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Reducing complex physics to its phase dynamics

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The dynamics of a fluid flow about its limit cycle can be analysed through phase reduction analysis – an approach that distils a high-dimensional dynamical system to its scalar phase dynamics. This technique provides insights into phase sensitivity, revealing the mechanisms that advance or delay phase dynamics. The phase-based reduced-order model derived from this approach serves as a foundation for identifying lock-on conditions and designing flow control techniques. Recent work by Sumanasiri *et al.* (*J. Fluid Mech.* vol. **1020**, 2025, R4) applied phase reduction analysis to the fluid–structure interaction problem of aerofoil flutter in a free stream. Their analysis systematically changed the stiffness of the structural dynamics to decipher the phase dynamics mechanism of flutter. Moreover, they considered the use of optimised heaving motion to suppress the emergence of flutter. Their approach opens new avenues for modifying flow physics through innovative modifications of material properties and structural dynamics.

Key words: low-dimensional models, flow-structure interactions

1. Introduction

Limit cycle oscillations are observed across a wide range of phenomena, including chemical reactions, human behaviour, circadian rhythms, and fluid flows. To shed light on the mechanism of phase synchronisation about the limit cycles, there have been extensive research efforts in physics and biology (Kuramoto 1984; Pikovsky, Rosenblum & Kurths 2001; Ermentrout & Terman 2010). The examination of phase synchronisation can be facilitated by the phase reduction analysis (Nakao 2016), which is a theoretical approach that reduces a high-dimensional dynamical system about a limit cycle to a single scalar phase variable (or a small number of variables) to describe the oscillatory dynamics. This technique is immensely powerful in explaining synchronisation dynamics of various complex systems.

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Let us consider a dynamical system for a state variable $\boldsymbol{q}(\boldsymbol{x}, t)$ that satisfies

$$\dot{\boldsymbol{q}} = \boldsymbol{F}(\boldsymbol{q}). \tag{1.1}$$

If this system admits a time-periodic solution $\boldsymbol{q}_0(\boldsymbol{x}, t) = \boldsymbol{q}_0(\boldsymbol{x}, t + T)$ with period T, the associated phase variable $\theta(t)$ for the limit cycle can be established. This phase variable can be defined in an appropriately chosen phase space. We can extend $\theta(t)$ to be a function of the state variable $\boldsymbol{q}$ in the basin of attraction of the limit cycle such that $\theta = \Theta(\boldsymbol{q}(\boldsymbol{x}, t))$. In this case, the phase function satisfies

$$\dot{\theta}(t) = \dot{\Theta}(\boldsymbol{q}(\boldsymbol{x}, t)) = \omega_n, \tag{1.2}$$

where $\omega_n = 2\pi/T$ is the natural frequency of the limit cycle oscillation.

The influence of small external forcing $\epsilon \boldsymbol{p}(\boldsymbol{x}, t)$ on the phase variable can be examined for

$$\dot{\boldsymbol{q}} = \boldsymbol{F}(\boldsymbol{q}) + \epsilon \boldsymbol{p}(\boldsymbol{x}, t), \tag{1.3}$$

where $\epsilon \ll 1$. Under this forcing, the phase variable evolves as

$$\dot{\theta}(t) = \dot{\Theta}(\boldsymbol{q}(\boldsymbol{x}, t)) \approx \nabla_{\boldsymbol{q}_0}\Theta \cdot \dot{\boldsymbol{q}} = \nabla_{\boldsymbol{q}_0}\Theta \cdot \left[\boldsymbol{F}(\boldsymbol{q}_0) + \epsilon \boldsymbol{p}(\boldsymbol{x}, t)\right] = \omega_n + \epsilon \boldsymbol{Z}(\theta) \cdot \boldsymbol{p}(\boldsymbol{x}, t). \tag{1.4}$$

In (1.4), we linearised the dynamics about the limit cycle and defined $\boldsymbol{Z}(\theta) \equiv \nabla_{\boldsymbol{q}_0}\Theta$, known as the phase sensitivity function (PSF). This PSF can be determined computationally or experimentally from an impulse response, or computationally through adjoint analysis.

Note that (1.2) and (1.4) reduce a high-dimensional system with variable $\boldsymbol{q}$ to its phase dynamics described by a single variable θ, effectively capturing the oscillatory dynamics. This PSF provides significant insights into the dynamics. The PSF reveals the sensitivity of the phase variable in response to a perturbation over time and space, and allows us to find the phase synchronisation condition, which is widely known as the Arnold tongue on the frequency-amplitude map. Moreover, the phase-based model can be used to design control schemes to alter the oscillatory dynamics of the system (Monga *et al.* 2019), as hinted by the inner product between the PSF and the external input $\boldsymbol{p}$ in altering $\dot{\theta}$ in (1.4). There are also networked oscillator-based models that combine multiple limit cycle representations.

2. Phase-reduction analysis of fluid flows

For fluid flows, oscillatory behaviour is prevalent in many forms, including vortex shedding, buffeting, fluttering, and flow-induced vibrations. The governing equation (1.1) for fluids can be the Navier–Stokes equations or any other equations that describe the flow physics of interest. In vortical flows, the PSF reveals the timing for the system to be receptive to phase advancement or delay, which can be directly related to the acceleration or delay of vortex formation, respectively. Phase reduction analysis has been used to examine phase synchronisation between flow unsteadiness and unsteady heat injection (Kawamura & Nakao 2015), flow control inputs (Taira & Nakao 2018; Nair *et al.* 2021), and body motion (Loe *et al.* 2021; Khodkar, Klamo & Taira 2021; Kim *et al.* 2024). While the phase variable can be defined in many ways that are suitable for analysing the given problem, the force coefficient and its time derivative have been widely used.

Phase-reduction analysis is not limited to single-component systems. It can also be applied to multi-physics problems. In the recent paper by Sumanasiri *et al.* (2025), the fluid–structure interaction problem of flutter in low-Reynolds-number flow is computationally analysed. They study the occurrence of flutter for a lightweight NACA

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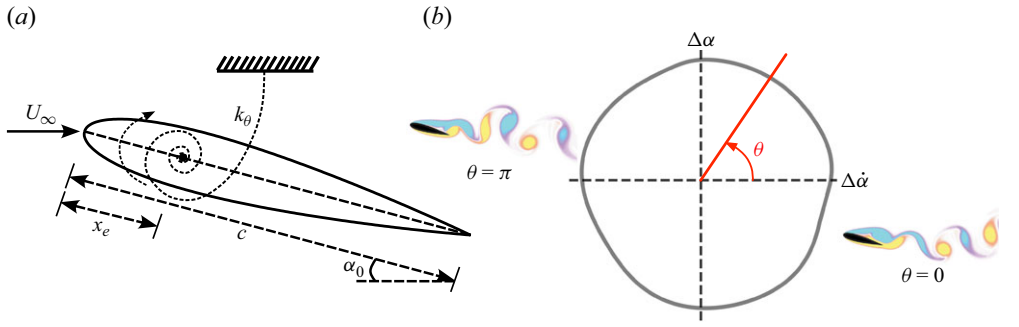


Figure 1. (*a*) Flutter problem studied by Sumanasiri, Sahu & Nair (2025). (*b*) Phase variable $\theta(t)$ defined by the pitch angle and its time derivative with the representative flow fields shown. Figures courtesy of the authors.

0015 aerofoil that is free to pitch about its one-third chord location at a chord-based Reynolds number of 1000, as illustrated in figure 1. By characterising the phase response function (reported as the PSF in their figures 3*c* and 3*d*) through impulsive modification of the spring stiffness, they find that the phase dynamics is most receptive to changes between the maximum pitching amplitude and the return to the equilibrium position. They also observe interesting departures of the flutter dynamics from the basin of attraction, as evident from the peaks appearing in the phase response functions. In their work, they make extensive connections between the insights gained from phase reduction and the flow physics by incorporating modal analysis and vortical force analysis.

To prevent the aerofoil pitching dynamics from departing from the basin of attraction, Sumanasiri *et al.* (2025) further considered introducing heaving. They quantified the PSF for heaving motion and determined a phase-based control strategy that disrupts the coherence between the structural motion and unsteady aerodynamic forcing. They succeeded with this approach and were able to suppress the violent flutter. This overall suppression of flutter was achieved not only by directly applying linear phase-reduction analysis but also by cleverly leveraging physical insights gained from analysing the nonlinear vortex dynamics around the aerofoil. They have also incorporated energy transfer assessment between the flow and structural dynamics to extend their findings.

Moreover, the work by Sumanasiri *et al.* (2025) is refreshing in light of the growing interest in using novel structural properties to modify the dynamics of fluid flows around bodies. The impulsive modification of material properties requires additional examination to theoretically quantify its parametric effects on the phase dynamics. Such an examination invites new research initiatives to further advance fluid–structure interaction research. In fact, metamaterials may play a role in fluid–structure interaction in a passive or active manner (Bertoldi *et al.* 2017; Coulais, Sounas & Alù 2017). By understanding the sensitivity of the phase dynamics at precise instances in relationship to dynamical events, novel metamaterial-based control strategies as well as structural designs may be developed. The formulation and examination process presented by Sumanasiri *et al.* (2025) will likely stimulate these new research activities.

3. Summary and outlook

The recent work by Sumanasiri *et al.* (2025) demonstrated the use of phase-reduction analysis on a fluid–structure interaction problem involving flutter. By combining the insights from phase-reduction analysis with modal analysis, the authors offered deep

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insights into the mechanism of synchronisation (desynchronisation) and their use in suppressing undesirable flutter oscillations. Their study encourages future applications of similar approaches to complex multi-physics problems in which oscillatory dynamics play a key role.

Compression of high-dimensional dynamical systems to their phase dynamics can also be systematically aided by the use of machine-learning-based techniques. Recently, nonlinear autoencoders have been used to uncover latent-variable dynamics around limit cycles (Fukami, Nakao & Taira 2024; Yawata *et al.* 2025). The discovery of low-dimensional representations through modern data-driven techniques can facilitate the phase-based modelling and control efforts not only for oscillatory flows, but also for transient flows (Fukami *et al.* 2024). These methods can serve as a foundation to help develop phase-based models for complex multi-frequency turbulent flows.

There are open questions regarding how phase reduction analysis can be applied to turbulent flows with dominant oscillation frequencies. Since such flows do not possess an exact limit cycle, the application of phase reduction analysis requires extensions to quantify the uncertainty in phase and amplitude around the notional cyclic dynamics. This may be achieved by combining phase reduction analysis with modal analysis, such as the dynamic mode decomposition or resolvent analysis, as considered by Sumanasiri *et al.* (2025). Furthermore, establishing systematic approaches to analyse energy transfers between oscillatory modes or subsystems can serve as an important tool for studying complex multi-component systems. Developments in phase-based analysis have the potential to offer novel reduced-order models and analysis techniques that can not only provide insights into unsteady physical mechanisms, but also serve as a foundation for aerodynamic and structural design optimisation, as the work by Sumanasiri *et al.* (2025) foreshadows.

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