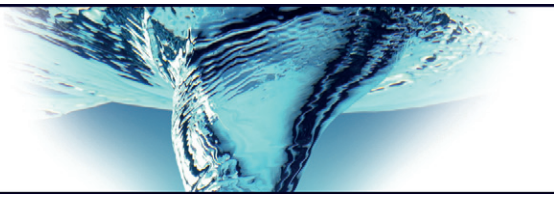
J. Fluid Mech. (2019), *vol.* 858, R1, doi:10.1017/jfm.2018.841



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Reconciling turbulent burning velocity with flame surface area in small-scale turbulence

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(Received 26 June 2018; revised 7 September 2018; accepted 12 October 2018;
first published online 5 November 2018)

A discrepancy between the enhancement in overall burning rate and the enhancement in flame surface area measured for high-intensity turbulence is addressed. In order to reconcile the two quantities, an additional contribution from the effective turbulent diffusivity is considered. This contribution is expected to arise in sufficiently intense turbulence from eddies smaller than the flamelet thickness. In the present work, the enhancement in diffusivity arising from these eddies is estimated based on a model energy spectrum; individual contributions from all turbulence length scales smaller the flamelet thickness are integrated over the corresponding portion of the spectrum. It is shown that diffusivity enhancement, estimated in this manner, is able to account for the measured discrepancy between the overall burning rate enhancement and flame surface area enhancement. The factor quantifying this discrepancy is formalized as a closed-form function of the Karlovitz number.

Key words: turbulent mixing, turbulent reacting flows, combustion

1. Introduction: burning velocity enhancement in turbulence

1.1. *Corrugated flamelets regime and Damköhler's first hypothesis*

Flamelet modelling of turbulent flame propagation relies on the first of two hypotheses laid out by Damköhler (1940). According to this hypothesis (DH1), the overall burning rate $\dot{m}$ quantified by the turbulent burning velocity s_T is linearly proportional to the turbulent surface area A_T of the propagating flame. This relationship is expressed as

$$\frac{s_T}{s_L} \sim \frac{A_T}{A} = \left(1 + \frac{\Delta A}{A}\right), \tag{1.1}$$

where s_L is the laminar burning velocity corresponding to the unburned mixture and A is the mean unwrinkled flame surface area. The expression on the right indicates that the enhancement in s_T with respect to s_L is related to the enhancement $\Delta A = A_T - A$

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https://doi.org/10.1017/jfm.2018.841 Published online by Cambridge University Press

of turbulent flame surface area beyond the unwrinkled area A, as determined by the isosurface of a suitable reaction progress variable.

DH1 strictly applies only when all turbulence length scales ℓ are much larger than the laminar flame thickness δ_L. In that case, the smallest (Kolmogorov) length scale η satisfies $\eta > \delta_L$. This regime of turbulent flame propagation – referred to as the corrugated flamelets (CF) regime (Peters 1999) – corresponds to a value less than unity of the Karlovitz number Ka defined (Bray & Cant 1991) as

$$\mathrm{Ka} \equiv \frac{\delta_L^2}{\eta^2} = \left(\frac{u'}{s_L}\right)^{3/2} \left(\frac{\delta_L}{\ell_0}\right)^{1/2}, \tag{1.2}$$

where the relation on the right expresses the definition in terms of the laminar burning velocity s_L, the laminar flame thickness, the integral length scale ℓ_0 and the turbulence intensity u', following Peters (1999). This equality relies on the assumption that the kinematic viscosity $\nu \sim s_L \delta_L$ and the Reynolds number $\mathrm{Re} \equiv u' \ell_0 / \delta_L \sim (\ell_0/\eta)^{4/3}$. Experiments have indicated persistently (Bradley 1992; Driscoll 2008) that DH1 remains valid in the CF regime, i.e. for $\mathrm{Ka} < 1$ or as long as $\eta > \delta_L$. The question is whether DH1 remains valid in the small-scale turbulence regime with $\eta < \delta_L$, for which $\mathrm{Ka} > 1$.

In recent years, two separate experimental investigations – conducted in Toronto (Yuen & Gülder 2013) and in Michigan (Wabel, Skiba & Driscoll 2017) – have disclosed a common result for a wide range of high Ka values. In these experiments, turbulent Bunsen flames are stabilized by an annular pilot methane–air flame or ethylene–air flame (Toronto) or by a co-flow of hot products produced by the same methane–air mixture (Michigan). The turbulent burning velocity s_T is calculated from the reactant inflow rate $\dot{m}$, assuming that the entire incoming fuel–air mixture is burned across the flame so that $\dot{m} = \rho A s_T$, where ρ is the density of the fuel–air mixture. Hence, the relative accuracy of resulting s_T/s_L measurements is equal to the accuracy of $\dot{m}$ measurements, which is within 5 % of the stated values (Yuen 2009; Wabel 2017). The total flame surface area A_T is measured using an appropriate isosurface of a marker: temperature (Toronto) or OH radical (Michigan), which is visualized using laser-induced Rayleigh scattering (Toronto) or laser-induced fluorescence (Michigan). The total flame surface area A_T is calculated using the method of Shepherd (1996): the flame surface density is determined from the marker isosurface and subsequently integrated over the flame brush, which is assumed to possess cylindrical symmetry. The accuracy of the resulting A_T/A measurements is related to errors arising from the pixel resolution as well as those arising from off-plane flame surface normals. In the Toronto measurements, the total error in flame surface area measurements is estimated to be approximately 15 %–20 % (Yuen 2009). In the Michigan measurements, the former is estimated to be up to 3 % and the latter is estimated to be approximately 18 % (Wabel 2017). In general, the error arising from off-plane flame surface normals are difficult to quantify. However, a recent Direct Numerical Simulation (DNS) study of turbulent jet flames (Wang, Hawkes & Chen 2016) has shown that the multiplicative correction factor needed on a two-dimensional flame surface area measurement tends to be unity near the inlet and approaches the value $4/\pi \approx 1.27$ in the downstream region (as derived by Veynante *et al.* 2010). The uncertainty in the Karlovitz numbers is related to the measurement of u'/s_L and ℓ_0/δ_L. In both experiments, these quantities were measured at the base of the flame using laser Doppler anemometry, with uncertainties of 1 %–3 %, leading to uncertainties in the Karlovitz number Ka of at most 4 %.

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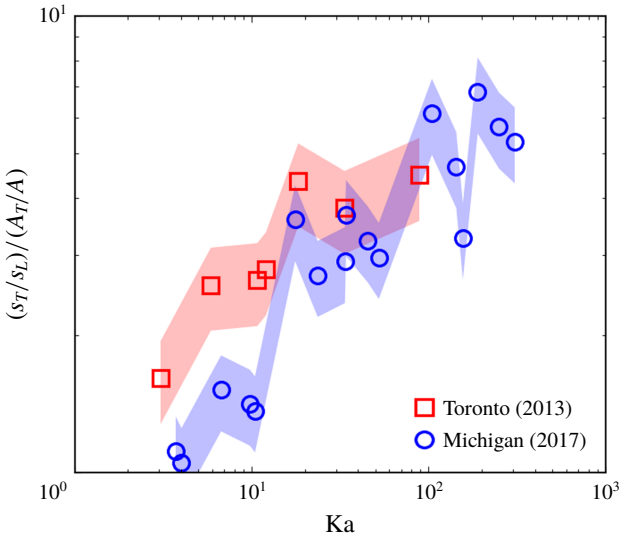


FIGURE 1. Measured disparity in the profiles of turbulent burning velocity enhancement s_T/s_L and turbulent surface area enhancement A_T/A obtained by the Toronto (Yuen & Gülder 2013) and Michigan (Wabel *et al.* 2017) experiments. Shaded regions in red and blue denote error bars for the Toronto and Michigan experiments, respectively.

The discrepancy between the profiles of s_T/s_L and A_T/A measured at Toronto and Michigan is shown in figure 1 by plotting the ratio $(s_T/s_L)/(A_T/A)$ as a function of Ka. For low values of Ka, the enhancement s_T/s_L is entirely accounted for by the flame surface area enhancement A_T/A, as per DH1. In other words, the ratio $(s_T/s_L)/(A_T/A)$ is close to unity for $\mathrm{Ka} \sim 1$. For higher values of Ka, the turbulent burning velocity enhancement s_T/s_L and the enhancement in flame surface area A_T/A show an increasing disparity, i.e. $(s_T/s_L)/(A_T/A) > 1$ for $\mathrm{Ka} \gg 1$. The Michigan experiments (blue circles in figure 1), which have examined a wider range of Ka values, indicate a disparity of up to one order of magnitude for $\mathrm{Ka} \sim 10^3$. At any given value of Ka, the experimental results from Toronto (red squares in figure 1) yield marginally higher disparities, yet the general behaviour observed is the same for both experiments: A_T/A fails to account for the overall enhancement s_T/s_L in turbulent burning velocity for $\mathrm{Ka} > 1$.

Why do the profiles of s_T/s_L and A_T/A diverge for $\mathrm{Ka} > 1$? What other mechanism, if not the area enhancement according to DH1, could possibly account for the enhancement s_T/s_L over and above the enhancement A_T/A? These open questions are addressed in the present work. In both experiments (Yuen & Gülder 2013; Wabel *et al.* 2017), a thickening effect is referred to at high Ka values, and the flame surface marker interface appears to get smeared with increasing Ka. This observation provides a clue towards the interpretation of a likely additional mechanism of s_T enhancement as follows.

1.2. *Thin reaction zones regime and Damköhler's second hypothesis*

In small-scale turbulence with $\mathrm{Ka} > 1$, the action of length scales $\ell < \delta_L$ may be considered. These scales may provide a further contribution to the enhancement of s_T over and above the area enhancement A_T/A. The second of Damköhler's two

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hypotheses (Damköhler 1940) presents a mechanism of s_T enhancement by turbulence length scales smaller than δ_L. According to Damköhler's second hypothesis (DH2), the overall burning velocity enhancement s_T/s_L in small-scale turbulence varies as a function of the enhancement D_T/D in the effective turbulent diffusivity D_T. This is expressed as

$$\frac{s_T}{s_L} = \sqrt{\frac{D_T}{D}} = \left(1 + \frac{\Delta D}{D}\right)^{1/2}, \tag{1.3}$$

where D is the molecular diffusivity in laminar flow. This dependence is an extension to turbulent flames of the dependence formulated for the laminar burning velocity by Zel'dovich & Frank-Kamenetskiĭ (1938). As written previously for DH1 (1.1), the expression on the right shows that the turbulent burning velocity enhancement s_T/s_L is related to the turbulent diffusivity enhancement $\Delta D = D_T - D$ over and above the molecular diffusivity D.

Damköhler (1940) had originally envisioned DH2 to apply strictly to turbulence with all length scales ℓ much smaller than the laminar flame thickness δ_L. This implies that the largest (integral) length scale satisfies $\ell_0 < \delta_L$ in the regime corresponding to DH2. Such a regime, however, is seldom obtained in practice because the integral length scale ℓ_0 of an experimentally generated turbulent flow is typically several times greater than the laminar flame thickness δ_L of the unburned mixture (Driscoll 2008). Hence, an intermediate regime between those corresponding to DH1 and DH2 is more relevant to practical conditions. In this intermediate regime, termed the thin reaction zones (TRZ) regime (Peters 1999), some turbulence length scales are smaller than δ_L, while the remaining length scales are larger than δ_L. Hence, for the TRZ regime, the integral length scale $\ell_0 > \delta_L$ and the Kolmogorov scale $\eta < \delta_L$, and the Karlovitz number satisfies $\mathrm{Ka} > 1$. Figure 2 shows the regimes corresponding to the Toronto and Michigan experiments (Yuen & Gülder 2013; Wabel *et al.* 2017) on a Borghi diagram. It is evident that most experiments correspond to the TRZ regime with $\mathrm{Ka} > 1$; some experiments from the Michigan data (blue circles in figure 2) even correspond to $\mathrm{Ka} > 100$. The additional mechanism that accounts for the enhancement $(s_T/s_L)/(A_T/A)$ over and above the area enhancement A_T/A must, therefore, be pertinent to the TRZ regime.

Since some turbulence length scales are smaller than the laminar flame thickness in the TRZ regime, these smaller scales may be surmised to affect the diffusive processes in the preheat zone of the flamelet (without disturbing the reaction zone). The action of these small scales $\ell < \delta_L$ may be expected to be over and above the action of large scales $\ell > \delta_L$, which act to enhance the flame surface area (without disturbing the processes internal to the flamelet). The related enhancement of s_T due to turbulence small scales has been estimated (Zimont 1979) from the turbulent diffusivity expected at the scale of the broadened flame brush thickness using semiempirical arguments. Unfortunately, no comparisons with experimental data were presented alongside. Ronney & Yakhot (1992) applied a similar approach using the Kolmogorov energy spectrum to compare the enhancement in s_T predicted for the TRZ regime with experimental data of Abdel-Gayed, Bradley & Lawes (1987). While a 'fair' agreement with experiments was found, it was assumed the small scales lie within the inertial subrange, i.e. the energy spectrum and the semiempirical formulation of enhanced diffusivity did not include a model of the dissipation range. More recently, Gülder (2007) has provided yet another estimate for the diffusive enhancement of s_T due to turbulence small scales by taking intermittency into account. Assuming that the small-scale diffusive effects dominate at the Taylor

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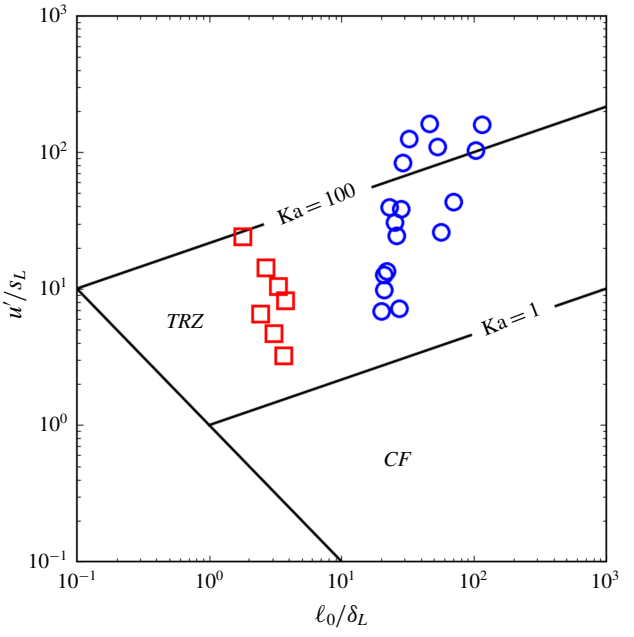


FIGURE 2. Turbulent flame propagation regimes in the Toronto (red squares) and Michigan (blue circles) experiments.

length scale λ, the related enhancement in diffusivity is derived as a function of the Taylor-scale Reynolds number $\mathrm{Re}_\lambda \equiv u'\lambda/\nu$. Limited experimental data was addressed in this work with regard to the range of Karlovitz numbers. Following a slightly different approach, Lee & Huh (2012) have provided expressions of s_T enhancement spanning a range of turbulence conditions based on estimates of the mean curvature and turbulent diffusivity at the flame brush leading edge. This approach leads to a reasonable agreement with experimental data (Lee, Lee & Huh 2014), but requires closure of the mean curvature term under varying turbulence conditions and tuning of the constants involved. These attempts have, therefore, left a gap for an accurate but simplified formulation that addresses the relatively recent experiments, which span a wider range of Karlovitz numbers.

Along the lines of the studies mentioned above, the present work provides a formulation for the contribution of diffusivity enhancement D_T/D towards the burning velocity enhancement s_T/s_L over and above the area enhancement A_T/A recorded in the TRZ regime. Following this concept, the overall turbulent burning velocity enhancement s_T/s_L in the Toronto and Michigan experiments (Yuen & Gülder 2013; Wabel *et al.* 2017) is written as a combination of DH1 and DH2 as follows

$$\frac{s_T}{s_L} = \underbrace{\left(1+\frac{\Delta A}{A}\right)}_{\textit{large-scale}} \underbrace{\left(1+\frac{\Delta D}{D}\right)^{1/2}}_{\textit{small-scale}}, \tag{1.4}$$

where the enhancement factor $\Delta A/A$ is attributed to turbulence large scales $\ell > \delta_L$, its effect directly measurable as an increase in flame area which is adopted from the Toronto and Michigan experiments. The enhancement factor $\Delta D/D$ is attributed to

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turbulence small scales $\ell < \delta_L$. Here, in contrast to the previous investigations (Zimont 1979; Ronney & Yakhot 1992; Gülder 2007; Lee & Huh 2012), the factor $\Delta D/D$ and its contribution to s_T/s_L is computed straightforwardly from a model energy spectrum that includes an exponential decay of energy at high wavenumbers (Pope 2000); this model spectrum is known to agree well with non-reacting experiments, and there is evidence of agreement with turbulent reacting flow experiments (Wang *et al.* 2007). Further, no assumption regarding the dominant scale of action of diffusivity enhancement is adopted, unlike Zimont (1979), Ronney & Yakhot (1992), or Gülder (2007). Rather, the total diffusive effect of all turbulence length scales ℓ smaller than the laminar flame thickness δ_L is estimated. The estimate obtained is directed towards reconciling the enhancement factors s_T/s_L and A_T/A measured in contemporary experiments (at Toronto and Michigan) which span a greater range of Karlovitz numbers than before. In line with previous studies, no model is included to account for the effect of flame quenching. The mathematical approach for estimating the enhancement factor $\Delta D/D$ is presented in § 2. This factor is compared with the disparity between DH1 and DH2 given by $(s_T/s_L)/(A_T/A)$ in § 3; encouraging results are obtained, and a discussion of ongoing and further work is provided in § 4.

2. Theory: diffusivity enhancement in small-scale turbulence

Prandtl's hypothesis is adopted to estimate the diffusivity enhancement arising from turbulence small scales. Accordingly, the turbulent diffusivity D_ℓ at a length scale ℓ in turbulent flow is assumed to be proportional to the product of the velocity u_ℓ and the length ℓ of the turbulent motion, i.e. $D_\ell = \ell u_\ell$. Subsequently, it is assumed that length scales $\ell < \delta_L$ contribute to the diffusivity enhancement ΔD. Hence, the overall diffusivity enhancement factor ΔD can be computed as the average

$$\Delta D = \int_{\delta_L^{-1}}^{\infty} \frac{u(\kappa)}{\kappa}\,\mathrm{d}\kappa, \tag{2.1}$$

over the range of wavenumbers corresponding to length scales $\ell = 0 \to \delta_L$ in the flow. This amounts to considering all wavenumbers $\kappa > \delta_L^{-1}$ towards the average diffusivity enhancement. (It may argued that the smallest length scale in turbulent flow is the Kolmogorov scale η, and the upper limit be taken as η^{-1} rather than ∞. Due to the rapid decay of energy at high wavenumbers, either choice leads to the same functional form in the final result.) A natural scheme of scaling is to use properties of the laminar flame: thickness δ_L and speed s_L. Thus, the molecular diffusivity $D = s_L\delta_L$, and the overall diffusivity enhancement factor $\Delta D/D$ is obtained as

$$\frac{\Delta D}{D} = \int_{1}^{\infty} \frac{\hat{u}(\hat{\kappa})}{\hat{\kappa}}\,\mathrm{d}\hat{\kappa}, \tag{2.2}$$

with the $\hat{}$ symbol denoting scaled variables: the scaled wavenumber $\hat{\kappa} = \kappa\delta_L$, and the scaled velocity $\hat{u}(\hat{\kappa}) = \sqrt{2\hat{E}(\hat{\kappa})}$, where $\hat{E}(\hat{\kappa})$ is the scaled energy spectrum. Presently, a model energy spectrum based on the Kolmogorov spectrum is adopted as presented by Pope (2000). This model includes inertial scale (von Karman 1948) and dissipation scale (Kraichnan 1959) spectra as follows

$$\hat{E}(\hat{\kappa}) = C\hat{\varepsilon}^{2/3}\hat{\kappa}^{-5/3}\hat{f}_L(\hat{\kappa})\hat{f}_\eta(\hat{\kappa}), \tag{2.3}$$

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where $C \sim 1.5$ is a parametric constant of order unity. The spectral functions $\hat{f}_L(\hat{\kappa})$ and $\hat{f}_\eta(\hat{\kappa})$ correspond to the energy spectrum profiles in the energy containing range and the dissipation range, respectively. For length scales $\ell < \delta_L$, the inertial range spectrum $\hat{f}_L(\hat{\kappa}) \to 1$ because $\hat{\kappa} L/\delta_L \gg 1$ in this range for a specified geometrical dimension L. For the dissipation range spectrum $\hat{f}_\eta(\hat{\kappa})$, an exponential profile is specified as follows

$$\hat{f}_\eta(\hat{\kappa}) \sim \mathrm{e}^{-\beta\hat{\kappa}\eta/\delta_L}, \tag{2.4}$$

where $\beta \sim 5.2$ is an empirically obtained parametric constant (Pope 2000). Hence, the overall diffusivity enhancement factor $\Delta D/D$ is written as

$$\frac{\Delta D}{D} = (2C)^{1/2}\hat{\varepsilon}^{1/3}\int_1^\infty \hat{\kappa}^{-11/6}\mathrm{e}^{-\beta\hat{\kappa}/(2\sqrt{\mathrm{Ka}})}\,\mathrm{d}\hat{\kappa}, \tag{2.5}$$

where the substitution $\delta_L/\eta = \sqrt{\mathrm{Ka}}$ has been made. The term $\hat{\varepsilon}$ is the scaled form of the turbulent kinetic energy dissipation rate ε, which is given by $\varepsilon = 15\nu(u'/\lambda)^2$ according to Taylor (1935), where λ is the Taylor length scale and ν is the kinematic viscosity. The scaled dissipation rate $\hat{\varepsilon}$ can be formulated as follows

$$\begin{aligned}\hat{\varepsilon} &= \varepsilon\frac{\delta_L}{s_L^3} = 15\nu\frac{\delta_L}{s_L^3}\left(\frac{u'}{\lambda}\right)^2 = 15\frac{\delta_L^2}{s_L^3}\frac{u'^2}{\ell_0^2}\frac{\mathrm{Re}}{10}\\ &= \frac{3}{2}\frac{u'^3\delta_L}{s_L^3\ell_0} = \frac{3}{2}\mathrm{Ka}^2,\end{aligned} \tag{2.6}$$

using the scaling relations $\nu \sim \delta_L s_L$ (Williams 1985) and $\lambda \sim \ell_0\sqrt{10/\mathrm{Re}}$ (Taylor 1935). With these substitutions in place, equations (2.5) simplifies to

$$\frac{\Delta D}{D} = C'\mathrm{Ka}^{2/3}\int_1^\infty \hat{\kappa}^{-11/6}\mathrm{e}^{-\beta\hat{\kappa}/(2\sqrt{\mathrm{Ka}})}\,\mathrm{d}\hat{\kappa}, \tag{2.7}$$

where $C' \equiv (2C)^{1/2}(3/2)^{1/3} \sim 1$. This reflects a closed-form dependence of the enhancement in diffusivity due to small scales $\eta < \ell < \delta_L$ on the Karlovitz number Ka. This is a novel result and provides an approach towards obtaining scaling relations for the diffusivity-related enhancement of s_T in the TRZ regime. With the change of variable $t = \beta\hat{\kappa}/(2\sqrt{\mathrm{Ka}})$, the integral is written as

$$\frac{\Delta D}{D} = C'\left(\frac{\beta}{2}\right)^{5/6}\mathrm{Ka}^{1/4}\int_{\beta/(2\sqrt{\mathrm{Ka}})}^\infty t^{-11/6}\mathrm{e}^{-t}\,\mathrm{d}t, \tag{2.8}$$

which has the form of the upper incomplete gamma function $\Gamma(s, x)$ given by

$$\Gamma(s, x) \equiv \int_x^\infty t^{s-1}\mathrm{e}^{-t}\,\mathrm{d}t = \Gamma(s) - \gamma(s, x), \tag{2.9}$$

where $\Gamma(s)$ is the gamma function and $\gamma(s, x)$ is the lower incomplete gamma function, expanded in terms of the lower limit x of the integral as

$$\gamma(s, x) = x^s\sum_{k=0}^\infty \frac{(-x)^k}{k!(s+k)}, \tag{2.10}$$

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where s is the exponent in the integrand. Substituting the values $s=-5/6$ and $x=1$ in $\Gamma(s,x)$, the solution of (2.8) is written as

$$\frac{\Delta D}{D}=C'\left(\frac{\beta}{2}\right)^{5/6}\mathrm{Ka}^{1/4}\left[\Gamma\left(-\frac{5}{6}\right)-\gamma\left(-\frac{5}{6},\frac{\beta}{2\sqrt{\mathrm{Ka}}}\right)\right], \tag{2.11}$$

which can be expanded in powers of Ka by applying (2.10) as follows

$$\begin{aligned}\frac{\Delta D}{D} &= C'\left(\frac{\beta}{2}\right)^{5/6}\mathrm{Ka}^{1/4}\left[\Gamma\left(-\frac{5}{6}\right)-\sum_{k=0}^{\infty}\frac{(-1)^k(\beta/2)^{k-5/6}}{k!\left(k-\frac{5}{6}\right)}\mathrm{Ka}^{((5/12)-(k/2))}\right]\\ &= C'[(6/5)\mathrm{Ka}^{2/3}+\Gamma(-5/6)(\beta/2)^{5/6}\mathrm{Ka}^{1/4}+3\beta\mathrm{Ka}^{1/6}-(3/28)\beta^2\mathrm{Ka}^{-1/3}+\cdots]\\ &= (6/5)C'\mathrm{Ka}^{2/3}+\Gamma(-5/6)(\beta/2)^{5/6}C'\mathrm{Ka}^{1/4}+O(\mathrm{Ka}^{1/6}),\end{aligned} \tag{2.12}$$

where C' and β are empirically derived constants. Therefore, the leading-order term for the overall enhancement in diffusivity scales as

$$\frac{\Delta D}{D}\sim\mathrm{Ka}^{2/3}, \tag{2.13}$$

which leads to the scaling law $s_T\sim\mathrm{Ka}^{1/3}$ for the contribution of diffusivity enhancement towards s_T enhancement for $\mathrm{Ka}\gg1$. A similar form has been observed in recent DNS of thermonuclear flames in the small-scale turbulence regime (Aspden, Woosley & Bell 2010). For the range of Ka values $1<\mathrm{Ka}<1000$ considered here, equation (2.8) may also be evaluated numerically for each given value of Ka, thereby providing a profile of variation of the integral over a range of Ka. Regardless of the method of evaluation of (2.8), the enhancement in diffusivity due to small-scale turbulence $\ell<\delta_L$ is obtained as a closed-form function of the Karlovitz number.

3. Result: burning velocity enhancement in small-scale turbulence

The values of $(s_T/s_L)/(A_T/A)$ measured in Toronto and Michigan for a range of Karlovitz numbers are shown once again in figure 3. The contribution of diffusivity enhancement D_T/D_L as estimated presently is shown alongside. Both the numerical evaluation $(1+\Delta D/D)^{1/2}$ of this contribution (solid line) and the analytically obtained leading-order term $\mathrm{Ka}^{1/3}$ (dashed line) of the solution to (2.8) address, to a significant degree, the disparity in burning velocity enhancement $(s_T/s_L)/(A_T/A)$. On close examination, it is seen that the numerical evaluation approximates particularly well the profile of disparity measured in Michigan (blue circles in figure 3) for low Karlovitz numbers $1<\mathrm{Ka}<10$. The Toronto experiments (red squares in figure 3) fall mostly in the intermediate range of Karlovitz numbers $10<\mathrm{Ka}<100$ and show higher disparities than the Michigan experiments. Here, the leading-order term $\mathrm{Ka}^{1/3}$ appears to address the disparity better. For high Karlovitz numbers in the range $100<\mathrm{Ka}<1000$, both the leading-order term and the numerical evaluation predict similar disparities measured to be of the order $s_T/s_L\sim\mathcal{O}(10)$. Hence, the contribution of the enhancement factor $\Delta D/D$ towards the enhancement s_T/s_L appears to be valid for the entire range of Karlovitz numbers in the measurements at Toronto and Michigan (Yuen & Gülder 2013; Wabel *et al.* 2017).

The reconciliation of DH1 and DH2 provided by the present formulation suggests that DH2 may indeed operate alongside DH1 in the TRZ regime as hinted by

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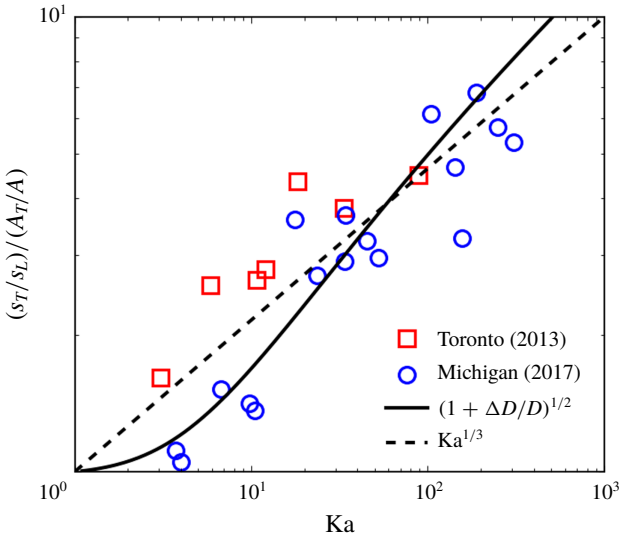


FIGURE 3. Measured disparity $(s_T/s_L)/(A_T/A)$ in the profiles of burning velocity enhancement and flame surface area enhancement as observed in the Toronto and Michigan experiments (symbols) and the enhancement factor (solid line) obtained from a numerical evaluation $I(\mathrm{Ka})$ of (2.8). Also shown is the leading-order term $\mathrm{Ka}^{1/3}$ (dashed line) obtained by solving (2.8) analytically using incomplete gamma functions.

previous investigations (Zimont 1979; Ronney & Yakhot 1992; Gülder 2007; Lee & Huh 2012). The operation of DH1 to enhance flame surface area is a consequence of turbulent length scales $\ell > \delta_L$, whereas the (simultaneous) operation of DH2 to enhance diffusivity is a consequence of turbulent length scales $\ell < \delta_L$. It is implicitly assumed that the enhancement due to DH1 does not interfere with the enhancement due to DH2, and that the operation of DH2 follows (1.3) for the entire range of D_T values expected here. Such a simultaneous operation of DH1 and DH2 appears to address the measured overall enhancement of s_T for high Ka values.

Estimation of the diffusive enhancement due to DH2 in terms of the Karlovitz number provides a means to reconcile existing measurements such as those considered here. Additional knowledge of the dependence of $\Delta A/A$ on relevant dimensionless groups such as the Karlovitz number could, in principle, aid the formulation of overall enhancement s_T/s_L in terms of such dimensionless groups. Yet, it must be noted that the current reconciliation of s_T/s_L with A_T/A does not depend on flamelet quenching: s_T/s_L as a function of Ka is formulated solely in terms of area enhancement of the flame surface and diffusivity enhancement in the flamelet preheat zone. While this is appropriate for addressing experimental data of Toronto (Gülder 2007) and Michigan (Wabel *et al.* 2017), further investigation is needed in order to address older experiments of Abdel-Gayed *et al.* (1987). Direct Numerical Simulation can provide necessary data to validate and improve the current formulation in this context. These ramifications warrant further investigation of the effective diffusivity enhancement in the Toronto and Michigan experiments with regard to a possible reconciliation between the different effects of Damköhler's first and second hypotheses (DH1 and DH2).

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4. Conclusions

Measurements in turbulent Bunsen flames have long showed a discrepancy between the burning velocity enhancement s_T/s_L and the flame surface area ratio A_T/A, which increases monotonically with the Karlovitz number Ka. It is shown that this discrepancy is commensurate with the effect of increase in diffusivity arising from the flux associated with turbulence small scales $\ell < \delta_L$, according to Damköhler's second hypothesis (DH2). The diffusivity enhancement D_T/D associated with these sub-δ_L scales is derived as a closed-form function of the Karlovitz number, which has implications with regard to subgrid scale modelling of turbulent premixed flames. Notwithstanding the assumptions involved in the current estimation of D_T/D, the degree of reconciliation offered by the approach suggests that further experiments and models should investigate the possibility of a diffusive contribution from small-scale turbulence at high Ka values.

Acknowledgements

The authors thank Dr A. Skiba for helping interpret the experimental data from Michigan. G.V.N. thanks Dr S. Cespedes for all the fun discussions on the integral. G.V.N. acknowledges the funding support of EPSRC grant EP/P022286/1.

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