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Study of ion-acoustic solitary wave structures in multi-component plasma containing positive and negative ions and *q*-exponential distributed electron beam

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Abstract

Using reductive perturbation technique, small-amplitude ion-acoustic solitary wave has been investigated in multi-component dense plasma, in which an electron beam propagates along the general streaming motion. The electrons in plasma have the *q*-exponential distribution. The positive and negative ions follow a regular Maxwellian distribution. It has been found that the positive and negative ion densities as well as the beam concentration have significant effect on the formation and properties of solitary structures. The streaming velocities of corresponding particles also have pronounced effect on the features of the solitons.

Keywords: Ion-acoustic solitary wave; Positive and negative ions; *q*-Exponential distributed electron beam

1. INTRODUCTION

There has been a lot of study on the topic because of the fact that there lies the possibility of long-distance propagation of electron beams through the atmosphere and the potential use of intense beams for plasma heating in both scientific and industrial fields. The motion of charged particles is of more interest rather than the atomic processes involved in the creation of plasmas; therefore, we will limit our discussion to the effects on the motion of charged particles. The models used herein assume pre-formed plasmas and do not address the involved process of ionization by the beam. Mostly electron beam is injected in a plasma in the form of pulses. The ideal plasma responds immediately to a pulsed beam, providing complete neutralization of the beam space charge and current. In many cases, the electron beam can be considered as a plasma, if it satisfies the properties of a plasma-like quasi-neutrality, which happens when the ion or electron beam is space charge compensated, particularly under low-energy beam transport. In such case, one can calculate plasma density by classical formula. However, most of these beam electrons have velocity comparable to the speed of light, and therefore relativistic dynamics of the particle must be considered. In the early 80s, a scheme for accelerating electrons, employing a bunched relativistic electron beam in a cold plasma, was analyzed by Chen *et al*., (1984).

Under such circumstances, a lot of wave mode may be generated like electro-acoustic waves, electron plasma waves, dust-acoustic waves, ion-acoustic waves (IAWs), etc. In this paper, we discuss about the IAWs in such a multi-component plasma. In the physics of plasmas an IAW is a type of longitudinal oscillation of the ions as well as the electrons in plasma, similar to sound (acoustic) waves travelling in neutral gas. Since the waves propagate through positively charged ions, IAWs can interact with their electromagnetic fields, as well as by way of simple collisions. They commonly govern the evolution of mass density by way of particle density, for instance due to pressure gradients, on longer time scales, longer than the frequency corresponding to the relevant length scale. Such plasma wave modes can occur in an unmagnetized plasma or in a magnetized plasma parallel to the magnetic field. The kinetic energy of plasma particles does affect how they respond to beams. They can give rise to instabilities only if the ion and electron drift velocities are less than $\sqrt{2kT_e/m_e}$. The Maxwellian distribution is canonical distribution in the classical framework of Boltzmann–Gibbs (BG) statistical mechanics, applicable to thermal equilibrium condition. Under constrains of canonical

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ensemble, if we maximize the BG entropy, the Boltzmann exponential distribution of energy is derived. In the absence of any potential energy, this exponential distribution of energy gives the Maxwellian distribution of velocities. Hence, the total energy in this case is reduced to the kinetic energy.

Likewise the κ distribution is the canonical distribution in the non-extensive statistics and is applicable to both thermal equilibrium and non-equilibrium conditions. Here also we maximize the Tsallis entropy under similar constrains and a q-deformed exponential distribution of energy is obtained known as q-Maxwellian distribution velocity. Once we substitute the energies in terms of velocities, this is equivalent to a κ distribution in space and plasma physics, where κ and q-indices are related as $\kappa = 1/(q-1)$.

After a proper study of the literature available, we find that the study of propagation of a non-relativistic electron beam in a plasma in a strong magnetic field has been studied using electrostatic one-dimensional particle simulation models has been carried out by Okuda and Berchem (1987). Ion sound solitary waves with density depressions has been studied by Cairns *et al.*, (1995). Chattopadhyay *et al.*, (2000) has studied the propagation of ion-acoustic solitons in relativistic plasma containing cold ions and two temperature electrons, studies were also made with warm ions by Ghosh *et al.*, (2008). Studies on similar lines were also made by Mamun (1997, 1998); Bandopadhyay and Das (2000).

In ion-acoustic solitary waves with q-non-extensive electron velocity distribution had been studied by Tribeche *et al.*, (2010). They showed that for q-extensive index greater than unity, the lower limit of the acceptable Mach number is smaller than the Boltzmann counterpart, and for the range $-1 < q < 1$, the lower limit of the same is greater than the Boltzmann counterpart; this allows the possibility of the existence of subsonic ion-acoustic solitons. They showed the formation of rarefactive ion-acoustic solitary waves with relatively high Mach number.

Head-on collision of ion-acoustic solitary waves with q-non-extensive electron velocity distribution has been studied by Eslami *et al.*, (2011); by using KdV method, they showed that there is a phase shift in the colliding solitary waves. The importance of q-index in the formations and properties of ion-acoustic solitary waves has also been discussed. Roy *et al.*, (2012) used Sagdeev's potential technique and showed that the effect of ion temperature and q-parameter has significant role in large amplitude and width of solitary structures. Spatial distribution of electron number density for different values of q-parameter was studied by Lin *et al.*, (2015). Hafez *et al.*, (2016) studied the ion-acoustic solitary waves with non-extensive distributed electrons, positrons, and relativistic thermal ions in the presence of magnetic field. Such a plasma with relativistic heavy ion collision is reported in quark gluon plasma and astrophysical plasmas. In the presence of suprathermal electrons, electron-acoustic localized structure in a collisionless unmagnetized plasma containing cold and hot electrons where hot electrons have the κ distribution and ions are stationary have been investigated by Danehkar *et al.*, (2011). The hot electrons have a non-Maxwellian suprathermal distribution. They also showed that a change screening mechanism depends strongly on excess suprathermality. They used non-linear pseudo potential technique and found only negative polarity solitary waves to exist.

In Section 2, we provide the basic formulation for the particle dynamics. Section 3 deals with the linear and non-linear analysis. The next section discusses the result and finally we conclude with some remarks on further study.

2. BASIC FORMULATION

We consider non-linear propagation of IAWs in a plasma consisting ions (positive and negative), electron beam and q-exponential distributed electrons. The q-exponential distribution is a distribution function used in probability theory that is derived from the maximization of the Tsallis entropy under constraint conditions. The q-exponential is a generalization of the exponential distribution in a similar way that Tsallis entropy is a generalization of the standard Shannon entropy. In presence of an electron beam, the usual plasma wave modes, for example, ion-acoustic, ion-cyclotron, etc., are modified because of interaction of plasma waves with electron beam component. Accordingly, relativistic form of Euler's equation in hydrodynamics is incorporated. Under these conditions, the non-linear dynamics of the IAW in this plasma governed by the following equations:

$$\frac{\partial(\gamma_j n_j)}{\partial t} + \frac{\partial(\gamma_j n_j u_j)}{\partial x} = 0, \tag{1}$$

$$m_p\left[\frac{\partial}{\partial t} + u_p\frac{\partial}{\partial x}\right](\gamma_p u_p) = -Z_p e\frac{\partial \phi}{\partial x} - \frac{1}{n_p}\frac{\partial p_p}{\partial x}, \tag{2}$$

$$m_n\left[\frac{\partial}{\partial t} + u_n\frac{\partial}{\partial x}\right](\gamma_n u_n) = Z_n e\frac{\partial \phi}{\partial x} - \frac{1}{n_n}\frac{\partial p_n}{\partial x}, \tag{3}$$

$$m_b\left[\frac{\partial}{\partial t} + u_b\frac{\partial}{\partial x}\right](\gamma_b u_b) = e\frac{\partial \phi}{\partial x}. \tag{4}$$

The system closed by Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e(n_p Z_p - n_n Z_n - n_e - n_b). \tag{5}$$

The first equation is fluid continuity equation of j-species particles ($j = n, p, b$, respectively, stands for negative ions, positive ions, beam). Equations (2–4) are the equations of motion of positive ions, negative ions, and beam, respectively. Here m_p, m_n, and m_b are mass of ions (positive, negative), and beam; u_j is the fluid velocity of particles in this plasma with relativistic factor $\gamma_j = 1/\sqrt{1 - u_j^2/c^2} \approx (1 + u_j^2/2c^2)$,

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Z is the ion charge number, p_p and p_n are the ion pressure where $p_p = n_p kT_p$ and $p_n = n_n kT_n$ (k = Boltzmann constant), ϕ is the electrostatic potential, and e is the magnitude of electron charge.

The electron number density is assumed to be q-exponential distributed, as

$$n_e = [1 + (q-1)\phi]^{(q+1)/2(q-1)}, \tag{6}$$

where q is the exponent associated with the electron distribution. The positive and negative ions follow the standard Maxwellian distribution, respectively.

3. LINEAR AND NON-LINEAR ANALYSIS

Equations (1)–(5) may be presented in a reduced form, for convenience. For this we have scaled the time t and x by $\omega_{pj}^{-1} = \left(m_j/4\pi n_{j0} Z^2 e^2\right)^{1/2}$ and the Debye length $\lambda_{De} = (kT_e/4\pi n_{j0} Ze^2)^{1/2}$. Again we scaled the number densities of ions and beam (n_j) by their respective unperturbed densities n_{j0} and fluid velocities u_j by the ion-sound speed $c_s = (ZkT_e/m_j)^{1/2}$, the ion pressure p_j by $p_{j0} = n_{j0}\, kT_j$ and the electrostatic potential ϕ by kT_e/e.

Thus, Eqs (1)–(5) can write into the normalized form:

$$\frac{\partial(\gamma_j n_j)}{\partial t} + \frac{\partial(\gamma_j n_j u_j)}{\partial x} = 0, \tag{7}$$

$$\left[\frac{\partial}{\partial t} + u_p \frac{\partial}{\partial x}\right](\gamma_p u_p) = -\mu_p \frac{\partial \phi}{\partial x} - A\alpha n_p^{\alpha-2} \frac{\partial n_p}{\partial x}, \tag{8}$$

$$\left[\frac{\partial}{\partial t} + u_n \frac{\partial}{\partial x}\right](\gamma_n u_n) = \mu_n \frac{\partial \phi}{\partial x} - B\beta n_n^{\beta-2} \frac{\partial n_n}{\partial x}, \tag{9}$$

$$\left[\frac{\partial}{\partial t} + u_b \frac{\partial}{\partial x}\right](\gamma_b u_b) = \frac{\partial \phi}{\partial x}, \tag{10}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + \frac{n_b}{\rho} + \frac{Z_n n_n}{\sigma} - \frac{Z_p n_p}{\delta}, \tag{11}$$

where $A\alpha = \gamma_p T_p / Z_p T_e$ and $B\beta = \gamma_n T_n / Z_n T_e$, $\mu_j = m_e/m_j$ are the fractional ion to electron temperature and ratio of electron mass to ion mass, respectively. Here A, B, α, β are constants to suitably fit the pressure laws (in the general form $p = An^{\alpha}$); ρ, σ, δ are fractions deciding relative particle concentration. Assuming space–time dependence of the field variables as exp $i(kx - \omega t)$, we get dispersion relation as

$$\begin{aligned} k^2 = &-\left\{\left(\frac{q+1}{2}\right) + \frac{(3-q)(q+1)}{2}\phi^0\right\} \\ &+ \frac{\mu_n Z_n}{\sigma} \frac{\left[2u_n^{(0)} k(E-1) + kEu_n^{(0)} - 2\omega(E-1)\right]}{E^2 u_n^{(0)}\left(\omega - ku_n^{(0)}\right) - B\beta\left[2u_n^{(0)} k(E-1) + kEu_n^{(0)} - 2\omega(E-1)\right]} \\ &+ \frac{\mu_p Z_p}{\delta} \frac{\left[2u_p^{(0)} k(D-1) + kDu_p^{(0)} - 2\omega(D-1)\right]}{D^2 u_p^{(0)}\left(\omega - ku_p^{(0)}\right) - A\alpha\left[2u_p^{(0)} k(D-1) + kDu_p^{(0)} - 2\omega(D-1)\right]} \\ &+ \frac{1}{\rho} \frac{\left[2u_b^{(0)} k(F-1) - kFu_b^{(0)} - 2\omega(F-1)\right]}{F^2 u_b^{(0)}\left(\omega - ku_b^{(0)}\right)}, \end{aligned} \tag{12}$$

where

$$\left(1 + \frac{u_p^{(0)^2}}{2c^2}\right) = D, \tag{13}$$

$$\left(1 + \frac{u_n^{(0)^2}}{2c^2}\right) = E, \tag{14}$$

$$\left(1 + \frac{u_b^{(0)^2}}{2c^2}\right) = F. \tag{15}$$

The derivation of the dispersion relation is given in the Appendix.

From above expression, it is clear that phase velocity is affected by ρ, σ, δ, and q.

The propagation equation of the IAWs may investigate by introducing new coordinates ξ and τ defined as:

$$\xi = \varepsilon^{1/2}(x - Mt), \tau = \varepsilon^{3/2} t. \tag{16}$$

Here, M is Mach number and ε is a small parameter measuring the weakness of dispersion and non-linearity.

Now perturbation expansion for these variables:

$$\begin{pmatrix} u_p \\ u_n \\ u_b \end{pmatrix} = \begin{pmatrix} u_p^0 \\ u_n^0 \\ u_b^0 \end{pmatrix} + \varepsilon \begin{pmatrix} u_p^1 \\ u_n^1 \\ u_b^1 \end{pmatrix} + \varepsilon^2 \begin{pmatrix} u_p^2 \\ u_n^2 \\ u_b^2 \end{pmatrix} + \cdots, \tag{17}$$

$$\begin{pmatrix} n_p \\ n_n \\ n_b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \varepsilon \begin{pmatrix} n_p^0 \\ n_n^0 \\ n_b^0 \end{pmatrix} + \varepsilon^2 \begin{pmatrix} n_p^1 \\ n_n^1 \\ n_b^1 \end{pmatrix} + \cdots, \tag{18}$$

$$\phi = \phi^0 + \varepsilon^1 \phi^1 + \varepsilon^2 \phi^2 + \varepsilon^3 \phi^3 + \cdots. \tag{19}$$

By using Eq. (16) and (17–19), in the set of Eq. (7–11) collecting the terms of lowest order ($\varepsilon^{3/2}$), we obtain

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$$n_p^{(1)} = G \cdot u_p^{(1)}, n_n^{(1)} = H \cdot u_n^{(1)}, n_b^{(1)} = I \cdot u_b^{(1)}, n_p^{(1)} = -\frac{\mu_p}{J} \cdot \phi^{(1)}, n_n^{(1)} = -\frac{\mu_n}{L} \cdot \phi^{(1)}, n_b^{(1)} = -\frac{1}{N} \cdot \phi^{(1)} \quad (20)$$

where

$$G = \frac{\left(u_p^1 - M\right)u_p^0/c^2 + \left(1 + u_p^{(0)2}/2c^2\right)}{\left(u_p^0 - M\right)\left(1 + u_p^{(0)2}/2c^2\right)},$$

$$H = \frac{\left(u_n^1 - M\right)u_n^0/c^2 + \left(1 + u_n^{(0)2}/2c^2\right)}{\left(u_n^0 - M\right)\left(1 + u_n^{(0)2}/2c^2\right)},$$

$$I = \frac{\left(u_b^1 - M\right)u_b^0/c^2 + \left(1 + u_b^{(0)2}/2c^2\right)}{\left(u_b^0 - M\right)\left(1 + u_b^{(0)2}/2c^2\right)},$$

$$J = \frac{\left(u_p^0 - M\right)\left\{\left(1 + u_p^{(0)2}/2c^2\right) + u_p^{(0)2}/c^2\right\}}{G} + A\alpha,$$

$$L = \frac{\left(u_n^0 - M\right)\left\{\left(1 + u_n^{(0)2}/2c^2\right) + u_n^{(0)2}/c^2\right\}}{H} + B\beta,$$

$$N = \frac{\left(u_b^0 - M\right)\left\{\left(1 + u_b^{(0)2}/2c^2\right) + u_b^{(0)2}/c^2\right\}}{I}.$$

Replacing the different quantities in first order in Eq. (20) into a single variable ϕ^1 after manipulation, we derive the non-linear equation for propagation of ion-acoustic solitary wave in the aforesaid plasma system, as described in the following:

$$\frac{\partial \phi^1}{\partial \tau} + P\phi^1 \frac{\partial \phi^1}{\partial \xi} + Q\frac{\partial^3 \phi^1}{\partial \xi^3} = 0. \quad (21)$$

Equation (A16) is a well-known KdV equation. Coefficients P and Q are defined as:

$$P = 2\left\{\frac{X_p Z_p}{\delta} - \frac{X_n Z_n}{\sigma} - \frac{X_b}{\rho} - \frac{(3-q)(q+1)}{2}\right\} \text{ and}$$

$$Q = \left\{\frac{Y_p Z_p}{\delta} - \frac{Y_n Z_n}{\sigma} - \frac{Y_b}{\rho}\right\}.$$

Here

$$X_p = \left[\frac{A\alpha(\alpha-1)\mu_p}{2J^2} - \left(u_p^0 - M\right)\frac{3\mu_p u_p^0}{2c^2J^2G^2} - \frac{\mu_p}{2J^2G^2}\left(1 + \frac{3u_p^{0^2}}{2c^2}\right)\right] - A\alpha X'_p,$$

$$X_n = \left[\frac{B\beta(\beta-1)\mu_n}{2L^2} - \left(u_n^0 - M\right)\frac{3\mu_n u_n^0}{2c^2L^2H^2} - \frac{\mu_n}{2L^2H^2}\left(1 + \frac{3u_p^{0^2}}{2c^2}\right)\right] - B\beta X'_n,$$

$$X_b = \left[-\left(u_b^0 - M\right)\frac{3u_p^0}{2c^2N^2I^2} - \frac{1}{2N^2I^2}\left(1 + \frac{3u_b^{0^2}}{2c^2}\right)\right],$$

$$Y_b = \left[\frac{(u_b^0 - M)(1 + 3u_b^{0^2}/2c^2)Y'_b}{I} - \left(1 + \frac{3u_b^{0^2}}{2c^2}\right)\frac{1}{N}\right],$$

$$Y_p = \left[\frac{\left(u_p^0 - M\right)\left(1 + 3u_p^{0^2}/2c^2\right)Y'_p}{G} - \left(1 + \frac{3u_p^{0^2}}{2c^2}\right)\frac{\mu_p}{J}\right],$$

$$Y_n = \left[\frac{\left(u_n^0 - M\right)\left(1 + 3u_n^{0^2}/2c^2\right)Y'_n}{H} - \left(1 + \frac{3u_p^{0^2}}{2c^2}\right)\frac{\mu_n}{L}\right].$$

Here the quantities X'_p, X'_n, Y'_p, and Y'_n are

$$X'_p = \frac{\left(-\mu_p^2\left(1 + u_p^{0^2}/2c^2\right)/J^2G\right) - \left(\mu_p^2\left(u_p^0 - M\right)/2c^2J^2G^2\right) - \left(\mu_p^2 u_p^0\left(2u_p^0 - M\right)/2c^2J^2G\right)}{\left(u_p^0 - M\right)\left(1 + u_p^{0^2}/2c^2\right)},$$

$$X'_n = \frac{\left(-\mu_n^2\left(1 + u_n^{0^2}/2c^2\right)/L^2H\right) - \left(\mu_n^2(u_n^0 - M)/2c^2L^2H^2\right) - \left(\mu_n^2 u_n^0(2u_n^0 - M)/2c^2L^2H\right)}{\left(u_n^0 - M\right)\left(1 + u_n^{0^2}/2c^2\right)},$$

$$Y'_p = \frac{\left(2u_p^0\mu_p/2c^2JG\right) + \mu_p\left(\left(1 + u_p^{0^2}/2c^2\right)/J\right)}{\left(u_p^0 - M\right)\left(1 + u_p^{0^2}/2c^2\right)},$$

$$Y'_b = \frac{\left(-\left(2u_n^0/2c^2NI\right)\right) - \left(\left(1 + u_b^{0^2}/2c^2\right)/N\right)}{(u_b^0 - M)\left(1 + u_b^{0^2}/2c^2\right)},$$

$$Y'_n = \frac{\left(-\left(2u_n^0\mu_n/2c^2LH\right)\right) - \mu_n\left(\left(1 + u_n^{0^2}/2c^2/L\right)\right)}{\left(u_n^0 - M\right)\left(1 + u_n^{0^2}/2c^2\right)}.$$

In order to derive the solution of Eq. (21), we use the transformation $\eta = \xi - U\tau$ and using the boundary conditions: $\phi \to 0$, $d^2\phi/d\xi^2 \to 0$, $\xi \to \infty$, and integrating and simplifying Eq. (21), we obtain the following solution:

$$\phi = \phi_m \sec h^2\left(\eta/\Delta\right) \quad (22)$$

where $\phi_m = 3M/A$ and $\Delta = \sqrt{4B/M}$ are the amplitude and width of the solitary waves accordingly.

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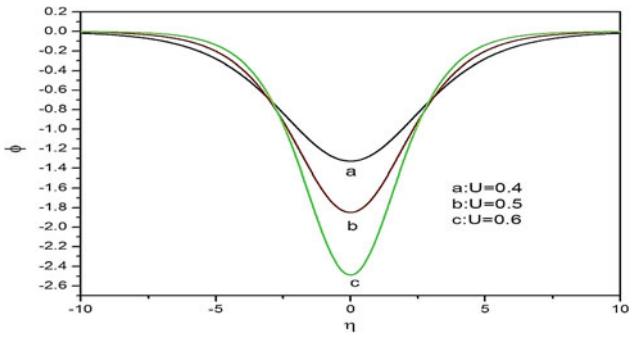


Fig. 1. Solitary wave structure for different U (Mach number).

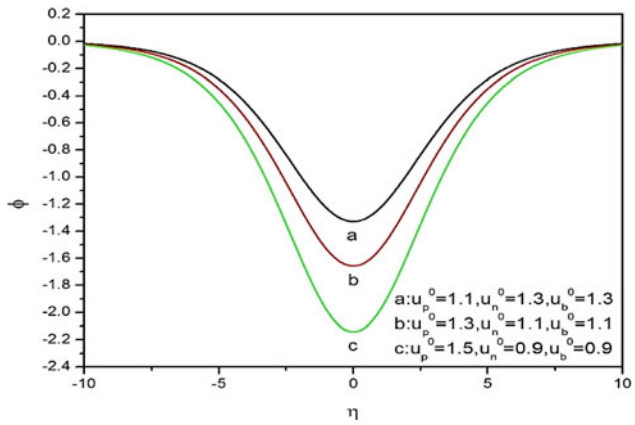


Fig. 3. Solitary wave structure for different streaming velocity of positive ion (u_b^0).

4. RESULTS

The numerical study of the above problem has been made by feeding the solutions with values of parameters that aptly fit the situation. It has been analytically found and graphically demonstrated that the ion-acoustic solitary structures become deeper with increasing Mach number (U) as shown in Figure 1. Figure 2 shows that the solitary wave structure for different streaming velocity for the beam. It has been found that the solitons become narrower and shorter with increasing beam-streaming velocity. However, the waves grow deeper when the positron streaming velocity increases (Fig. 3).

It has been found that for negative values of q, the amplitude of the rarefactive solitons and its width increases with the increase in q value (Fig. 4). This is in accordance with the findings of Tribeche *et al*., (2010). Similar findings had been reported by Danehkar *et al*., (2011) in which the IAWs were found behave likewise using Sagdeevs' method. Hafez *et al*., (2016) analyze the formulation and characteristics of compressive solitary structures with relativistic ions; however, in our work, we considered the electron to acquire relativistic speed, so we found the formulation of rarefactive solitary structures. According to Hafez, for weakly relativistic on ion-acoustic solitary wave, the depth and width of solitary profile decrease with decrease in the value of q which is similar to our findings.

The results can be thus interpreted as the wave advances, the initially smooth perturbation becomes steeper and oscillations appear almost negligible; their amplitude increases with increasing distance traversed by the wave, and the width of the wave front, starting with a certain instant, remains constant. The solitary structures are rarefactive, that is, for ions there will be a fall in density of ions. This may be put forward like this that as the positive ion velocity increases compared with negative ion and electron beam velocity, the positive ions tend to come close and due to Columbic repulsion move apart thus causing a fall in density of positive ions.

5. REMARKS

Thermal electron beam has many applications in both scientific and industrial sectors. It is applied for remelting, welding, surface layer treatment, perforation, surface structuring, micro treatment, evaporation, etc. Thus, the quality and property of electron beam is a determining factor in the process. The findings of this research paper may be quite helpful in

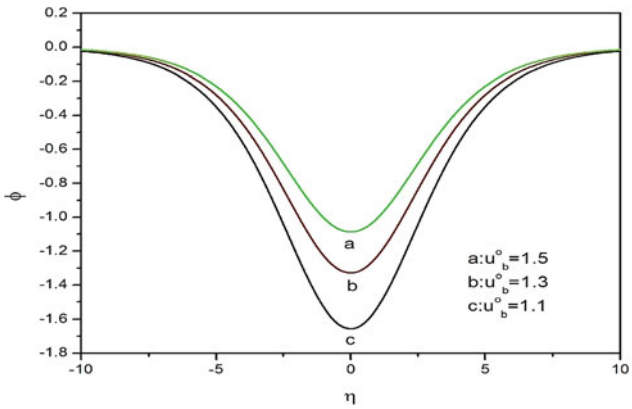


Fig. 2. Solitary wave structure for different streaming velocity for the beam (u_b^0).

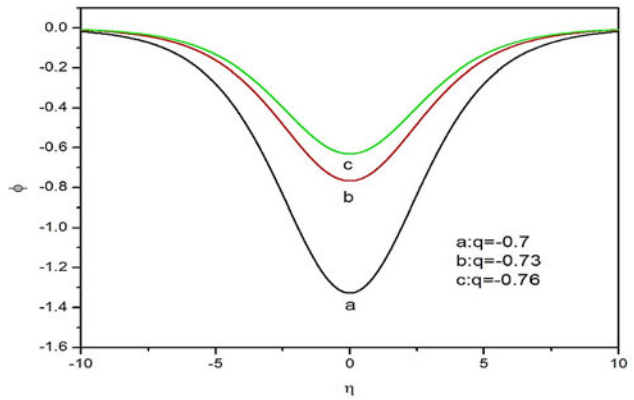


Fig. 4. Solitary wave structure for different values of non-extensive parameter (q).

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controlling such processes and finding the unwanted features that might creep in.

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APPENDIX

The normalized basic equations are

$$\frac{\partial(\gamma_j n_j)}{\partial t}+\frac{\partial(\gamma_j n_j u_j)}{\partial x}=0, \tag{A1}$$

$$\left[\frac{\partial}{\partial t}+u_p\frac{\partial}{\partial x}\right](\gamma_p u_p)=-\mu_p\frac{\partial\phi}{\partial x}-A\alpha n_p^{\alpha-2}\frac{\partial n_p}{\partial x}, \tag{A2}$$

$$\left[\frac{\partial}{\partial t}+u_n\frac{\partial}{\partial x}\right](\gamma_n u_n)=\mu_n\frac{\partial\phi}{\partial x}-B\beta n_n^{\beta-2}\frac{\partial n_n}{\partial x}, \tag{A3}$$

$$\left[\frac{\partial}{\partial t}+u_b\frac{\partial}{\partial x}\right](\gamma_b u_b)=\frac{\partial\phi}{\partial x}, \tag{A4}$$

$$\frac{\partial^2\phi}{\partial x^2}=n_e+\frac{n_b}{\rho}+\frac{Z_n n_n}{\sigma}-\frac{Z_p n_p}{\delta}. \tag{A5}$$

The perturbation expansion of the field variables are given by

$$\begin{pmatrix}u_p\\u_n\\u_b\\\phi\end{pmatrix}=\begin{pmatrix}u_p^0\\u_n^0\\u_b^0\\\phi^0\end{pmatrix}+\varepsilon\begin{pmatrix}u_p^{(1)}\\u_n^{(1)}\\u_b^{(1)}\\\phi^{(1)}\end{pmatrix}+\varepsilon^2\begin{pmatrix}u_p^{(2)}\\u_n^{(2)}\\u_b^{(2)}\\\phi^{(2)}\end{pmatrix}+\cdots, \tag{A6}$$

$$\begin{pmatrix}n_p\\n_n\\n_b\end{pmatrix}=\begin{pmatrix}1\\1\\1\end{pmatrix}+\varepsilon\begin{pmatrix}n_p^{(1)}\\n_n^{(1)}\\n_b^{(1)}\end{pmatrix}+\varepsilon^2\begin{pmatrix}n_p^{(2)}\\n_n^{(2)}\\n_b^{(2)}\end{pmatrix}+\cdots. \tag{A7}$$

The electrons have the *q*-extensive distributions is given by,

$$\begin{aligned}n_e&=[1+(q-1)\phi]^{(q+1)/2(q-1)}\\&=\left[\left\{1+\left(\frac{q+1}{2}\right)\phi^0+\frac{(3-q)(q+1)}{4}\phi^{0^2}\right\}\right.\\&\quad+\varepsilon\left\{\left(\frac{q+1}{2}\right)\phi^{(1)}+\frac{(3-q)(q+1)}{2}\phi^0\phi^{(1)}\right\}\\&\quad+\varepsilon^2\left\{\left(\frac{q+1}{2}\right)\phi^{(2)}+\frac{(3-q)(q+1)}{2}\phi^0\phi^{(2)}\right.\\&\quad\left.\left.+\frac{(3-q)(q+1)}{4}\phi^{(1)^2}\right\}\right].\end{aligned} \tag{A8}$$

Assuming space–time dependence of the field variables as $e^{i(kx-\omega t)}$ we get,

$$\frac{\partial}{\partial x}\equiv ik, \tag{A9}$$

$$\frac{\partial}{\partial t}\equiv -i\omega. \tag{A10}$$

Now,

$$n_p^{\alpha-2}=1+\varepsilon(\alpha-2)n_p^{(1)}+\varepsilon^2(\alpha-2)n_p^{(2)}+\cdots, \tag{A11}$$

$$n_n^{\beta-2}=1+\varepsilon(\beta-2)n_n^{(1)}+\varepsilon^2(\beta-2)n_n^{(2)}+\cdots. \tag{A12}$$

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From momentum equations, we get by linearizing in powers of ε,

$$-i\omega\gamma_p u_p^{(1)} + iku_p^0\gamma_p u_p^{(1)} = -\mu_p ik\phi^{(1)} - A\alpha ikn_p^{(1)},$$

or

$$k[\gamma_p u_p^{(0)} u_p^{(1)} + \mu_p\phi^{(1)} + A\alpha n_p^{(1)}] = \omega\gamma_p u_p^{(1)}. \tag{A13}$$

Similarly,

$$k[\gamma_n u_n^{(0)} u_n^{(1)} - \mu_n\phi^{(1)} + B\beta n_n^{(1)}] = \omega\gamma_n u_n^{(1)}, \tag{A14}$$

and

$$k[\gamma_b u_b^{(0)} u_b^{(1)} - \phi^{(1)}] = \omega\gamma_n u_n^{(1)}. \tag{A15}$$

From continuity equation of positron by linearizing ε we get,

$$i\omega\left\{n_p^{(1)}\left(1+\frac{u_p^{(0)^2}}{2c^2}\right)+\frac{2u_p^{(0)}u_p^{(1)}}{2c^2}\right\} = ik\left\{2\frac{u_p^{(0)^2}u_p^{(1)}}{2c^2}+n_p^{(1)}u_p^{(0)}\left(1+\frac{u_p^{(0)^2}}{2c^2}\right)+u_p^{(1)}\left(1+\frac{u_p^{(0)^2}}{2c^2}\right)\right\},$$

or

$$n_p^{(1)}A'\left(k-\omega u_p^{(0)}\right) = \left\{\frac{-2\omega(A'-1)}{2c^2u_p^{(0)}}+2k(A'-1)+kA'\right\}u_p^{(1)}, \tag{A16}$$

where

$$\left(1+\frac{u_p^{(0)^2}}{2c^2}\right) = A' = D. \tag{A17}$$

Therefore, from Eqs (A13) and (A16) we get,

$$u_p^{(1)} = \frac{Du_p^{(0)}\left(\omega-ku_p^{(0)}\right)}{\left[2u_p^{(0)}k(D-1)+kDu_p^{(0)}-2\omega(D-1)\right]}n_p^{(1)} \tag{A18}$$

Putting this value on the result of momentum equation we get,

Hence,

$$-\frac{Z_pn_p}{\delta} = -\frac{\mu_pZ_p}{\delta}\frac{\left[2u_p^{(0)}k(D-1)+kDu_p^{(0)}-2\omega(D-1)\right]}{D^2u_p^{(0)}\left(\omega-ku_p^{(0)}\right)-A\alpha\left[2u_p^{(0)}k(D-1)+kDu_p^{(0)}-2\omega(D-1)\right]}\phi^{(1)}. \tag{A19}$$

Similarly,

$$\frac{Z_nn_n}{\sigma} = -\frac{\mu_nZ_n}{\sigma}\frac{\left[2u_n^{(0)}k(E-1)+kEu_n^{(0)}-2\omega(E-1)\right]}{E^2u_n^{(0)}\left(\omega-ku_n^{(0)}\right)-B\beta\left[2u_n^{(0)}k(E-1)+kEu_n^{(0)}-2\omega(E-1)\right]}\phi^{(1)}, \tag{A20}$$

where

$$\left(1+\frac{u_n^{(0)^2}}{2c^2}\right) = E, \tag{A21}$$

$$\frac{n_b}{\rho} = -\frac{1}{\rho}\frac{\left[2u_b^{(0)}k(F-1)-kFu_b^{(0)}-2\omega(F-1)\right]}{F^2u_b^{(0)}\left(\omega-ku_b^{(0)}\right)}\phi^{(1)}, \tag{A22}$$

where

$$\left(1+\frac{u_b^{(0)^2}}{2c^2}\right) = F. \tag{A23}$$

Putting this value in Poisson's equation, we will have

$$-k^2\phi^{(1)} = n_e^{(1)}+\frac{n_b^{(1)}}{\rho}+\frac{Z_nn_n^{(1)}}{\sigma}-\frac{Z_pn_p^{(1)}}{\delta}$$

$$-k^2\phi^{(1)} = \left\{\left(\frac{q+1}{2}\right)\phi^{(1)}+\frac{(3-q)(q+1)}{2}\phi^0\phi^{(1)}\right\}-\frac{\mu_nZ_n}{\sigma}\frac{\left[2u_n^{(0)}k(E-1)+kEu_n^{(0)}-2\omega(E-1)\right]}{E^2u_n^{(0)}\left(\omega-ku_n^{(0)}\right)-B\beta\left[2u_n^{(0)}k(E-1)+kEu_n^{(0)}-2\omega(E-1)\right]}\phi^{(1)}$$
$$-\frac{\mu_pZ_p}{\delta}\frac{\left[2u_p^{(0)}k(D-1)+kDu_p^{(0)}-2\omega(D-1)\right]}{D^2u_p^{(0)}\left(\omega-ku_p^{(0)}\right)-A\alpha\left[2u_p^{(0)}k(D-1)+kDu_p^{(0)}-2\omega(D-1)\right]}\phi^{(1)}-\frac{1}{\rho}\frac{\left[2u_b^{(0)}k(F-1)-kFu_b^{(0)}-2\omega(F-1)\right]}{F^2u_b^{(0)}\left(\omega-ku_b^{(0)}\right)}\phi^{(1)}.$$

$$k^2 = -\left\{\left(\frac{q+1}{2}\right)+\frac{(3-q)(q+1)}{2}\phi^0\right\}+\frac{\mu_nZ_n}{\sigma}\frac{\left[2u_n^{(0)}k(E-1)+kEu_n^{(0)}-2\omega(E-1)\right]}{E^2u_n^{(0)}\left(\omega-ku_n^{(0)}\right)-B\beta\left[2u_n^{(0)}k(E-1)+kEu_n^{(0)}-2\omega(E-1)\right]}$$
$$+\frac{\mu_pZ_p}{\delta}\frac{\left[2u_p^{(0)}k(D-1)+kDu_p^{(0)}-2\omega(D-1)\right]}{D^2u_p^{(0)}\left(\omega-ku_p^{(0)}\right)-A\alpha\left[2u_p^{(0)}k(D-1)+kDu_p^{(0)}-2\omega(D-1)\right]}+\frac{1}{\rho}\frac{\left[2u_b^{(0)}k(F-1)-kFu_b^{(0)}-2\omega(F-1)\right]}{F^2u_b^{(0)}\left(\omega-ku_b^{(0)}\right)}.$$

This is the linear dispersion relation.

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