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Optimal location of a robot path when considering velocity performance

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SUMMARY

A method for searching the best location of a path in the robot workspace considering the velocity performance of the robot is presented. After a thorough investigation of the robot performance indices, a new measure for the velocity efficiency of a robot moving its end-effector along a path is introduced. This measure is an approximation of the minimum of the Manipulator Velocity Ratio (MVR) along the path. The minimum is calculated from a finite set of MVRs determined at specified points of the path using an algorithm for an approximate motion of the end effector along the path.

The introduced measure is used as the objective function in an optimisation problem, where an optimal location of the path is searched. The objective function is procedural and non continuous, so a Genetic Algorithm is used to search the space for the optimal location of the path. The proposed method is tested using a simulated PUMA-like robot, which has to move its end-effector along a straight-line segment. At the end of the paper, the results obtained in these tests are presented and discussed.

1. INTRODUCTION

The design of the layout of a robot workcell is a very important engineering task, which is performed mostly by trial-and-error and intuition. Using CAD and/or Robot Simulation Systems, the designer can obtain considerable support in order to put the jigs and fixtures and locate the robot tasks in the workspace of the robot. These systems offer powerful 3D graphics for a realistic representation of the objects, collision detection and off-line programming, but do not provide special tools for the determination of the optimal or, at least, sub-optimal location for a robot task. Therefore, the workcell designer moves the fixtures in trial locations into the workcell and uses off-line programming searches for the best location, taking into account design criteria such as low cycle time, reduced wear and energy consumption, or avoiding obstacles and/or joint velocity limits. It is well known, that the capabilities of the robot depend on the robot configuration; for example, the velocity efficiency or the mechanical advantage of the robot varies considerably in the robot workspace. Therefore, in applications of trajectory planning and generation such as painting, welding, cutting (especially in laser cutting), it is very important to search for the location of the trajectory, where the velocity efficiency of the robot would be the maximum. In these cases, the shape of the path is known since it has been specified in the design of the manufactured part.

A lot of research efforts have been devoted to minimise the cycle time of robot tasks and this problem involved a variation of formulations. In some cases,[1–4] the minimisation of the cycle time is presented as a version of the well-known Travelling Salesman Problem (TSP), where the task of the robot is to visit a number of known fixed points. The end-effector of the robot can visit these points in any order, since there are tasks where the order is not critical such as the spot welding. The TSP problem has been adapted to the characteristics of the robots mainly by replacing the distance between the points with the time required to move between two successive points in the configuration space of the robot. Another characteristic taken into account is that the time required to move from one point to the next is not equal to the time required to move back.

In the design of a robotic workcell, the optimal position of the base of the robot has been studied by a number of researchers.[5,6] Dissanayake and Gal[5] proposed a technique to optimise the location of the robot, the sequence of the tasks and the configuration of the robot at each task in order to minimise the cycle time. In another approach,[6] the optimal position of the robot is found by minimising the distance between the space effectively reached by the robot and the target space if no overlapping occurs between these spaces. In the case of overlapping, the volume of the target space not reached by the robot is minimised.

In the above two categories of methods for minimising the cycle time, the capabilities of the robot such as the velocity efficiency were not taken into account. In some methods[7–11] for trajectory planning or kinematic control of redundant robots, indices or measures characterising the efficiency of the robot are taken into account. These measures are related to the Jacobian matrix, so they are configuration dependant. In some of these papers, an integral cost criterion is optimised instead of an instantaneous criterion as it has been used in the well-known methods based on extended or pseudo-inverse Jacobian. In these papers, the location of the path in the Cartesian space is pre-specified, while the trajectory in the joint space is determined using the proposed methods.

The optimisation of the robot task location based on the manipulator efficiency attracted little attention of the robotic research community.[12–14] An algorithm has been proposed by Nelson and Donath[12] for optimising the location of an assembly task in a manipulator's workspace. In their

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algorithm, a gradient function of manipulability has been developed based on an explicit determination of the manipulability function in joint space. The formulated problem of optimisation has been solved by a modified steepest descent procedure, so the explicit calculation of det(J(q)) and $\vartheta|\det(J(q))|/\vartheta q$ for a non redundant manipulator is required. Therefore, this method cannot be used when the determinant of the Jacobian is non-analytic or non-continuous. Fardanesh and Rastegar[13] proposed an approach for optimal placement of a prescribed task in the workspace of a manipulator workspace. The optimisation criterion is the cycle time needed to visit a set of points with specified relative positions, while the location of the set of points in Cartesian coordinates is not pre-specified but is determined by the proposed approach. Aspragathos[14] proposed a method based on Genetic Algorithms to locate a continuous path in the location with the higher manipulability. An optimisation problem is formulated where an approximation of the mean value of the manipulability is estimated using a bisection algorithm for a path described by a straight-line segment or by a 3D curve is calculated. The optimisation problem has been formulated in a procedural way and the objective function is non-linear and non-continuous, so its solution is based on a simple Genetic Algorithm.

The present paper is an extension of the procedure in reference [14], since a new optimisation criterion is introduced to characterise the location of path where the velocity performance of the robot is the best. The proposed criterion is based upon the orientation of the manipulability ellipsoid relatively to the path. It is well known that if the major axis of the manipulability ellipsoid is parallel to the path then the velocity performance of the manipulator is the best. Therefore, an objective function is formulated to represent the relative orientation of the manipulability ellipsoid to the path. This objective function is not analytical but it is procedural and since the search space is quite complicated a Genetic Algorithm is used to search for the optimum of the formulated objective function.

2. DEFINITION OF THE OPTIMAL LOCATION OF A PATH

Let us assume that the robot of an industrial workcell has been selected, and the shape of a path to be followed by the tip of the robot tool has been defined. This path could represent the welding seam, the pattern to be followed in glue applications or in cut processes. In these cases, it is very important to reduce the motion time by exploiting the robot transmission capabilities at a maximum. Therefore, a location of the path where the velocity performance of the robot is the best, has to be searched, before any minimum time motion algorithm is applied.

From an engineering point of view, it is desired to locate a task into the workspace where the required joint velocities are the lowest for a given velocity of the end-effector. In this case, the energy consumption and the wear of the robot actuators are reduced. Alternatively, it is required to determine the location of a path, where the maximum end-effector velocity would be obtained under the constraints of the upper limits of the joint actuators. Therefore as the best location of the path can be considered the location, where the configurations of the robot along the path require the lower joint velocities over the other locations of the path. This qualitative definition of the best location of a robot path has to be translated to a quantitative definition of an optimisation problem.

For the definition of the optimisation problem it is necessary to define a velocity performance or velocity efficiency measure along a path for robot manipulators. The proposed velocity efficiency measure has been defined after a thorough investigation of the robot performance indices that appeared in the literature.

2.1. Review of the dexterity measures

It is well known that the end-effector velocity is related to the joint velocity through the Jacobian matrix J, and the proposed measures of the robot velocity efficiency are connected to the properties of this matrix. Yoshikawa[15] introduced the following performance measure called manipulability

$$w = \det J \tag{1}$$

for the case of non-redundant manipulators, while for the redundant manipulators the manipulability index is given by

$$w = \sqrt{\det(JJ^T)} \tag{2}$$

Alternatively, the product of the singular values of the Jacobean matrix gives the manipulability index

$$w = \sigma_1 \sigma_2 \ldots \sigma_n \tag{3}$$

where σ_i are the singular values of the Jacobian matrix, and n is the number of the degrees-of-freedom of the robot. The manipulability index can be considered as a measure of the distance between the manipulator configuration and the singular ones. It represents the average motion capabilities of the robot at its configuration, where the manipulability index is determined. Using this as a criterion for the robot velocity efficiency it cannot be concluded which direction of the movement is the best, because it does not include any information concerning the shape of the manipulability ellipsoid.

When the manipulability index is equal to zero, the corresponding configuration is singular. However, this index cannot be used safely as a practical measure of the Jacobian ill conditioning. Instead the matrix condition number has been recommended for its accuracy of estimation. Based on this remark, another dexterity measure is proposed,[16] which is reciprocal to the condition number of the Jacobian matrix

$$k(J) = \frac{\sigma_{\max}}{\sigma_{\min}} \tag{4}$$

where $\sigma_{\max}$, $\sigma_{\min}$ are the maximum and minimum singular values of the Jacobian matrix respectively. Since the maximum and minimum singular values of the Jacobian matrix correspond to the major and minimum axis of the manipulability ellipsoid, this dexterity measure gives an indication of the shape of the ellipsoid. When the condition

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number is equal to unity, the robot is at "isotropic" points, where the robot end-effector can move with the same performance to any direction. In other words, the ellipsoid is transformed into a circle and it presents a directional uniformity. From the definition of the condition number, it can be concluded that the greater the condition number the closer the arm to a singular configuration. The condition number measures the roundness of the ellipsoid but does not give any indication of its volume.

A third index is equal to the minimum singular value of the Jacobian matrix[17] and can be used to set an upper bound on the required joint velocities. The minimum singular value dominates the variation of the manipulability index and the reciprocal of the condition number, since its value changes more sharply close to singularities than the rest of the singular values.

The Cartesian velocity of a body includes components of different nature, namely three elements for the translation velocity and three elements for the angular velocity. Therefore the elements of the Jacobian matrix are of different units and thus these indices could be meaningless. Cosellin[18] and Angeles[19] proposed alternative definitions of the Jacobian matrix to overcome this problem. Another approach to resolve this problem is to normalise the velocities of the joints and of the end-effector, which is followed in this paper.

The mutual orientation of the generalised kinematic ellipsoid (manipulability ellipsoid), the generalised dynamic ellipsoid and the generalised stiffness ellipsoid is given by the Hybrid Manipulator Measures (HMM) introduced by Ermolov and Podurajev.[20] The HMM are frame-invariant and can be used to optimise the trajectory placement within the workspace in the case of constrained motions.

Dubey and Luh[21] used the Manipulator Velocity Ratio (MVR) in control algorithms of redundant robots as a performance criterion to be maximised along a desired trajectory. The Manipulator Velocity Ratio is defined as the ratio of the end-effector velocity norm to the joint velocity vector norm

$$r_v = \sqrt{\frac{\underline{\dot{x}}_v^T \underline{\dot{x}}_v}{\underline{\dot{q}}_v^T \underline{\dot{q}}_v}} \tag{4}$$

where $\underline{\dot{x}}_v$ and $\underline{\dot{q}}_v$ are the weighted vectors of the end-effector and joint velocities, respectively. By replacing the well known equation

$$\underline{\dot{q}}_v = J_v^+ \underline{\dot{x}}_v \tag{5}$$

where J_v^+ is the inverse or the pseudo-inverse of the weighted Jacobian J_v for non-redundant or redundant manipulators, respectively, the following relation for the MVR it can be obtained

$$r_v = \sqrt{\frac{\underline{\dot{x}}_v^T \underline{\dot{x}}_v}{(J_v^+ \underline{\dot{x}}_v)^T (J_v^+ \underline{\dot{x}}_v)}} \tag{6}$$

If $\underline{u}_v$ is a unit vector along $\underline{\dot{x}}_v$, then Eq. (6) can be replaced by the following relation for the MRV:

$$r_v = \frac{1}{\sqrt{\underline{u}_v^T (J_v J_v^T)^{-1} \underline{u}_v}} \tag{7}$$

It was proved that for a given configuration, the end points of the vectors $r_v \underline{u}_v$ form the surface of an ellipsoid with principal axes $\sigma_1 \underline{u}_1, \ldots, \sigma_1 \underline{u}_i, \ldots, \sigma_n \underline{u}_n$, where $\underline{u}_i$ is an eigenvector of the $J_v J_v^T$, and that

$$\sigma_n \le r_v \le \sigma_1 \tag{8}$$

The above reviewed measures or indices depend on the configuration of the robot therefore they characterise locally the performance of the robot and can be used as an instantaneous or local criterion. Integral measures are proposed[7,8] to overcome the local and instantaneous nature of these indices. Martin et al.[7] chose to minimise the following path integral:

$$\int_{t_0}^{t_f} \left(\frac{1}{2} \underline{\dot{q}}^T W^{-1} \underline{\dot{q}} + g(\underline{q}) \right) dt \tag{9}$$

subject to the constraints of the kinematic equations of the robot:

$$\underline{x} = f(\underline{q}) \tag{10}$$

W is a diagonal matrix whose elements reflect the relative cost in terms of energy or time, of utilising each joint axis, and $g(\underline{q})$ is a function of the robot configuration such as the manipulability measure or the distance of some obstacle.

Uchiyama et al.[8] proposed the following integral measure for optimal trajectory planning:

$$\int_{P_0}^{P_1} \frac{ds}{\det J} \tag{11}$$

where P_0 and P_1 are the end points of the desired trajectory in the Cartesian space, and ds is the differential distance along the trajectory.

From the review of the proposed dexterity measures so far the following conclusions can be derived:

(i) The manipulability index, the reciprocal of the condition number, and the minimum singular value, or any combination of them can represent the motion ability of the robot. Using these criteria the ability of the robot to move in a given direction cannot be derived.
(ii) The MVR is an index of the robot ability to move in a given direction at a specific configuration.
(iii) The Hybrid Manipulator Measures are suitable for combined tasks, where velocity and applied force are specified simultaneously.
(iv) These integral measures proved superior compared to the instantaneous ones, but they are used to determine a trajectory in the joint space for given boundaries in the Cartesian space. However, even if the manipulability of the robot is taken into account in these

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integral measures, the directional variation of the robot velocity efficiency is not considered.

(v) The integral measures are time consuming and can fail when the integrated function presents large variations. A very high value of the integrated index may give a poor performance along the rest of the path.

2.2. *The proposed optimisation criterion*

From the investigation of the proposed measures, it can be concluded that the velocity ratio (MVR) depends not only on the robot configuration but also on the end-effector velocity vector. In addition, it could be proved that for a given configuration the MVR presents a measure of the deviation between the end-effector velocity and the major axis of the ellipsoid. This can be understood better by the following example, where a 3D ellipsoid is used to illustrate the relation between the principal axes of the ellipsoid and the end-effector velocity. In Figure 1, a 3D ellipsoid is shown, where the vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ correspond to the major, secondary and minor axes of the 3D ellipsoid, respectively.

If $|\underline{\dot{q}}|=1$ then

$$\left(\frac{V_x}{a}\right)^2+\left(\frac{V_y}{b}\right)^2+\left(\frac{V_z}{c}\right)^2=1 \tag{12}$$

where $\underline{V}=[V_x, V_y, V_z]^T$ is the vector of the translational velocity of the end-effector referenced to the Cartesian system defined by the axes of the manipulability ellipsoid. Since the elements of the end-effector velocity vector are equal to the projection of the velocity vector to the principal axes of the ellipsoid, Eq. (12) can be written as:

$$|\underline{V}|=\frac{1}{\sqrt{\left(\frac{\underline{u}^T\underline{a}}{a^2}\right)^2+\left(\frac{\underline{u}^T\underline{b}}{b^2}\right)^2+\left(\frac{\underline{u}^T\underline{c}}{c^2}\right)^2}} \tag{13}$$

where $\underline{u}$ is the unit vector along the end-effector velocity. When the direction of the end-effector velocity coincides with the major axis of the ellipsoid, the magnitude of the end-effector velocity is maximum, since the scalar products of $\underline{u}$ with the vectors corresponding to the other two axes become equal to zero due to the orthogonality of the ellipsoid axes. For the same reason, the end-effector velocity becomes minimum when its direction coincides with the minor axis.

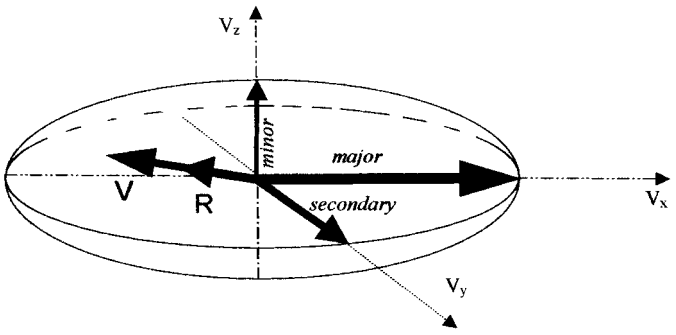


Fig. 1. The axes of the 3D manipulability ellipsoid and the end-effector velocity.

From the presentation of the MVR, it can be concluded that it includes information concerning the ability of the robot to move the end-effector along a specific direction, and it gives the velocity efficiency of the robot in this direction. Therefore, it is the most suitable criterion for the free motion of a robot; however, it is a local and instantaneous criterion. In the following, various forms of global criteria for the movement of a robot along a path with its maximum velocity efficiency are suggested and compared. The first is the minimum of the MVR and the second is the mean value of the MVR along a path.

The problem of estimating an optimal location of a desired robot path of a given shape is defined as follows: Determine the location of the path represented by a curve in 3D space so that the minimum of the Manipulator Velocity Ratio (MVR) along the path is maximum. The minimax form of optimisation gives the best worst-case performance throughout the workspace. Therefore, the location of the path is determined where the robot presents high performance at every point of the path.

The method is presented using a straight-line segment as the desired shape of the path for simplicity and then it can be extended to any 3D curve. The local Cartesian co-ordinate frame $\{F_S\}$ and the length of the segment define the position and the orientation of the straight-line segment. As it is illustrated in Figure 2, the starting frame $\{F_S\}$ coincides with the staring point of the path, while the x-axis is along the path and the z-axis is perpendicular to the straight-line segment.

The frame $\{F_S\}$ is defined in a parametric form by $\underline{x}_s=[x\ y\ z\ \vartheta\ \phi\ \psi]^T$,where x, y, z are the Cartesian co-ordinates of the starting point of the segment defined in the universal frame $\{U\}$ and ϑ, φ, ψ are the Euler angles of the frame $\{F_S\}$ referenced to the frame $\{U\}$. Alternatively, the location of the frame $\{F_S\}$ can be defined through the transformation

$$^{U}T_{F_S}(\underline{q}_s),\quad \underline{q}_s=[q_1\ \ldots\ q_n]^T \text{ and } \underline{q}_S^L\le\underline{q}_S\le\underline{q}_S^U \tag{14}$$

where $\underline{q}_S$ is the vector of the joint co-ordinates of the manipulator, when the frame of the tool coincides with the frame $\{F_S\}$. For simplicity, it is supposed that the tool is perpendicular to the segment. The Cartesian or the joint co-ordinates defining the local frame $\{F_S\}$ are the variables of the objective function representing the minimum value of

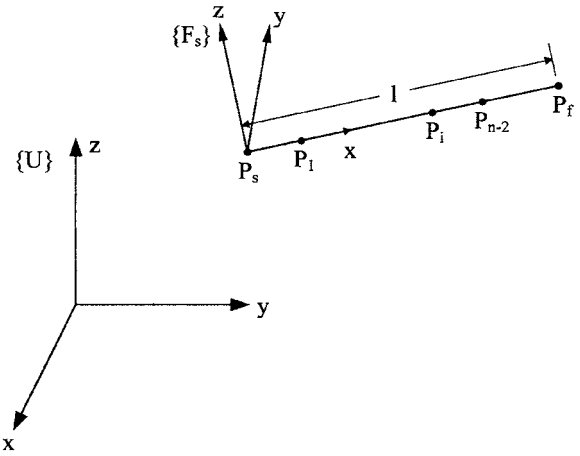


Fig. 2. The intermediate points of the straight-line segment.

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the MVR along the path. The global measure, which is the objective function, is approximated in a procedural way based on Taylor's[22] bounded deviation method for planning and execution of straight-line manipulator trajectories. It is well known, that given two configuration frames of the hand, the starting frame $\{F_S\}$ and the frame $\{F_f\}$ corresponding to the final point of the path, the Taylor's algorithm calculates the corresponding joint co-ordinates $\underline{q}_S$ and $\underline{q}_f$. Subsequently, the Cartesian co-ordinates of the joint midpoint are calculated and are compared to the Cartesian configuration corresponding to the midpoint of the straight-line segment. If the two configurations "deviate" more than an allowed amount, the final point configuration is replaced by the Cartesian midpoint configuration and the algorithm is applied recursively to this straight-line segment. This generation of midpoints terminates, when the deviation is smaller than the specified amount. The arm motion is linearly interpolated between successive midpoints. Taylor proposed the Euclidean metric as a position deviation function. This technique reduces substantially the real-time computation.

Let suppose that the Taylor's algorithm determines P_i $(i=1, k-2)$ intermediate knot points, then a global index for the MVR along the path is given by

$$r_m=\min\{(r_v)_i, (r_v)_s, (r_v)_f\}, \quad for \quad i=1, k-2 \tag{15}$$

where $(r_v)_i$, $(r_v)_s$, and $(r_v)_f$ are the MVR indices at an intermediate point, the starting point, and the final point of the segment, respectively. The calculated minimum MVR along the path is an approximation of the real minimum of the MVR along the path, and it depends upon the permitted maximum position deviation used in Taylor's algorithm. This dependence is complicated enough so it is difficult to estimate the error. However, using the proposed approximation, a considerable amount of computational time is saved.

It is well known that the Taylor algorithm converges quickly and a deviation can be obtained as small as it is desired. The relation between the position deviation and the respective deviation of the MVR is very complicated and it can be assumed that this function is incremental at least for small position deviation as it concluded experimentally. However, near a singular point for a small position deviation a very large deviation of the corresponding MVRs is expected. Since the proposed optimisation algorithm pushes the location of the path far from singular points then such problems are avoided.

Every intermediate knot position orientation of the tool on the straight-line segment is given as a function of the parameters defining the frame at the starting position orientation by

$${}^{U}T_{F_i}={}^{U}T_{F_S}(\underline{q}_S)[L_i \; 0 \; 0]^T \tag{16}$$

where L_i is the distance of the ith intermediate point from the origin of the coordinate system $\{F_S\}$

The Cartesian co-ordinates $\underline{x}_S$ can replace the joint co-ordinates $\underline{q}_S$, but in the following the joint coordinates are used. Solving the inverse kinematic problem at every intermediate and the final position/orientation of the tool, the joint co-ordinates of the manipulator are determined and then the corresponding Jacobian matrix

$$J(\underline{q}_i({}^{U}T_{F_i}(\underline{q}_s)) \tag{17}$$

For a genetic algorithm search, it is not necessary to obtain an analytic formula for the path MVR index so the inverse kinematic problem can be solved numerically if it is impossible to obtain an analytic solution. Using Eq. (17) the MVR at every intermediate knot point and at the two end points of the segment are calculated as functions of $\underline{q}_S$, and finally the measure r_m is calculated using Eq. (15). So at the end of the analysis the MVR index along the path is determined as a function of the joint coordinates of the frame $\{F_S\}$.

The number and the position of the intermediate knot points depends upon the location of the path into the workspace so that the objective function is non-continuous.

In a previous work by Aspragathos,[22] the Taylor's method has been generalised to be applicable to a Cartesian trajectory represented by any 3D curve. In this case too, the algorithm calculates enough intermediate points in order to generate a bounded position deviation trajectory of the tip of the manipulator hand. Therefore, the proposed formulation of the index r_m can be applied to a 3D curve using the generalised Taylor's method.

For comparison and final selection of the best objective function in order to search for the optimal location of a path from the point of the velocity efficiency of the robot, an index equal to the average of the MVR along the path is also proposed.

$$r_a=((r_v)_s+(r_v)_f+\sum_{i=1}^{i=k-2}(r_v)_i)/k \quad for \quad i=1, k-2 \tag{18}$$

In this paper, two additional velocity efficiency indices based on the robot manipulability are investigated and compared. The index w_m presents the minimum manipulability along the path and it is defined by

$$w_m=\min(w_i, w_s, w_f) \quad for \quad i=1, k-2 \tag{19}$$

where w_i, w_s, and w_f are the manipulability indices at an intermediate point, the starting point, and the final point of the straight-line segment, respectively. The forth measure of the velocity efficiency along the path, which is investigated in this work, is the path manipulability defined by Aspragathos.[14] This measure represents the approximate average of the manipulability along the path and it is given by

$$w_a=\left(\sum_{i=1}^{i=k-2} w_i+w_s+w_f\right)\Big/k \tag{20}$$

From the presented analysis of the proposed indices concerning the performance of the robot along a path, the following conclusions can be derived:

(i) The definition of the objective function based on the minimum of a local index along the path is better than an objective function representing the mean value,

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since in the mean value index the result can be dominated by a very high value of the local index at a point of the path.

(ii) The bisection method used to determine the local indices in the intermediate points of the path saves computational time compared to that of integral indices.

(iii) The MVR index gives a measure of the robot velocity performance in the direction of the path, while the manipulability index gives an average of the robot velocity performance and the reciprocal of the conditioning number represents the degree of isotropy.

Based on these conclusions the measure r_m is proposed as the most suitable objective function, in order to determine a location of the path where the robot presents the best velocity performance. To strengthen this qualitative justification of the proposed measure, the presented four indices r_m, r_a, w_m, and w_a are compared experimentally in the section of the results of this paper and it is verified that the first index gives more coherent relation between the index and the required maximum joint velocity along the path. Therefore, the first index is used in the following as an objective function in searching for the optimal location of a path in the robot workspace.

By maximising the global MVR index r_m subject to the constraints of optimisation variables given by Eq. (14), one can find the location of the path where the MVR of the robot is the higher. This optimisation problem cannot be solved using a classical gradient method (see Cooper and Steinberg[23]) because the objective function has a procedural nature and in the most general case is non-analytic, non-continuous and non-differentiable. The selection of the Genetic Algorithms to solve this problem is based on the advantages of the GAs in search of very complex and unstructured environments.

3. THE GENETIC ALGORITHM SOLUTION

Genetic Algorithms (GA) are powerful domain independent search mechanisms, which emulate the process of genetic evolution found in nature as a means of progressing towards the optimum. Combining an artificial survival of the fittest with special genetic operators, GA provides a robust search mechanism that is suitable for a variety of search problems. A Genetic Algorithm works simultaneously from a number of points scattered in the search space and examines many local optima in parallel, thus the probability to determine the global optimum is high, when it works in a multi-modal space. Genetic Algorithms require only the objective function, while in many optimisation methods auxiliary information is necessary. For example, the gradient methods need the derivatives of the objective function.

In the following, a solution to the formulated optimisation problem based on a simple Genetic Algorithm is introduced. There are at least three basic reasons that make GAs attractive in solving such a problem: First, they are proven to provide a robust search in complex spaces by finding nearly global optima. In general, the bisection recursive method used to define the knot points under bounded position deviation provides a non-continuous, multi-modal and very complex non-linear search space. On the other hand, classical optimisation methods depending upon restrictive requirements of continuity and derivative existence, and due to their inherent local scope of search, are unsuitable for the solution of this problem. Second, GAs do not require an analytic form of the objective function and of the constraints. In our case the objective function and the constraints are procedural. Third, GAs allow additional constraints to be specified easily, so the introduced method can be extended to work in an environment cluttered with obstacles or other constraints.

The simple GA used in the presented optimisation problem is composed of the following three operators: reproduction, crossover and mutation. This simple GA has been proved effective for a variety of optimisation problems. A critical aspect in designing a GA is the basic mechanism that links the GA to the real problem, which has to be solved. This mechanism is twofold: First, an evaluation of the function (fitness) that returns a measure of the expected goodness of the solution is defined, and, second, a way of encoding solutions to the real problem on artificial chromosomes is proposed.

GAs are essentially unconstrained search procedures within the given representation space. Goldberg[24] proposed a penalty function concept to convert the constrained problem to an unconstrained one. Another formulation is to design and apply special recombination operators that maintain feasibility. In the GA used to solve the proposed optimisation problem, a different approach introduced by Nearchou and Aspragathos[25] has been adopted, so when a candidate solution violates the constraints, the fitness function takes a value equal to zero and has almost zero possibility to survive and no hope to get copies in the next generations.

Therefore the fitness function related to the objective function given by Eq. (15) and the joint coordinate constraints (see Eq. (14)) is given by

$$fitness = \begin{cases} r_m(\underline{q}_S), & \text{if} \quad \underline{q}_S^L \le \underline{q}_S \le \underline{q}_S^U \\ 0, & \text{if} \quad \underline{q}_S \le \underline{q}_S^L \text{ or } \underline{q}_S^U \le \underline{q}_S \end{cases} \tag{21}$$

The variables of the objective function are the joint coordinates q_S determining the local frame $\{F_S\}$, which defines the position and the orientation of the path. This set of variables is coded as a finite length binary string, which represents one chromosome of the population.

In this work, the chromosome selected is an m-bit string with the following syntax: the (m/n)-bits correspond to the first joint of the robot, the next (m/n)-bits correspond to the second joint, etc. Therefore the ith joint co-ordinate can be calculated from the values of the corresponding (m/n)-bit using the following mapping:

$$q_i = q_i^L + [\underline{Bitvalue}/2^{m/n}]q_i^U \tag{22}$$

where $\underline{Bitvalue}$ is the decimal number corresponding to the string of the ith joint, and q_i^L, q_i^U are the limits of the ith joint variable.

It must be underlined that a single m-bit string represents one of the 2^m alternative solutions to the problem. The value

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m/n, which denotes a number of bits that correspond to a parameter of the original optimization problem, is not selected arbitrarily but depends on the adequate "resolution" while preventing the search space from becoming too large for easy and effective searching. The GA generates successive populations of the strings in the form of natural selection. The purpose of the searching is to find the best-fit string (chromosome). Concerning the convergence of the algorithm, the following termination condition is used: (average fitness)/(best fitness)$>\varepsilon$, where ε is an arbitrarily selected real number close to one and represents the degree of the uniformity of the chromosome values (alleles).

4. DISCUSSION OF THE RESULTS

In this paper, two categories of simulated experiments are carried out using a PUMA-like robot. The first category includes the experiments for the investigation of the suitability of the proposed measures for the determination of the location of the path where the velocity efficiency of the robot is the best. In the second category, the efficiency of the proposed method for estimating the location of a straight-line path with the higher minimum MVR along the path is demonstrated.

4.1. Comparison of the proposed measures

For an arbitrary location of a straight-line segment the end-effector of the robot moves with constant velocity along this path using the Resolved Motion Rate Control.[25] For this location of the path the four measures are determined and the maximum joint velocity appeared during the motion along the path is calculated. The joint velocities are normalised to obtain a common base for comparison. This experiment is repeated for a large number of randomly selected locations of the same straight-line segment where the end-effector moves with the same constant velocity. The results of these experiments are presented in the diagrams of Figures 3 to 6.

Figure 3 illustrates the relation between the maximum joint velocity $\dot{q}_i^{\max}$ and the minimum of the MVR along the path r_m. It is clear that the points of the diagram presenting the pairs $\{\dot{q}_i^{\max}, r_m\}$ are very well correlated and a very close

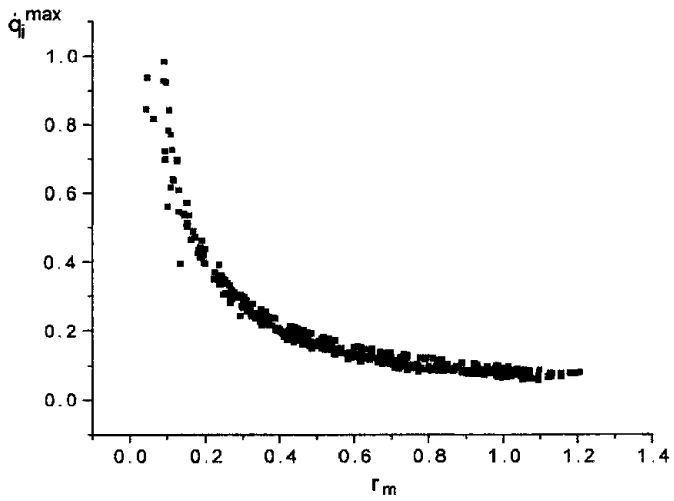


Fig. 3. The relation between the maximum joint velocity $\dot{q}_i^{\max}$ and the minimum of the MVR along the path r_m.

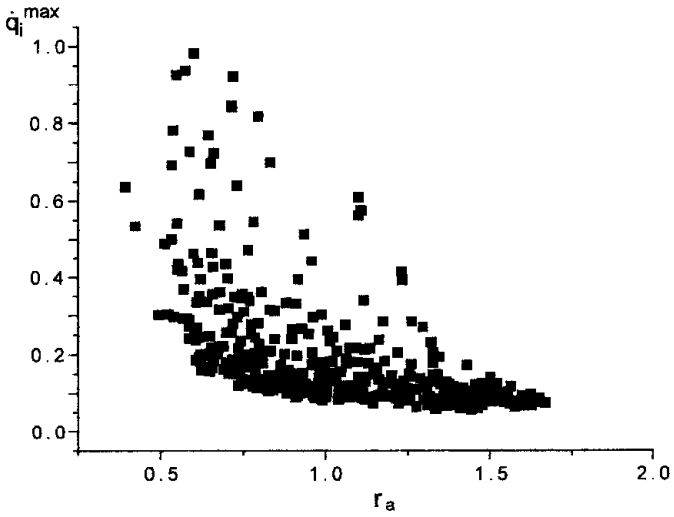


Fig. 4. The maximum joint velocity $\dot{q}_i^{\max}$ versus the average of the MVR along the path r_a.

to reciprocal relation can be concluded. The scattering of the points is quite limited. In Figure 4, the maximum joint velocity $\dot{q}_i^{\max}$ versus the average of the MVR along the path r_a is shown, for all the randomly selected locations of the path. For low and medium values of the measure r_a the scattering of the points is much greater than for high values of r_a.

Figure 5 presents the relation between the maximum joint velocity $\dot{q}_i^{\max}$ and the minimum of the manipulability along the path w_m, for all the randomly selected locations of the path. The high scattering of the points appears for low values of the measure w_m. The higher scattering appears in the last diagram shown in Figure 6, where the relation between maximum joint velocity $\dot{q}_i^{\max}$ and the average manipulability w_a is presented.

From the comparison of the results presented in Figures 3 to 6, it can be concluded that the measure of the velocity efficiency along a path r_m defined by the minimum MVR is superior, since statistically it corresponds to the lowest maximum joint velocity for the same location of the path and the same constant end-effector velocity.

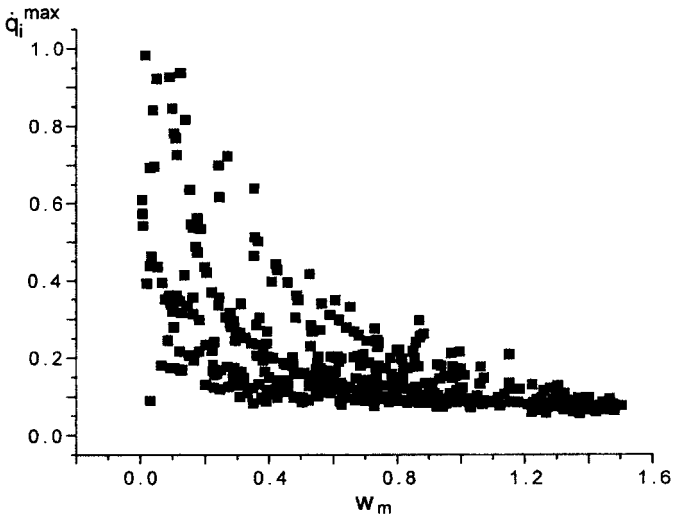


Fig. 5. The relation between the maximum joint velocity $\dot{q}_j^{\max}$ and the minimum of the manipulability along the path w_m.

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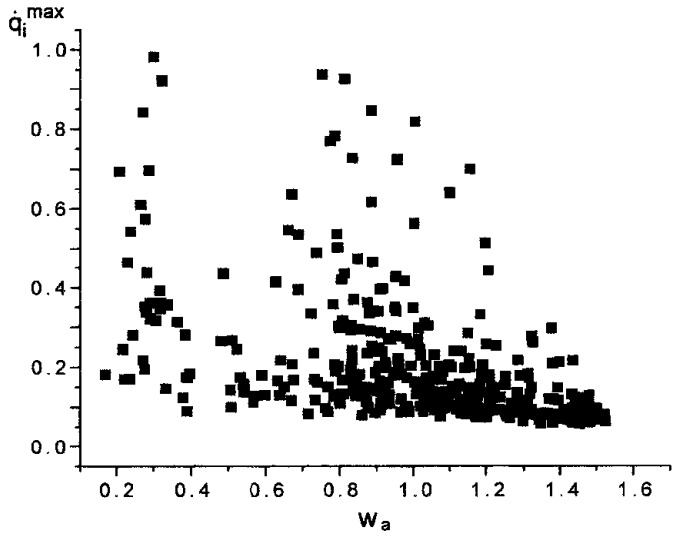


Fig. 6. The relation between maximum joint velocity $\dot{q}_i^{\max}$ and the average manipulability w_a along the path.

4.2. Search for the best location of the path

The second category of experiments includes the demonstration of the efficiency of the proposed method in searching for the best location of a straight-line path of length L. Suppose that the simulated PUMA-like robot has to move the end-effector along the path presented by a straight-line segment with the tool axis perpendicular to the path. It is required to search for a location of the straight-line segment into the workspace of the robot where the velocity efficiency of the robot will be the best. The velocity efficiency of the robot along the path is defined by the fitness function given by Eq. (21).

The Genetic Algorithm was run several times and some of the results are presented in Table I. In the first column of the Table I, the maximum of the r_m (the minimum of the MVR along the path) obtained by searching the workspace of the robot is presented. In the second column, the maximum joint velocity $\dot{q}_i^{\max}$ is shown, which is obtained by moving the end-effector along the path with constant velocity using the Resolved Motion Rate Control.[25] The third and fourth columns shows the Cartesian coordinates of the starting and end point of the path in the determined optimal location, respectively.

From the results shown in the Table I, the following conclusions can be derived: The maximum fitness obtained by the runs of the Genetic Algorithm ranges between 1.28 and 1.16, due to the low accuracy of the Genetic Algorithms. However, all the results are very close to the maximum of the fitness function, so the solutions obtained by the proposed method are considered as sub-optimal ones. Even for the first three cases, where the obtained maximum of r_m is equal to 1.28, the obtained locations of the path differ considerably. This means that there are more than one areas in the workspace of the robot, where a sub-optimal location can be found. This conclusion helps the designer of a workcell, because there are more alternative "very good" solutions for the location of a task.

Table I.

r_m	$\dot{q}_i^{\max}$	Starting point	Final point
1.28	0.68	(1.69, 0.02, –0.32)	(1.57, –0.92, –0.00)
1.28	0.73	(1.18, –1.44, –0.56)	(0.27, –1.58, –0.94)
1.28	0.69	(1.45, –1.25, –0.25)	(1.65, –0.40, –0.75)
1.275	0.68	(–0.32, 1.79, 0.30)	(–1.14, 1.24, 0.47)
1.26	0.69	(0.95, –1.52, –0.13)	(1.49, –0.71, –0.34)
1.25	0.77	(0.16, 1.93, 0.53)	(–0.58, 1.51, 1.05)
1.24	0.78	(1.5, 0.2, 0.74)	(1.63, 0.86, –0.01)
1.22	0.72	(0.68, 1.42, 0.21)	(1.48, 0.89, –0.05)
1.20	0.79	(–0.93, 1.4, –0.57)	(–1.6, 1.05, 0.09)
1.16	0.60	(–0.93, –1.62, –0.59)	(0.04, –1.87, –0.55)

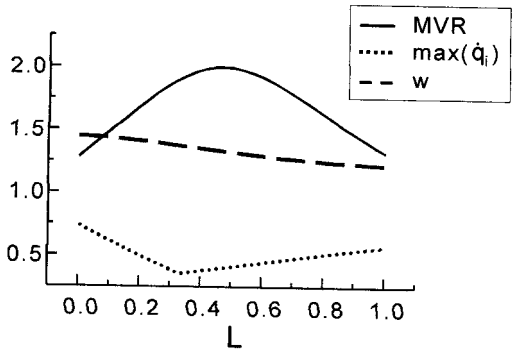


Fig. 7. Variation of the MVR, of the maximum joint velocity and of the manipulability along the path for the 2nd case of the Table I.

For further investigation of the results obtained by the proposed method, the variation of the MVR (solid line), of the maximum between the absolute of the joints velocity (dotted line), and the manipulability index (dashed line) along the path are presented in the diagrams of Figures 7 and 8 corresponding to the 2nd and 6th case of the Table I respectively. The presented diagrams show that the variation of the MVR presents the minimum value at one of the ends of the path and the maximum MVR around the middle of the path. The maximum between the absolute velocities of the joints for each configuration of the robot moving its end-effector with constant velocity presents a minimum around the middle of the straight-line path. This correspondence between the maximum of the MVR and the minimum of the maximum joint velocity was expected from the definition of the MVR. Similar results are obtained in all the runs of the GA, so the proposed method can be considered as quite robust even the accuracy of the results is not high.

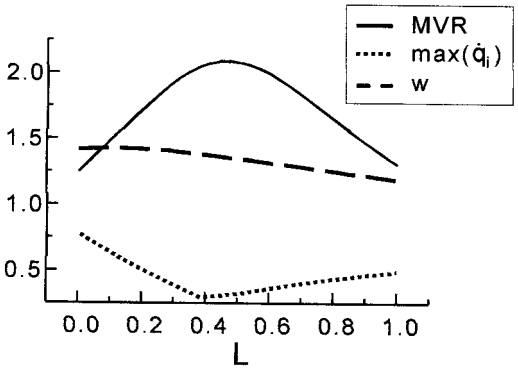


Fig. 8. Variation of the MVR, of the maximum joint velocity and of the manipulability along the path for the 6th case of the Table I.

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The proposed method can be implemented and incorporated in software packages for robot simulators and for robotic workcell design to help the designer in the layout of the workcell. The designer can locate the path in the determined location using the proposed algorithm and then he can apply another minimum time algorithm to reduce the cycle time of the task by exploiting the velocity transmission capabilities of the robot.

5. CONCLUSIONS

An approach for the determination of an optimal or sub-optimal location of a path into the workspace of a robot is presented. The optimisation criterion is based on the robot velocity efficiency or the capability of velocity transmission of the robotic mechanism. A global measure describing the velocity efficiency of the robot along the path is introduced. Among the four suggested global measures, which are compared experimentally, the minimum Manipulator Velocity Ratio (MVR) was proved superior. This result was expected since the MVR depends on the direction of the velocity end-effector, so this criterion is a measure of the deviation of the path direction to the direction of the manipulability ellipsoid.

The proposed measure is calculated in a procedural way, which does not guarantee the continuity of the objective function of the formulated optimisation problem. Mainly for this reason, a Genetic Algorithm is used to search the complex space for a global optimal or sub-optimal location of the path. By successive runs different sub-optimal solutions are determined in the workspace of a PUMA-like robot. The maximum values of the proposed criterion are very close to each other while the corresponding locations are scattered into the workspace.

After a thorough verification and validation, the proposed algorithm can be incorporated into software packages for robot simulation. Using this algorithm the best location of a task can be determined, which is a considerable help in optimal design of the layout of a robotic workcell. Therefore the proposed algorithm can save design time and cost, and reduce the energy consumption and wear of the robot or the cycle time of a task.

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