MHD DOUBLE DIFFUSIVE NATURAL CONVECTION FLOW OVER EXPONENTIALLY ACCELERATED INCLINED PLATE

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ABSTRACT

An investigation of unsteady MHD double diffusive natural convection flow of a viscous, incompressible, electrically conducting, heat absorbing, radiating and chemically-reactive fluid past an exponentially accelerated moving inclined plate in a fluid-saturated porous medium, when the temperature of the plate and the concentration at the surface of the plate have ramped profiles, is carried out. Exact solutions for the fluid velocity, fluid temperature and the species concentration, under Boussinesq approximation, are obtained in closed form by the Laplace transform technique. The expressions for the shear stress, rate of heat transfer and the rate of mass transfer at the plate are also derived. Numerical evaluations of the fluid velocity, fluid temperature and the species concentration are performed and displayed graphically whereas those of the shear stress, rate of heat transfer and the rate of mass transfer at the plate are presented in tabular form for various values of the pertinent flow parameters.

Keywords: Thermal radiation, Chemical reaction, Ramped temperature, Ramped surface concentration.

1. INTRODUCTION

Study of magnetohydrodynamic natural convection flow with heat and mass transfer in porous and non-porous media is of considerable importance due to its varied and wide applications in many engineering and industrial processes viz. geothermal energy exploration, building insulation, desert coolers, materials processing etc. Keeping in view the importance of such study, several researchers investigated magnetohydrodynamic natural convection flow in porous and non-porous media with heat and mass transfer near bodies with different geometries. Some notable research studies on the topic are due to Hossain and Mandal [1], Jha [2], Ibrahim *et al.* [3], Eldabe *et al.* [4] and Gorla and Chamkha [5, 6].

Hydromagnetic natural convection flows under the influence of thermal radiation have gained considerable attention of researchers due to its frequent occurrence in many physical, scientific and industrial problems. Heat exchanger, internal combustor, glass production, furnace design, cosmical flight aerodynamics, rocket propulsion systems and spacecraft re-entry vehicles are some of the examples where radiation effects are greatly significant. It is interesting to note that the presence of radiation term makes energy equation non-linear. Therefore, to solve the energy equation with radiative heat transfer, some reasonable approximations are proposed to linearize the equation e.g. Cogley approximation [7] and Rosseland approximation [8]. Takhar *et al.* [9] investigated the effect of thermal radiation on hydromagnetic natural convection flow over a semi-infinite vertical plate under the action of buoyancy force. Azam [10] analyzed the effect of radiation on MHD mixed convection flow past a semi-infinite moving vertical plate when the temperature differences are very high. Mbeledogu and Ogulu [11] discussed the unsteady MHD natural convection heat and mass transfer flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer. Palani and Abbas [12] made a numerical study to analyze the combined effects of magnetic field and radiation on

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free convection flow past an isothermal vertical plate. Prakash *et al.* [13] studied the effect of radiation on MHD free convection flow past an impulsively started infinite vertical plate taking Dufour effects into account. Hussanan *et al.* [14] considered unsteady MHD natural convection flow of a radiating fluid over an oscillating vertical plate with constant mass diffusion and Newtonian heating. Recently, Vedavati *et al.* [15] discussed the heat and mass transfer effects on hydromagnetic natural convection flow through a porous medium in the presence of thermal radiation.

There has been a keen interest of researchers in studying hydromagnetic fluid flow problems in porous and non-porous media taking into consideration the effect of temperature dependent heat source/sink which may have strong influence on heat transfer characteristics of several physical problems of practical interest viz. convection in Earth's mantle, post-accident heat removal, fire and combustion modeling, fluids undergoing exothermic and/or endothermic chemical reaction, development of metal waste from spent nuclear fuel etc. In light of these applications, several researchers investigated the hydromagnetic natural convection flow of a viscous, incompressible and heat generating/absorbing fluid near bodies with different geometries under different conditions. Mention may be made of the research studies on the topic by Vajravelu and Nayfeh [16], Chamkha and Khaled [17], Chamkha [18], Alam *et al.* [19], Ravikumar *et al.* [20] and Seth *et al.* [21]. Moreover, the combined effects of heat generation/ absorption and thermal radiation on Magnetohydrodynamic natural convection flow play a crucial role in controlling the heat transfer and may have important applications in the industry. Keeping in view the importance of such study, Chamkha [22], Seddeek [23], Saha *et al.* [24], Reddy *et al.* [25], Vieru *et al.* [26] and Uddin *et al.* [27] considered the effects of radiation and heat generation/absorption on hydromagnetic natural convection flow past a flat plate considering different aspects of the problem.

It has been observed that combined heat and mass transfer flow with chemical reaction is significant in many engineering and industrial processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler. Therefore, it has gained considerable attention of researchers in the past few years. Anjalidevi and Kandasamy [28] investigated the effect of chemical reaction, heat and mass transfer on laminar boundary layer flow along a semi-infinite horizontal plate. Postelnicu [29] studied the influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous medium considering Soret and Dufour effects. Patil and Kulkarni [30] discussed the effect of chemical reaction on free convection flow of a polar fluid through a porous medium in the presence of internal heat generation. Ibrahim *et al.* [31] considered the chemical reaction and radiation absorption effects on unsteady hydromagnetic free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Pal and Talukdar [32] analyzed the chemical reaction and buoyancy effects on MHD mixed convection flow with heat and mass transfer in a porous medium taking thermal radiation and Ohmic heating into account. Ali *et al.* [33] investigated the effect of chemical reaction on hydromagnetic free convection heat and mass transfer flow past a vertical plate embedded in a fluid saturated porous medium. Khan *et al.* [34] made a numerical study of Magnetohydrodynamic laminar boundary layer flow past a wedge under the influence of thermal radiation, heat generation and chemical reaction. Seth *et al.* [35] studied the effect of chemical reaction on unsteady hydromagnetic natural convection flow of a thermally radiating fluid past a vertical plate considering Newtonian heating into account. Tripathy *et al.* [36] analyzed the heat and mass transfer effect on MHD flow past a moving vertical plate through porous medium in the presence of heat source and chemical reaction. Gorla *et al.* [37] considered the effect of chemical reaction and heat source on MHD natural convection flow past an infinite vertical plate in a rotating frame.

In all the above investigations, the solution of the problems is obtained under different thermal conditions which are continuous and well defined. However, there are so many practical problems which involve non-uniform or discontinuous thermal conditions. Some of the industry based applications include nuclear heat transfer control, materials processing, turbine blade heat transfer, electronic circuits and sealed gas-filled enclosure heat transfer operations. Taking into consideration this fact several researchers investigated natural convection flow with ramped wall temperature. Some of the relevant research studies are due to Chandran *et al.* [38], Nandkeolyar and Das [39], Mohamad *et al.* [40], Samiulhaq *et al.* [41], Rajesh and Chamkha [42], Kundu *et al.* [43], Das *et al.* [44], Khalid *et al.* [45], Seth and Sarkar [46] and Seth *et al.* [47].

In the present problem we propose to study an unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting, optically thick radiating, temperature dependent heat absorbing and chemically reactive fluid with combined heat and mass transfer near an exponentially accelerated moving infinite inclined plate embedded in a fluid saturated porous medium when the temperature of the plate and concentration at the surface of the plate have ramped profiles. As per author's knowledge, this problem has not yet received attention of the researchers. This study may have bearings on the problems of practical interest viz. inclined plate clarifiers, oil-water separator, sedimentation equipment such as inclined plate settlers, in fabrication of thin film, photo voltaic devices, air conditioning system regarding to building heat transfer, nuclear heat transfer, material processing, electronic circuit, sealed gas filled enclosure etc.

2. FORMULATION OF THE PROBLEM

Consider unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting, temperature dependent heat absorbing, optically

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thick radiating and chemically reactive fluid with combined heat and mass transfer near an inclined plate embedded in a fluid saturated porous medium. The Cartesian co-ordinate system is chosen in such a way that $x'-$ axis is along the plate which is inclined at angle θ with the vertical direction and $y'-$ axis is normal to the plane of the plate in the fluid. Fluid is permeated by uniform transverse magnetic field B_0 which is applied in a direction parallel to y' axis. Initially i.e. at time $t' \le 0$ both the fluid and plate are at rest and maintained at uniform temperature T'_∞ and uniform concentration C'_∞. At time $t' > 0$ plate is exponentially accelerated with velocity $U_0 e^{a't'}$ in the $x'-$ direction (U_0 and a' being uniform velocity and plate acceleration coefficient respectively). Temperature of the plate is raised or lowered to $T'_\infty + (T'_w - T'_\infty)t'/t_0$ and the level of concentration at the surface of the plate is raised or lowered to $C'_\infty + (C'_w - C'_\infty)t'/t_0$ when $0 < t' \le t_0$ Thereafter i.e. at $t' > t_0$ plate is maintained at uniform temperature T'_w and level of concentration at surface of the plate is preserved at uniform concentration C'_w It is assumed that there exists a homogeneous chemical reaction of first order with constant rate K'_2 between the diffusing species and the fluid. Geometry of the problem is displayed in Fig. 1.

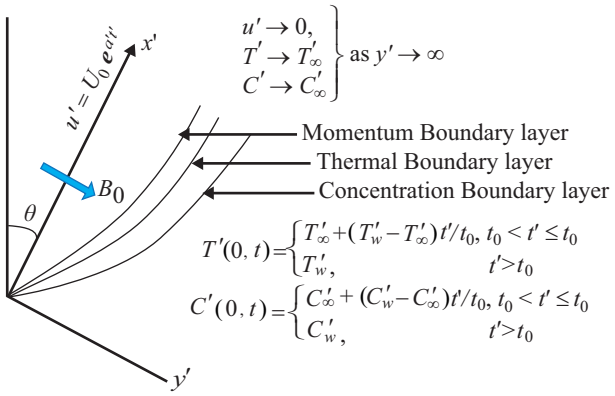


Fig. 1 Geometry of the Problem.

Since plate is of infinite extent along x' and z' directions and is electrically non-conducting, all physical quantities depend on y' and t' only. Therefore, the equation of continuity $\nabla . \vec{q} = 0$ and solenoidal relation for magnetic field $\nabla . \vec{B} = 0$, respectively, reduce to $\frac{\partial v'}{\partial y} = 0$ and $\frac{\partial B'}{\partial y} = 0$. Thus we obtain, v' = Constant and B'_y = Constant, where $\vec{q} = (u', v', w')$ and $\vec{B} = (B'_x, B'_y, B'_z)$ are fluid velocity and induced magnetic field respectively.

Since plate is non-porous and a uniform magnetic field of strength B_0 is applied in y- direction, fluid velocity v' and magnetic field B'_y may be assumed as $v' = 0$ and $B'_y = B_0$, throughout the region. Also, there is no flow of fluid in z- direction, therefore, $w' = 0$. Thus fluid velocity $\vec{q} = (u', 0, 0)$.

It is assumed that the induced magnetic field generated by fluid motion is negligible in comparison to the applied one. This assumption is valid for metallic liquids and partially ionized fluids which are commonly used in various industrial processes. Thus, $\vec{B} = (0, B_0, 0)$.

Keeping in view the assumptions made above, the governing equations for unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting, temperature dependent heat absorbing, optically thick radiating and chemically reactive fluid through a fluid saturated porous medium with combined heat and mass transfer, under Boussinesq approximation, are given by

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \sigma \frac{B_0^{\,2}}{\rho} u' - \frac{\nu}{K_1} u' + g\beta_T (T' - T'_\infty)\cos\theta + g\beta_C (C' - C'_\infty)\cos\theta, \quad (1)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - g\beta_T (T' - T'_\infty)\sin\theta - g\beta_C (C' - C'_\infty)\sin\theta \quad (2)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r'}{\partial y} - Q_0 (T' - T'_\infty), \quad (3)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K'_2 (C' - C'_\infty), \quad (4)$$

where u', T', C', K_1, K'_2, k, c_p, Q_0, D, q'_r, ν, σ, ρ, g, β_T and β_C are, respectively, fluid velocity, fluid temperature, species concentration, permeability of porous medium, coefficient of chemical reaction, thermal conductivity, specific heat at constant pressure, heat absorption co-efficient, molecular diffusivity, radiating flux vector, kinematic co-efficient of viscosity, electrical conductivity, fluid density, acceleration due to gravity, co-efficient of thermal expansion and co-efficient of expansion for species concentration.

The pressure gradient term $-\frac{1}{\rho}\frac{\partial p}{\partial x}$ is absent in equation (1) because fluid flow is induced due to the movement of plate as well as buoyancy forces acting in x- direction. Equation (2) implies that fluid pressure p varies in y- direction due to presence of buoyancy forces acting in y- direction. This means that change in fluid pressure from plate to the edge of the boundary layer varies due to change in thermal and solutal buoyancy forces.

The initial and boundary conditions for fluid flow are

$$u' = 0,\ T' = T'_\infty,\ C' = C'_\infty \quad \text{for } y' \ge 0 \text{ and } t' \le 0, \quad (5a)$$

$$u' = U_0 e^{a't'} \quad \text{at } y' = 0 \text{ and } t' > 0 \quad (5b)$$

$$\left.\begin{aligned} T' &= T'_\infty + (T'_w - T'_\infty)t'/t_0, \\ C' &= C'_\infty + (C'_w - C'_\infty)t'/t_0 \end{aligned}\right\} \quad \text{at } y' = 0 \text{ and } 0 < t' \le t_0 \quad (5c)$$

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$$T' = T'_w,\ \ C' = C'_w \qquad \text{at } y' = 0 \text{ and } t' > t_0 \tag{5d}$$

$$u' \to 0,\ T' \to T'_\infty,\ C' \to C'_\infty \qquad \text{as } y' \to \infty \text{ and } t' > 0 \tag{5e}$$

For an optically thick radiating fluid, in addition to emission there is also self-absorption and usually the absorption co-efficient is wavelength dependent and large. Therefore, adopting Rosseland approximation, radiating flux vector is expressed as

$$q'_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'}, \tag{6}$$

where k^* is mean absorption co-efficient and σ^* is Stefan-Boltzmann constant.

Assuming small temperature difference between fluid temperature within the boundary layer T' and free stream temperature T'_∞, T'^4 is expanded in Taylor series about free stream temperature T'_∞ to linearize equation (5). T'^4 assumes the following form after neglecting second and higher order terms in Taylor series

$$T'^4 \cong 4T'^4_\infty T' - 3T'^4_\infty . \tag{7}$$

Using (6) and (7) in (2), we obtain

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma^* T'^3_\infty}{3k^*\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho c_p}(T' - T'_\infty). \tag{8}$$

In order to non-dimensionalize equations (1), (3) and (8), the following non-dimensional variables and parameters are introduced

$$\left.\begin{aligned} &y = y'/U_0 t_0, u = u'/U_0, t = t'/t_0, a = a'\nu/U_0^2, T = (T' - T'_\infty)/(T'_w - T'_\infty), C = (C' - C'_\infty)/(C'_w - C'_\infty),\\ &G_r = g\beta_T \nu(T'_w - T'_\infty)/U_0^3, G_c = g\beta_C \nu(C'_w - C'_\infty)/U_0^3, K = K_1 U_0^2/\nu^2, K_2 = \nu K'_2/U_0^2, M = \sigma B_0^2 \nu/\rho U_0^2,\\ &N = 16\sigma^* T'_\infty/3kk^*, P_r = \nu\rho c_p/k, \phi = \nu Q_0/\rho c_p U_0^2 \text{ and } S_c = \nu/D \end{aligned}\right\} \tag{9}$$

Equations (1), (3) and (8) with the use of (9), are presented in non-dimensional form as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u + G_r T\cos\theta + G_c C\cos\theta, \tag{10}$$

$$\frac{\partial T}{\partial t} = \frac{(\mathrm{N}+1)}{P_r}\frac{\partial^2 T}{\partial y^2} - \phi T, \tag{11}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - K_2 C, \tag{12}$$

where K, M, N, a, G_r, G_c, P_r, ϕ, K_2 and S_c are, respectively, permeability parameter, magnetic parameter, radiation parameter, plate acceleration parameter, thermal Grashof number, solutal Grashof number, Prandtl number, heat absorption parameter, chemical reaction parameter and Schmidt number.

It may be noted that the characteristic time t_0 is defined according to the non-dimensional process mentioned above as

$$t_0 = \nu/U_0^2 \tag{13}$$

Making use of (9) initial and boundary conditions (5a) to (5e), in non-dimensional form, are given by

$$u = 0,\ T = 0,\ C = 0, \qquad \text{at } y \geq 0 \text{ and } t \leq 0, \tag{14a}$$

$$u = e^{at} \qquad \text{at } y = 0 \text{ and } t > 0, \tag{14b}$$

$$T = t,\ C = t \qquad \text{at } y = 0 \text{ and } 0 < t \leq 1, \tag{14c}$$

$$T = 1,\ C = 1 \qquad \text{at } y = 0 \text{ and } t > 1 \tag{14d}$$

$$u \to 0, T \to 0, C \to 0 \qquad \text{as } y \to \infty \text{ for } t > 0. \tag{14e}$$

Equations (10) to (12) subject to the initial and boundary conditions (14a) to (14e) are solved by Laplace transform technique. The exact solutions for fluid temperature $T(y,t)$ species concentration $C(y,t)$ and fluid velocity $u(y,t)$ are obtained and are presented in the following form after simplification

$$T(y,t) = P(y,t) - P(y,t-1)H(t-1) \tag{15}$$

$$C(y,t) = Q(y,t) - Q(y,t-1)H(t-1), \tag{16}$$

$$u(y,t) = f_1(y,a,p_1,t,1) + G(y,t) + G(y,t-1)H(y,t-1), \tag{17}$$

where

$$P(y,t) = f_3(y,\alpha,\phi,t), \quad Q(y,t) = f_3(y,\beta,K_2,t),$$

$$G(\mathrm{y},\mathrm{t}) = -\eta_1\left[\frac{1}{2\xi_1}\{f_2(y,\xi_1,p_1,t,1) - f_2(y,\xi_1,\phi,t,\alpha)\} + \frac{1}{2\xi_1^2}\{f_1(y,-\xi_1,p_1,t,1) - f_1(y,-\xi_1,\phi,t,\alpha)\}\right]$$
$$-\eta_2\left[\frac{1}{2\xi_2}\{f_2(y,\xi_2,p_1,t,1) - f_2(y,\xi_2,K_1,t,\beta)\} + \frac{1}{2\xi_2^2}\{f_1(y,-\xi_2,p_1,t,1) - f_1(y,-\xi_2,K,t,\beta)\}\right].$$

Expression for f_1, f_2 and f_3 are provided in Appendix-I.

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3. SOLUTION IN THE CASE OF UNIT SCHMIDT NUMBER

It is worth mentioning that solutions (16) and (17) for species concentration and fluid velocity respectively, are not valid when Schmidt number $S_c = 1$. Since Schmidt number represents relative potency of momentum diffusivity and chemical molecular diffusivity of fluid, $S_c = 1$ corresponds to the fluids having same order of magnitude of boundary layer thicknesses for both momentum and concentration boundary layers. There are some fluids which belong to this category [48]. Solving equations (10) and (12) with $S_c = 1$ by Laplace transform Technique, we obtain the solutions for species concentration $C(y,t)$ and fluid velocity $u(y,t)$ which are expressed as:

$$C(y,t) = Q_1(y,t) - Q_1(y,t-1)H(t-1), \tag{18}$$

$$u(y,t) = f_1(y,a,p_1,t,1) + G_1(y,t) + G_1(y,t-1)H(t-1), \tag{19}$$

where

$$Q_1(y,t) = f_3(y,1,K_2,t),$$

$$G_1(\mathrm{y},\mathrm{t}) = -\alpha_1 \cos\theta\left[\left\{f_2(y,\alpha_2^2,p_1,t,1) - f_2(y,\alpha_2^2,\phi,t,\alpha)\right\} + \frac{1}{\alpha_2^2}\times\left\{f_1(y,-\alpha_2,p_1,t,1) - f_1(y,-\alpha_2,\mathrm{K}_2,t,1)\right\}\right.$$
$$\left. + \frac{1}{\beta_2^2}\left\{f_1(\mathrm{y},-\beta_2,p_1,\mathrm{t},1) - f_1(\mathrm{y},-\beta_2,\mathrm{K}_2,\mathrm{t},1)\right\}\right],$$

$$p_1 = M + \frac{1}{K},\ \alpha = \frac{N+1}{P_r},\ \beta = \frac{1}{S_c},\ \eta_1 = \frac{\alpha G_r \cos\theta}{\alpha\text{-}1},\ \eta_2 = \frac{\beta G_c \cos\theta}{\beta\text{-}1},\ \xi_1 = \frac{\phi - p_1\alpha}{1-\alpha},\ \xi_2 = \frac{\mathrm{K} - p_1\beta}{1-\beta},\ a_1 = N+1,$$
$$\alpha_1 = \frac{G_r}{1-P_r/a_1},\ \alpha_2 = \frac{P_r\phi - \gamma a_1}{P_r - a_1},\ \beta_1 = \frac{G_c}{1-S_c},\ \beta_2 = \frac{K_2 S_c - \gamma}{S_c - 1},$$

$H(t\text{-}1)$ and $erfc(x)$ are, respectively, unit step function and complementary error function.

3.1 Solution in the Case of Isothermal Plate with Uniform Surface Concentration

In order to highlight the effects of ramped temperature and ramped surface concentration on fluid flow, it may be worthwhile to compare the fluid flow near an accelerated moving inclined plate with ramped temperature and ramped surface concentration with the one near an accelerated moving vertical plate with uniform temperature and uniform surface concentration. Keeping in view the assumptions made above, the solutions for fluid temperature, species concentration and fluid velocity for natural convection flow past an exponentially accelerated moving inclined isothermal plate with uniform surface concentration are obtained and are expressed in the following form

$$T(y,t) = f_1(y,0,\phi,t,\alpha), \tag{20}$$

$$C(y,t) = f_1(y,0,K_2,t,\alpha), \tag{21}$$

$$u(y,t) = f_1(y,a,p_1,t,1) - \frac{\eta_3}{\xi_3}\left[\left\{f_1(y,0,p_1,t,1) + f_1(y,-\xi_3,\phi,t,\alpha)\right\} - \left\{f_1(y,-\xi_3,p_1,t,1) + f_1(y,0,\phi,t,\alpha)\right\}\right]$$
$$- \frac{\eta_4}{\xi_4}\left[\left\{f_1(y,0,p_1,t,1) + f_4(y,-\xi_4,K_2,t,S_c) - \left\{f_1(y,-\xi_4,p_1,t,1) + f_4(y,0,K_2,t,S_c)\right\}\right\}\right], \tag{22}$$

where

$$\eta_3 = \frac{\alpha G_r \cos\theta}{\alpha - 1},\ \xi_3 = \frac{\phi - p_1\alpha}{1-\alpha},\ \eta_4 = \frac{G_c \cos\theta}{1 - S_c} \text{ and } \xi_4 = \frac{K_2 S_c - p_1}{S_c - 1}.$$

Expression for f_4 is provided in Appendix – I.

3.2 Shear Stress, Rate of Heat Transfer and Rate of Mass Transfer at the Plate

Expressions for the shear stress at the plate τ, rate of heat transfer at the plate N_u and rate of mass transfer at the plate S_h, are derived and are presented in the following form after simplification.

3.3 For Plate with Ramped Temperature and Ramped Surface Concentration

$$\tau = -f_5(p_1,a,1,t) + G_2(y,t) + G_2(y,t-1)H(t-1), \tag{23}$$

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$$N_u = \left(\frac{1}{2\sqrt{\alpha\phi}} + t\sqrt{\frac{\phi}{\alpha}}\right)\left[Erfc\left(\sqrt{t\phi}\right) - 1\right] + \sqrt{\frac{t}{\pi\alpha}}\, e^{-t\phi}, \tag{24}$$

$$S_h = -\left[\frac{te^{-K_2 t}}{\sqrt{\pi t\beta}} + \sqrt{\frac{K_2}{\beta}}\left(t + \frac{1}{2\beta}\right)\left\{1 - Erfc\left(\sqrt{K_2 t}\right)\right\}\right]. \tag{25}$$

3.4 For Plate with Uniform Temperature and Uniform Surface Concentration

$$\tau = \frac{-e^{at}}{2} f_6(p_1, a, 1, t, 1) - \frac{\eta_3}{\xi_3}\left[\left\{f_6(p_1, 0, 1, t, 1) - f_7(\phi, 0, \alpha, t, 1)\right\} + e^{-\xi_3 t}\left\{f_6(\phi, -\xi_3, \alpha, t, 1) - f_6(p_1, -\xi_3, 1, t, 1)\right\}\right]$$
$$-\frac{\eta_4}{\xi_4} \times \left[\left\{f_6(p_1, 0, 1, t, 1) - f_7(K_2, 0, 1, t, S_c)\right\} + e^{-\xi_4 t}\left\{f_6(K_2, -\xi_4, 1, t, S_c) - f_6(p_1, -\xi_4, 1, t, 1)\right\}\right], \tag{26}$$

$$N_u = f_6(\phi, 0, \alpha, t, 1), \tag{27}$$

$$S_h = \sqrt{\frac{K_2}{\beta}}\left\{1 - erfc\left(\sqrt{K_2 t}\right)\right\} + \frac{e^{-K_2 t}}{\sqrt{\pi\beta t}}, \tag{28}$$

where

$$G_2(y,t) = \eta_1\left[\frac{e^{-t\xi_1}}{\xi_1^2}\left\{-f_6(\phi, -\xi_1, \alpha, t, 1) + f_5(p_1, -\xi_1, 1, t)\right\} + \frac{1}{\xi_1}\left\{-f_5(\phi, \xi_1, \alpha, t) + f_5(p_1, \xi_1, 1, t)\right\}\right]$$
$$+\eta_2\left[\frac{e^{-t\xi_2}}{\xi_2} \times \left\{-f_6(K_1, -\xi_2, \beta, t, 1) + f_5(p_1, \xi_2, 1, t)\right\} + \frac{1}{\xi_2^2}\left\{-f_5(K_1, \xi_2, \beta, t) + f_5(p_1, \xi_2, 1, t)\right\}\right].$$

Expressions for f_5, f_6 and f_7 are presented in Appendix-I.

4. RESULTS AND DISCUSSION

In order to gain a perspective of the physics of the flow regime, numerical computations are conducted from analytical solutions (15) to (17) and (20) to (22) for fluid velocity, fluid temperature and species concentration and from the analytical expressions (23) to (28) for shear stress, rate of heat transfer and rate of mass transfer at the plate, for various values of the physical parameters that describe the flow characteristics. The numerical results are illustrated in Figs. 2 to 14 along with the Tables 1 to 3. In the present computation, the values of Prandtl number P_r and magnetic parameter M have been fixed at 0.71 (Ionized Air) and 5 respectively.

Figures 2 to 8 depict the graphs of fluid velocity u under the influence of time t, plate acceleration parameter a, angle of inclination θ, thermal Grashof number G_r, solutal Grashof number G_c, radiation parameter N and heat absorption parameter ϕ, within the momentum boundary layer for both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration. It is observed from these figures that fluid velocity corresponding to isothermal plate with uniform surface concentration is higher than that of ramped temperature plate with ramped surface concentration. It is evident from Fig. 2 that there is an increase in u on increasing time t. This observation suggests that fluid velocity is getting accelerated with the progress of time. It is revealed from Fig. 3

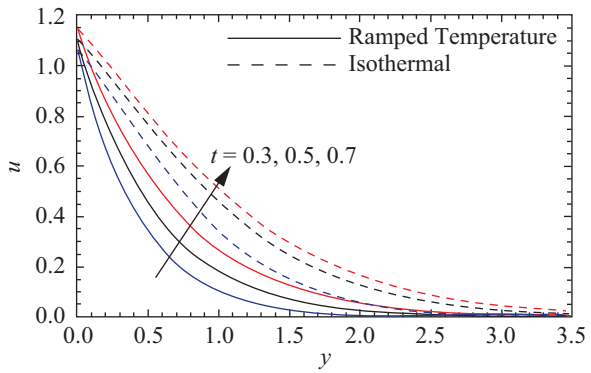


Fig. 2 Velocity profiles for $a = 0.2$, $\theta = 45°$, $N = 3$, $\phi = 3$, $G_r = 10$, $G_c = 3$ and $S_c = 0.6$.

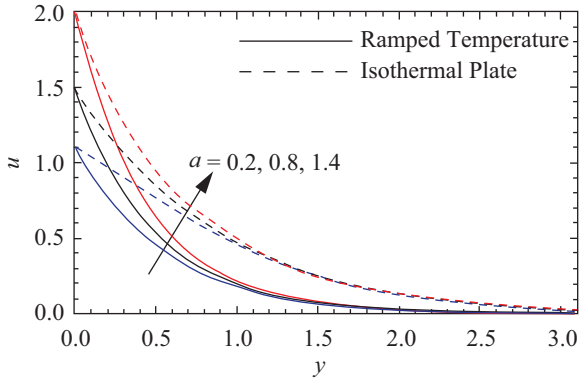


Fig. 3 Velocity profiles for $\theta = 45°$, $N = 3$, $\phi = 3$, $G_r = 10$, $G_c = 3$, $S_c = 0.6$ and $t = 0.5$.

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that u increases on increasing plate acceleration parameter a in the region near the plate and the effect of a is almost negligible in the region away from the plate. This observation suggests that higher plate velocity results in accelerated fluid motion in the region near the moving plate which is exponentially accelerated. It is inferred from Fig. 4 that u is getting decelerated on increasing the angle of inclination θ. This is because the effect of buoyancy force is getting reduced due to multiplication of the term $\cos\theta$ in the buoyancy force term. Since $\cos\theta$ assumes maximum value when $\theta = 0$ and it is getting reduced as θ increases. Due to this reason strength of

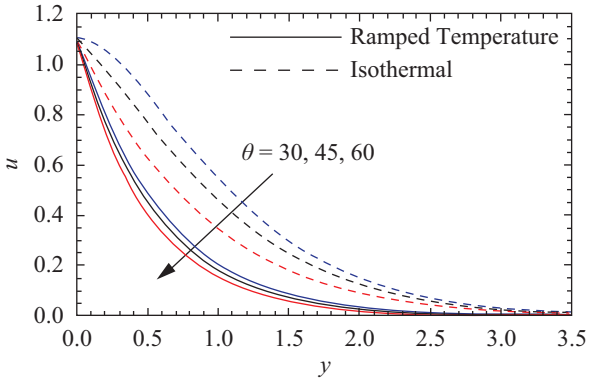


Fig. 4 Velocity profiles for $a = 0.2$, $N = 3$, $\phi = 3$, $G_r = 10$, $G_c = 3$, $S_c = 0.6$ and $t = 0.5$.

buoyancy forces is getting reduced on increasing angle of inclination of the plate. It is evident from the analysis of Figs. 5 and 6 that the effects of thermal Grashof number G_r and solutal Grashof number G_c, are to increase fluid velocity u. Since G_r presents the relative strength of thermal buoyancy force to viscous force and G_c is the measure of solutal buoyancy force to viscous force, as G_r and G_c increase, thermal and solutal buoyancy forces become stronger. This implies that thermal as well as solutal buoyancy forces tend to accelerate fluid velocity within boundary layer region. Figs. 7 and 8 uniquely establish that u increases on increasing radiation parameter N whereas it decreases on increasing heat absorption parameter ϕ. In other words, fluid velocity of a highly radiating and lesser heat absorbing fluid is

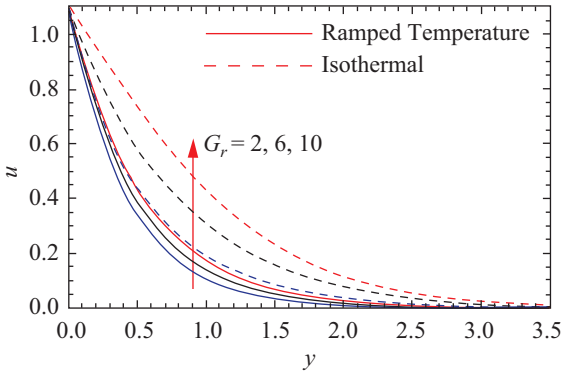


Fig. 5 Velocity profiles for $a = 0.2$, $\theta = 45°$, $N = 3$, $\phi = 3$, $G_c = 3$, $S_c = 0.6$ and $t = 0.5$.

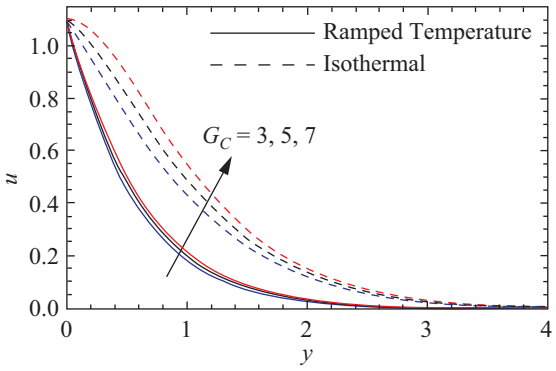


Fig. 6 Velocity profiles for $a = 0.2$, $\theta = 45°$, $N = 3$, $\phi = 3$, $G_r = 10$, $S_c = 0.6$ and $t = 0.5$.

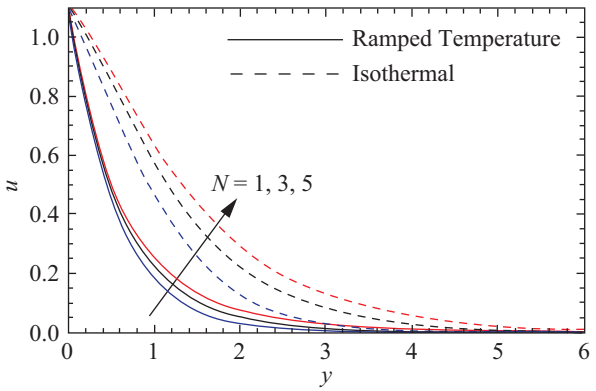


Fig. 7 Velocity profiles for $a = 0.2$, $\theta = 45°$, $\phi = 3$, $G_r = 10$, $G_c = 3$, $S_c = 0.6$ and $t = 0.5$.

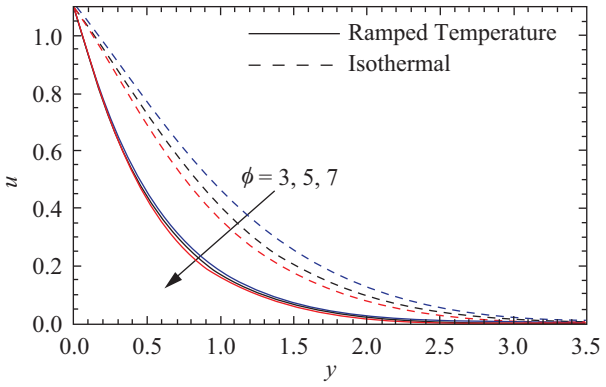


Fig. 8 Velocity profiles for $a = 0.2$, $\theta = 45°$, $N = 3$, $G_r = 10$, $G_c = 3$, $S_c = 0.6$ and $t = 0.5$.

greater than that of lesser radiating and highly heat absorbing fluid. This fact is justified because an increase in Radiation parameter N results in the rise of fluid temperature as observed from Fig. 10, by virtue of which the thermal buoyancy force gets stronger. This increased thermal buoyancy force results in accelerated fluid motion. Likewise, from Fig. 11, we observe a fall in the fluid temperature upon increasing heat absorption parameter ϕ and this fall in the fluid temperature weakens the thermal buoyancy force and, therefore, the fluid velocity is getting decelerated.

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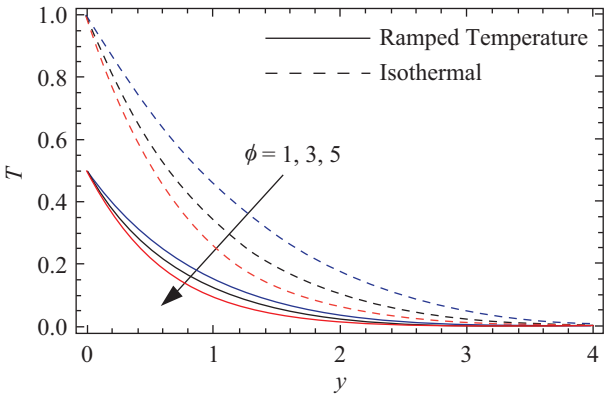


Fig. 9 Temperature Profiles for $N = 3$ and $t = 0.5$.

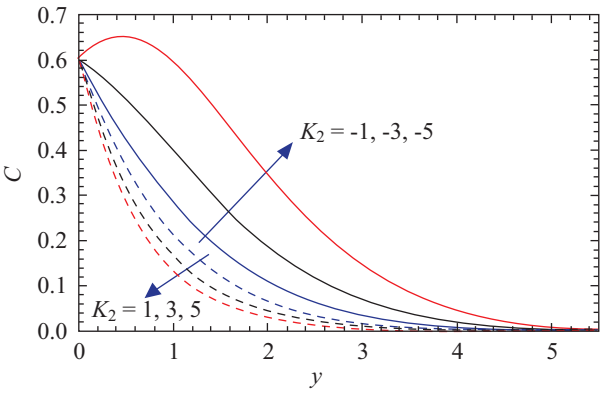


Fig. 12 Concentration profiles for $S_c = 0.32$ and $t = 0.5$.

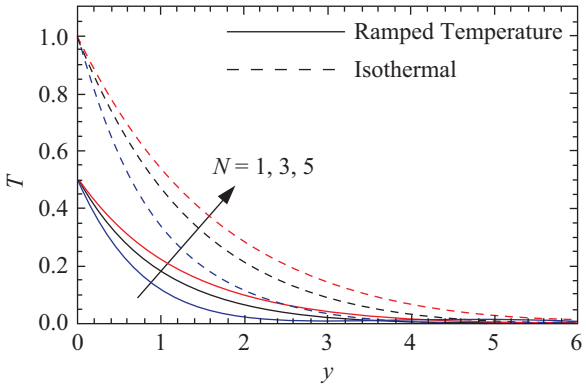


Fig. 10 Temperature Profiles for $\phi = 3$ and $t = 0.5$.

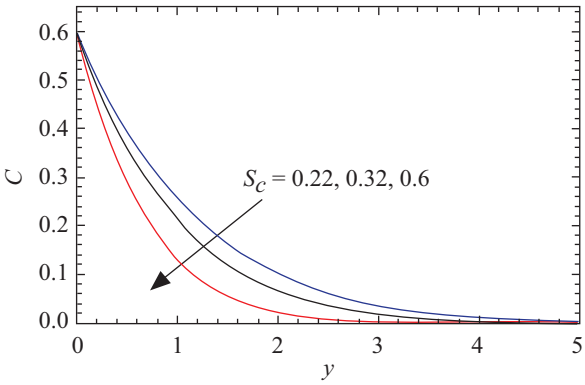


Fig. 13 Concentration profiles for $K_2 = 1$ and $t = 0.5$.

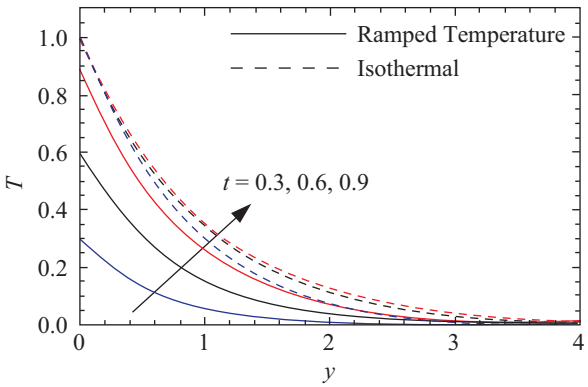


Fig. 11 Temperature Profiles for $N = 3$ and $\phi = 3$.

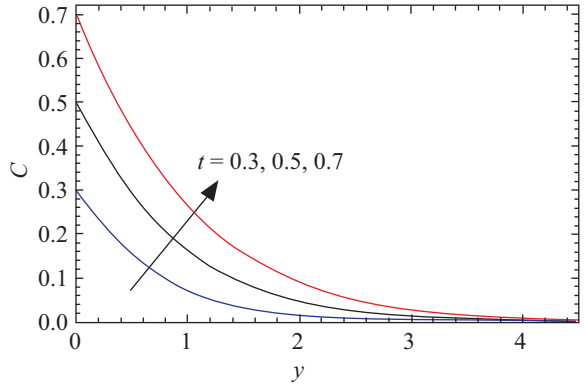


Fig. 14 Concentration profiles for $K_2 = 1$ and $S_c = 0.32$.

Figures 9 to 11 have been plotted to analyze the behavior of fluid temperature T with respect to heat absorption parameter ϕ, radiation parameter N and time t within the thermal boundary layer. We see that, an increase in N or t results in a significant rise in fluid temperature T whereas the effect of ϕ on T, is opposite to that of N or t. Physically speaking, thermal radiation has a tendency to enhance the fluid temperature whilst heat absorption affects the fluid temperature adversely. As time progresses, the temperature of the fluid is getting enhanced.

To analyze the characteristics of species concentration C within the concentration boundary layer region, the numerical results are presented in Figs. 12 to 14 for various values of chemical reaction parameter K_2, Schmidt number S_c and time t. Fig. 12 is plotted for positive as well as negative values of K_2. We observe from Fig. 12 that, an increase in the positive values of K_2 causes C to decrease comprehensively. On the other hand, an increase in the negative value of K_2 results in the enhancement of C. Since positive value of K_2 corresponds to a chemical reaction of destructive kind whereas negative value of K_2 implies a generative chemical reaction. Therefore an increase in the positive value of K_2 results in the consumption of species with a better rate and consequently the thickness of concentra-

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tion boundary layer decreases whereas an increase in the negative value of K_2 catalyzes the generative chemical reaction which is why the thickness of concentration boundary layer increases. As per Figs. 13 and 14, we see that C decreases on increasing S_c whereas it increases on increasing t. Physically speaking, an increase in S_c means a decrease in the molecular diffusion. Hence the imposition of high chemical molecular diffusion causes the species concentration to rise comprehensively. The concentration of species is getting enhanced with the progress of time.

The behavior of the shear stress at the plate τ under the influence of G_r, G_c, ϕ, a and t are presented in Table 1. It is evident from Table 1 that, for both ramped temperature plate with ramped concentration and isothermal plate with uniform surface concentration, shear stress at the plate τ decreases on increasing G_r, G_c and N whereas it increases upon increasing ϕ, a and t. This implies that thermal buoyancy force, solutal buoyancy force and thermal radiation tend to reduce shear stress at the plate whereas heat absorption and plate acceleration tend to enhance shear stress at the plate for both ramped temperature plate with ramped concentration and isothermal plate with uniform surface concentration. Shear stress at the plate is getting enhanced with the progress of time for both ramped temperature plate with ramped concentration and isothermal plate with uniform surface concentration.

The numerical values of rate of heat transfer at the plate $-N_u$ are provided in Table 2 for various values of ϕ, N and t. It is clear from Table 2 that N_u decreases on increasing N whereas it increases on increasing ϕ for both ramped temperature and isothermal plate. Also the rate of heat transfer at the plate N_u increases for ramped temperature whereas it decreases for isothermal plate, with the progress of time. This implies that, for both ramped temperature and isothermal plates, rate of heat transfer at the plate is getting enhanced due to heat absorption whereas it is adversely affected by thermal radiation. Rate of heat transfer at the plate is getting enhanced for ramped temperature plate whereas it is getting reduced for isothermal plate, with the progress of time.

Numerical values of rate of mass transfer at the plate $-S_h$ are presented in Table 3 for different values of K_2, t

Table 1 Shear Stress at the Plate τ

G_r	G_c	ϕ	N	a	t	Ramped temperature plate with ramped surface concentration	Isothermal plate with uniform surface concentration
2	5	3	3	0.2	0.5	2.4246	1.6457
6	5	3	3	0.2	0.5	2.1115	0.5175
10	5	3	3	0.2	0.5	1.7983	0.2315
10	**3**	3	3	0.2	0.5	1.9531	0.5266
10	**5**	3	3	0.2	0.5	1.7983	0.4175
10	**7**	3	3	0.2	0.5	1.6436	0.2315
10	5	**3**	3	0.2	0.5	1.8725	0.3496
10	5	**5**	3	0.2	0.5	1.9025	0.4524
10	5	**7**	3	0.2	0.5	1.9280	0.5342
10	5	3	**1**	0.2	0.5	1.9531	0.5266
10	5	3	**3**	0.2	0.5	1.8725	0.3496
10	5	3	**5**	0.2	0.5	1.8312	0.2603
10	5	3	3	**0.2**	0.5	2.3315	0.5801
10	5	3	3	**0.8**	0.5	3.0317	1.2802
10	5	3	3	**1.4**	0.5	3.8915	2.1400
10	5	3	3	0.2	**0.3**	2.3315	0.5801
10	5	3	3	0.2	**0.5**	2.4531	0.5866
10	5	3	3	0.2	**0.7**	2.5737	0.5918

Table 2 Rate of Heat Transfer at the Plate $-N_u$

ϕ	t	N	Ramped Temperature	Isothermal Plate
3	0.5	1	0.6837	1.0521
		3	0.4834	0.7439
		5	0.3947	0.6074
3	0.3	3	0.4699	1.0961
	0.5		0.6837	1.0521
	0.7		0.8926	1.0394
3		3	0.6837	1.0521
5			0.7986	1.3375
7			0.9006	1.5778

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Table 3 Rate of Mass Transfer at the Plate -S_h.

K_2	t	S_c	Ramped Surface Concentration	Uniform Surface Concentration
3	0.5	0.22	0.5800	0.8282
		0.32	0.6995	0.9989
		0.60	0.9578	1.3678
3	0.3	0.22	0.5074	0.8628
	0.5		0.5800	0.8283
	0.7		0.7143	0.8183
1	0.5	0.22	0.5472	0.5472
3			0.5800	0.8283
5			0.6440	1.0531

and S_c. It is revealed form Table 3 that S_h increases on increasing either K_2 or S_c in both the cases i.e when concentration of species at surface of the plate has a ramped profile and when it has a uniform concentration at surface of the plate. On increasing time t, the value of S_h increases when concentration of species at surface of the plate has a ramped profile and it decreases when species concentration at surface of the plate is uniform. Since S_c is ratio of momentum diffusivity to chemical molecular diffusivity, S_c decreases when chemical molecular diffusion increases. This implies that, in both the cases i.e. when concentration of species at surface of the plate has a ramped profile and when it has a uniform concentration at surface of the plate, rate with which the mass is being transferred from surface of the plate to the fluid, is getting enhanced on increasing the rate of chemical reaction and momentum diffusion whereas it is getting reduced on imposition of high chemical molecular diffusivity. As time progresses, when the concentration of species at the plate has a ramped profile, the rate of mass transfer at the plate is getting enhanced whereas it is getting reduced when the concentration of species at the plate is uniform.

5. CONCLUSIONS

The noteworthy results for the problem under consideration are summarized below.

- For both ramped temperature plate with ramped surface concentration and isothermal plate with uniform surface concentration:
  - Plate acceleration parameter, thermal buoyancy force, solutal buoyancy force and thermal radiation tend to accelerate the fluid velocity whereas angle of inclination and heat absorption have reverse effect on it. Fluid velocity is getting accelerated with the progress of time.
  - Thermal radiation tends to enhance the fluid temperature whereas heat absorption has reverse effect on it. Fluid temperature is getting enhanced with the progress of time.
  - Thermal buoyancy force, solutal buoyancy force and thermal radiation have tendency to reduce shear stress at the plate whereas heat absorption and plate acceleration parameter have reverse effect on it. Shear stress at the plate is getting reduced with the progress of time.
- Rate of heat transfer at the plate is getting enhanced on increasing heat absorption whereas its tendency is reversed on increasing thermal radiation for both ramped temperature and isothermal plate. For ramped temperature plate, rate of heat transfer at the plate is getting enhanced whereas it is getting reduced for isothermal plate with the progress of time.
- Rate of mass transfer at the plate is getting enhanced on increasing chemical molecular diffusivity and rate of chemical reaction in both the cases i.e. when concentration of species at the surface of the plate has a ramped profile and when it is uniform. As time progresses, when concentration of species at the plate has a ramped profile, rate of mass transfer at the plate is getting enhanced whereas it is getting reduced when concentration of species at the surface of the plate is uniform.

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APPENDIX-I:

$$f_1(c_1,c_2,c_3,c_4,c_5)=\frac{e^{c_2c_4}}{2}\left[e^{c_1\sqrt{\frac{c_3+c_2}{c_5}}}\,Erfc\left(\frac{c_1}{2\sqrt{c_4c_5}}+\sqrt{(c_3+c_2)c_4}\right)+e^{-c_1\sqrt{\frac{c_3+c_2}{c_5}}}\,Erfc\left(\frac{c_1}{2\sqrt{c_4c_5}}-\sqrt{(c_3+c_2)c_4}\right)\right],$$

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$$f_2(c_1,c_2,c_3,c_4,c_5)=\left[\left(c_4-\frac{1}{c_2}+\frac{c_1}{2\sqrt{c_3c_5}}\right)e^{c_1\sqrt{\frac{c_3}{c_5}}}\,Erfc\left(\frac{c_1}{2\sqrt{c_4c_5}}+\sqrt{c_3c_4}\right)+\left(c_4-\frac{1}{c_2}-\frac{c_1}{2\sqrt{c_3c_5}}\right)e^{-c_1\sqrt{\frac{c_3}{c_5}}}\,Erfc\left(\frac{c_1}{2\sqrt{c_4c_5}}-\sqrt{c_3c_4}\right)\right],$$

$$f_3(c_1,c_2,c_3,c_4)=\frac{1}{2}\left[\left(c_4+\frac{c_1}{2\sqrt{c_2c_3}}\right)e^{c_1\sqrt{\frac{c_3}{c_2}}}\,Erfc\left(\frac{c_1}{2\sqrt{c_2c_4}}+\sqrt{c_3c_4}\right)+\left(c_4-\frac{c_1}{2\sqrt{c_2c_3}}\right)e^{-c_1\sqrt{\frac{c_3}{c_2}}}\,Erfc\left(\frac{c_1}{2\sqrt{c_2c_4}}-\sqrt{c_3c_4}\right)\right],$$

$$f_4(c_1,c_2,c_3,c_4,c_5)=\frac{e^{c_2c_4}}{2}\left[e^{c_1\sqrt{(c_3+c_2)c_5}}\,Erfc\left(\frac{c_1}{2}\sqrt{\frac{c_5}{c_4}}+\sqrt{(c_3+c_2)c_4}\right)+e^{-c_1\sqrt{(c_3+c_2)c_5}}\,Erfc\left(\frac{c_1}{2}\sqrt{\frac{c_5}{c_4}}-\sqrt{(c_3+c_2)c_4}\right)\right],$$

$$f_5(c_1,c_2,c_3,c_4)=\frac{1}{2\sqrt{c_1c_3}}\left\{1-Erfc\left(\sqrt{c_1c_4}\right)\right\}+\left(c_4-\frac{1}{c_2}\right)\frac{e^{-c_1c_4}}{\sqrt{\pi c_3c_4}}+\sqrt{c_1}\left(c_4-\frac{1}{c_2}\right)\left\{1-Erfc\left(\sqrt{c_1c_4}\right)\right\},$$

$$f_6(c_1,c_2,c_3,c_4,c_5)=\sqrt{\frac{(c_1+c_2)c_5}{c_3}}\left\{1-Erfc\left(\sqrt{(c_1+c_2)c_4}\right)\right\}+\frac{e^{-c_4(c_1+c_2)}\sqrt{c_5}}{\sqrt{\pi c_3c_4}},$$

$$f_7(c_1,c_2,c_3,c_4)=\sqrt{\frac{c_1}{c_3}}\left(c_4-\frac{1}{c_2}+\frac{1}{2c_1}\right)\left\{Erfc\left(\sqrt{c_1c_4}\right)-1\right\}+\left(c_4-\frac{1}{c_2}\right)\frac{e^{-\frac{c_1c_4}{c_3}}}{\sqrt{\pi c_4}}.$$

NOMENCLATURE

a	Plate acceleration parameter
B_0	uniform magnetic field
C'	species concentration
c_p	specific heat at constant pressure
D	molecular diffusivity
g	acceleration due to gravity
G_r	thermal Grashof number
G_c	solutal Grashof number
k	thermal conductivity of the fluid
K_1	permeability of porous medium
K	permeability parameter
K_2'	coefficient of chemical reaction
K_2	chemical reaction parameter
M	magnetic parameter
N	radiation parameter
P_r	Prandtl number
Q_0	heat absorption coefficient
q_r'	radiating flux vector
S_c	Schmidt number
T'	fluid temperature
t_0	characteristic time
U_0	uniform velocity
u'	fluid velocity
	Greek Symbols
β_T	coefficient of thermal expansion
β_C	coefficient of expansion for species concentration
σ	electrical conductivity
ρ	fluid density
σ^*	Stefan-Boltzmann constant
ν	kinematic coefficient of viscosity
ϕ	heat absorption parameter
θ	angle of inclination

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