} = \lambda_g B_{n_p} - 2\frac{\omega}{k}a^2,$$

$$B_{v_n} = \lambda_g B_{n_n} - 2\frac{\omega}{k}b^2. \tag{35}$$

Finally, substituting the above derived expressions into $l = 1$ component of the third order ($n' = 3$) part of the reduced equation and after a long but straightforward analytical calculations, we obtain the following nonlinear Schrödinger equation (*NLSE*):

$$i\frac{\partial\phi}{\partial\tau} + P\frac{\partial^2\phi}{\partial\xi^2} + Q|\phi|^2\phi = 0, \tag{36}$$

where

$$\begin{aligned}
P = &-\frac{\delta^2}{(1-\alpha\epsilon_z)}\frac{\omega}{k^4}(\lambda_g - \omega/k)^2\left[a^3\left(2\lambda_g\frac{\omega}{k} - \frac{\omega^2}{k^2} - \frac{\sigma_p}{\delta Z_p}\right)\right.\\
&\left.-\frac{\alpha\eta^2}{\epsilon_z}b^3\left(2\lambda_g\frac{\omega}{k} - \frac{\omega^2}{k^2} - \frac{\sigma_n}{\eta\delta Z_p}\right)\right]\\
&-\frac{(\lambda_g - \omega/k)^2}{2\omega} + \frac{(\lambda_g - \omega/k)}{2k},
\end{aligned} \tag{37}$$

and

$$\begin{aligned}
Q = &\frac{(\lambda_g - \omega/k)}{2k}\frac{a^2\delta}{(1-\alpha\epsilon_z)}\left[\frac{3\omega}{2k}(A_{v_p} + B_{v_p}) + (A_{n_p} + B_{n_p})\right.\\
&\left.\times\left(\frac{\omega^2}{k^2} - \frac{\sigma_p}{2\delta Z_p}\right) + a^2\frac{\sigma_p}{\delta Z_p}\right]\\
&+\frac{(\lambda_g - \omega/k)}{2k}\frac{\alpha\eta b^2\delta}{(1-\alpha\epsilon_z)}\left[\frac{3\omega}{2k}(A_{v_n} + B_{v_n}) + (A_{n_n} + B_{n_n})\right.\\
&\left.\times\left(\frac{\omega^2}{k^2} - \frac{\sigma_n}{2\eta\delta Z_p}\right) + b^2\frac{\sigma_n}{\eta\delta Z_p}\right]\\
&-\frac{(\lambda_g - \omega/k)}{2k}\left[A_\phi + B_\phi + \frac{1+3\beta}{2}\right].
\end{aligned} \tag{38}$$

In the *NLSE* (36), we have replaced $\phi_1^{(1)}$ by ϕ for the sake of notational convenience.

4. Stability analysis and discussion

In the standard stability analysis, we linearize around the monochromatic wave solution of the NLSE and modulation on the wave amplitude takes place in the propagation direction. Therefore, we separate the amplitude ϕ into two parts as follows:

$$\phi = [\Phi_0 + \delta\phi(\zeta)]e^{(-i\Lambda\tau)}, \tag{39}$$

where $\zeta = K\xi - \Omega\tau$ is the modulation phase and $0 < K \ll k$ and $\Omega \ll \omega$ are, respectively, the wavenumber and frequency of modulation. Φ_0 is the amplitude of pump carrier wave, the amplitude perturbation $\delta\phi \ll \Phi_0$ and small amplitude perturbation and Λ is a nonlinear frequency shift.

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

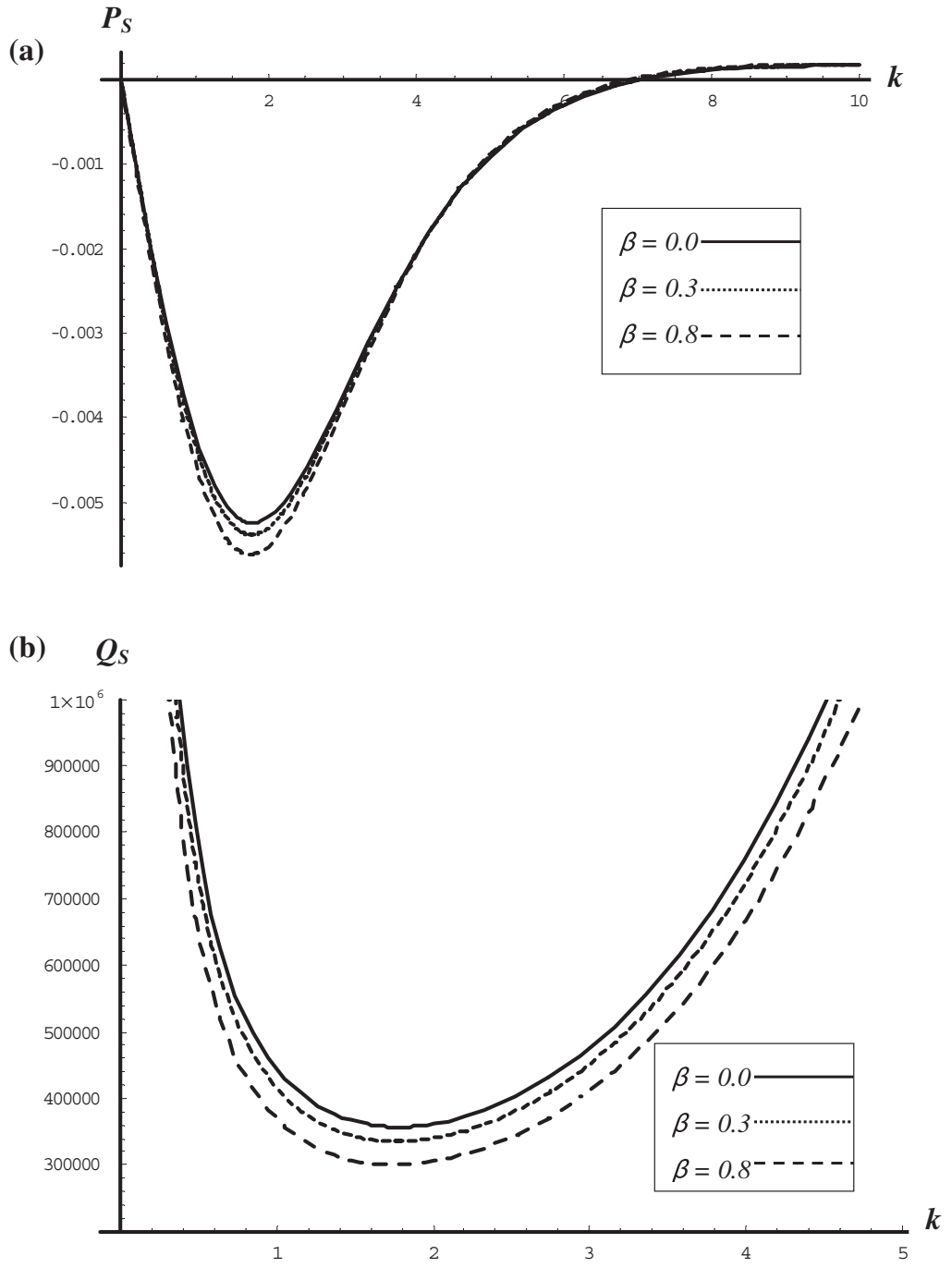


Figure 1. (a). Variation of P_S as a function of k for three different values of β with $\alpha = 0.1$, $\eta = 1$, $\epsilon_z = Z_p = Z_n = 1$, $\sigma_p = 0.1$ and $\sigma_n = 0.01$. (b). Variation of Q_S as a function of k for three different values of β with $\alpha = 0.1$, $\eta = 1$, $\epsilon_z = Z_p = Z_n = 1$, $\sigma_p = 0.1$ and $\sigma_n = 0.01$.

Substituting (39) into (36) and collecting the terms of same order, we obtain

$$\Delta = -Q|\Phi_0|^2, \tag{40}$$

and

$$i\frac{\partial \delta\phi}{\partial \tau} + P\frac{\partial^2 \delta\phi}{\partial \zeta^2} + Q|\Phi_0|^2(\delta\phi + \delta\phi^*) = 0, \tag{41}$$

where $\delta\phi^*$ is complex conjugate of $\delta\phi$. On assuming that the amplitude perturbation varies as $exp[\imath(K\xi - \Omega\tau)]$ and following the standard procedure (Amin et al., 1998), we get

$$\Omega^2 = PK^2(PK^2 - 2Q|\Phi_0|^2). \tag{42}$$

Equation (42) is nonlinear dispersion relation for the amplitude modulation. It is apparent from this relation that $\Omega^2 > 0$ for all $K > 0$ when PQ < 0. In this case Ω is real and waves are stable. However, when PQ > 0, $\Omega^2 < 0$ then $K^2 < \frac{2Q}{P} \mid \Phi_0 \mid^2$ and waves are modulationally unstable. The maximum growth rate is obtained for $K = \sqrt{|\frac{Q}{P}|}|\Phi_0|$ and is given by

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

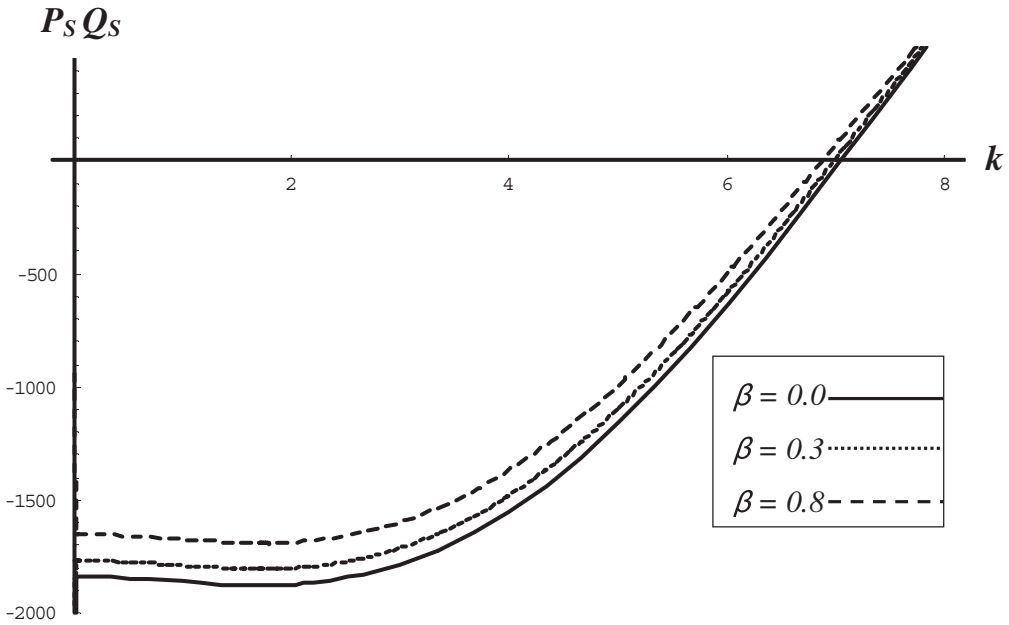


Figure 2. Variation of $P_S Q_S$ as a function of k for three different values of β with $\alpha = 0.1$, $\eta = 1$, $\epsilon_z = Z_p = Z_n = 1$, $\sigma_p = 0.1$ and $\sigma_n = 0.01$.

$\gamma_{max} = Im(\Omega)_{max} = Q|\Phi_0|^2$. It may be noticed that instability sets in for perturbation wavelength $\lambda > \lambda_c$, where $\lambda_c = \frac{2\pi}{K_c}$ and $K_c = \sqrt{|\frac{P}{Q}|}|\Phi_0|$.

Since dispersion relation (15) is a quadratic equation, it has two wave modes *viz*. fast and slow ion-acoustic modes. We here study the instability characteristics of both modes.

4.1. Slow Mode

Firstly, we study the stability profiles of slow ion-acoustic waves for which the expressions for ω/k obtained from (15) and λ_g form (23) are used. The expressions for P and Q given by (37) and (38) show that the coefficients of dispersion term P and nonlinear term Q are related to the relative positive temperature $\sigma_p (= T_p/T_e)$, negative temperature $\sigma_n (= T_n/T_e)$, $\delta (= \frac{\eta+\alpha\epsilon_z^2}{\eta(1-\alpha\epsilon_z)})$, relative density $\alpha (= n_n^{(0)}/n_p^{(0)})$, positive charge Z_p, charge ratio ϵ_z, mass ratio η and of course wave number k. Therefore, we expect that these parameters would affect the stability criteria over a wide range of parameter space which is more pronounced in multi-component plasma and results so obtained will also be compared with those of fast ion-acoustic mode. For this purpose, we have chosen following set of parameters for slow ion-acoustic mode corresponding to (H^+, H^-) plasma (Mishra and Chhabra, 1996; Gill et al. 2004):

$\alpha = 0.1$, $\epsilon_z = Z_p = Z_n = 1$, $\eta = 1$, $\sigma_p = 0.1$, $\sigma_n = 0.01$ and for three different values of β.

We have plotted the graphs of P_S and Q_S for the above mentioned parameters in Figs. 1(a) and (b). From graphs in Fig. 1(b), it is found that Q_S is always positive. However, P_S as a function of k is negative upto certain value of $k = k_s$ beyond which it becomes positive. For the stable propagation of the slow ion-acoustic mode, we have $P_S Q_S < 0$, and for unstable wave propagation we have $P_S Q_S > 0$. Thus for $k < k_s$ (threshold), we find that slow ion-acoustic mode is modulationally stable. However, for $k > k_s$, the mode becomes unstable. It implies that there are two types of stationary solutions *viz*. (i) $P_S Q_S > 0$ and (ii) $P_S Q_S < 0$. For the former case, we have envelope soliton given by

$$\phi = \sqrt{\left|\frac{2\gamma}{Q_S}\right|} sech\left(\left|\frac{\gamma}{P_S}\right|\zeta\right) e^{i\gamma\tau}, \tag{43}$$

where γ is constant (Kakutani and Sugimoto 1974).

For second case, *i. e.* $P_S Q_S < 0$, we obtain modulationally slow ion-acoustic mode with special solution known as envelope dark soliton given by

$$\phi = \sqrt{\left|\frac{\gamma}{Q_S}\right|} \tan h\left(\left|\frac{\gamma}{2P_S}\right|\zeta\right) e^{i\gamma\tau}. \tag{44}$$

To elucidate results for further discussion on possible localized stationary structures of the *NLSE*, we have plotted $P_S Q_S$ as a function of k for the above set of parameters in Fig. 2. Clearly, the graph divides the space into two parts, $P_S Q_S > 0$ and $P_S Q_S < 0$, thus exhibiting the stable and unstable regions of parameter space. Also, the threshold k_s is observed to decrease with increase in β. Earlier investigations show that relative temperatures σ_p and σ_n play crucial role on the mode structure of the multi-component plasma (Mishra and Chhabra 1996; Gill et al. 2004), we have also investigated their role in the propagation characteristics. To highlight this aspect, we have numerically computed $P_S Q_S$ as a function of k under different conditions for σ_p, σ_n and results are plotted in Fig. 3(a). Other parameters are taken as $\alpha = 0.1$, $\beta = 0.1$, $\epsilon_z = Z_p = Z_n = 1$, $\eta = 1$. The solid curve

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

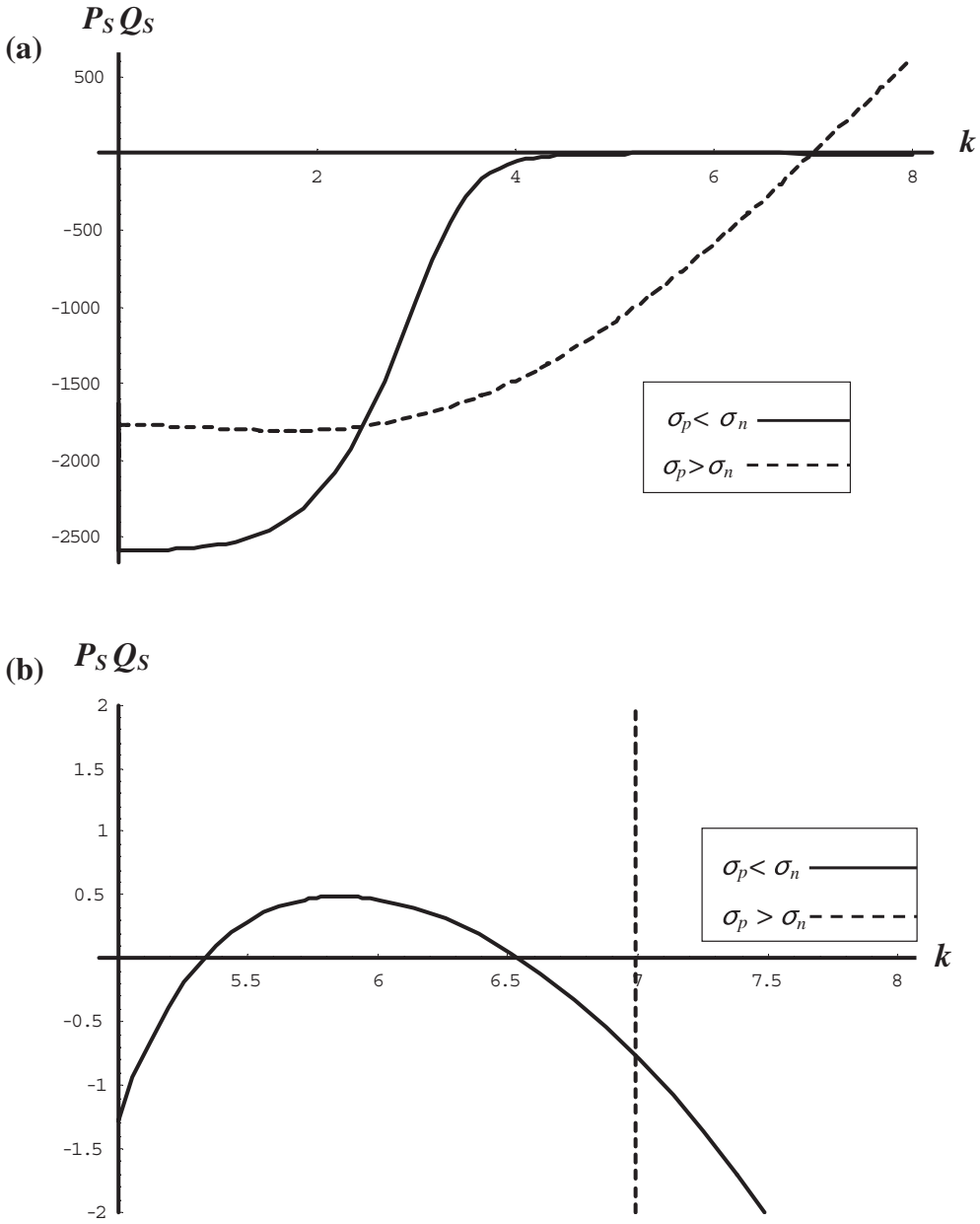


Figure 3. (a). Variation of $P_S Q_S$ as a function of k for two different values of σ_p and σ_n with $\alpha = 0.1$, $\beta = 0.3$ $\eta = 1$ and $\epsilon_z = Z_p = Z_n = 1$. Solid curve for $\sigma_p < \sigma_n$ ($\sigma_p = 0.01$ and $\sigma_n = 0.1$) and dotted curve is for $\sigma_p > \sigma_n$ ($\sigma_p = 0.1$ and $\sigma_n = 0.01$). (b). A more closer view of Fig. 3(a).

corresponds to $\sigma_p < \sigma_n$ while opposite holds for dotted curve. For $\sigma_p > \sigma_n$, we have $P_S Q_S >$ or < 0 leading two types of envelope or dark solitons. However, for $\sigma_p < \sigma_n$, the situation becomes conspicuously interesting and peculiar behaviour is observed in certain range of k values as shown succinctly in Fig. 3(b). By highlighting the particular portion of $P_S Q_S$ *versus* k graph, we have three regions which alternate between stable, unstable and stable solutions, *i.e.* for $k < 5.3$ and > 6.6, we have dark solitons whereas for $k > 5.3$ and < 6.6, the modulationally unstable slow ion-acoustic mode exhibiting envelope solitons. To understand as how stability profile depends on $\alpha = (n_n^{(0)}/n_p^{(0)})$, we have plotted $Q_S = 0$ contours for α–k space. The unstability region decreases when $\sigma_p < \sigma_n$ and increases for $\sigma_p > \sigma_n$, as shown in Fig. 4(a). In a similar contour graph for $\sigma_p - k$ is shown in Fig. 4(b). It is observed that increase of negative ion temperature decreases the threshold k_s.

4.2. Fast Mode

In earlier investigation of Mishra et al. (1993), P_F, the coefficient of dispersive term is independent of σ_p, σ_n, η, ϵ_z, α and k [see (28) of Mishra et al. (1993)]. However, here P_S explicitly depends on these parameters. Thus width and amplitude are significantly modified by variation of these parameters. However, when $\sigma_p = \sigma_n = 0$, our expression for P_F reduces to that of Mishra et al. (1993). In order to study and compare the stability properties of fast ion-acoustic mode with those of slow ion-acoustic mode, we have plotted P_F and Q_F as a

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

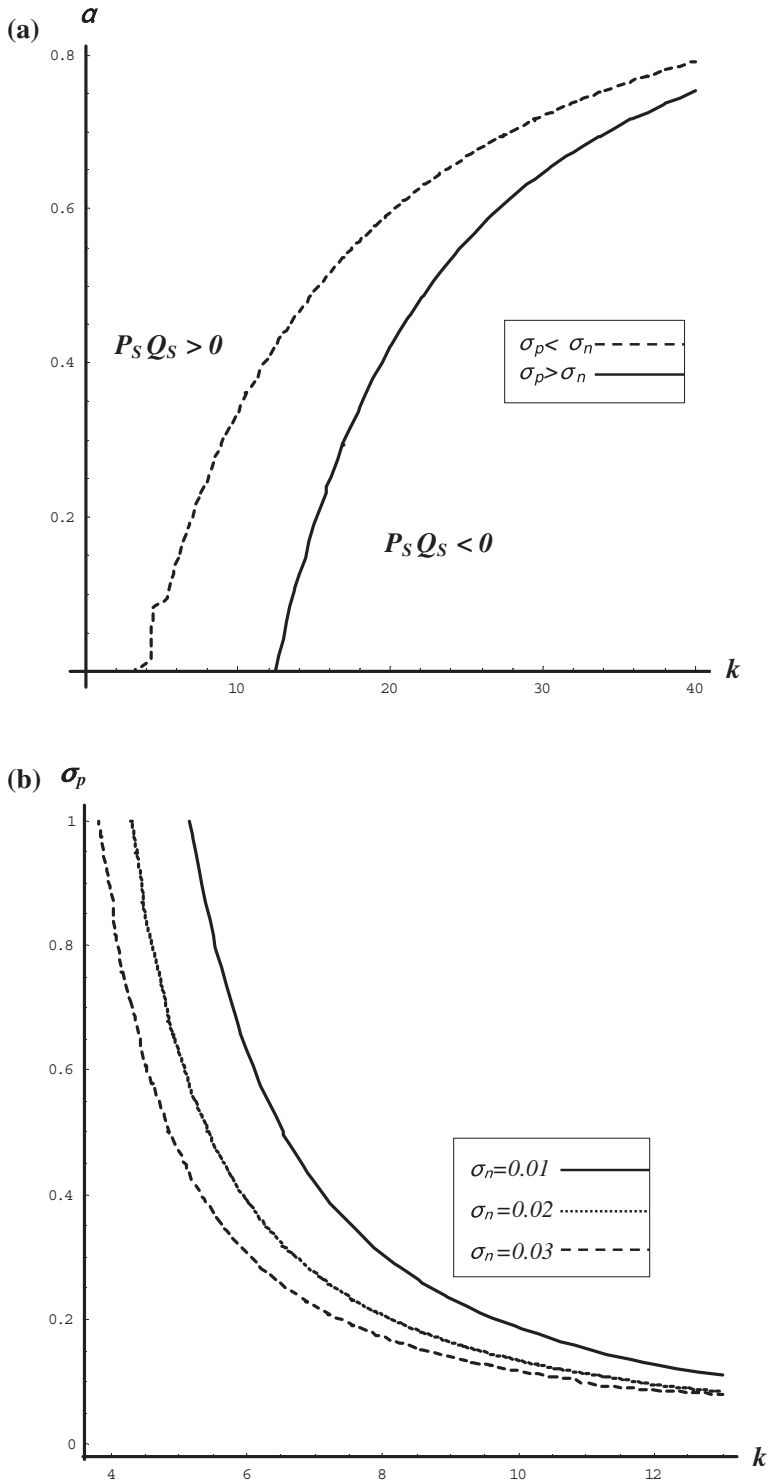


Figure 4. (a). Plot of $Q_S = 0$ in $(\alpha - k)$ plane for two different values of σ_p and σ_n with $\beta = 0.1$ $\eta = 1$ and $\epsilon_z = Z_p = Z_n = 1$. Solid curve for $\sigma_p > \sigma_n$ ($\sigma_p = 0.1$ and $\sigma_n = 0.01$) and dotted curve is for $\sigma_p < \sigma_n$ ($\sigma_p = 0.01$ and $\sigma_n = 0.1$). (b). Plot of $Q_S = 0$ in $(\sigma_p - k)$ plane for three different values of σ_n with $\alpha = 0.1$, $\beta = 0.1$ $\eta = 1$ and $\epsilon_z = Z_p = Z_n = 1$.

function of k for three different values of β and plotted their graphs in Figs. 5(a) and (b). The other parameters chosen for H^+H^- plasma are as follows:

$\alpha = 0.1$, $\epsilon_z = Z_p = Z_n = 1$, $\eta = 1$, $\sigma_p = 0.1$, $\sigma_n = 0.01$ and for three different values of β.

P_F is negative over a very large range of k that becomes zero and finally changes its sign to positive. The effect of β significantly increases the the absolute magnitude of P_F. We find that P_F and Q_F are only negative corresponding to $k = k_f \approx 2.6$, thus leading to modulationally fast ion-acoustic stable mode. For very small $k = k_f$, there is decrease of threshold k_f with increase in non-thermal parameter, particularly for $\beta = 0.3$ and $k < 0.1$ we have stable propagation. One may

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

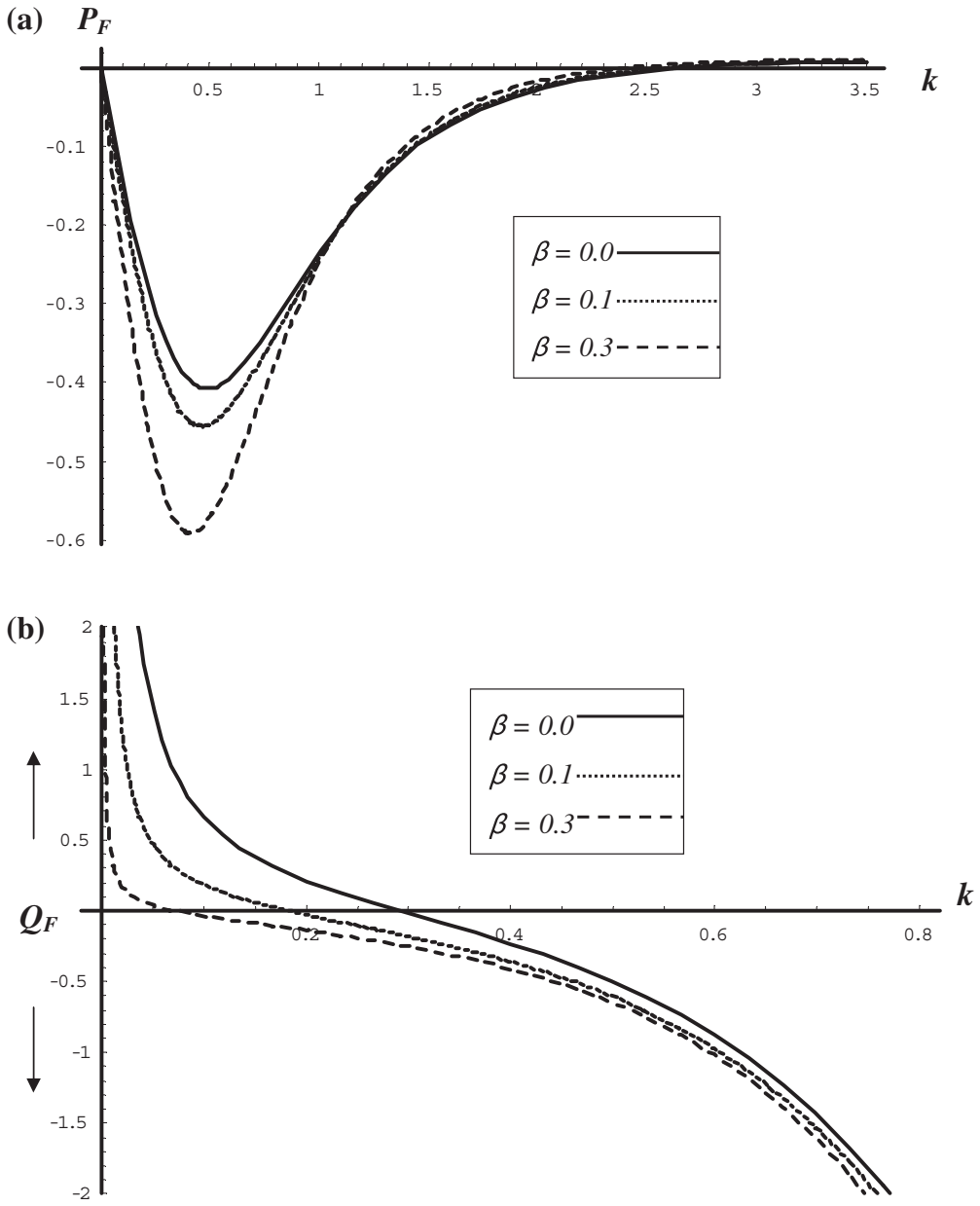


Figure 5. (a). Variation of P_F as a function of k for three different values of β with $\alpha = 0.1$, $\eta = 1$, $\epsilon_z = Z_p = Z_n = 1$, $\sigma_p = 0.1$ and $\sigma_n = 0.01$. (b). Variation of $Q_F = 0$ as a function of k for three different values of β with $\alpha = 0.1$, $\eta = 1$, $\epsilon_z = Z_p = Z_n = 1$, $\sigma_p = 0.1$ and $\sigma_n = 0.01$.

note that magnitude of $| P_F |$ increases significantly with increase in β. Thus there are two types of stationary solutions. (i) $P_F Q_F > 0$ and (ii) $P_F Q_F < 0$. For the unstable first case, we have unstable mode with the following envelope solutions given by:

$$\phi = \sqrt{\left|\frac{2\gamma}{Q_F}\right|} sech\left(\left|\frac{\gamma}{P_F}\right|\zeta\right) e^{i\gamma\tau}. \tag{45}$$

In the second case, $P_F Q_F < 0$, we have special solution known as envelope dark soliton given by

$$\phi = \sqrt{\left|\frac{\gamma}{Q_F}\right|} \tan h\left(\left|\frac{\gamma}{2P_F}\right|\zeta\right) e^{i\gamma\tau}, \tag{46}$$

where γ is constant.

As obvious from (45) and (46), width and amplitude of the soliton varies with P_F and Q_F. We further note that width is proportional to $|P_F|$ and wave amplitude is inversely proportional to $|Q_F|$. Figures 5(a) and (b) exhibit the variation of P_F and Q_F for different β. Apparently, it is observed from Fig. 5(a) that soliton width for fast ion-acoustic mode increases with β. That is, the width of the wave packet of multi-component plasma containing non-thermal electrons is greater than that in plasma with isothermal electrons. However, increase in β leads to faster decrease in Q_F resulting with subsequent increase in peak amplitude [Fig. 5(b)].

For the sake of comparison of stability properties of slow and fast ion-acoustic mode, we have plotted $P_F Q_F$ as a function of k, both for $\sigma_p > \sigma_n$ and

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

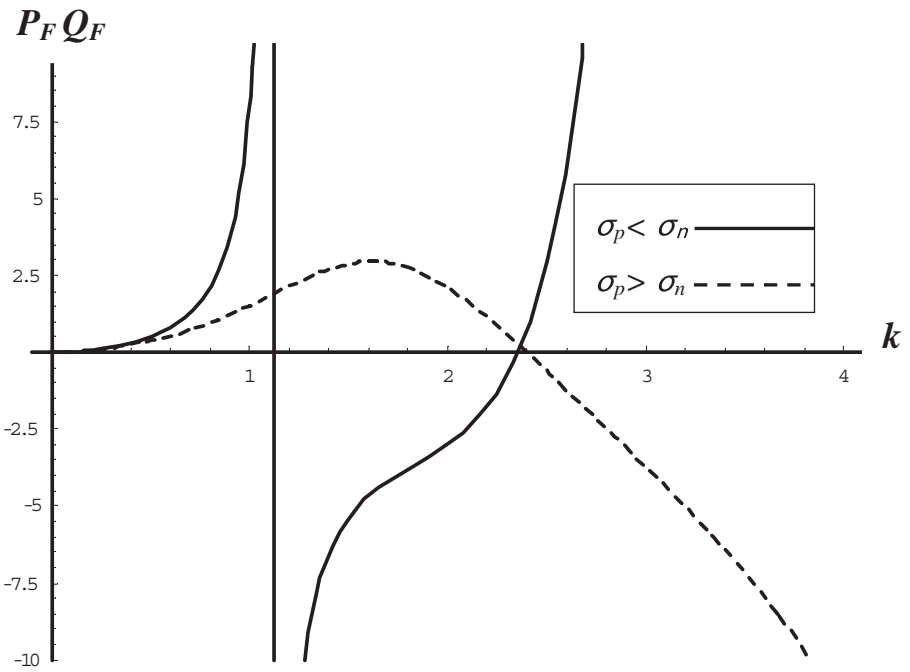


Figure 6. Variation of $P_F Q_F$ as a function of k for two different values of σ_p and σ_n with $\alpha = 0.1$, $\beta = 0.3$ $\eta = 1$ and $\epsilon_z = Z_p = Z_n = 1$. Solid curve for $\sigma_p < \sigma_n$ ($\sigma_p = 0.01$ and $\sigma_n = 0.1$) and dotted curve is for $\sigma_p > \sigma_n$ ($\sigma_p = 0.1$ and $\sigma_n = 0.01$).

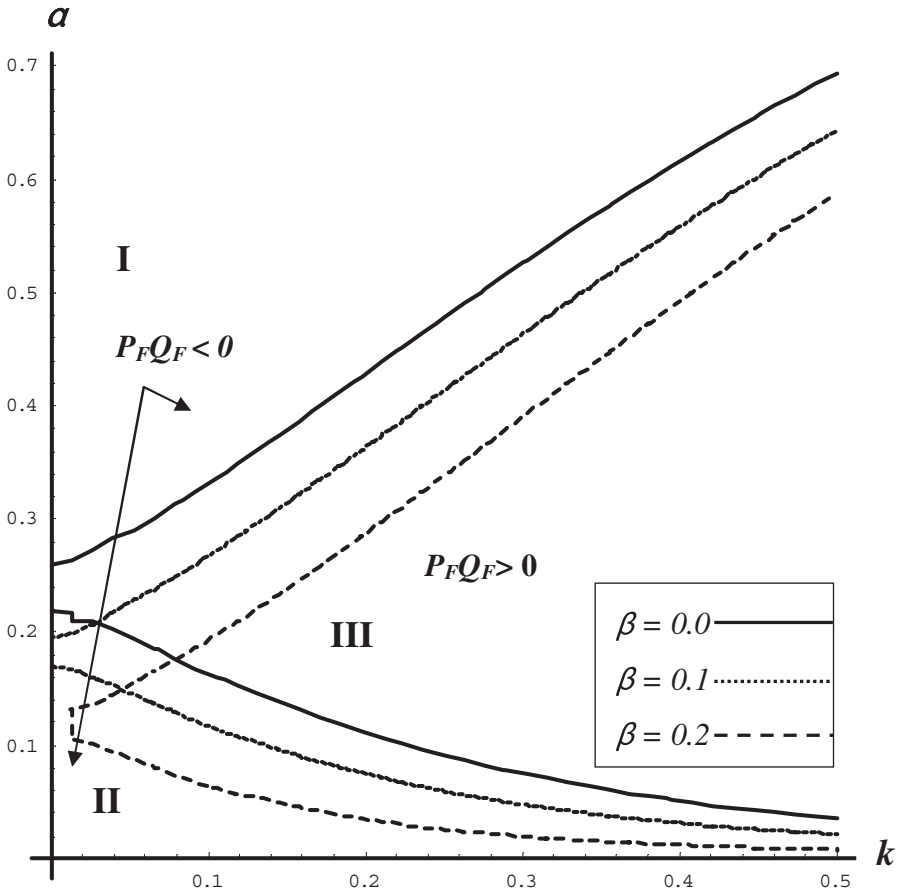


Figure 7. Plot of $Q_F = 0$ in $(\alpha - k)$ plane for three different values of β with $\alpha = 0.1$, $\eta = 1$ and $\epsilon_z = Z_p = Z_n = 1$. Regions I and II correspond to $P_F Q_F < 0$ (stable) and region III corresponds to $P_F Q_F > 0$ (unstable).

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

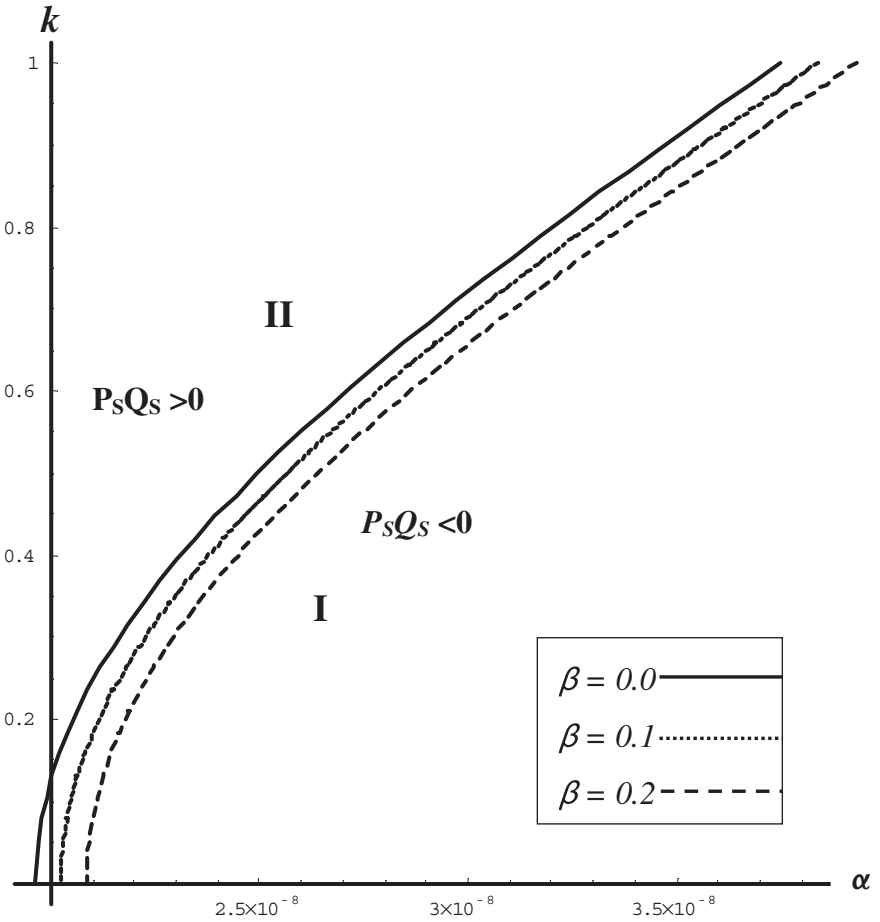


Figure 8. Plot of $Q_F = 0$ in $(k - \alpha)$ plane for three different values of β with $\alpha = 0.1$, $\eta = 1$ and $\epsilon_z = Z_p = Z_n = 1$. Region I corresponds to $P_F Q_F < 0$ (stable) and region II corresponds to $P_F Q_F > 0$ (unstable).

$\sigma_p < \sigma_n$ and the results are shown in Fig. 6. Some interesting observations are noteworthy, *e.g.* for $\sigma_p < \sigma_n$, we obtain unstable envelope solitons upto certain k. However, there is a band of k values where singularity is observed, and so, solutions cannot be possible without taking higher order nonlinear and dispersion effects. Beyond stop band, we again obtain both dark and envelope solitons for fast ion-acoustic mode. But in case of $\sigma_p > \sigma_n$, unstable envelope solitons are obtained upto $k \approx 2.3$, beyond which modulationally stable fast ion-acoustic dark solitons exist. This is in contrast to stability characteristics obtained for slow IAW.

Figure 7 shows the contours of Q_F in the parameter space α-k. As mentioned in the earlier discussion, Q_F has different signs under different parameter regimes. The parameter space is divided into three different regions labeled as I, II and III. While the upper (I) and lower (II) regions correspond to stable region for wave packet propagation, the central region (III) depicts the unstable mode ($P_F Q_F > 0$). Two important observations from the Fig. 7 are made:

(i) While stable upper region (I) increases with β, lower region (II) decreases with β. (ii) In contrast to slow mode [Fig. 8] only two contour regions in the parameter space are observed.

5. Conclusions

In this paper, we have investigated the stability properties of ion-acoustic waves in a multi-component plasma consisting of positive and negative ions with non-thermal electrons. Using reductive perturbation, dispersion relation is obtained. This corresponds to two modes, *viz.* slow ion-acoustic mode and fast ion-acoustic mode. We have derived nonlinear Schrödinger equation which governs the dynamics of modulational instability. We have studied the stability characteristics of both modes. Some of the important results are summarized as follows:

(i) Coefficients of nonlinear and dispersive terms depend on the number of parameters such as σ_p, σ_n, η, α, ϵ_z and k. Slow ion-acoustic mode is modulationally stable upto certain range of k_s and envelope solitons are obtained.

(ii) For $k > k_s$, slow ion-acoustic wave is modulationally unstable possessing envelope solitons.

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

(iii) The threshold k_s for slow ion-acoustic mode decreases with increase in non-thermal parameter β.

(iv) When $\sigma_p > \sigma_n$, both dark and envelope solitons are obtained. However, for $\sigma_p < \sigma_n$, we obtain alternatively stable, unstable and stable solutions.

(v) Region of stability decreases with increase in σ_p.

(vi) For $\sigma_p < \sigma_n$, there is stop band of k-values where no solutions exist. For k below the stop band, fast ion-acoustic wave is modulationally unstable. Far from stop band, both unstable as well as stable solitons are observed. However, for $\sigma_p > \sigma_n$, unstable and stable characteristics are observed.

(vii) In case of fast ion-acoustic mode, the threshold k_f decreases with increase in non-thermal parameter β.

(viii) For a given β=0.2, $k_f > 0.2$ and < 2.7, the fast ion-acoustic mode is modulationally stable. However, for $k > 2.7$, this mode is unstable.

(ix) For the fast ion-acoustic mode, the contour map of $\alpha - k$ shows that parameter space is divided into three regions, two of which are stable and one unstable. On the other hand, for slow IA mode, only two regions, *i.e.* unstable and stable in parameter space are obtained.

References

Abdullaev, F. K., Bouketir, A., Messikh, A. and Umarov, B. A. 2007 Modulational instability and discrete breathers in the discrete cubic-quintic nonlinear Schrödinger equation. *Physica* D **232**, 54–61.

Aggrawal, G. P. 2007 *Nonlinear Fibre Optics*. San Diego, CA: Academic Press.

Amin, M. R., Morfill, G. E. and Shukla, P. K. 1998 Modulational instability of dust-ion-acoustic waves. *Phys. Rev.* E **58**, 6517–6523.

Bailung, H., Chutia, J. and Nakamura, Y. 1996 Propagation of solitary ion wavepacket in multi-component plasma with negative ions. *Chaos Solitons Fractals* **7**, 21–24.

Bailung, H. and Nakamura, Y. 1993 Observation of modulational instability in a multicomponent plasma with negative ions. *J. Plasma Phys.* **2**, 231–242.

Ballav, M. and Chowdhury, A. R. 2007 A generalized nonlinear Schrödinger equation and optical soliton in a gradient index cylindrical media. *Chaos Solitons Fractals* **31**, 794–803.

Benjamin, T. B. and Feir, J. E. 1967 The disintegration of wave trains in deep water Part I. *J. Fluid Mech.* **27**, 417–430.

Bharuthram, R. 1992 Arbitrary amplitude double layers in a multi-species electron positron plasma. *Astrophys Space Sci.* **189**, 213–222.

Bilbault, J. M., Marquie, P. and Michaux, B. 1995 Modulational instability of two counterpropagating waves in an experimental transmission line. *Phys. Rev.* E **51**, 817–820.

Cairns, R. A., Mamun, A. A., Bingham, R., Bostrom, R., Dendy, R. O., Nairn, C. M. C. and Shukla, P. K. 1995 Electrostatic solitary structures in nonthermal plasmas. *Geophys. Res. Lett.* **22**, 2709–2712.

Chhabra, R. S. and Sharma, S. R. 1986 Modulational instability of obliquely modulated ion-acoustic waves in a two ion plasma. *Phys Fluids.* **29**, 128–132.

Dauxois, T. and Peyrard, M. 2006 *Physics of Solitons*. Cambridge, UK: Cambridge University Press.

Davidson, R. C. 1972 *Methods in Nonlinear Plasma Theory*. New York and London: Academic Press, pp. 37.

Davydov, A. S. 1985 *Solitons in Molecular Systems*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Duan, W. S. 2006 3+1 dimensional envelope waves and its stability in magnetized dusty plasma. *Chaos Solitons Fractals* **27**, 926–929.

El-Dib, Y. O. 2000 Instability of parametrically second- and third-subharmonic resonances governed by nonlinear Schrödinger equations with complex coefficients. *Chaos Solitons Fractals* **11**, 1773–1787.

El-labany, S. K., Krim, M. S. A., El-Warraki, S. A. and El-Taibany, W. F. 2003 Modulational instability of a weakly relativistic ion acoustic wave in a warm plasma with nonthermal electrons. *Chin. Phys. Soc.* **12**, 759–764.

El-Shewy, E. K. 2007 Higher order solution of an electron-acoustic solitary waves with nonthermal electrons. *Chaos, Solitons Fractals* **34**, 628–638.

Ganapathy, R., Malomed, B. A. and Porsezian, K. 2006 Modulational instability and generation of pulse trains in asymmetric dual-core nonlinear optical fibers. *Phys. Lett.* A **354**, 366–372.

Gill, T. S., Bala, P., Kaur, H. and Saini, N. S. 2004 Ion-acoustic solitons and double-layers in a plasma consisting of positive and negative ions with non-thermal electrons. *Eur. Phys. J.* D **31**, 91–100.

Gill, T. S., Kaur, H., Bansal, S., Saini, N. S. and Bala, P. 2007 Modulational instability of electron-acoustic waves: an application to auroral zone plasma. *Eur. Phys. J.* D **41**, 151–156.

Gill, T. S., Kaur, H. and Saini, N. S. 2006 Small amplitude electron-acoustic solitary waves in a plasma with nonthermal electrons. *Chaos, Solitons Fractals* **30**, 1020–1024.

Griffiths, S. D., Grimshaw, R. H. J. and Khusnutdinova, K. R. 2006 Modulational instability of two pairs of counter-propagating waves and energy exchange in a two-component system. *Physica* D **214**, 1–24.

Guio, P., Borve, S., Daldorff, L. K., Lynov, J. P., Michelsen, P., Pecseli, H. L. and Rasmussen, J. J., Saeki, K. and Trulsen. 2003 Phase space vortices in collisionless plasmas. *J. Nonlinear Process. Geophys.* **10**, 75–86.

Hertzberg, M. P., Cramer, N. F. and Vladimirov, S. V. 2004 Modulational and decay instabilities of Alfven waves in a multicomponent plasma. *J. Geophys. Res.* **109**, A02103(1–10).

Infeld, E. and Rowlands, G. 1990 *Nonlinear Waves, Solitons and Chaos*. Cambridge, UK: Cambridge University Press.

Johansson, M. 2006 Discrete nonlinear Schrödinger approximation of a mixed Klein-Gorden/Fermi-Pasta-Ulam chain: modulational instability and a statistical condition for creation of thermodynamic breathers. *Physica* D **216**, 62–70.

Jukui, X. 2003 Modulational instability of ion-acoustic waves in a plasma consisting of warm ions and non-thermal electrons. *Chaos, Solitons Fractals* **18**, 849–853.

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

Kakutani, T. and Sugimoto, N. 1974 Krylov-Bogoliuobov-Mitropolsky method for nonlinear wave modulation. *Phys. Fluids* **17**, 1617–1625.

Kourakis, I., Esfandyari, A., Mehdipoor, M. and Shukla, P. K. 2006 Modulated electrostatic modes in pair plasmas: modulaional stability profile and envelope excitations. *Phys. Plasmas* **13**, 052117(1–9).

Kourakis, I. and Shukla, P. K. 2004b Modulated wave packets and enveope solitary structures in complex plasmas. *IEEE Trans. Plasma Sci.* **32**, 573–581.

Kourakis, I. and Shukla, P. K. 2004a Oblique amplitude modulation of dust-acoustic plasma waves. *Phys. Scripta* **69**, 316–327.

Kourakis, I. and Shukla, P. K. 2005c Exact theory for localized envelope modulated electrostatic wavepackets in apace and dusty plasmas. *Nonlinear Process. Geophys.* **12**, 407–423.

Kourakis, I., Shukla, P. K., Marklund, M. and Stenflo, L. 2005b Modulational instability criteria for two-component Bose-Einstein condensates. *Eur. Phys. J.* B **46**, 381–384.

Kourakis, I., Shukla, P. K. and Morfill, G. 2005a Modulational instability and localized excitations involving two nonlinearly coupled upper-hybrid waves in plasmas. *New J. Phys.* **7**, 153(1–18).

Kourakis, I., Verheest, F. and Cramer, N. 2007 Nonlinear perpendicular propagation of ordinary mode electromagnetic wave packets in pair plasmas and electron-positron-ion plasmas. *Phys. Plasmas* **14**, 022306.

Law, K. J. H., Kevrekidis, P. G., Frantzeskakis, D. J. and Bishop, A. R. 2008 Emergence of unstable modes in an expanding domain for energy-conserving wave equations. *Phys. Lett.* A **372**, 658–664.

McFadden, J. P., Carlson, C. W., Ergun, R. E., Mozer, F. S., Muschietti, L., Roth, I. and Moebius, E. 2003 FAST observations of ion solitary waves. *J. Geophys. Res.* **108**, 8018.

Mishra, M. K. and Chhabra, R. S. 1996 Ion-acoustic compressive and rarefactive solitons in a warm multicomponent plasma with negative ions. *Phys. Plasmas* **3**, 4446–4454.

Mishra, M. K., Chhabra, R. S. and Sharma, S. R. 1993 Modulational instability of ion-acoustic waves in a plasma with negative ions. *Phys. Rev.* E **48**, 4642–4647.

Mishra, M. K., Chhabra, R. S. and Sharma, S. R. 1994 Stability of oblique modulation of ion-acoustic waves in a multicomponent plasma. *Phys Plasmas* **1**, 70–75.

Mishra, M. K., Chhabra, R. S. and Sharma, S. R. 1995 Nonlinear oblique modulation of ion-acoustic waves in a multicomponent plasma with negative ions *Phys. Rev.* E **51**, 4790–4795.

Mishra, A. P. and Chowdhury, A. R. 2003 Modulated ion-acoustic wave near criticalness—a new nonlinear Schrödinger equation in a collisional plasma. *FIZIKA* A **11**, 163–176.

Mishra, A. P. and Chowdhury, A. R. 2006a Dust-acoustic waves in a self-gravitating complex plasma with trapped electrons and nonisothermal ions. *Eur. Phys. J.* D **37**, 105–113.

Mishra, A. P. and Chowdhury, A. R. 2006 Modulational instability of dust acoustic waves in a dusty plasma with nonthermal electrons and ions. *Eur. Phys. J.* D **39**, 49–57.

Ndzana, F., Mohamadou, A. and Kofane, T. C. 2007 Modulational instability in the cubic-quintic nonlinear Schrödinger equation through the variational approach. *Opt. Commun.* **275**, 421–428.

Nohara, B. T., Arimoto, A. and Saigo, T. 2008 Governing equations of envelopes created by nearly bichromatic waves and relation to the nonlinear Schrödinger equation. *Chaos Solitons Fractals* **35**, 942–948.

Pottelette, R., Ergun, R. E., Truemann, R. A., Bethomier, M., Carlson, C. W., McFadden, J. P. and Roth, I. 1999 Modulated electron-acoustic waves in auroral density cavities:FAST observations. *Geophys. Res. Lett.* **26**, 2629–2632.

Rapti, Z., Trombettoni, A., Kevrekidis, P. G., Frantzeskakis, D. J., Malomed, B. A. and Bishop, A. R. 2004 Modulational instabilities and domain walls in coupled discrete nonlinear Schrödinger equations. *Phys. Lett.* A **330**, 95–106.

Remoissenet, M. 1994 *Waves Called Solitons*. Berlin, Germany: Springer-Verlag.

Saeki, K., Michelsen, P., Pecseli, H. L. and Rasmussen, J. J. 1979 Formation and caoalescence of electron solitary holes. *Phys. Rev. Lett.* **42**, 501–504.

Sahu, B. and RoyChoudhury, R. 2006 Modulational instability of dust acoustic waves in a dusty plasma with nonthermal electrons and ions. *Phys. Plasmas* **13**, 072302(1–5).

Salahuddin, M. 2002 Ion-acoustic envelope solitons in electron-positron-ion plasmas. *Phys. Rev.* E **66**, 036407(1–4).

Shivamogi, B. K. 1988 *Introduction to Nonlinear Fluid-Plasma Waves*. Doredrecht, The Netherlands: Kluwer Academic Press.

Shukla, P. K., Stenflo, L. and Fedele, R. 2001 Modulational instability of two colliding Bose-Einstein condensates. *Phys. Scripta* **64**, 553.

Singh, S. V. and Lakhina, G. S. 2004 Electron-acoustic solitary waves with nonthermal distribution of electrons. *Nonlinear Process Geophys.* **11**, 275–279.

Swider, W. 1988 *Ionospheric Modeling* (ed. J. N. Korenkov). Basel, Switzerlsnd: Birkhauser, pp. 403.

Tan, B. and Boyd, J. P. 2001 Stability and long time evolution of the periodic solutions to the two coupled nonlinear Schrödinger equations. *Chaos Solitons Fractals* **12**, 721–734.

Tang, R. A. and Jukui, X. 2003 Stability of oblique modulation of dust-acoustic waves in a warm dusty plasma with dust charge variation. *Phys. Plasmas* **10**, 3800–3803.

Tang, R. A. and Xue, J. K. 2004 Nonthermal electrons and warm ion effects on oblique modulation of ion-acoustic waves. *Phys. Plasmas* **11**, 3939–3944.

Theocharis, G., Rapti, Z., Kevrekidis, P. G., Frantzeskakis, D. J. and Konotop, V. V. 2003 Modulational instability of Gross-Pitaevskii-type equations in 1+1 dimensions. *Phys. Rev.* A **67**, 063610(1–8).

Tstovich, V. and Wharton, C. B. 1978 Laboratory electron-positron plasma. A new research object. *Comments Plasma Phys. Control. Fusion* **4**, 91–100.

Tsukabayashi, I. and Nakamura, Y. 1990 Abstract no. 139200. *Phys. Absr.* **93**, 1388.

Verheest, F., Hellberg, M. A., Gray, G. J. and Mace, R. L. 1995 Electrostatic solitons in multispecies electron-positron plasmas. *Astrophys. Space Sci.* **239**, 125–139.

Vladimirov, S. V., Tsytovich, V. N., Popel, S. I. and Khakimov, F. K. 1995 *Modulational Interactions in Plasmas*. Norwell, MA.: Kluwer Academic Press.

Washimi, H. and Taniuti, T. 1966 Propagation of ion-acoustic solitary waves of small amplitude. *Phys. Rev. Lett.* **17**, 996–998.

Watanabe, W. 1977 Self-modulation of a nonlinear ion wave packet. *J. Plasma Phys.* **17**, 487–501.

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press

Whitam, G. B. 1967 Nonlinear-dispersion of water waves. *J. Fluid Mech.* **27**, 399–412.

Xue, J. 2004a Modulational instability of multi-dimensional dust ion-acoustic waves. *Phys. Lett*. A **330**, 390–395.

Xue, J. 2004 Propagation of nonplanar dust-acoustic envelope solitary waves in a two-ion-temperature dusy plasma. *Phys. Plasmas* **11**, 1860–1865.

Xue, J. 2005 Modulational instability of the trapped Bose-Einstein condensates. *Phys. Lett*. A **341**, 527–531.

Xue, J. and He, L. 2003 Modulational instability of cylindrical and spherical dust ion-acoustic waves. *Phys. Plasmas* **10**, 339–342.

Yemele, D. and Marquie, P. 2003 Chaotic-like behaviour of modulated waves in a nonlinear discrete LC transmission line. *Chaos Solitons Fractals* **15**, 465–473.

Zhu, S. 2007 Exact solutions for the high order dispersive cubic-quintic nonlinear Schrödinger equation by extended hyperbolic auxiliary equation method. *Chaos Solitons Fractals* **34**, 1608–1612.

https://doi.org/10.1017/S0022377812000013 Published online by Cambridge University Press