J. Plasma Physics (2010), *vol.* 76, *part* 1, *pp.* 87–99. © Cambridge University Press 2009
doi:10.1017/S0022377809990055

Surface waves in magnetized quantum electron-positron plasmas

A. P. MISRA[1], N. K. GHOSH[2] and P. K. SHUKLA[3]

[1] Department of Mathematics, Visva-Bharati University, Santiniketan-731 235, India
(apmisra@visva-bharati.ac.in)

[2] Gangapuri Siksha Sadan, Purba Putiary, Kolkata-700 093, India

[3] Institut für Theoretische Physik IV and Centre for Plasma Science and Astrophysics, Fakultät für Physik and Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany
(ps@tp4.rub.de)

(Received 6 March 2009 and accepted 17 April 2009, first published online 5 June 2009)

Abstract. The dispersion properties of electrostatic surface waves propagating along the interface between a quantum magnetoplasma composed of electrons and positrons, and vacuum are studied by using a quantum magnetohydrodynamic plasma model. The general dispersion relation for arbitrary orientation of the magnetic field and the propagation vector is derived and analyzed in some special cases of interest (viz. when the magnetic field is directed parallel and perpendicular to the boundary surface). It is found that the quantum effects facilitate the propagation of electrostatic surface modes in a dense magnetoplasma. The effect of the external magnetic field is found to increase the frequency of the quantum surface wave. The existence of a singular wave on the boundary surface is also proved, and its properties are analyzed numerically. It is shown that the new wave characteristics appear due to the Rayleigh type of the wave.

1. Introduction

The subject of electron-positron (e-p) plasmas, which are composed of particles of the same mass but opposite charge, is not only important in astrophysics, but also relevant in laboratory experiments. Phenomena related to e-p plasmas are frequently encountered in the early universe [1, 2], in the bipolar outflows (jets) in active galactic nuclei [3, 4], near the polar caps of fast rotating neutron stars [5], in the inner regions of the acceleration disks surrounding black holes [6], at the center of our own galaxy [7], in intense laser fields [8], as well as in semiconductor plasmas [9]. Recent advances in creating non-relativistic e-p plasmas in laboratories opened up not only the understanding of various physical phenomena, but also the development of plasma sources in microwave discharges using bulk or surface waves [10]. In such plasmas, there are various new linear and nonlinear wave modes and structures [11–15]. When the magnetized pair plasmas are extremely dense, the dispersion caused by quantum electron tunneling can also play an important role [16, 17].

The propagation of surface waves on the interface between a vacuum and a quantum plasma has attracted much attention because of wide applications in

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

many areas such as laser physics, plasma spectroscopy, plasma technology, and surface science [18–22]. Recent developments of surface wave phenomena in quantum plasmas include Alfvén surface modes in a dusty e-p plasma with spin-1/2 effects [12], surface Langmuir oscillations in semi-bounded quantum plasmas [23], electrostatic/electromagnetic surface modes in a quantum plasma half-space [24], etc. In the classical case, the pioneer works for bounded gyrotropic plasmas are by Fainberg and Gorabatenko [25], Allis et al. [26] for gas discharge plasmas and by Legendy [27] for semiconductor plasmas. Moreover, the effects of density gradients and a finite temperature on the dispersion properties of surface waves on a plasma half-space were investigated in the early days by Kaw et al. [28].

It is well known that in classical bounded gyrotropic plasmas there are two normal modes with different transverse wave decay constants $\Gamma_1(\omega, k)$ and $\Gamma_2(\omega, k)$. On the boundary curve of a complex zone where the two constants are complex conjugate, the equality of the real parts of $\Gamma_1(\omega, k)$ and $\Gamma_2(\omega, k)$ is satisfied. Existence of such complex zones in classical gyrotropic plasmas has been proved by Ivanov et al. [29, 30]. They showed that the points of the complex-zone boundary are neither degenerate waves nor ones with zero field components. They also proved that the intersection points of the complex-zone loop with the dispersion curve describe singular waves.

In this work we study the dispersion properties of electrostatic surface waves along an interface between a semi-bounded quantum magnetoplasma consisting of electron and positrons, and vacuum. By using the quantum magnetohydrodynamic model, we derive the general dispersion relation and carry out a numerical analysis in some special cases of interest. The condition for the existence of a singular wave is given and its appearance is described by the points of intersection of the boundary curve and the singular wave dispersion curve for a fixed set of parameters.

2. Basic equations and dispersion relation

We consider a homogeneous semi-infinite magnetized quantum electron-positron plasma filling the half-space $z > 0$, bounded by vacuum ($z < 0$). The external magnetic field lies in the x–z plane making an angle θ with the z axis. The propagation vector lies in the x–y plane by making an angle φ with the y axis (see Fig. 1).

The dynamics of electrons ($\alpha = \mathrm{e}$) and positrons ($\alpha = \mathrm{p}$) are governed by the equations

$$\frac{\partial n_\alpha}{\partial t} + \nabla.(n_\alpha \mathbf{v}_\alpha) = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\mathbf{v}_\alpha = \frac{q_\alpha}{m}(\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B}) - \frac{1}{mn_\alpha}\nabla P_\alpha + \frac{\hbar^2}{2m^2}\nabla\left(\frac{1}{\sqrt{n_\alpha}}\nabla^2\sqrt{n_\alpha}\right), \tag{2}$$

$$\nabla \cdot \mathbf{E} = \sum_\alpha \frac{q_\alpha n_\alpha}{\epsilon_0}, \tag{3}$$

where we have included the quantum force (tunneling) associated with the Bohm potential (last term in (2)) and the quantum statistical effects via the equation of state [31]

$$P_\alpha = \frac{1}{5}\frac{mV_{F_\alpha}^2 n_\alpha^{5/3}}{n_0^{2/3}}, \tag{4}$$

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

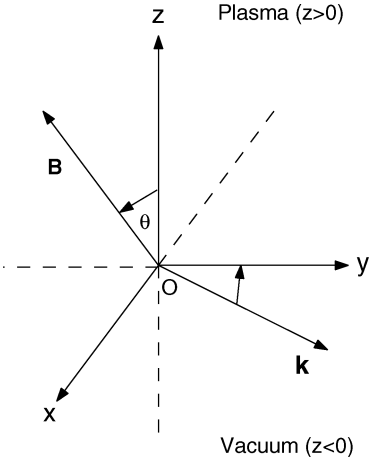


Figure 1. Schematic diagram of a quantum plasma half-space. The plasma–vacuum interface is at $z = 0$. The external magnetic field **B** lies in the x–z plane by making an angle θ with the z axis in the anticlockwise sense. The propagation vector **k** lies in the x–y plane by making an angle φ with the y axis in the anticlockwise sense.

which pertains to a three-dimensional zero-temperature Fermi gas. In (1)–(3), n_α, $\mathbf{v}_\alpha$ represent the particle number density (with equilibrium value n_0) and velocity; m is the particle mass, **E** is the electric field defined by $\mathbf{E} = -\nabla\phi^{\rm p}$, where $\phi^{\rm p}$ is the electrostatic plasma potential, $\hbar$ is Planck's constant divided by 2π. Also, q_α is the particle charge with $q_{\rm e} = -e$ for an electron and $q_{\rm p} = e$ for a positron. In (4), $V_{F_\alpha} \equiv \sqrt{2k_B T_{F\alpha}/m}$ is the Fermi thermal velocity of an α-species particle with $T_{F\alpha} \equiv \hbar^2(3\pi^2)^{2/3} n_{\alpha 0}^{2/3}/(2mk_B)$ being the Fermi temperature. Notice that electron–positron pair annihilation can be neglected in the derivation of (1)–(3). To neglect the annihilation effect, we must have $\omega_{\rm p}^{-1} \ll \tau_{\rm ann}$, where $\omega_{\rm p}^2 = n_0 e^2/\varepsilon_0 m$ is the plasma frequency and $\tau_{\rm ann}$ is the annihilation time given for a non-relativistic plasma by [32] $\tau_{\rm ann} = 4\Theta/[3(1+6\Theta)n_{\rm e,p}\sigma_\tau c]$. The Thomson cross section of the electron is defined as $\sigma_\tau = 6.65 \times 10^{-25}$ cm^2 and c is the speed of light in vacuum. The normalized thermal energy $\Theta(= k_B T_F/mc^2)$ is such that [32] $\zeta^2 \leqslant \Theta \leqslant 1$, or in terms of temperature $3 \times 10^5 < T_F({\rm K}) < 5.9 \times 10^9$, where $\zeta = 7.29 \times 10^{-3}$ is the fine-structure constant. From the above inequalities with $n_{\rm e0} = n_{\rm p0} = n_0$, we obtain $\Theta > 2.66 \times 10^{-19} n_0^{1/2}$ and the range of the number density for which the pair annihilation can be neglected as $3.9 \times 10^{28} < n_0({\rm cm}^{-3}) < 1.4 \times 10^{37}$.

Linearizing the set of (1)–(4) on letting $n_\alpha = n_0 + n_1$ and assuming $\mathbf{v}_\alpha, \phi^{\rm p}$ as first-order quantities, and **B** as $\mathbf{B}_0$, the zeroth-order quantity, we obtain

$$\frac{\partial n_{\alpha 1}}{\partial t} + n_0 \nabla \cdot \mathbf{v}_\alpha = 0, \tag{5}$$

$$m^2 n_0 \frac{\partial \mathbf{v}_\alpha}{\partial t} = q_\alpha m n_0(-\nabla\phi^{\rm p} + \mathbf{v}_\alpha \times \mathbf{B}_0) - \frac{1}{3} m^2 V_{F\alpha}^2 \nabla n_{\alpha 1} + \frac{\hbar^2}{4}\nabla(\nabla^2 n_{\alpha 1}), \tag{6}$$

$$\nabla^2 \phi^{\rm p} = -\frac{e}{\varepsilon_0}(n_{\rm p1} - n_{\rm e1}). \tag{7}$$

The linearized Poisson equation in vacuum for the perturbed wave potential $\phi^{\rm v}$ is

$$\nabla^2 \phi^{\rm v} = 0. \tag{8}$$

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

Assuming time variation of the field quantities of the form $\sim\exp(-i\omega t)$, we obtain from (6) that

$$\mathbf{v}_\alpha = \frac{iq_\alpha}{m\omega}(\mathbf{v}_\alpha \times \mathbf{B}_0) + i\nabla\bigg(- \frac{q_\alpha}{m\omega}\phi^{\rm p} - \frac{V_{F\alpha}^2}{3n_0\omega}n_{\alpha 1} + \frac{\hbar^2}{4m^2 n_0\omega}\nabla^2 n_{\alpha 1}\bigg). \tag{9}$$

Equation (9) can be expressed as

$$\mathbf{v}_\alpha = \frac{iq_\alpha B_0}{m\omega}(\mathbf{v}_\alpha \times \mathbf{b}) + \mathbf{u}_\alpha, \tag{10}$$

where $\mathbf{b} = \mathbf{B}_0/B_0$ and $\mathbf{u}_\alpha$ is given by

$$\mathbf{u}_\alpha = i\nabla\bigg(- \frac{q_\alpha}{m\omega}\phi^{\rm p} - \frac{V_{F\alpha}^2}{3n_0\omega}n_{\alpha 1} + \frac{\hbar^2}{4m^2 n_0\omega}\nabla^2 n_{\alpha 1}\bigg). \tag{11}$$

Now taking the scalar and vector product of (10) with $\mathbf{b}$, we obtain for $\alpha = \mathrm{e}$ that

$$\mathbf{b}\cdot\mathbf{v}_{\rm e} = \mathbf{b}\cdot\mathbf{u}_{\rm e}, \tag{12}$$

$$\mathbf{b}\times\mathbf{v}_{\rm e} = \frac{i\Omega}{\omega}\mathbf{b}\times(\mathbf{b}\times\mathbf{v}_{\rm e}) + \mathbf{b}\times\mathbf{u}_{\rm e}, \tag{13}$$

where $\Omega = eB_0/m$. If we now solve (10) for $\mathbf{b}\times\mathbf{v}_{\rm e}$ and use the results of (12) and (13), we obtain

$$\frac{-i\omega}{\Omega}(\mathbf{v}_{\rm e} - \mathbf{u}_{\rm e}) = \frac{i\Omega}{\omega}(\mathbf{bb}\cdot\mathbf{v}_{\rm e} - \mathbf{v}_{\rm e}) + \mathbf{b}\times\mathbf{u}_{\rm e}, \tag{14}$$

which leads to

$$\bigg(1 - \frac{\omega^2}{\Omega^2}\bigg)\mathbf{v}_{\rm e} = \mathbf{bb}\cdot\mathbf{u}_{\rm e} - \frac{\omega^2}{\Omega^2}\mathbf{u}_{\rm e} - \frac{i\omega}{\Omega}\mathbf{b}\times\mathbf{u}_{\rm e}. \tag{15}$$

Taking the divergence of (15) and using the results $\nabla\cdot\mathbf{v}_{\rm e} = i\omega n_{\rm e1}/n_0$ and $\nabla\cdot(\mathbf{b}\times\mathbf{u}_{\rm e}) = 0$, we obtain for $\alpha = e$, and similarly for $\alpha = p$, the combined equation

$$\omega\bigg(1 - \frac{\omega^2}{\Omega^2}\bigg)\frac{n_{\alpha 1}}{n_0} = \mathbf{b}\cdot\nabla\bigg[\mathbf{b}\cdot\nabla\bigg(\frac{q_\alpha\phi^{\rm p}}{m\omega} - \frac{v_F^2 n_{\alpha 1}}{3n_0\omega} + \frac{\hbar^2}{4n_0\omega m^2}\nabla^2 n_{\alpha 1}\bigg)\bigg]$$
$$- \frac{\omega^2}{\Omega^2}\nabla^2\bigg(\frac{q_\alpha\phi^{\rm p}}{m\omega} - \frac{v_F^2 n_{\alpha 1}}{3n_0\omega} + \frac{\hbar^2}{4n_0\omega m^2}\nabla^2 n_{\alpha 1}\bigg). \tag{16}$$

Equations (7) and (16) can be combined to form a sixth-order differential equation in $\phi^{\rm p}$. Assuming the plasma potential to vary as $\tilde{\phi}^{\rm p}\exp(i\kappa_x x + i\kappa_y y - i\omega t)$, and dropping the tilde notation, we obtain the following equation for the plasma potential $\phi^{\rm p}$:

$$A\frac{d^4\phi^{\rm p}}{d\bar{z}^4} + B\frac{d^3\phi^{\rm p}}{d\bar{z}^3} + C\frac{d^2\phi^{\rm p}}{d\bar{z}^2} + D\frac{d\phi^{\rm p}}{d\bar{z}} + F\phi^{\rm p} = 0, \tag{17}$$

where $\bar{z} = zk$ is the normalized z-coordinate, and we have neglected the very slow non-local variations ($k^2\partial^4/\partial z^4 \gg k\partial^5/\partial z^5, \partial^6/\partial z^6$). The coefficients A, B, C, D, F are given by:

$$A = \frac{ek^4}{m\omega\omega_{\rm p}^2}\bigg[\frac{\hbar^2 k^2}{4m^2}\bigg(\sin^2\theta\sin^2\varphi + 2\cos^2\theta - 3\frac{\omega^2}{\Omega^2}\bigg) + \frac{V_{F\rm e}^2}{3}\bigg(\cos^2\theta - \frac{\omega^2}{\Omega^2}\bigg)\bigg], \tag{18}$$

$$B = i\frac{\varepsilon_0 k^4}{en_0\omega}\bigg(\frac{\hbar^2 k^2}{2m^2} + \frac{V_{F\rm e}^2}{3}\bigg)\sin 2\theta\sin\varphi, \tag{19}$$

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

$$C = \frac{ek^2}{m\omega}\left[\frac{\hbar^2k^4}{4m^2\omega_{\rm p}^2}\left(2\sin^2\theta\sin^2\varphi - \cos^2\theta + 3\frac{\omega^2}{\Omega^2}\right) + 2\left(\frac{\omega^2}{\Omega^2} - \cos^2\theta\right)\right] + \frac{ek^2}{m\omega}\left[\frac{\omega^2}{\omega_{\rm p}^2}\left(1 - \frac{\omega^2}{\Omega^2}\right) + \frac{k^2V_{F\rm e}^2}{3\omega_{\rm p}^2}\left(2\frac{\omega^2}{\Omega^2} - \cos^2\theta + \sin^2\theta\sin^2\varphi\right)\right] \tag{20}$$

$$D = -i\frac{ek^2}{m\omega}\left[2 + \frac{k^2}{\omega_{\rm p}^2}\left(\frac{\hbar^2k^2}{4m^2} + \frac{V_{F\rm e}^2}{3}\right)\right]\sin 2\theta\sin\varphi, \tag{21}$$

$$F = \frac{ek^2}{m\omega}\left[\left(2 + \frac{k^2}{\omega_{\rm p}^2}\left(\frac{\hbar^2k^2}{4m^2} + \frac{V_{F\rm e}^2}{3}\right)\right)\sin^2\theta\sin^2\varphi - \frac{\omega^2}{\omega_{\rm p}^2}\left(1 - \frac{\omega^2}{\Omega^2}\right)\right], \tag{22}$$

where $k^2 = k_x^2 + k_y^2$ and $V_{F\rm e} = V_{F\rm p} = V_F$. (Since particles have the same equilibrium number density n_0, $T_{F\rm e} = T_{F\rm p} = T_F$, say.)

Equation (17) is solved by assuming that

$$\phi^{\rm p}(\bar{z}) = e^{\Gamma\bar{z}}, \tag{23}$$

where $\Gamma = \Gamma_R + i\Gamma_I$ is, in general, a complex quantity. Substituting (23) in (17), we obtain the following characteristic equation for Γ:

$$A\Gamma^4 + B\Gamma^3 + C\Gamma^2 + D\Gamma + F = 0, \tag{24}$$

where Γ is normalized by k. Since the plasma occupies the half-space $z > 0$, and we are considering surface modes that have a maximum amplitude at the boundary and decay as we move away from the surface in either direction, the general solution of (17) can be written as

$$\phi^{\rm p}(\bar{z}) = A_1e^{\Gamma_1\bar{z}} + A_2e^{\Gamma_2\bar{z}}, \tag{25}$$

where A_1, A_2 are constants to be determined by the appropriate boundary conditions. In (24), the coefficients A, C, F are real, while B, D are purely imaginary quantities. For such an equation it can be shown that the complex roots of the equation occur as $\Gamma_1, -\Gamma_1^*, \Gamma_2, -\Gamma_2^*$, where '$*$' denotes the complex conjugate. In order that the solutions are consistent with the boundary conditions, to be stated later, from the four complex roots of the biquadratic equation in Γ we choose those roots Γ_1 and Γ_2 that have negative real parts.

For the vacuum, the wave equation for the potential $\phi^{\rm v}$ is given by

$$\frac{d^2\phi^{\rm v}}{d\bar{z}^2} - \phi^{\rm v} = 0, \tag{26}$$

which has the solution

$$\phi^{\rm v}(\bar{z}) = C_1e^{\bar{z}} + C_2e^{-\bar{z}}, \tag{27}$$

where we choose $C_2 = 0$, so that we have exponentially decaying solutions as we move into the vacuum away from the surface $z = 0$. Thus we need three boundary conditions to determine the three unknowns A_1, A_2 and C_1.

(i) The tangential component of the electric field is continuous across the interface, i.e. $d\phi^{\rm p,v}/dx = 0 = d\phi^{\rm p,v}/dy$, or $\phi^{\rm p} = \phi^{\rm v}$ (= constant) at $z = 0$.

(ii) The normal component of the displacement vector ($\mathbf{D} = \epsilon\mathbf{E}$) is continuous across the interface (as the plasma has hot electron and positron components, there is no surface electric charge density), i.e., $\epsilon E_z^{\rm p} = E_z^{\rm v}$, or $\epsilon\, d\phi^{\rm p}/dz = d\phi^{\rm v}/dz$ at $z = 0$, where $\epsilon = 1 - \omega_{\rm p}^2/\omega^2$ is the plasma dielectric response.

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

(iii) For the hot components (electrons and positrons) the normal velocity components are zero (mirror reflection condition), i.e., $v_{ez} = v_{pz} = 0$ at $z = 0$.

Using the above three boundary conditions we obtain after some manipulation the following dispersion relation for the quantum surface modes:

$$(\Gamma_1 - \Gamma_2)\,\mathrm{Dis}(\omega, k) = 0, \tag{28}$$

where

$$\mathrm{Dis}(\omega,k) = \bar{A}[\Gamma_1\Gamma_2(\Gamma_1+\Gamma_2)-(\Gamma_1^2+\Gamma_1\Gamma_2+\Gamma_2^2)]+\bar{B}(\Gamma_1\Gamma_2-\Gamma_1-\Gamma_2)-(\bar{C}+\bar{D}). \tag{29}$$

The coefficients $\bar{A}, \bar{B}, \bar{C}$ and $\bar{D}$ are given by

$$\bar{A} = \frac{ek^3}{m\omega\omega_p^2}\left(1 - \frac{\Omega^2\cos^2\theta}{\omega^2}\right)\left(\frac{\hbar^2k^2}{2m^2} + \frac{V_{Fe}^2}{3}\right), \tag{30}$$

$$\bar{B} = -\frac{e}{m}\frac{k^3\Omega^2\sin\theta}{\omega^3\omega_p^2}\left(\frac{\omega}{\Omega}\cos\varphi + i\cos\theta\sin\varphi\right)\left(\frac{\hbar^2k^2}{2m^2} + \frac{V_{Fe}^2}{3}\right), \tag{31}$$

$$\bar{C} = -\frac{ek}{m\omega}\left(1 - \frac{\Omega^2\cos^2\theta}{\omega^2}\right)\left[2 + \frac{k^2}{\omega_p^2}\left(\frac{\hbar^2k^2}{4m^2} + \frac{V_{Fe}^2}{3}\right)\right], \tag{32}$$

$$\bar{D} = k\sin\theta\frac{e\Omega^2}{m\omega^3}\left(\frac{\omega}{\Omega}\cos\varphi + i\cos\theta\sin\varphi\right)\left[2 + \frac{k^2}{\omega_p^2}\left(\frac{\hbar^2k^2}{4m^2} + \frac{V_{Fe}^2}{3}\right)\right]. \tag{33}$$

Here Γ_1 and Γ_2 are the solutions of the biquadratic equation (24). Note that for arbitrary values of θ and φ, when the solutions of (24) are, in general, complex, we choose those values of Γ that have negative real parts, so that the desired solution describes wave fields that damp as we move away from the surface to the interior of the plasma. Since the analytical expressions for Γ_1 and Γ_2 are complicated for arbitrary values of θ and φ, we consider, in particular, the cases $\theta = \pi/2$ and $\varphi = 0$, $\theta = \pi/2$ and $\varphi = \pi$, $\theta = 0$ and any φ, and $\theta = \varphi = \pi/2$, which are discussed in the next section. Note that in all of these cases the coefficients B and D in (24) are identically zero, and (24) then gives rise to solutions of the form $\Gamma_{1,2} = \left(-C \pm \sqrt{C^2 - 4AF}\right)/2A$. The dispersion relation (28) is similar to that obtained in [33]. However, in this treatment surface wave propagation is studied in the classical plasma half-space with ambient magnetic field. The ions are considered immobile, forming only the background plasma. Moreover, the case $\Gamma_1 = \Gamma_2$ is treated there in an incorrect manner. When $\Gamma_1 = \Gamma_2$, one has singular waves, which has different dispersion relation [29,30] as can be seen in the next section. Note that from Eq. (28) one cannot recover the classical results as in [33]. Also, one cannot recover the previous quantum surface waves as in [24], by simply disregarding the positron dynamics. In this work, the surface waves were considered electromagnetic and electrostatic as the limiting case.

Without the quantum effects, i.e., V_F^2, $\hbar^2k^2 \to 0$, the dispersion relation for the surface waves propagating in a cold electron-positron plasma (classical) can be recovered. In this case the characteristic equation (24) reduces to

$$C\Gamma^2 + D\Gamma + F = 0, \tag{34}$$

for which the solution for the plasma potential (with the negative real part of Γ_1) can be considered as

$$\phi^p(\bar{z}) = A_1 e^{\Gamma_1\bar{z}}. \tag{35}$$

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

The two solutions (27) and (35) should be connected at $z = 0$ through the appropriate boundary conditions. (a) The tangential component of the electric field is continuous across the interface, i.e. $\phi^{\rm p} = \phi^{\rm v}$ at $z = 0$. (b) The normal component of the displacement vector is discontinuous (since a cold fluid forms a surface charge) across the interface, i.e., $\epsilon\, d\phi^{\rm p}/dz + \sigma/\varepsilon_0 = d\phi^{\rm v}/dz$ at $z = 0$. The surface charge density σ can be obtained by integrating the charge conservation equation for both electrons and positrons over an infinitesimally flat pill box across the interface [34] in the form

$$\frac{\partial \sigma}{\partial t} + \hat{z} \cdot en_0(\mathbf{v}_{\rm p} - \mathbf{v}_{\rm e}), \tag{36}$$

which, upon using (1) for v_{ez} and v_{pz}, yields

$$\sigma = \frac{en_0}{i\omega}(v_{\rm pz} - v_{\rm ez}) = -\frac{2n_0e^2}{m\omega^2}A_1\Gamma_1. \tag{37}$$

The dispersion relation in this case is obtained by using the above boundary conditions (a) and (b) as

$$\Gamma_1\left(1 - \frac{2}{\bar{\omega}^2}\right) = 1, \tag{38}$$

where we use $\bar{\Omega} \equiv \Omega/\omega_{\rm p}$, $\bar{\omega} \equiv \omega/\omega_{\rm p}$ and $\Gamma_1 = -\sqrt{D^2 - 4CF}/2C$ with $D^2 - 4CF > 0$. The dispersion relation (38) describes the propagation of classical surface modes in a cold electron-positron plasma. In particular, for $\theta = \pi/2$ and $\varphi = 0$ or π, the dispersive modes exist for $\bar{\omega} < \sqrt{2}$ and are given by

$$\left(1 - \frac{2}{\bar{\omega}^2}\right) + \left(1 + 2/\bar{\Omega}^2\left(1 - \frac{\bar{\omega}^2}{\bar{\Omega}^2}\right)\right)^{1/2} = 0. \tag{39}$$

We note that no real dispersive mode exists in the case where $\theta = 0$.

3. Results and discussions

3.1. Surface wave

Since it is very difficult to find a general solution of (28) analytically, we analyze the dispersion relation (when $\Gamma_1 \neq \Gamma_2$; the case of $\Gamma_1 = \Gamma_2$ will be considered later) in some special cases of interest, viz. for $\theta = \pi/2, 0$ and $\varphi = 0, \pi, \pi/2$.

The kind of dispersion curves given in (28) depends strongly on the ambient magnetic field $\mathbf{B}_0$ via the gyrofrequency Ω, the quantum parameter $H \equiv \hbar\omega_{\rm p}/4k_BT_F$ as well as the angles θ and φ. The dispersion curves differ qualitatively not only at weakly ($\Omega < \omega_{\rm p}$) and strongly ($\Omega > \omega_{\rm p}$) magnetized cases, but also in the case where the quantum effect plays a crucial role. This is one of the purposes of the present work.

When $\theta = \pi/2$ and $\varphi = 0$, i.e., the magnetic field lies in the plane of interface and is directed toward the propagation vector, i.e., $k\|\hat{y}\|\mathbf{B}_0$, the main distinction of the dispersion curves at different values of H, e.g., $H = 0.01$ (for which $n_0 \sim 2.5 \times 10^{38}$ m^{-3}), $H = 0.04$ (for which $n_0 \sim 6.5 \times 10^{34}$ m^{-3}) for a fixed $\bar{\Omega} = 0.2$ (see Fig. 2) is their behavior at large $K \equiv kV_F/\omega_{\rm p}$. This value of K becomes smaller with increasing values of the magnetic field strength. Both in weakly and strongly magnetized plasmas the dispersion curves, being distinct, have similar behaviors: the frequency increases with increasing K for a given set of parameters. Also, as the magnetization increases the wave frequency increases. We note that the frequency

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

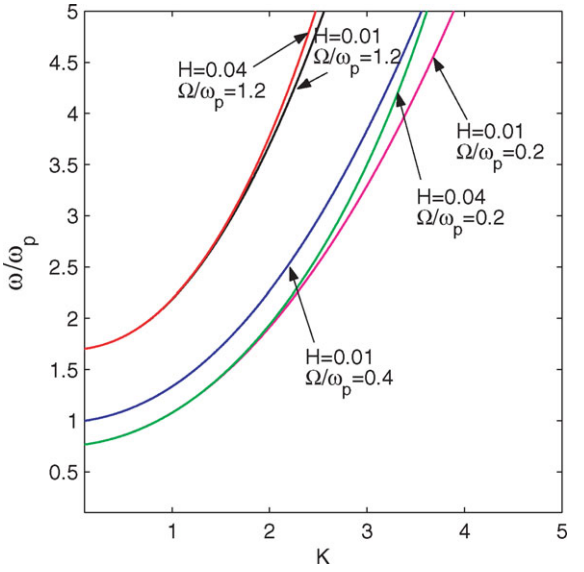


Figure 2. Plot of normalized frequency ($\omega/\omega_{\rm p}$) versus the normalized wave number ($K \equiv kV_F/\omega_{\rm p}$) for the propagation of surface waves ($\Gamma_1 \neq \Gamma_2$, (28)) for $\theta = \pi/2, \varphi = 0$. The dispersion is shown for weakly ($\Omega/\omega_{\rm p} = 0.4 < 1$) and strongly ($\Omega/\omega_{\rm p} = 1.2 > 1$) magnetized plasmas for different values of $H = 0.01$ (for which $n_0 \sim 2.5 \times 10^{38}$ m^{-3}) and $H = 0.04$ (for which $n_0 \sim 6.5 \times 10^{34}$ m^{-3}) (a colour version of this figure is available online at www.journals.cambridge.org/pla).

region of the surface wave exceeds the plasma frequency $\omega_{\rm p}$, which may be true in the case of a quantum plasma [24] or a classical gyrotropic plasma [33].

For the case where $\theta = \pi/2$ and $\varphi = \pi$, the dispersion curves behave quite differently (see Fig. 3). In the case of weakly or strongly magnetized cases, the curves mainly appear at large K. For a fixed value of $\bar{\Omega} = 0.2$, the wave frequency decreases with K for $\omega < \omega_{\rm p}$, and increases for $\omega > \omega_{\rm p}$. The curves are well separated with increasing magnetic field strength, and appear at large K for larger values of $\bar{\Omega}$. These branches are characteristic of a quantum plasma and propagate along the positive and negative y-direction.

Next we consider the case where $\theta = 0$ and for any φ, i.e., when the magnetic field is normal to the plane of interface. In this case it is clear from Fig. 4 that the root $\omega = \Omega$ is a solution of the dispersion equation $\mathrm{Dis}(\omega, k) = 0$. The other branch (above the line $\omega = \Omega$) starting with the same value of $\omega = \Omega$ get widely separated. The frequencies of these branches increase with increasing the values of H (both in weakly and strongly magnetized cases). Also, for a fixed H they increase as K increases. The surface modes exist with frequencies ω above the plasma frequency $\omega_{\rm p}$ in a strongly magnetized plasma (Fig. 4(b)), whereas they exist in weakly magnetized case for both $\omega < \omega_{\rm p}$ and $\omega > \omega_{\rm p}$ (Fig. 4(a)).

Another interesting feature can be observed when $\theta = \varphi = \pi/2$. In this case the propagation vector is perpendicular to the magnetic field, i.e., $k \| \hat{z}$ and $\mathbf{B}_0 || \hat{x}$. Then the modes (called *ordinary* modes, if they exist) involving particle motion strictly parallel to the magnetic field are unaffected by the magnetic field, while the other modes may depend on the magnetic field (called *extraordinary* mode). Such modes which depend on the magnetic field are exhibited in Fig. 5 both for

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

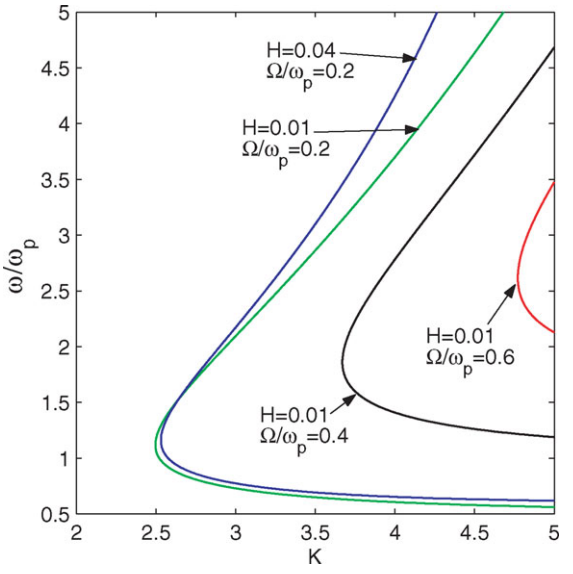


Figure 3. Plot of normalized frequency ($\omega/\omega_{\rm p}$) versus the normalized wave number ($K \equiv kV_F/\omega_{\rm p}$) for the propagation of surface waves ($\Gamma_1 \neq \Gamma_2$, (28)) for $\theta = \pi/2, \varphi = \pi$. The dispersion is shown for weakly ($\Omega/\omega_{\rm p} = 0.4 < 1$) magnetized plasmas only for different values of $H = 0.01$ (for which $n_0 \sim 2.5 \times 10^{38}$ m^{-3}) and $H = 0.04$ (for which $n_0 \sim 6.5 \times 10^{34}$ m^{-3}). In the strongly magnetized case ($\Omega/\omega_{\rm p} > 1$), the surface modes appear at higher values of K (not shown in the figure) (a colour version of this figure is available online at www.journals.cambridge.org/pla).

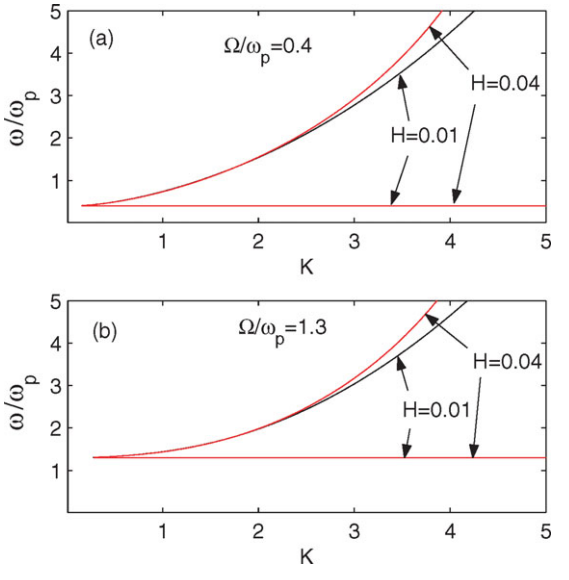


Figure 4. Plot of normalized frequency ($\omega/\omega_{\rm p}$) versus the normalized wave number ($K \equiv kV_F/\omega_{\rm p}$) for the propagation of surface waves ($\Gamma_1 \neq \Gamma_2$, (28)) for $\theta = 0$ and for any φ in (a) weakly ($\Omega/\omega_{\rm p} = 0.4 < 1$) and (b) strongly ($\Omega/\omega_{\rm p} = 1.3 > 1$) magnetized cases for $H = 0.01$ and 0.04 (a colour version of this figure is available online at www.journals.cambridge.org/pla).

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

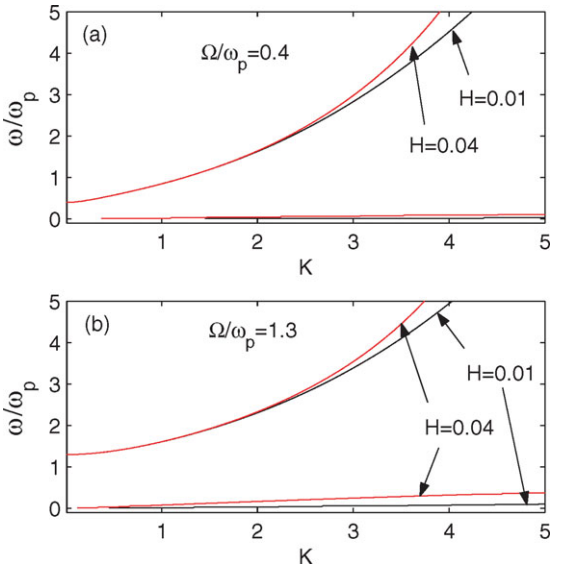


Figure 5. As Fig. 4, but for $\theta = \varphi = \pi/2$. In this case the surface modes depend on the magnetic field also (called *extraordinary* mode). It is seen that the two roots are well separated. The upper curves correspond to the modes (upper hybrid frequency) where $\omega \gtrsim \Omega$ and the lower ones correspond to modes (lower hybrid frequency) with $\omega \ll \Omega$ (a colour version of this figure is available online at www.journals.cambridge.org/pla).

weakly and strongly magnetized cases for different values of H. It is seen that the wave frequency increases with increasing the values of H and Ω.

3.2. Singular wave

When $\Gamma_1 = \Gamma_2$ $(= \Gamma)$ is satisfied, i.e., the roots of the characteristic equation (24) are multiple ones, the plasma potential ϕ^{p} cannot be solved in the form of (25). In this case the general solution of (5) can be written as

$$\phi^{\mathrm{p}}(\bar{z}) = (A_1 + A_2\bar{z})e^{\Gamma\bar{z}}, \tag{40}$$

where we take the negative real part of the complex root Γ. Moreover, (24) has two equal roots under the condition

$$\frac{D^2}{B^2} = \frac{F}{A}, \tag{41}$$

which is obtained from the relations among the roots and the coefficients of (24), assuming its roots as Γ, $-\Gamma^*$, Γ, $-\Gamma^*$. The curves along which $\Gamma_1 = \Gamma_2$ is called a singular curve and along it one must use the solution of the type of (40). The boundary curve which $\Gamma_1 = \Gamma_2$ is a result of the gyrotropic character of the plasma and it appears in magnetized plasmas [29, 30]. The dispersion relation in this case can readily be written as

$$\bar{A}(2\Gamma - 3)\Gamma^2 + \bar{B}(\Gamma - 2)\Gamma - (\bar{C} + \bar{D}) = 0. \tag{42}$$

The boundary curve (where the condition for $\Gamma_1 = \Gamma_2$ is satisfied) and the dispersion curve of the singular wave (42) are plotted on the same Figs 6 and 7 for different sets of parameter values in weakly ($\bar{\Omega} < 1$) and strongly ($\bar{\Omega} > 1$)

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

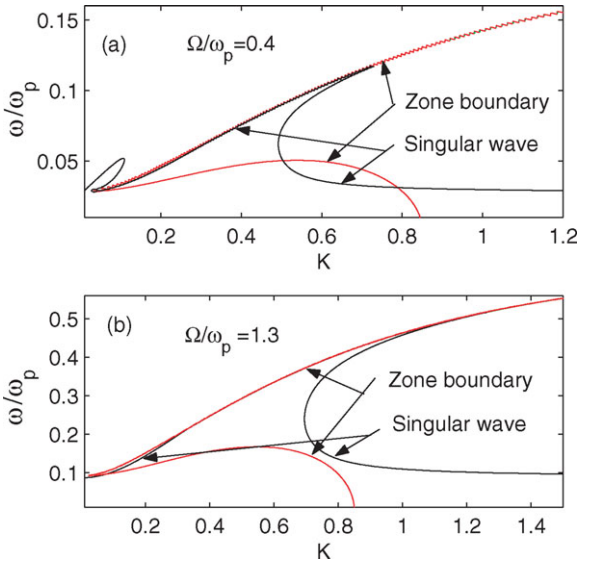


Figure 6. Plot of normalized frequency versus the normalized wave number for the propagation of singular surface waves ($\Gamma_1 = \Gamma_2$, (41) and (42)) for $\theta = 1.5$ and $\phi = 2$. The upper panel (a) is for weakly ($\Omega < \omega_p$) and the lower panel (b) for strongly ($\Omega > \omega_p$) magnetized cases for a fixed value of $H = 0.03$. The intersection points describe the singular waves (a colour version of this figure is available online at www.journals.cambridge.org/pla).

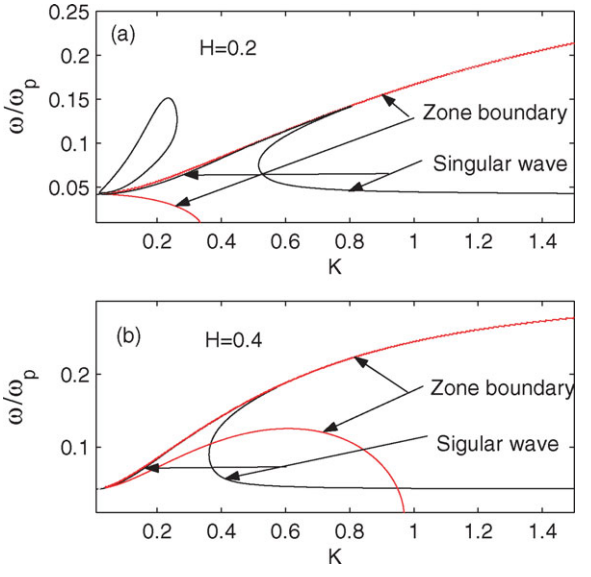


Figure 7. Plot of normalized frequency versus the normalized wave number for the propagation of singular surface waves ($\Gamma_1 = \Gamma_2$, (41) and (42)) for $\theta = 1.5$ and $\phi = 2$. In weakly magnetized plasmas ($\Omega/\omega_p = 0.6$), the upper panel (a) is for $H = 0.02$ and the lower panel (b) for $H = 0.04$. The intersection points describe the singular waves (a colour version of this figure is available online at www.journals.cambridge.org/pla).

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

magnetized cases. The plot of the singular wave dispersion intersects the boundary curve at some points which clearly describe the singular waves in our quantum plasma system. We find that there can be two, three or more points of intersection depending on the value of $\bar{\Omega}$. The number of intersection points increases in the case of weakly magnetized plasmas (see, for example, Fig. 6(a)) depending on the value of H. We note in Fig. 7(a) that for $H = 0.02$ and $\bar{\Omega} = 0.6$, there are not less than three intersection points. Thus, changing the magnetic filed as well as the particle number density region, the different points of the boundary curve with all frequencies in $0 < \omega < \omega_p$ have a possibility to become points of the singular wave dispersion curves. However, to cover the whole range one has to change the values of θ and φ as well as other parameters Ω and H. Note that the real dispersion curves (where $\Gamma_1 \neq \Gamma_2$) cannot be plotted in the same figures as those for the singular ones, because the condition for $\Gamma_1 = \Gamma_2$ is not satisfied for $\theta = \pi/2$ and $\theta = 0$ cases. In those cases one has to consider the characteristic equation (24) for arbitrary values of θ and φ, which is very difficult even numerically, and beyond the scope of the present work.

4. Conclusions

In this paper, we have investigated the dispersion properties of electrostatic surface modes propagating at the interface between a magnetized electron-positron quantum plasma and vacuum. The general dispersion relation is derived for arbitrary orientation of the magnetic field and propagation vector, and is analyzed numerically in some special cases, viz. when the magnetic field is directed parallel or perpendicular to the boundary surface. It is found that the quantum effects facilitate the propagation of electrostatic surface modes. The existence of a singular wave on the boundary curve is established and its properties are analyzed numerically. The present result is of practical use for understanding the dispersion properties of quantum electron-positron plasma oscillations at the dense plasma–vacuum interface.

Acknowledgements

This work was partially supported by the Special Assistance Program (SAP), University Grants Commission (UGC), Govt. of India, through sanction letter no. F.510/8/DRS/2004 (SAP-I) and partial research grant from Visva-Bharati University.

References

[1] Misner, W., Thorne, K. and Wheeler, J. A. 1973 *Gravitation*. San Francisco: Freeman, p. 763.

[2] Rees, J. M. 1983 *The Very Early Universe* (ed. G. W. Gibbons, S. W. Hawking and S. Siklas). Cambridge: Cambridge University Press.

[3] Miller, H. R. and Witta, P. J. 1987 *Active Galactic Nuclei*. Berlin: Springer, p. 202.
Goldreich, P. and Julian, W. H. 1969 *Astrophys. J.* **157**, 869.

[4] Begelman, M. C., Blandford, R. D. and Rees, M. J. 1984 *Rev. Mod. Phys.* **56**, 255.

[5] Michel, F. C. 1982 *Rev. Mod. Phys.* **54**, 1.
Yu, M. Y., Shukla, P. K. and Stenflo, L. 1986 *Astrophys. J.* **309**, L63.

[6] Rees, M. J. 1971 *Nature* (London) **229**, 312.

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press

[7] Burns, M. L. 1983 *Positron-Electron Pairs in Astrophysics*. New York: American Institute of Physics.

[8] Berezhiani, V., Tskhakaya, D. D. and Shukla, P. K. 1992 *Phys. Rev.* A **46**, 6608.
Marklund, M. and Shukla, P. K. 2006 *Rev. Mod. Phys.* **78**, 591.

[9] Shukla, P. K., Rao, N. N., Yu, M. Y. and Tsintsadze, N. L. 1986 *Phys. Rep.* **135**, 1.

[10] Ferreira, C. M. 1992 *Microwave Discharges: Fundamentals and Applications*. New York: Plenum Press.

[11] Shukla, P. K. and Stenflo, L. 2006 *J. Plasma Phys.* **72**, 605.

[12] Misra, A. P. 2007 *Phys. Plasmas* **14**, 064501.

[13] Shukla, P. K., Rao, N. N., Yu, M. Y. and Tsintsadze, N. L. 1986 *Phys. Rep.* **138**, 1.

[14] Roy, K., Misra, A. P. and Chatterjee, P. 2008 *Phys. Plasmas* **15**, 032310.

[15] Eliasson, B. and Shukla, P. K. 2006 *Phys. Rep.* **422**, 225.

[16] Gardner, C. L. and Ringhofer, C. 1995 *Phys. Rev.* E **53**, 157.
Manfredi, G. 2005 *Fields Inst. Commun.* **46**, 263.
Shukla, P. K. 2006 *Phys. Lett.* A **352**, 242.
Marklund, M., Brodin, G., Stenflo, L. and Liu, C. S. 2008 *Eur. Phys. Lett.* **84**, 17006.

[17] Jung, Y. F. 2001 *Phys. Plasmas* **8**, 3842.
Shukla, P. K. and Eliasson, B. 2006 *Phys. Rev. Lett.* **96**, 245001.
Shukla, P. K. and Eliasson, B. 2006 *Phys. Rev. Lett.* **99**, 096401.
Shaikh D. and Shukla P. K. 2007 *Phys. Rev. Lett.* **99** 125002.
Shukla, P. K. 2009 *New J. Phys.* **5**, 92.

[18] Alexandrov, A. F., Bogdankevich, L. S. and Rukhadze, A. A. 1984 *Principles of Plasma Electrodynamics*. Berlin: Springer.

[19] Stenflo, L. 1996 *Phys. Scripta* **T63**, 59.

[20] Aliev, Yu. M., Schlüter, H. and Shivarova, A. 2000 *Guided-Wave-Produced Plasmas*. Berlin: Springer.

[21] Stenflo, L. and Yu, M. Y. 2003 *Phys. Plasmas* **10**, 912.

[22] Lee, H. J. 2005 *Phys. Plasmas* **12**, 094701.

[23] Chang, I. S. and Jung, Y. D. 2008 *Phys. Lett.* A **9**, 372.

[24] Lagar, M., Shukla, P. K. and Smolyakov, A. 2007 *Phys. Plasmas* **14**, 124501.

[25] Fainberg, B. Ya and Gorbatenko, M. F. 1959 *ZhTF* **29**, 549.

[26] Allis, W. P., Buchsbaum, S. J. and Bers, A. 1963 *Waves in Anistropic Plasma*. MIT Press: Cambridge, MA.

[27] Legendy, C. R. 1964 *Phys. Rev.* A **135**, 1713.

[28] Kaw, P. K. and Mcbride, J. B. 1970 *Phys. Fluids* **13**, 1784.

[29] Ivanov, S. T., Bogdanov, A. T. and Alexov, E. G. *Phys. Scripta* **42**, 458.
Ivanov, S. T. and Nikolaev, N. I. 1996 *J. Phys. D: Appl. Phys.* **29**, 1007.

[30] Ivanov, S. T., Nikolaev, N. I. and Thomae, R. W. 1998 *Phys. Scripta* **57**, 645.

[31] Landau, L. D. and Lifshitz, E. M. 1998 *Statistical Physics*. Oxford: Oxford University Press.

[32] Svensson, R. 1982 *Astrophys.* **258**, 335.

[33] Janaki, M. S. and Dasgupta, B. 1998 *Phys. Plasmas* **5**, 4163.

[34] Sedov, L. I. 1972 *A Course in Continuum Mechanics*. Groningen, Netherlands: Wolters-Noordhoff, Vol. II, ch. 7.

https://doi.org/10.1017/S0022377809990055 Published online by Cambridge University Press