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Pitch-angle scattering rates in planetary magnetospheres

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Abstract. The mechanism of gyroresonant scattering of charged particles by small-amplitude, broadband electromagnetic waves is examined. By means of quasi-linear theory, a simple expression for the resonant pitch-angle diffusion coefficient $D_{\alpha\alpha}$ is developed, which is valid for parallel-propagating, small-amplitude, (R-mode or L-mode) electromagnetic waves of general spectral density. We calculate the average diffusion coefficient with respect to the pitch-angle α, namely $\langle D_{\alpha\alpha}\rangle$, which provides a practical estimate for particle scattering rates as a function of particle kinetic energy. The results for $\langle D_{\alpha\alpha}\rangle$, corresponding to a Gaussian wave frequency spectrum, are used to estimate scattering rates for several types of wave–particle interactions in the terrestrial and Jovian magnetospheres.

1. Introduction

Wave–particle interactions in plasmas may be described by the quasi-linear theory of weak plasma turbulence, the foundations of which were developed in the 1960s (e.g., Kennel and Engelmann 1966; Lerche 1968). In quasi-linear theory, cyclotron-resonant wave–particle interactions generally give rise to both pitch-angle diffusion and energy diffusion in the particle phase space. The time evolution of the particle distribution function is described by a kinetic equation involving resonant (pitch-angle, energy, mixed) diffusion coefficients for specified forms of the wave spectra (e.g., Kennel and Engelmann 1966; Lyons 1974; Albert 1999; Horne et al. 2003). Motivation for studying cyclotron resonant wave–particle interactions comes not least from their role in influencing the particle dynamics in the Earth's magnetosphere. Wave–particle interactions also have diverse applications in other planetary magnetospheres, and in other space and cosmic plasmas.

The importance of pitch-angle diffusion in radiation belt physics was recognized in the landmark papers by Kennel and Petschek (1966) and Kennel (1969). The Earth's dipole magnetic field acts like a magnetic mirror, with the result that particles bounce back and forth between conjugate mirror points in the northern and southern hemispheres. The mirroring, which involves reflection of a charged particle in a region of converging field lines, follows directly from the application of the first adiabatic invariant of the particle motion. Particles with sufficiently small

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equatorial pitch angles bounce to sufficiently high magnetic latitude that they impact the atmosphere and are lost by absorption. Such a range of pitch angles defines a 'loss cone' in which particle trapping is not possible. The remainder of velocity space constitutes the quasi-trapped region. As a result of wave-induced pitch-angle diffusion, particles can be scattered from the quasi-trapped region into the loss cone, and are thereby lost by precipitation. Particle precipitation is a fundamental process in the microphysics of planetary magnetospheres, e.g., in the production of aurorae. Precipitating particles, in particular relativistic electrons, are also known to change the chemical composition of the Earth's middle atmosphere.

In Sec. 2, using quasi-linear theory, we develop a formula for the rate of pitch angle diffusion $D_{\alpha\alpha}$ applicable to particles in cyclotron resonance with parallel-propagating (R-mode or L-mode) electromagnetic waves. We then calculate the average rate of pitch-angle scattering $\langle D_{\alpha\alpha}\rangle$, with respect to pitch-angle α, as a function of particle kinetic energy. The theory in this paper follows from recent work of Summers and Thorne (2003) that concerned, in particular, the pitch-angle scattering of relativistic electrons by electromagnetic ion cyclotron (EMIC) waves during geomagnetic storms. Following the presentation of our theory in Sec. 2, we also briefly describe the concepts of strong and weak pitch-angle diffusion. In Sec. 3 we apply our results to particular examples of pitch-angle scattering in the terrestrial and Jovian magnetospheres. The applications chosen relate to important, and still not fully-resolved, current problems in space plasma physics. Finally, in Sec. 4 we briefly comment on our results.

2. Theory

2.1. General formulation

We start with the formula for the resonant pitch-angle diffusion coefficient,

$$D_{\alpha\alpha} = \frac{\pi}{2}\frac{|\Omega_\sigma|}{\gamma}\frac{kW(k)}{W_0}, \tag{2.1}$$

modified from Summers and Thorne (2003). The modification comprises the insertion of the previously omitted factor $\pi/2$, and the generalization to an arbitrary particle species specified by σ. Derivation of (2.1) is based on quasi-linear theory, and applies to small-amplitude, broadband electromagnetic waves propagating parallel to an assumed uniform background magnetic field; k is the resonant wavenumber, and $W(k)$ is the wave magnetic field spectral density given by

$$W_{\rm tot} = \int_{k_1}^{k_2} W(k)\,dk, \tag{2.2}$$

where $W_{\rm tot} = (\Delta B)^2/8\pi$ is the total wave magnetic energy density, and ΔB is the wave amplitude. The background magnetic field energy density is $W_0 = B_0^2/8\pi$ where B_0 is the magnetic field strength; $|\Omega_\sigma| = |q_\sigma|B_0/(m_\sigma c)$ is the particle gyrofrequency where $|q_\sigma|$ is the magnitude of the particle charge, m_σ is the particle rest mass and c is the speed of light. The Lorentz factor is $\gamma = (1 - v^2/c^2)^{-1/2}$ where v is the particle speed. In result (2.1) the resonant wavenumber k is given by

$$\frac{ck}{|\Omega_\sigma|} = \frac{1}{p|\cos\alpha|}, \tag{2.3}$$

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where $p = \gamma v/c$ is the normalized particle momentum and α is the particle pitch-angle. If desired, $D_{\alpha\alpha}$ can be expressed as an explicit function of α by substituting for k from (2.3) into (2.1). Relation (2.3) is obtained from the relativistic Doppler gyroresonance condition,

$$\omega - kv\cos\alpha = N|\Omega_\sigma|/\gamma, \tag{2.4}$$

by setting the cyclotron harmonic $N = \pm 1$, and omitting the (frequency) ω-term. Omission of the ω-term, which produces significant simplifications in the theory, can only be rigorously justified if $\omega/|\Omega_\sigma| \ll 1$.

Writing the kinetic energy E_k of a particle in units of its rest energy $m_\sigma c^2$ as $E = E_k/(m_\sigma c^2) = \gamma - 1$, we find it convenient to express the reduced gyroresonance condition (2.3) in the form

$$E(E+2)\cos^2\alpha = \frac{\Omega_\sigma^2}{c^2k^2}, \tag{2.5}$$

for which the relation $p^2 = E(E+2)$ has been used.

From (2.5), since the waveband considered has wavenumbers in the range $k_1 \leqslant k \leqslant k_2$, it follows that the kinetic energy E and pitch angle α of particles in resonance with the waves must satisfy the restriction

$$\frac{\Omega_\sigma^2}{c^2k_2^2} \leqslant E(E+2)\cos^2\alpha \leqslant \frac{\Omega_\sigma^2}{c^2k_1^2}. \tag{2.6}$$

We now define

$$E_1 = \left(1 + \frac{\Omega_\sigma^2}{c^2k_1^2}\right)^{1/2} - 1, \quad E_2 = \left(1 + \frac{\Omega_\sigma^2}{c^2k_2^2}\right)^{1/2} - 1, \tag{2.7}$$

and

$$\alpha_1 = \cos^{-1}\left(\frac{|\Omega_\sigma|}{ck_1}\frac{1}{[E(E+2)]^{1/2}}\right), \quad \alpha_2 = \cos^{-1}\left(\frac{|\Omega_\sigma|}{ck_2}\frac{1}{[E(E+2)]^{1/2}}\right). \tag{2.8}$$

Then, from (2.6), it first follows that if $E < E_2$, particles cannot resonate with the waves, i.e. E_2 is the minimum resonant energy. Secondly, if $E_2 < E < E_1$, it follows that wave–particle resonance takes place over pitch angles such that $0 \leqslant \alpha \leqslant \alpha_2$. Thirdly, if $E > E_1$, then resonance takes place over pitch angles in the range $\alpha_1 \leqslant \alpha \leqslant \alpha_2$. In this paper, since we are concerned, in particular, with resonant particles that are scattered into the atmospheric loss cone in a planetary magnetosphere, we restrict our attention to the case $E_2 < E < E_1$, for which the loss cone is included within the cone of pitch angles over which resonance occurs.

In practice, it is often the case that a wave magnetic field spectrum is given (e.g., from observations) as a function of wave frequency. In order to transform from wavenumbers in the range $k_1 \leqslant k \leqslant k_2$ to wave frequencies in the corresponding range $\omega_1 \leqslant \omega \leqslant \omega_2$, we write

$$\int_{k_1}^{k_2} W(k)\,dk = \int_{\omega_1}^{\omega_2} W(k(\omega))\frac{dk}{d\omega}\,d\omega = \int_{\omega_1}^{\omega_2} \tilde{W}(\omega)\,d\omega, \tag{2.9}$$

where $k_1 = k(\omega_1)$ and $k_2 = k(\omega_2)$, and then (2.1) becomes

$$D_{\alpha\alpha} = \pi\frac{|\Omega_\sigma|}{\gamma W_0}\frac{k^2\tilde{W}}{(d(k^2)/d\omega)}. \tag{2.10}$$

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In (2.10), $\tilde{W}$ is the wave frequency spectrum and k^2 is an explicit function of ω obtained from the wave dispersion equation.

We now define the average resonant pitch-angle diffusion coefficient,

$$\langle D_{\alpha\alpha}\rangle = \frac{1}{\alpha_2}\int_0^{\alpha_2} D_{\alpha\alpha}\, d\alpha, \quad \text{for } E_2 < E < E_1, \tag{2.11}$$

where $D_{\alpha\alpha}$ takes the form of (2.1) or (2.10), as desired; $\langle D_{\alpha\alpha}\rangle$ is a function of particle kinetic energy E and represents an inverse timescale for pitch-angle scattering.

If the wave spectrum is given in the form $W = W(k)$, then, from (2.1), (2.3) and (2.11), we find that $\langle D_{\alpha\alpha}\rangle$ can be expressed as

$$\langle D_{\alpha\alpha}\rangle = \frac{\pi}{2}\frac{1}{\alpha_2}\frac{|\Omega_\sigma|}{\gamma W_0}\int_{k_3}^{k_2} \frac{W(k)\, dk}{\left(E(E+2)c^2k^2/\Omega_\sigma^2 - 1\right)^{1/2}}, \tag{2.12}$$

where

$$\frac{c^2k_3^2}{\Omega_\sigma^2} = \frac{1}{E(E+2)}. \tag{2.13}$$

It should be noted that, although the integrand in (2.12) is singular at the lower limit of integration k_3, the integral is well-defined and straightforward to evaluate by efficient numerical techniques.

Alternatively, if the given wave spectrum is in the form $\tilde{W} = \tilde{W}(\omega)$, then from (2.3), (2.10) and (2.11), we deduce that

$$\langle D_{\alpha\alpha}\rangle = \frac{\pi}{2}\frac{1}{\alpha_2}\frac{|\Omega_\sigma|}{\gamma W_0}\int_{\omega_3}^{\omega_2} \frac{\tilde{W}(\omega)\, d\omega}{((E(E+2)/u^2)(\omega/\Omega_\sigma)^2 - 1)^{1/2}}, \tag{2.14}$$

where $u = \omega/(ck)$ is the dimensionless wave phase speed, regarded as a function of frequency ω, and ω_3 satisfies the relations

$$\frac{E(E+2)}{u_3^2}\left(\frac{\omega_3}{\Omega_\sigma}\right)^2 = 1, \quad u_3 = \frac{\omega_3}{ck(\omega_3)}. \tag{2.15}$$

In (2.14), the integrand is singular at $\omega = \omega_3$, but the improper integral is again well-defined and readily evaluated numerically.

Since a wave dispersion relation can be expressed in the form of k^2 as an explicit function of ω, then the integral in (2.12) can always be expressed as an integral with respect to ω. The integral in (2.14) cannot, however, always be expressed as an integral with respect to k. In certain particular cases, it is possible to calculate $\langle D_{\alpha\alpha}\rangle$ directly as a proper integral with respect to α, using (2.11), if it is desired to avoid the improper integrals occurring in (2.12) and (2.14).

We now consider two special cases for the wave spectrum.

2.2. Kolmogorov spectrum

Assuming a Kolmogorov wave spectrum $W = W(k) \propto k^{-q}$, where $q(>1)$ is the spectral index, we deduce from (2.2) that

$$W(k) = (q-1)W_{\text{tot}}\left(\frac{1}{k_1^{q-1}} - \frac{1}{k_2^{q-1}}\right)^{-1}\frac{1}{k^q}. \tag{2.16}$$

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It is convenient in this case to express $\langle D_{\alpha\alpha}\rangle$ as an integral with respect to α, using (2.3), (2.11) and (2.16). The result is

$$\langle D_{\alpha\alpha}\rangle = \frac{\pi}{2}\frac{1}{\alpha_2}\frac{|\Omega_\sigma|}{\gamma}\frac{(q-1)R}{(1-(k_1/k_2)^{q-1})}\left(\frac{ck_1}{|\Omega_\sigma|}\right)^{q-1}[E(E+2)]^{(q-1)/2}\int_0^{\alpha_2}(\cos\alpha)^{q-1}\,d\alpha, \tag{2.17}$$

where

$$R = \frac{W_{\rm tot}}{W_0} = \frac{(\Delta B)^2}{B_0^2} \tag{2.18}$$

is the ratio of the turbulent wave energy to the ambient magnetic field energy.

Rather than give an analytical expression for the integral in (2.17) corresponding to a general form of q, we choose to take a specific example, namely $q = 4$, and thereby evaluate $\langle D_{\alpha\alpha}\rangle$ as a simple explicit function of E. We find

$$\langle D_{\alpha\alpha}\rangle = \frac{\pi|\Omega_\sigma|}{\cos^{-1}\left[\left([1/E(E+2)]\left(\Omega_\sigma^2/c^2k_2^2\right)\right)^{1/2}\right]}\frac{R}{(1-(k_1/k_2)^3)}\left(\frac{ck_1}{|\Omega_\sigma|}\right)^3 \times\frac{[E(E+2)]^{3/2}}{(E+1)}\left(1-\frac{1}{E(E+2)}\frac{\Omega_\sigma^2}{c^2k_2^2}\right)^{1/2}\left(1+\frac{1}{2E(E+2)}\frac{\Omega_\sigma^2}{c^2k_2^2}\right). \tag{2.19}$$

2.3. Gaussian spectrum

We assume the Gaussian wave spectrum,

$$\tilde{W}(\omega) \propto \exp\{-[(\omega-\omega_m)/\delta\omega]^2\}, \tag{2.20}$$

where ω_m is the centre frequency, with $\omega_1 = \omega_m - \delta\omega$ and $\omega_2 = \omega_m + \delta\omega$. Then from (2.2) and (2.9) it follows that

$$\tilde{W}(\omega) = \frac{W_{\rm tot}}{\nu\delta\omega}e^{-[(\omega-\omega_m)/\delta\omega]^2}, \tag{2.21}$$

where $\nu = \sqrt{\pi}\,\mathrm{erf}(1)$ (≈ 1.49), and erf is the error function. Substitution of (2.21) into (2.14) yields the result

$$\langle D_{\alpha\alpha}\rangle = \frac{\pi}{2}\frac{1}{\alpha_2}\frac{|\Omega_\sigma|}{\gamma}\frac{R}{\nu\delta\omega}\int_{\omega_3}^{\omega_2}\frac{e^{-[(\omega-\omega_m)/\delta\omega]^2}\,d\omega}{((E(E+2)/u^2)(\omega/\Omega_\sigma)^2-1)^{1/2}}, \tag{2.22}$$

where R is again given by (2.18).

In Sec. 3 we apply (2.22) to calculate pitch-angle scattering rates corresponding to several types of wave–particle interactions in the terrestrial and Jovian magnetospheres.

2.4. Strong and weak diffusion

It is useful to compare $\langle D_{\alpha\alpha}\rangle$, in any given application, with the rate of 'strong' pitch-angle diffusion. The notion of strong diffusion was developed by Kennel and Petschek (1966) and Kennel (1969). In the strong diffusion regime, particles diffuse across the loss cone in less than a quarter-bounce period, with the result that the precipitation mechanism saturates and the particle flux is driven isotropic. The particle precipitation rate is then independent of the amplitude of the scattering waves, and depends only on the particle bounce time and the geometric size of the loss cone. For a planet with a dipole magnetic field, the rate of strong pitch-angle

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diffusion can be written approximately as

$$D_{\mathrm{SD}} = \frac{0.205}{L^4}\left[\frac{4L}{4L-3}\right]^{1/2}\frac{[E(E+2)]^{1/2}}{(E+1)}\frac{c}{R_{\mathrm{p}}}, \tag{2.23}$$

where L is the magnetic shell parameter, R_{p} is the radius of the planet and, as above, E is the particle kinetic energy and c is the speed of light. Expression (2.23) is a generalization to an arbitrary planet, with a dipole magnetosphere, of a result given by Summers and Thorne (2003) for the Earth.

If the particle injection source is weak, then the diffusion rate is slow and the 'weak' pitch-angle diffusion regime results. In weak diffusion there are few particles in the loss cone at any given time, and the rate of particle precipitation equals the rate of pitch-angle diffusion. The concepts of strong and weak diffusion are neatly summarized by the leaky bucket analogy illustrated in Fig. 12 of Kennel (1969).

3. Applications

The dispersion equation for R-mode electromagnetic waves (including whistlers) propagating parallel to a uniform background magnetic field is

$$\frac{1}{u^2} = 1 - \frac{1}{\alpha^* X}\left(\frac{1}{X-1} + \frac{1}{X/\epsilon + 1}\right), \tag{3.1}$$

where

$$\alpha^* = \Omega_{\mathrm{e}}^2/\omega_{\mathrm{pe}}^2, \tag{3.2}$$

and $X = \omega/|\Omega_{\mathrm{e}}|$ is the normalized wave frequency; $u = \omega/(ck)$ is the normalized wave phase speed, as above. Equation (3.1) applies to small-amplitude waves propagating in a cold electron–proton plasma, is based on linear theory and can be found in many texts (e.g., Stix 1992). $|\Omega_{\mathrm{e}}| = eB_0/(m_{\mathrm{e}}c)$ is the electron gyrofrequency, where e is the electronic charge and m_{e} the electron rest mass; $\omega_{\mathrm{pe}} = (4\pi N_0 e^2/m_{\mathrm{e}})^{1/2}$ is the electron plasma frequency, where N_0 is the electron number density, and $\epsilon = m_{\mathrm{e}}/m_{\mathrm{p}}$, where m_{p} is the proton rest mass. The parameter α^*, defined by (3.2), depends for its value on the ratio B_0^2/N_0, and plays a significant role in cold plasma theory.

We further require the dispersion relation for electromagnetic ion cyclotron waves propagating in a multi-ion plasma, specifically containing hydrogen (H^+), helium (He^+) and oxygen (O^+) ions. The relevant relation is the L-mode version of (3.1), augmented on the right-hand side by terms corresponding to helium and oxygen ions, similar in form to the already-present term for hydrogen ions. In dimensionless form, the required dispersion equation is

$$\frac{1}{u^2} = 1 - \frac{1}{\alpha^* X}\left(\frac{1}{X+1} + \frac{\eta_1}{X/\epsilon - 1} + \frac{\eta_2}{4X/\epsilon - 1} + \frac{\eta_3}{16X/\epsilon - 1}\right), \tag{3.3}$$

where $\eta_1 = N_1/N_0$, $\eta_2 = N_2/N_0$ and $\eta_3 = N_3/N_0$ denote the fractional composition of the ion species in the plasma, with N_1, N_2 and N_3 equal to the hydrogen, helium and oxygen ion number densities, respectively. Charge neutrality in the plasma requires that $\eta_1 + \eta_2 + \eta_3 = 1$.

We now consider a number of applications of our theory. In each of the following applications, we assume that the wave spectrum is Gaussian. Accordingly, to

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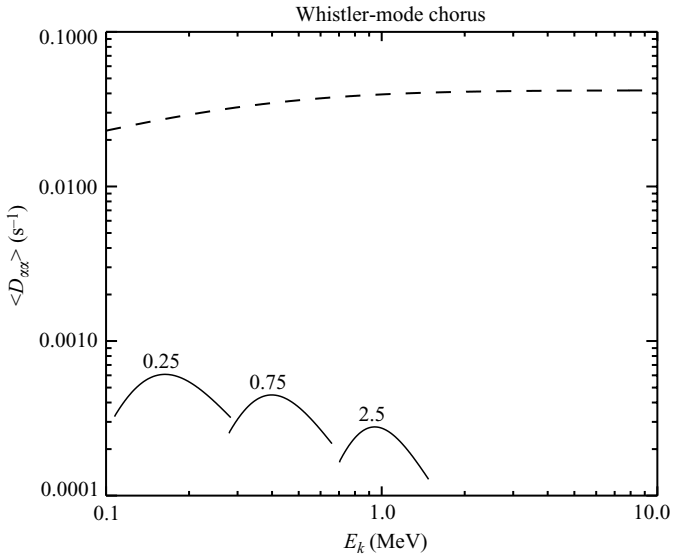


Figure 1. Average pitch-angle diffusion coefficient $\langle D_{\alpha\alpha}\rangle$ as a function of electron kinetic energy, for electrons in resonance with whistler-mode chorus outside the Earth's plasmasphere at $L = 4$, for wave amplitude $\Delta B = 25$ pT and Gaussian frequency spectrum with centre $\omega_m = 0.25\,|\Omega_e|$ and semi-bandwidth $\delta\omega = 0.1\,|\Omega_e|$; the parameter $\alpha^* = \Omega_e^2/\omega_{pe}^2$ labels the curves. The dashed curve represents the rate of strong pitch-angle diffusion (2.23).

calculate local pitch-angle scattering rates we use result (2.22) together with the appropriate form of dispersion equation (3.1) or (3.3).

3.1. Electrons interacting with whistler-mode chorus outside the Earth's plasmapause

Whistler-mode chorus emissions are electromagnetic waves generated inside the Earth's magnetosphere, but outside the plasmapause, predominantly near the geomagnetic equator, in the frequency range $\omega/|\Omega_e| = 0.1$–0.5. For parallel propagation, the waves are right-hand circularly polarized. The intensity of chorus waves depends on location and substorm activity (e.g., Meredith et al. (2001) and references therein). While the mechanism of stochastic acceleration by chorus-mode waves is considered to be a leading candidate for generating relativistic (>1 MeV) electrons during magnetic storms (Summers and Ma 2002; Meredith et al. 2002; Summers et al. 2002; Horne et al. 2003), pitch-angle scattering by chorus waves is also expected to contribute to precipitation losses of energetic electrons from the outer radiation belt. In Fig. 1 we plot the average pitch-angle diffusion coefficient $\langle D_{\alpha\alpha}\rangle$ as a function of electron kinetic energy, for a Gaussian wave spectrum with $X_1 = 0.15$ and $X_2 = 0.35$ (corresponding to centre $\omega_m = 0.25\,|\Omega_e|$ and semi-bandwidth $\delta\omega = 0.1\,|\Omega_e|$). We set $L = 4$, $|f_e| = |\Omega_e|/2\pi = 13.7$ kHz, $R = 2.6 \times 10^{-9}$ (corresponding to wave amplitude $\Delta B = 25$ pT), and choose $\alpha^* = 0.25$, 0.75, 2.5 as representative values for the outer radiation belt outside the plasmasphere. The dashed curve representing the rate of strong diffusion D_{SD} (given by (2.23)) is shown for comparison with the profiles for $\langle D_{\alpha\alpha}\rangle$. Clearly, chorus-mode waves corresponding to the chosen parameter values cannot produce strong diffusion scattering. The pitch-angle scattering rates shown in Fig. 1 represent local values

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at the equator, for each given value of α^*, corresponding to a uniform magnetic field. In order to apply the results to the Earth's dipole magnetic field, further averaging must be carried out with respect to both the bounce motion in the magnetic mirror geometry, and the azimuthal gradient drift motion of the electrons. These combined averaging processes generally can be expected to produce pitch-angle scattering rates that are at least an order of magnitude lower than the local equatorial values. Therefore, from Fig. 1 we deduce that timescales for pitch-angle scattering of several-hundred-keV electrons by chorus waves are typically 1 day or more. Such time scales indicate that chorus waves can contribute non-negligibly to energetic electron precipitation loss from the outer zone, in particular during intense geomagnetic storms, even though in the weak diffusion regime.

3.2. Electrons interacting with whistler-mode hiss near Io's orbit in Jupiter's magnetosphere

In an analysis of data from the *Pioneer 10* flyby, Fillius et al. (1976) showed that pitch-angle scattering plays a significant role in determining the distribution of energetic electrons in the inner magnetosphere of Jupiter. Moreover, Fillius et al. (1976) attributed electron losses to pitch-angle scattering into the planetary loss cone. The first direct observations of whistler-mode waves at Jupiter were by *Voyager 1* in 1979 (see the summary by Kurth et al. (1985) and references therein), thereby suggesting the existence of atmospheric lightning at Jupiter. Scarf et al. (1979) provided an initial quantitative analysis of pitch-angle scattering in the Io torus region, based on *Voyager 1* wave observations, and estimated electron precipitation lifetimes. A detailed survey of wave–particle interaction processes in the Jovian magnetosphere was provided by Thorne (1983). Jovian synchrotron radiation, which has been observed for decades, is generated by relativistic electrons trapped in the radiation belt. Complete understanding of synchrotron emissions requires accurate modelling of the electron distributions. This necessitates a detailed accounting of all forms of atmospheric loss, including pitch-angle scattering, as pointed out by Wang et al. (2002). Whistler-mode waves at Jupiter typically comprise broadband hiss that is strongly enhanced in the vicinity of the orbit of Io, and with a maximum spectral intensity near a few hundred hertz. At Io we put $L = 6$, $B_0 = 1900$ nT and $|f_e| = |\Omega_e|/2\pi = 53.2$ kHz. We consider α^* in the range $0.02 \leqslant \alpha^* \leqslant 0.2$, which corresponds to the approximate density range $200 \leqslant N_0 \leqslant 1600$ cm^{-3}. For whistlers in the frequency range 100 Hz–10 kHz we set $X_1 = 0.0019$, $X_2 = 0.0181$ (or $X_m = 0.01$, $\delta X = 0.0081$). In Fig. 2 we show $\langle D_{\alpha\alpha} \rangle$ curves for this waveband, for $\alpha^* = 0.02$, 0.06, 0.2, and wave amplitude $\Delta B = 8$ pT. For these parameters pitch-angle scattering rates for electrons in the energy range 200 keV to several MeV are well below the strong diffusion rate and, after additional bounce-averaging has been carried out, the scattering time scales will be at least of the order of days.

3.3. Electrons interacting with electromagnetic ion cyclotron waves inside the Earth's plasmasphere

Electromagnetic ion cyclotron (EMIC) waves are low-frequency (0.1–5.0 Hz) waves, sometimes referred to as Pc1 pulsations, that are excited inside the Earth's high-density plasmasphere, in particular in a spatially localized zone along the duskside plasmapause. EMIC waves are intensified during geomagnetic storms, with typical wideband amplitudes in the range 1–10 nT. Dependent on various factors, including

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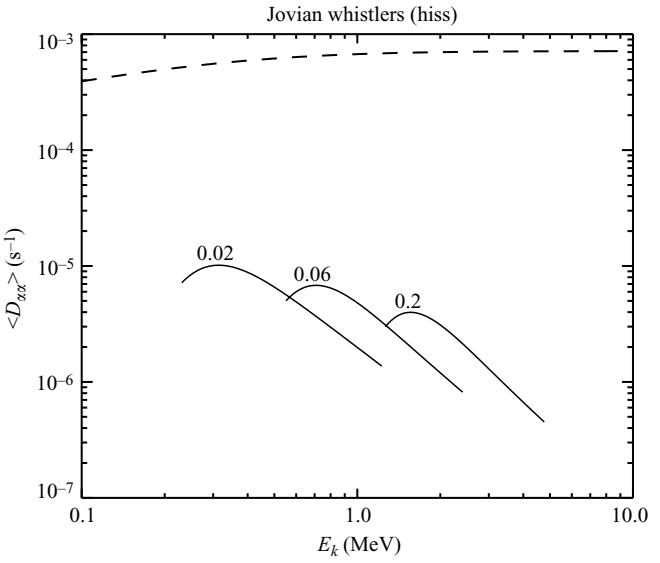


Figure 2. Average pitch-angle diffusion coefficient $\langle D_{\alpha\alpha} \rangle$ as a function of electron kinetic energy, for electrons in resonance with whistler-mode hiss in the Jovian magnetosphere near Io, at $L = 6$, for wave amplitude $\Delta B = 8$ pT and Gaussian frequency spectrum with centre $\omega_m = 0.01\,|\Omega_e|$ and semi-bandwidth $\delta\omega = 0.0081\,|\Omega_e|$; the parameter $\alpha^* = \Omega_e^2/\omega_{pe}^2$ labels the curves. The dashed curve represents the rate of strong pitch-angle diffusion (2.23).

ion composition, the intensity of geomagnetic activity and the location relative to the plasmapause, EMIC waves may be excited in three distinct bands below the hydrogen (H^+), helium (He^+) and oxygen (O^+) ion gyrofrequencies. During magnetic storms, relativistic (>1 MeV) electrons move in near circular orbits about the Earth and traverse the duskside plasmapause where EMIC waves are preferentially excited. Summers and Thorne (2003) demonstrated that pitch-angle scattering by EMIC waves can cause significant precipitation of relativistic electrons during different phases of a storm, under appropriate conditions. In the upper, middle and lower panels of Fig. 3, we show $\langle D_{\alpha\alpha} \rangle$ curves for electron scattering by H^+ band, He^+ band and O^+ band EMIC waves, respectively, in each case at $L = 4$, and for wave amplitude $\Delta B = 1$ nT. In each panel we set $\alpha^* = 10^{-3}$, 3×10^{-3}, 10^{-2} to represent the expected range of densities in the plasmasphere. It should be noted that the density drops from about $10^3\,cm^{-3}$ inside the plasmapause to about 1–10 cm^{-3} outside the plasmapause, at $L = 4$. Thus the plasma with which the EMIC waves interact just inside the plasmapause has a value of α^* that is orders of magnitude smaller than that which supports the whistler-mode chorus just outside the plasmapause, discussed in Sec. 3.1. To construct Fig. 3 we use parameter values for the ion composition and wavebands taken from Summers and Thorne (2003). The parameter values for H^+ band waves ($\eta_1 = 0.85$, $\eta_2 = 0.1$, $\eta_3 = 0.05$, $X_m = 0.6\epsilon$, $\delta X = 0.1\epsilon$) typify EMIC waves generated during the sudden commencement phase of a magnetic storm. From Fig. 3 (upper panel), we see that, even after bounce-averaging and drift-averaging, electron scattering rates should exceed that for strong diffusion. Scattering time scales of several hours for electrons in the energy range of a few hundred keV to a few MeV imply that H^+ band waves could account for the rapid depletion of the relativistic electron population in the

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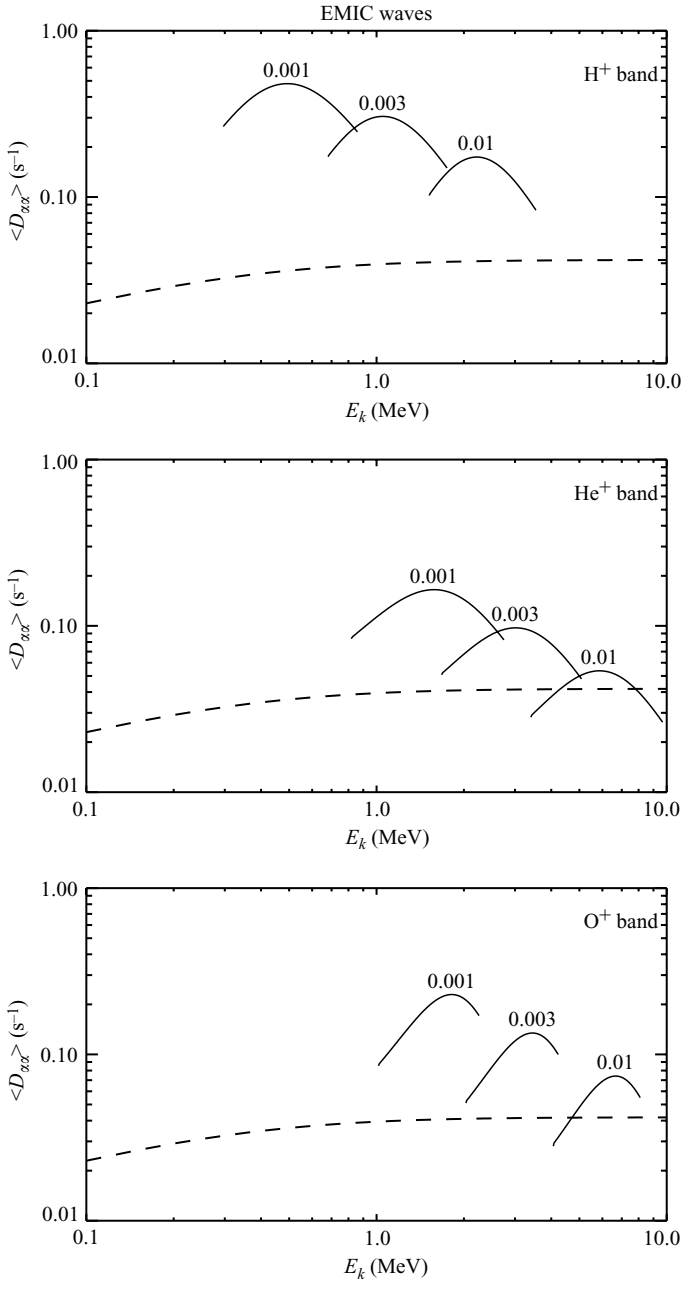


Figure 3. Average pitch-angle diffusion coefficient $\langle D_{\alpha\alpha} \rangle$ as a function of electron kinetic energy, for electrons in resonance with EMIC waves inside the Earth's plasmasphere, at $L = 4$, for wave amplitude $\Delta B = 1$ nT, in a multi-ion (H^+, He^+, O^+) plasma, for Gaussian frequency spectra (ion compositions and waveband parameters given in text); the parameter $\alpha^* = \Omega_e^2/\omega_{pe}^2$ labels the curves. The dashed curve represents the rate of strong pitch-angle diffusion (2.23).

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outer radiation belt during the initial phases of a storm. He^+ band parameters ($\eta_1 = 0.7,\ \eta_2 = 0.2,\ \eta_3 = 0.1,\ X_m = 3\epsilon/16,\ \delta X = \epsilon/32$) typify the main phase and recovery phase of a storm. Scattering rates for He^+ band waves (Fig. 3, middle panel) are less than those for H^+ band waves generally. However, rates are close to strong diffusion levels, and we deduce that electrons with energies exceeding a few MeV could be removed from the outer radiation belt over a time scale of several hours by He^+ band scattering. Fig. 3 (lower panel), which corresponds to O^+ band waves excited during the main phase of a storm ($\eta_1 = 0.6,\ \eta_2 = 0.2,\ \eta_3 = 0.2,\ X_m = 9\epsilon/160,\ \delta X = \epsilon/320$), shows that O^+ band waves are likely to scatter significantly only the multi-MeV electron population.

3.4. Ring current protons interacting with electromagnetic ion cyclotron waves

The Earth's ring current contains ions flowing toroidally around the Earth, centred in the equatorial plane, and extending approximately from $L = 3$ to $L = 8$; see Daglis and Kozyra (2002) for a review of outstanding problems of ring current dynamics. The ring current has a fundamental role in magnetospheric physics, particularly in magnetic storm research, primarily because changes in the ring current cause changes in the Earth's surface magnetic field. The processes governing the growth and decay of the ring current are not completely understood. Scattering of ring current protons into the loss cone by EMIC waves was suggested by Kennel and Petschek (1966) and Cornwall et al. (1970) as an important ring current loss mechanism. Jordanova et al. (1996) examined the effects of heavy ions on the pitch-angle diffusion coefficients for proton resonance with EMIC waves, and calculated the lifetimes of ring current protons. Soraas et al. (1999) reported observations of enhanced precipitation of several-hundred-keV protons in localized regions at mid-latitudes during the April 1981 geomagnetic storm, and suggested that scattering of protons into the loss cone was due to cyclotron resonance with EMIC waves. Direct evidence from wave and particle data acquired from the Dynamic Explorer (DE1) satellite shows that EMIC waves indeed scatter protons into the atmospheric loss cone (Erlandson and Ukhorskiy 2001). Walt and Voss (2001) reported losses of ring current protons due to strong diffusion scattering during the August 1998 storm, with proton trapping lifetimes of less than one hour. In Fig. 4 we show profiles of $\langle D_{\alpha\alpha} \rangle$ for protons in resonance with EMIC waves in a multi-ion plasma, for the hydrogen, helium and oxygen wavebands specified in Fig. 3. In each panel we set $L = 4$, $\Delta B = 1$ nT and $\alpha^* = 10^{-3}, 3 \times 10^{-3}, 10^{-2}$. From Fig. 4, we deduce that local scattering rates for protons in the energy range 1–50 keV typically far exceed the strong diffusion rate. Since EMIC waves are typically confined to a narrow spatial region in the duskside plasmapause, high energy electrons interact with these waves only for a small fraction (a few percent) of their orbits about the Earth. Nevertheless, after bounce-averaging has also been carried out, our calculations show that time scales for proton scattering can be of the order of several hours, corresponding to a significant decay of ring current protons.

4. Comments

From quasi-linear theory, we have developed an explicit expression for the average pitch-angle diffusion coefficient $\langle D_{\alpha\alpha} \rangle$, with respect to pitch-angle α, for particles in resonance with small-amplitude, parallel-propagating electromagnetic waves of general spectral density. The expression provides a practical estimate of local

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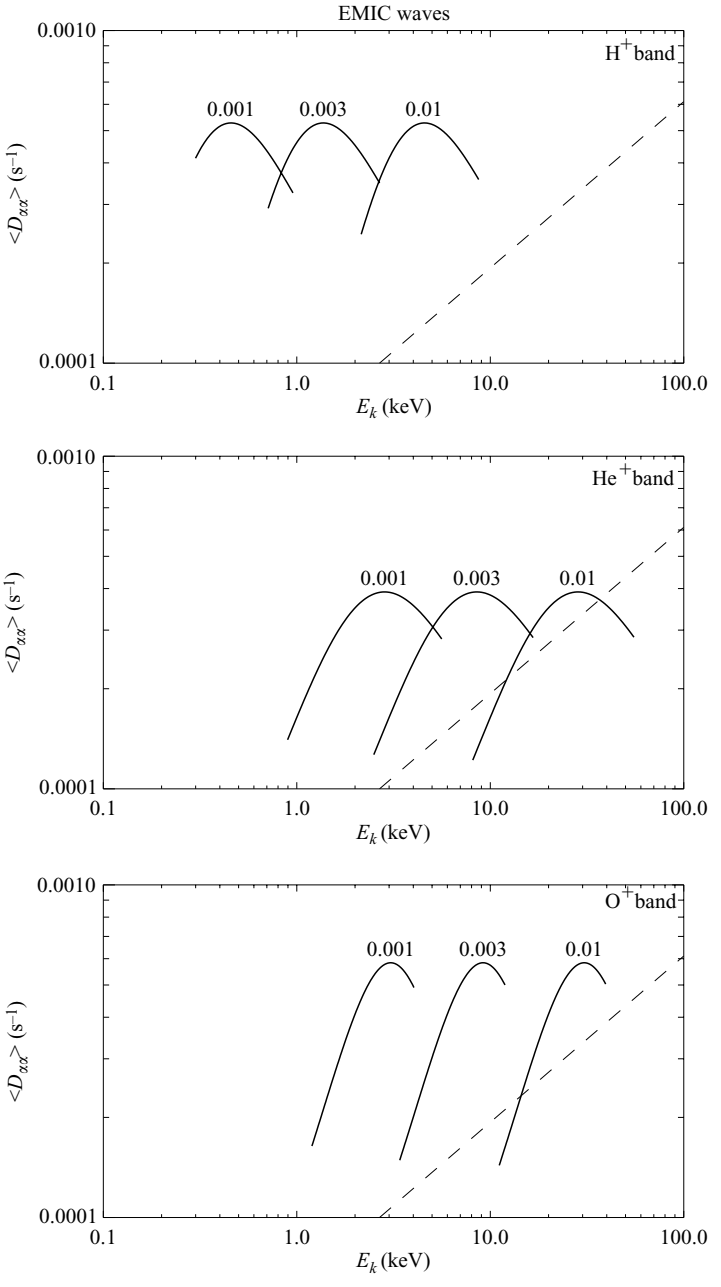


Figure 4. Average pitch-angle diffusion coefficient $\langle D_{\alpha\alpha}\rangle$ as a function of proton kinetic energy, for ring current protons in resonance with the EMIC wavebands specified for Fig. 3, in a multi-ion (H^+, He^+, O^+) plasma, at $L = 4$, with wave amplitude $\Delta B = 1$ nT; the parameter $\alpha^* = \Omega_e^2/\omega_{pe}^2$ labels the curves. The dashed curve represents the rate of strong pitch-angle diffusion (2.23).

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pitch-angle scattering rates as a function of particle kinetic energy, and is straightforward to evaluate numerically. To apply the theory developed in this paper to planetary magnetospheres, the local scattering rates need to be adjusted to take account of bounce-averaging and drift-averaging appropriate to a dipole magnetic field. It would be particularly useful to extend the results presented here to include oblique wave propagation. This would necessitate inclusion of the effects of higher-order harmonic scattering.

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