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Numerical simulation of an electromagnetic field by parallelized 3D AIBO-FDTD

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Abstract. The finite-difference time-domain (FDTD) method is useful in solving three-dimensional (3D) electromagnetic problems. However, it contains a numerical instability because it is a complete explicit method, and huge amounts of memory are required in the case of 3D analysis. The implicit FDTD method called the Alternating Implicit Block Overlapped (AIBO)-FDTD method was proposed for two-dimensional (2D) electromagnetic analysis. In our study, the 3D AIBO-FDTD method is developed by the modification of the 2D method. This method is suitable for parallel processing since the computational domain can be effectively partitioned. This property is especially compatible with implementations using distributed-memory multiprocessor systems such as PC clusters, which provide a huge memory space that accommodates a large problem. Although this method seems numerically stable since the implicit scheme is included, our result shows that the numerical instability occurs when the time step exceeds Courant's stability condition. This condition is still more relaxed than the conventional FDTD method. Furthermore, we demonstrated that parallel processing is an effective technique for improving the performance of the 3D AIBO-FDTD method, while this method is slower than FDTD when it is run on one processor.

1. Introduction

The finite-difference time-domain (FDTD) method provides the solution of the Maxwell equation directly. Since the grain size on parallel processing is coarse, it is suitable for parallel processing. However, this method contains numerical instability due to the complete explicit method. Moreover, since huge memory is required in the case of three-dimensional (3D) analysis, and it takes much time, parallel processing becomes a useful technique.

In this paper, the Alternating Implicit Block Overlapped (AIBO)-FDTD method is applied to the 3D electromagnetic analysis, and the numerical stability is investigated. Furthermore, the 3D AIBO-FDTD method is implemented on multiprocessor using MPI to utilize much memory, and it is evaluated on the speedup using a PC cluster for parallel processing.

2. AIBO-FDTD method

The AIBO-FDTD method [1] is an improvement of the Alternating Direction Implicit (ADI)-FDTD method [2] to reduce the dependency in calculation for parallel processing.

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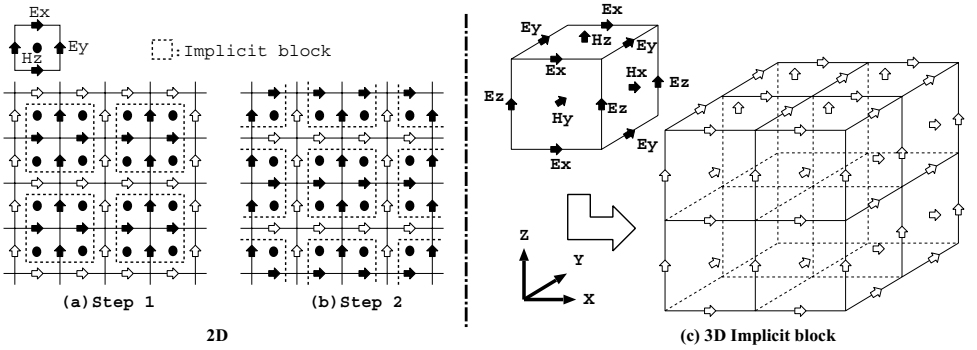


Figure 1. Calculation domains in the AIBO-FDTD method.

Figures 1(a) and (b) show the calculation domains and steps of the two-dimensional (2D) AIBO-FDTD method. The white arrows indicate the explicit values, and the black arrows and black circles indicate the implicit values. On each step, each implicit computational domain forms a square block. Here, this block is called an implicit block.

First, as in Fig. 1(a), the explicit values are calculated explicitly, then the implicit values are obtained using the explicit values implicitly. Next, in Step 2, the implicit blocks are shifted to a diagonal direction to overlap the implicit blocks of the previous step. Therefore, the explicit values are calculated using the implicit values obtained in Step 1, and similar computations are performed for the implicit values.

Since there is no dependency between each implicit block in the same step, a group of implicit blocks can be assigned to each task for parallel processing. Moreover, the communication between processors is performed only once within each step.

In fact, since the simulation field is 3D, the shape of the implicit block becomes a cube consisting of eight unit cells, as shown in Fig. 1(c). The values on the surface of the implicit block are calculated explicitly so that the calculation domains, i.e. the implicit blocks can be separated each other, and the values inside the implicit block are obtained implicitly.

The calculation in Step 1 produces the results at the $(n+1/2)$th time step using the results of the nth time step. Next, in Step 2, the explicit values are calculated using the implicit values obtained in Step 1, the similar computations are performed for the implicit values. The calculation in Step 2 gives the results at the $(n+1)$th time step.

3. Numerical stability

We evaluate the numerical stability of the 3D AIBO-FDTD method. We assume Mur's first-order absorbing boundary condition. It is also assumed that the division number of the domain is $100\times100\times100$, and the input is a 1.5 GHz electromagnetic wave along the z-axis in the center of the field, i.e. $(0,0,z)$.

In the conventional FDTD method, the time step Δt depends on Courant's stability condition as shown in (3.1) because it is a complete explicit method:

$$c\Delta t < \frac{1}{\sqrt{(1/\Delta x)^2+(1/\Delta y)^2+(1/\Delta z)^2}} \tag{3.1}$$

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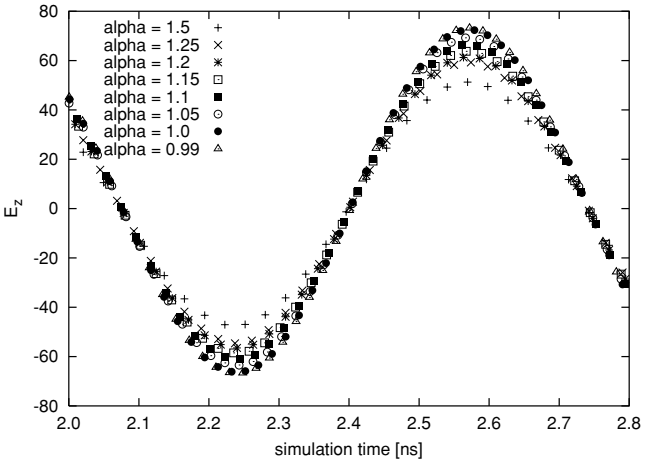


Figure 2. E_z at the observation point (10, 10, 0) in time domain on 3D AIBO-FDTD method.

Since the AIBO-FDTD method is a quasi implicit method, it is considered that the condition of Δt is relaxed from Courant's condition in the 2D case [1]. However, in the 3D case, since the number of the explicit values on the surface of the implicit block is more than that of the implicit values inside the implicit block, the property of the explicit method seems dominant.

Accordingly, we investigate the stability using the various values of α as shown below:

$$\Delta t = \frac{\alpha}{c \times \sqrt{(1/\Delta x)^2 + (1/\Delta y)^2 + (1/\Delta z)^2}} \tag{3.2}$$

$$(\alpha = 0.99, 1.0, 1.05, 1.1, 1.15, 1.2, 1.25, 1.5)$$

Figure 2 shows the z-component of electric field at the observation point (10, 10, 0) in time domain with α shown in (3.2). As shown in Fig. 2, the values in the case of $\alpha > 1.0$ are smaller than the correct values at $\alpha = 0.99$, whose Δt satisfies Courant's condition. Hence, in the 3D AIBO-FDTD method, the time step in simulation is restricted by the Courant's condition although it is slightly more relaxed than that which restricts the conventional FDTD method.

4. Evaluation

The 3D AIBO-FDTD is evaluated using MPI to examine the adaptability for parallel processing on a PC cluster. The problem described above is partitioned into cubic (or rectangular) domains to be assigned to multiple processors, parallelized using MPI, and executed on a PC cluster. MPI is a standard message-passing library suitable for distributed-memory multiprocessor systems or PC clusters. The performance is evaluated in the environment shown in Table 1.

Figure 3 shows the speedup ratio as a function of the number of processors. This result shows the effect of the parallelization using the AIBO-FDTD method. However, in the case of 4PU, 8PU, and 16PU, the speedup ratio is smaller than 2PU. For that reason, the ratio of the communication time to execution time

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Table 1. Evaluation environment (specification of the PC cluster).

Number of nodes	Eight
Processor	Dual Intel Xeon 2.4 GHz
Memory	1 GB per node
OS	FreeBSD 5.1-Release
Network	1 Gbps Ethernet
MPI	LAM 6.5.9

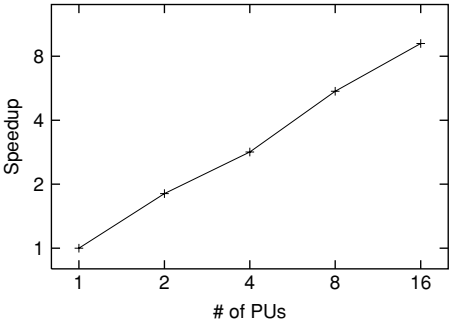


Figure 3. The speedup ratio as a function of the number of PUs.

becomes relatively larger. Although the communication ratio is 10% using 2PU, it becomes 12%, 27%, 48% with 4PU, 8PU, 16PU, respectively. This degradation of the performance by the communication disappears as the size of problem increases since an amount of the calculation increases relatively.

5. Conclusion

In this paper, we have investigated the 3D AIBO-FDTD method for parallel processing. The AIBO-FDTD method is a quasi implicit method; however, the numerical instability occurs when the time step exceeds the Courant's stability condition since the explicit calculations are dominant in the 3D AIBO-FDTD method.

Moreover, we demonstrated that parallel processing can effectively improve the performance of the 3D AIBO-FDTD method. 2PU demonstrated the best performance: 1.8 times faster than 1PU in the case of $100 \times 100 \times 100$ division number.

As future work, we will develop a novel quasi implicit FDTD method for 3D fields.

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