

FIGURE 3

Hence, exactly 15 distinct tiles that tessellate can be formed on a square template with identical asymmetric edges. Thirteen are isohedral and two are anisohedral.

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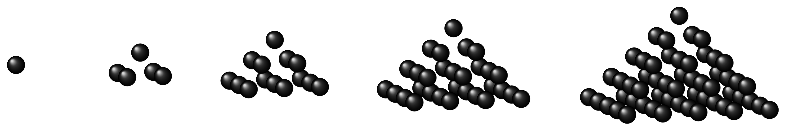
ROBERT SCHENK

424 W Jackson St., Rensselaer, IN 47978 USA

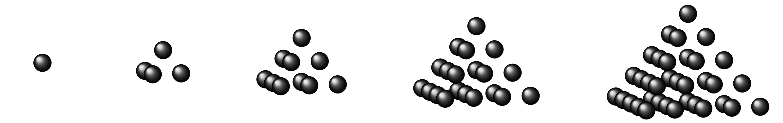
e-mail: *schenk@saintjoe.edu*

100.35 Proof without words: the average of square pyramidal and triangular is tetrahedral

Let P_n represent the number of dots in a square pyramid with n levels and T_n represent the number of dots in a triangular pyramid with n levels both illustrated below.



$P_1 = 1$ $P_2 = 5$ $P_3 = 14$ $P_4 = 30$ $P_5 = 55$



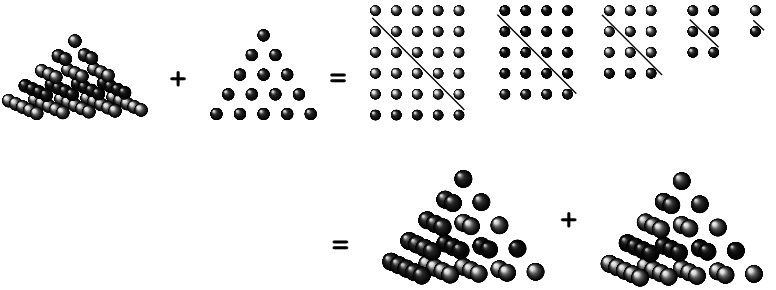
$T_1 = 1$ $T_2 = 4$ $T_3 = 10$ $T_4 = 20$ $T_5 = 35$

We call these the square pyramidal and tetrahedral numbers, respectively. Also, let $t_n = 1 + 2 + \ldots + n$ represent the nth ordinary triangular numbers.

Theorem: The nth tetrahedral number is the average of the nth square pyramidal number and the nth triangular number, i.e. $T_n = \dfrac{P_n + t_n}{2}$.

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Proof:



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TOM EDGAR
Mathematics Department, Pacific Lutheran University, Tacoma, WA, USA
e-mail: *edgartj@plu.edu*

100.36 Another look at higher vertical motion

Introduction

Suppose a projectile is fired vertically into space with initial height h and velocity v_0. Assuming that the acceleration due to gravity is the constant g and there is no air resistance, the motion is governed by

$$\frac{d^2y}{dt^2} = -g. \tag{1}$$

The velocity v and the distance above the ground y are given by

$$\frac{dy}{dt} = -gt + v_0, \text{ and } y = -\tfrac{1}{2}gt^2 + v_0t + h.$$

Here we assume that the projectile does not go very high, so that we can have a constant gravitational force. But how high is 'not very high'? This question cannot be answered by the above analysis. In this paper we will discover how to replace the differential equation (1) by a more accurate relation

$$\frac{d^2y}{dt^2} = -g + \left(\frac{h}{R}\right)\frac{2gy}{h}. \tag{2}$$

In (2), R is the radius of the planet or moon near the surface of which the motion of the particle mass m is taking place and g is the acceleration at the surface of this heavenly body. Because R is very large, the dimensionless factor $\frac{h}{R}$ is very small. This small correction allows us to calculate the motion of the projectile when it reaches heights that are a bit too high for (1) to be satisfactory. We will solve (2) and obtain an improvement in the velocity as

$$\frac{dy}{dt} = -gt + v_0 + \frac{h}{R}\left(2gt + \frac{v_0g}{h}t^2 - \frac{g^2}{3h}t^3\right) \tag{3}$$

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