REFLECTION OF PLANE WAVES IN A ROTATING TEMPERATURE-DEPENDENT THERMOELASTIC SOLID WITH DIFFUSION

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ABSTRACT

The governing equations of a model of rotating generalized thermoelastic diffusion in an isotropic medium with temperature-dependent mechanical properties are formulated in context of Lord-Shulman theory of generalized thermoelasticity. The modulus of elasticity is taken as a linear function of reference temperature. The solution of the governing equations indicates the existence of four coupled plane waves in *x*-*z* plane. The reflection of plane waves from the free surface of a rotating temperature-dependent thermoelastic solid half-space with diffusion is considered. The required boundary conditions are satisfied by the appropriate potentials for incident and reflected waves in the half-space to obtain a system of four non-homogeneous equations in the reflection coefficients. The expressions for energy ratios of the reflected waves are also obtained. The reflection coefficients and energy ratios are found to depend upon the angle of incidence, reference temperature, thermodiffusion and rotation parameters. Aluminum material is modeled as the half-space to compute the absolute values of the reflection coefficients and the energy ratios. Effects of temperature dependence and rotation parameters on the reflection coefficients and energy ratios are shown graphically for a certain range of the angle of incidence of the incident plane wave.

Keywords: Generalized thermoelasticity, Plane waves, Reflection, Rotation, Diffusion.

1. INTRODUCTION

The classical dynamical coupled theory of thermoelasticity was extended to generalized thermoelasticity theories by Lord and Shulman [1] and Green and Lindsay [2]. These theories consider heat propagation as a wave phenomenon rather than a diffusion phenomenon and predict a finite speed of heat propagation. Wave propagation in thermoelasticity has many applications in various engineering fields. The reflection and refraction problems in generalized thermoelasticity are discussed by many authors [3-7].

Thermodiffusion in an elastic solid is due to coupling of the fields of temperature, mass diffusion and that of strain. Heat and mass exchange with the environment during thermodiffusion in an elastic solid. Following the coupled thermoelastic model of Nowacki [8-10], Sherief *et al*. [11] presented a theory of generalized thermoelastic diffusion, which allows the finite speed of propagation of waves. Singh [12,13] studied the wave propagation in thermoelastic solid with diffusion in context of Lord-Shulman and Green-Lindsay theories. Abo-Dahab and Singh [14] studied the effects of magnetic field on wave propagation in thermoelastic media with diffusion. Aouadi [15-17] studied various other kinds of problems with thermoelastic diffusion.

The problems on waves and vibration become more significant in the field of seismology, when we study the problems with additional parameters, *e.g.* viscosity, temperature, porosity, micro-rotation, anisotropy, diffusion, rotation, *etc*. Recently, Othman and Kumar [18] studied the influence of temperature dependence on the reflection of plane waves in magneto-thermoelasticity. The study of wave propagation in a rotating temperature-dependent thermoelastic solid medium with diffusion has not been attempted so far and it will be an interesting mechanical problem with its applications in determining the stability of the medium. In the present paper, the governing equations of temperature-dependent thermoelasticity with rotation and diffusion

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are solved to show the existence of four plane waves in the medium. For incident plane wave, the reflection coefficients and energy ratios of the reflected waves are obtained at free surface of the half-space of medium. The influences of rotation and temperature dependence on the reflections coefficients and energy ratios of reflected waves are studied numerically.

2. FORMULATION OF THE PROBLEM

We consider a thermoelastic half space rotating uniformly with an angular velocity $\mathbf{\Omega} = \Omega \vec{n}$ where $\vec{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion has two additional terms, centripetal acceleration $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})$ due to time varying motion only and Coriolis acceleration $2\mathbf{\Omega} \times \dot{\mathbf{u}}$, where $\mathbf{u}$ is the dynamic displacement vector, which are absent in the non-rotating media. Following Singh [12], the governing equations for an isotropic, homogenous elastic solid with generalized thermo-diffusion at constant temperature T_0 in the absence of body forces are

(i) The constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta_1 \Theta - \beta_2 C] , \tag{1}$$

$$\mathrm{P}^* = -\beta_2 e_{kk} + bC - a\Theta , \tag{2}$$

(ii) The equation of motion

$$\begin{aligned} &\mu u_{i,jj} + (\lambda+\mu) u_{j,ij} - \beta_1 \Theta_{,i} - \beta_2 C_{,i} \\ &= \rho\left[\ddot{u}_i + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}) + 2\mathbf{\Omega} \times \dot{\mathbf{u}}\right] , \end{aligned} \tag{3}$$

(iii) The equation of heat conduction

$$\rho C_E (\dot{\Theta} + \tau_0 \ddot{\Theta}) + \beta_1 T_0 (\dot{e} + \tau_0 \ddot{e}) + a T_0 (\dot{C} + \tau_0 \ddot{C}) = K^* \Theta_{,ii} , \tag{4}$$

(iv) The equation of mass diffusion

$$D\,\beta_2\, e_{,ii} + Da\Theta_{,ii} + (\dot{C} + \tau \ddot{C}) - DbC_{,ii} = 0 , \tag{5}$$

where

$$\beta_1 = (3\lambda+2\mu)\,\alpha_t , \quad \beta_2 = (3\lambda+2\mu)\,\alpha_c ,$$
$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) .$$

Now we consider temperature-dependent mechanical properties. For that we assume,

$$\begin{aligned} &E = E_0 f(T) , \quad \lambda = E_0 \lambda_0 f(T) , \quad \mu = E_0 \mu_0 f(T) , \\ &\beta_1 = \beta_{10} E_0 f(T) , \quad \beta_2 = \beta_{20} E_0 f(T) , \end{aligned} \tag{6}$$

where $\lambda_0 = \nu/(1+\nu)(1-2\nu)$, $\mu_0 = 1/2(1+\nu)$

We also consider the following non-dimensional variables

$$\begin{aligned} &x^* = C_0 \eta_0 x , \quad z^* = C_0 \eta_0 z , \quad u_i^* = C_0 \eta_0 u_i , \\ &t^* = C_0^2 \eta_0 t , \quad \tau^* = C_0^2 \eta_0 \tau , \quad \tau_0^* = C_0^2 \eta_0 \tau_0 , \\ &T^* = \frac{\beta_{10} E_0 \Theta}{\rho C_o^2} , \quad C^* = \frac{\beta_{20} E_0 C}{\rho C_o^2} , \quad \sigma_{ij}^* = \frac{\sigma_{ij}}{\rho C_o^2} , \\ &P^* = \frac{P}{\beta_{20} E_0} , \quad \mathbf{\Omega}^* = \frac{\mathbf{\Omega}}{\eta_0 C^2} , \quad C_o^2 = E_0(\lambda_0 + 2\mu_0)/\rho , \\ &\eta_0 = \rho C_E / K^* . \end{aligned} \tag{7}$$

With the help of relations given by (6) and (7) and after dropping the asters over the non-dimensional variables for convenience, the Eqs. (1) to (5) reduce to the following equations

$$\sigma_{ij} = \left[(2\beta-1)e\delta_{ij} + (1-\beta)(u_{i,j} + u_{j,i}) - T\delta_{ij} - C\delta_{ij}\right] f(T) , \tag{8}$$

$$\mathrm{P}^* = -f(T)e + \alpha_3 C - \alpha_1 T , \tag{9}$$

$$\begin{aligned} &\left[\ddot{\mathbf{u}}_i + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}) + 2\mathbf{\Omega} \times \dot{\mathbf{u}}\right] \\ &= \left[\beta e_{,i} + (1-\beta)\nabla^2 u_i - T_{,i} - C_{,i}\right] f(T) + \left[(2\beta-1)e f_{,i}\right. \\ &\left. + (1-\beta)(u_{i,j} + u_{j,i}) f_{,j} - T f_{,i} - C f_{,i}\right] , \end{aligned} \tag{10}$$

$$\nabla^2 T = (\dot{T} + \tau_0 \ddot{T}) + \varepsilon\delta_0 f(T)(\dot{e} + \tau_0 \ddot{e}) + \varepsilon\delta_0 \alpha_1 (\dot{C} + \tau_0 \ddot{C}) , \tag{11}$$

$$f(T) D^2 e + \alpha_1 D^2 T + \alpha_2 (\dot{C} + \tau \ddot{C}) - \alpha_3 D^2 C = 0 , \tag{12}$$

where,

$$\begin{aligned} &\alpha_1 = \frac{a\rho C_0^2}{E_0^2 \beta_{10} \beta_{20}} , \quad \alpha_2 = \frac{\rho C_0^2}{D E_0^2 \beta_{20}^2 \eta_0} , \\ &\alpha_3 = \frac{b\rho C_0^2}{E_0^2 \beta_{20}^2} , \quad \varepsilon = \frac{\beta_{10} E_0}{\rho C_E} , \\ &\delta_0 = \frac{\beta_{10} E_0 T_0}{\rho C_0^2} , \quad \beta = E_0(\lambda_0 + \mu_0)/\rho C_0^2 = 1/2(1-\nu) . \end{aligned}$$

Now, we assume that the modulus of elasticity as a linear function of reference temperature T_0. Therefore, we take $f(T) = (1 - \alpha^* T_0)$ in further analysis. We also restrict our study to x-z plane with $\boldsymbol{u}$ = (u_1, 0, u_2) and $\mathbf{\Omega}$ = (0, Ω, 0). Making use of the following displacement components u_1 and u_3 in terms of scalar potentials ϕ and ψ

$$u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z} , \quad u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} ,$$

the Eqs. (8) to (12) become

$$\sigma_{zz} = \frac{1}{\alpha}\left[\frac{\partial^2 \phi}{\partial z^2} + (2\beta-1)\frac{\partial^2 \phi}{\partial x^2} - 2(\beta-1)\frac{\partial^2 \psi}{\partial x \partial z} - T - C\right] , \tag{13}$$

$$\sigma_{zx} = \frac{(1-\beta)}{\alpha}\left[2\frac{\partial^2 \phi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2}\right] , \tag{14}$$

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$$\left(\alpha\frac{\partial^2}{\partial t^2}-\alpha\Omega^2-\nabla^2\right)\phi+T+C+\left(2\alpha\Omega\frac{\partial}{\partial t}\right)\psi=0\ , \quad (15)$$

$$\left(2\alpha\Omega\frac{\partial}{\partial t}\right)\phi+\left[(1-\beta)\nabla^2+\alpha\Omega^2-\frac{\partial^2}{\partial t^2}\right]\psi=0\ , \quad (16)$$

$$\nabla^2 T=\tau_m\frac{\partial T}{\partial t}+\varepsilon_1\,\tau_m\frac{\partial(\nabla^2\phi)}{\partial t}+\varepsilon_2\,\tau_m\frac{\partial C}{\partial t}\ , \quad (17)$$

$$\nabla^4\phi+\alpha\alpha_1\nabla^2 T+\alpha\alpha_2\,\tau_n\frac{\partial C}{\partial t}-\alpha\alpha_3\nabla^2 C=0\ , \quad (18)$$

where $\alpha=1/(1-\alpha^* T_0)$, $\tau_m=\left(1+\tau_0\frac{\partial}{\partial t}\right)$ and $\tau_n=\left(1+\tau\frac{\partial}{\partial t}\right)$.

The Eqs. (15) to (18) are coupled in ϕ, T, C and ψ. For a harmonic wave propagating in x- direction, where wave normal lies in x-z plane, and makes an angle θ with the negative z-axis, we consider the solution of the system of Eqs. (15) to (18) in the following form

$$\{\phi\ ,\ T\ ,\ C\ ,\ \Psi\}=\{\bar{\phi}_0\ ,\ \bar{T}_0\ ,\ \bar{C}_0\ ,\ \bar{\Psi}_0\}\ \exp\left[ik(x\ \sin\theta+z\ \cos\theta-Vt)\right]\ . \quad (19)$$

Making use of Eq. (19) in Eqs. (15) to (18), we obtain a system of four homogeneous equations in $\bar{\phi}_0$, $\bar{T}_0$, $\bar{C}_0$ and $\bar{\psi}_0$, which admits a non-trivial solution when ξ satisfies the following bi-quadratic equation

$$J\ \xi^4+L\xi^3+M\xi^2+N\xi+O=0\ , \quad (20)$$

where,

$$\xi=\mathrm{V}^2\ ,$$

$$J=\left[4\frac{\Omega^2}{\omega^2}-\Omega_0^2\right]p_2\ ,$$

$$L=\left(\Omega_0^2-4\frac{\Omega^2}{\omega^2}\right)\bar{p}(p_1\,\varepsilon_2+p_2+p_3)+\Omega_0\ p_2(\varepsilon_1-\beta+2)\ ,$$

$$M=-\left[\bar{P}^2 P_3\left(\Omega_0^2-4\frac{\Omega^2}{\omega^2}\right)+(1-\beta)p_2(\varepsilon_1+1)+\bar{p}\Omega_0\left[p_1\{\varepsilon_1+(2-\beta)\,\varepsilon_2\}+p_2(2-\beta)+p_3\left(\varepsilon_1+(2-\beta)+p_4(\varepsilon_2-1)/\bar{p}\right)\right]\right]\ ,$$

$$N=(1-\beta)\bar{p}\left[p_1(\varepsilon_1+\varepsilon_2)+p_2+p_3(\varepsilon_1+1)+p_4(\varepsilon_2-1)/\bar{p}\right]+\bar{p}^2\Omega_0\left[p_3(2-\beta)+p_4/\bar{p}\right]\ ,$$

$$O=-(1-\beta)\bar{p}^2(p_3-p_4/\bar{p})\ ,$$

$$d_1=\frac{K}{C_E\,\tau_m^1}\ ,\ d_2=\frac{\rho Db}{\tau_n^1}\ ,\ d_3=\frac{Da^2T_0}{C_E\,\tau_n^1}\ ,$$

$$d_4=\frac{\rho Da}{\tau_n^1}\ ,\ \varepsilon_1=\frac{\beta_1\,\beta_2\,T_0}{\rho\,C_E}\ ,\ \varepsilon_2=\frac{\rho D\beta_2^2}{\tau_n^1}\ ,$$

$$\tau_m^1=\frac{i}{\omega}+\tau_0\ ,\ \tau_n^1=\frac{i}{\omega}+\tau\ ,\ \bar{p}=\frac{\alpha}{\tau_m^1}\ ,$$

$$p_1=\frac{\alpha_1}{\tau_n^1}\ ,\ p_2=\frac{\alpha_2}{\tau_m^1}\ ,\ p_3=\frac{\alpha_3}{\tau_n^1}\ ,$$

$$p_4=\frac{1}{\tau_m^1\,\tau_n^1}\ ,\ \Omega_0=1+\frac{\Omega^2}{\omega^2}\ .$$

Equation (20) is a bi-quadratic equation in ξ. Each root corresponds to a wave velocity if V^2 is real and positive. The four values of ξ correspond to four coupled plane waves P_1, P_2, P_3 and P_4 propagating with velocities V_1, V_2, V_3 and V_4, respectively. Numerical computations of speeds for a particular material (Aluminum) show that P_1 wave is fastest.

3. BOUNDARY CONDITIONS AND REFLECTION COEFFICIENTS

We consider the stress-free surface of a thermoelastic solid half-space with diffusion as shown in Fig. 1. For an incident coupled plane wave P_1, we get four reflected coupled plane waves P_1, P_2, P_3 and P_4 in the thermoelastic solid half-space. The appropriate displacement potentials ϕ and ψ, temperature T and concentration C which satisfy the boundary conditions at thermally insulated free surface are taken in the following form

$$\begin{aligned}\phi=\ & A_0\exp\left[ik_1(x\ \sin\theta_0+z\ \cos\theta_0-V_1 t)\right]\\ &+A_1\exp\left[ik_1(x\ \sin\theta_1-z\ \cos\theta_1-V_1 t)\right]\\ &+A_2\exp\left[ik_2(x\ \sin\theta_2-z\ \cos\theta_2-V_2 t)\right]\\ &+A_3\exp\left[ik_3(x\ \sin\theta_3-z\ \cos\theta_3-V_3 t)\right]\\ &+A_4\exp\left[ik_4(x\ \sin\theta_4-z\ \cos\theta_4-V_4 t)\right]\ ,\end{aligned} \quad (21)$$

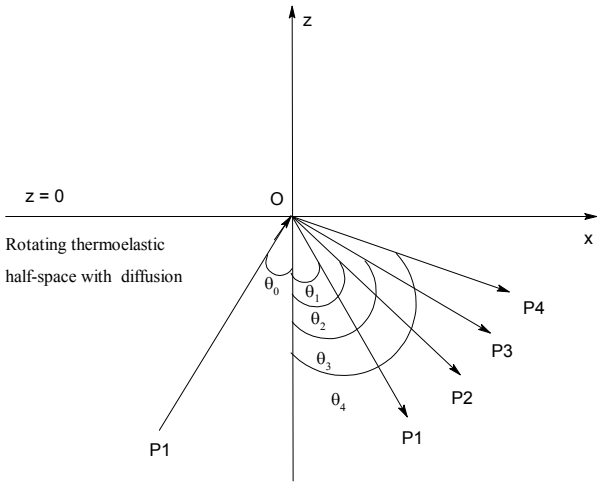


Fig. 1 Geometry of the problem showing incident and reflected waves

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$$
\begin{aligned}
T = {} & \varsigma_1 A_0 \exp\left[ik_1(x \sin\theta_0 + z \cos\theta_0 - V_1 t)\right] \\
& + \varsigma_1 A_1 \exp\left[ik_1(x \sin\theta_1 - z \cos\theta_1 - V_1 t)\right] \\
& + \varsigma_2 A_2 \exp\left[ik_2(x \sin\theta_2 - z \cos\theta_2 - V_2 t)\right] \\
& + \varsigma_3 A_3 \exp\left[ik_3(x \sin\theta_3 - z \cos\theta_3 - V_3 t)\right] \\
& + \varsigma_4 A_4 \exp\left[ik_4(x \sin\theta_4 - z \cos\theta_4 - V_4 t)\right] , \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
C = {} & \eta_1 A_0 \exp\left[ik_1(x \sin\theta_0 + z \cos\theta_0 - V_1 t)\right] \\
& + \eta_1 A_1 \exp\left[ik_1(x \sin\theta_1 - z \cos\theta_1 - V_1 t)\right] \\
& + \eta_2 A_2 \exp\left[ik_2(x \sin\theta_2 - z \cos\theta_2 - V_2 t)\right] \\
& + \eta_3 A_3 \exp\left[ik_3(x \sin\theta_3 - z \cos\theta_3 - V_3 t)\right] \\
& + \eta_4 A_4 \exp\left[ik_4(x \sin\theta_4 - z \cos\theta_4 - V_4 t)\right] , \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
\psi = {} & \chi_1 A_0 \exp\left[ik_1(x \sin\theta_0 + z \cos\theta_0 - V_1 t)\right] \\
& + \chi_1 A_1 \exp\left[ik_1(x \sin\theta_1 - z \cos\theta_1 - V_1 t)\right] \\
& + \chi_2 A_2 \exp\left[ik_2(x \sin\theta_2 - z \cos\theta_2 - V_2 t)\right] \\
& + \chi_3 A_3 \exp\left[ik_3(x \sin\theta_3 - z \cos\theta_3 - V_3 t)\right] \\
& + \chi_4 A_4 \exp\left[ik_4(x \sin\theta_4 - z \cos\theta_4 - V_4 t)\right] , \qquad (24)
\end{aligned}
$$

where wave normal of the incident wave P_1 makes an angle θ_0 with the negative direction of z- axis and reflected waves P_1, P_2, P_3 and P_4 make angles θ_1, θ_2, θ_3 and θ_4 with negative z- axis, and

$$
\varsigma_i = \left[\frac{\xi_i(\varepsilon_2\, p_4 + \varepsilon_1\, \bar{p} p_3 - \varepsilon_1\, \xi_i\, p_2)}{\xi_i(\varepsilon_2\, \bar{p} p_1 + \bar{p} p_3 - \xi_i\, p_2 + \bar{p} p_2) - \bar{p}^2 p_3} \right] ,
$$

$$
\eta_i = \left[\frac{-\xi_i\, p_4 + \bar{p} p_4 + \varepsilon\, \bar{p} p_1\, \xi_i}{\xi_i(\varepsilon_2\, \bar{p} p_1 + \bar{p} p_3 - \xi_i\, p_2 + \bar{p} p_2) - \bar{p}^2 p_3} \right] ,
$$

$$
\chi_i = -\frac{2i\xi_i \dfrac{\Omega}{\omega^2}}{\Omega_0\, \xi_i - (1-\beta)} , \quad \omega_i = k_i\, v_i , \quad (i = 1,\ 2,\ 3,\ 4) . \qquad (25)
$$

The required boundary conditions at free surface $z = 0$ are the vanishing of normal and tangential stresses and vanishing of heat flux and mass flux across the free surface, *i.e.*

$$
\sigma_{zz} = 0 , \quad \sigma_{zx} = 0 , \quad \frac{\partial T}{\partial z} = 0 , \quad \frac{\partial P}{\partial z} = 0 , \qquad (26)
$$

where

$$
\sigma_{zz} = \frac{1}{\alpha}\left[\frac{\partial^2 \phi}{\partial z^2} + (2\beta - 1)\frac{\partial^2 \phi}{\partial x^2} - 2(\beta - 1)\frac{\partial^2 \psi}{\partial x \partial z} - T - C \right] ,
$$

$$
\sigma_{zx} = \frac{(1-\beta)}{\alpha}\left[2\frac{\partial^2 \phi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right] ,
$$

$$
\frac{\partial P}{\partial z} = -\left(\frac{\partial^3 \phi}{\partial x^2 \partial z} + \frac{\partial^3 \phi}{\partial x^3} \right) - \alpha\alpha_1 \frac{\partial T}{\partial z} + \alpha\alpha_3 \frac{\partial C}{\partial z} .
$$

In order to satisfy the boundary conditions (26), the wave number k_1, k_2, k_3, k_4 and the incident and reflected angles are connected by the relation

$$
k_1 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3 = k_4 \sin\theta_4 , \qquad (27)
$$

which can also be written as (extended Snell's law)

$$
\frac{\sin\theta_0}{V_1} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\theta_3}{V_3} = \frac{\sin\theta_4}{V_4} , \qquad (28)
$$

In view of the relation (27), the potentials given by the Eqs. (21) ~ (24) satisfy the boundary condition (26) to obtain the following system of four non-homogenous equations as

$$
\Sigma\, a_{ij}\, Z_j = b_i ,\ Z_j = \frac{A_j}{A_0} , \quad (i, j = 1,\ 2,\ 3,\ 4) , \qquad (29)
$$

where,

$$
a_{11} = \left[2\beta \sin^2\theta_1 + \cos 2\theta_1 + (\beta - 1)\chi_1 \sin 2\theta_1 + \frac{\varsigma_1}{k_1^2} + \frac{\eta_1}{k_1^2} \right] \left(\frac{k_1}{k_1} \right)^2 ,
$$

$$
a_{12} = \left[2\beta \sin^2\theta_2 + \cos 2\theta_2 + (\beta - 1)\chi_2 \sin 2\theta_2 + \frac{\varsigma_2}{k_2^2} + \frac{\eta_2}{k_2^2} \right] \left(\frac{k_2}{k_1} \right)^2 ,
$$

$$
a_{13} = \left[2\beta \sin^2\theta_3 + \cos 2\theta_3 + (\beta - 1)\chi_3 \sin 2\theta_3 + \frac{\varsigma_3}{k_3^2} + \frac{\eta_3}{k_3^2} \right] \left(\frac{k_3}{k_1} \right)^2 ,
$$

$$
a_{14} = \left[2\beta \sin^2\theta_4 + \cos 2\theta_4 + (\beta - 1)\chi_4 \sin 2\theta_4 + \frac{\varsigma_4}{k_4^2} + \frac{\eta_4}{k_4^2} \right] \left(\frac{k_4}{k_1} \right)^2 ,
$$

$$
a_{21} = \left[\sin 2\theta_1 + \chi_1 \cos 2\theta_1 \right] \left(\frac{k_1}{k_1} \right)^2 ,
$$

$$
a_{22} = \left[\sin 2\theta_2 + \chi_2 \cos 2\theta_2 \right] \left(\frac{k_2}{k_1} \right)^2 ,
$$

$$
a_{23} = \left[\sin 2\theta_3 + \chi_3 \cos 2\theta_3 \right] \left(\frac{k_3}{k_1} \right)^2 ,
$$

$$
a_{24} = \left[\sin 2\theta_4 + \chi_4 \cos 2\theta_4 \right] \left(\frac{k_4}{k_1} \right)^2 ,
$$

$$
a_{31} = \cos\theta_1 \frac{\varsigma_1}{k_1^2} \left(\frac{k_1}{k_1} \right)^3 ,
$$

$$
a_{32} = \cos\theta_2 \frac{\varsigma_2}{k_2^2} \left(\frac{k_2}{k_1} \right)^3 ,
$$

$$
a_{33} = \cos\theta_3 \frac{\varsigma_3}{k_3^2} \left(\frac{k_3}{k_1} \right)^3 ,
$$

$$
a_{34} = \cos\theta_4 \frac{\varsigma_4}{k_4^2} \left(\frac{k_4}{k_1} \right)^3 ,
$$

$$
a_{41} = \cos\theta_1 \left[1 - \alpha\alpha_1 \frac{\varsigma_1}{k_1^2} + \alpha\alpha_3 \frac{\eta_1}{k_1^2} \right] \left(\frac{k_1}{k_1} \right)^3 ,
$$

$$
a_{42} = \cos\theta_2 \left[1 - \alpha\alpha_1 \frac{\varsigma_2}{k_2^2} + \alpha\alpha_3 \frac{\eta_2}{k_2^2} \right] \left(\frac{k_2}{k_1} \right)^3 ,
$$

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$$a_{43} = \cos\theta_3 \left[1 - \alpha\alpha_1 \frac{\varsigma_3}{k_3^2} + \alpha\alpha_3 \frac{\eta_3}{k_3^2}\right]\left(\frac{k_3}{k_1}\right)^3 ,$$

$$a_{44} = \cos\theta_4 \left[1 - \alpha\alpha_1 \frac{\varsigma_4}{k_4^2} + \alpha\alpha_3 \frac{\eta_4}{k_4^2}\right]\left(\frac{k_4}{k_1}\right)^3 ,$$

$$b_1 = -\left[2\beta\sin^2\theta_0 + \cos 2\theta_0 - (\beta - 1)\chi_1 \sin 2\theta_0 + \frac{\varsigma_1}{k_1^2} + \frac{\eta_1}{k_1^2}\right]\left(\frac{k_1}{k_1}\right)^2 ,$$

$$b_2 = [\sin 2\theta_0 - \chi_1 \cos 2\theta_0]\left(\frac{k_1}{k_1}\right)^2 ,$$

$$b_3 = \cos\theta_0 \frac{\varsigma_1}{k_1^2}\left(\frac{k_1}{k_1}\right)^3 ,$$

$$b_4 = \cos\theta_0 \left[1 - \alpha\alpha_1 \frac{\varsigma_1}{k_1^2} + \alpha\alpha_3 \frac{\eta_1}{k_1^2}\right]\left(\frac{k_1}{k_1}\right)^3 ,$$

and $Z_1 = A_1 / A_0, Z_2 = A_2 / A_0, Z_3 = A_3 / A_0, Z_4 = A_4 / A_0$ are the reflection coefficients (ratio of the amplitudes of the reflected wave to the amplitude of the incident wave) of the reflected P_1, P_2, P_3 and P_4 waves.

4. ENERGY RATIOS

Following Achenbach [19], the rate of energy transmission per unit area denoted by P^* is given as

$$P^* = \tau_{zz}\, \dot{u}_3 + \tau_{zx}\, \dot{u}_1 . \tag{30}$$

The expression of energy ratios $E_i (i = 1, 2, 3, 4)$ of reflected P_1, P_2, P_3 and P_4 are obtained as

$$E_1 = Q\left[\begin{array}{l}\left\{-2\beta\sin^2\theta_1 - \cos 2\theta_1 - (\beta-1)\chi_1 \sin 2\theta_1 - \frac{\varsigma_1}{k_1^2} - \frac{\eta_1}{k_1^2}\right\}\{-\cos\theta_1 + \chi_1 \sin\theta_1\} \\ +(1-\beta)\{\chi_1 \cos 2\theta_1 + \sin 2\theta_1\}\{\sin\theta_1 + \chi_1 \cos\theta_1\}\end{array}\right]\left(\frac{k_1^4 v_1}{k_1^4 v_1}\right) Z_1^2 , \tag{31}$$

$$E_2 = Q\left[\begin{array}{l}\left\{-2\beta\sin^2\theta_2 - \cos 2\theta_2 - (\beta-1)\chi_2 \sin 2\theta_2 - \frac{\varsigma_2}{k_2^2} - \frac{\eta_2}{k_2^2}\right\}\{-\cos\theta_2 + \chi_2 \sin\theta_2\} \\ +(1-\beta)\{\chi_2 \cos 2\theta_2 + \sin 2\theta_2\}\{\sin\theta_2 + \chi_2 \cos\theta_2\}\end{array}\right]\left(\frac{k_2^4 v_2}{k_1^4 v_1}\right) Z_2^2 , \tag{32}$$

$$E_3 = Q\left[\begin{array}{l}\left\{-2\beta\sin^2\theta_3 - \cos 2\theta_3 - (\beta-1)\chi_3 \sin 2\theta_3 - \frac{\varsigma_3}{k_3^2} - \frac{\eta_3}{k_3^2}\right\}\{-\cos\theta_3 + \chi_3 \sin\theta_3\} \\ +(1-\beta)\{\chi_3 \cos 2\theta_3 + \sin 2\theta_3\}\{\sin\theta_3 + \chi_3 \cos\theta_3\}\end{array}\right]\left(\frac{k_3^4 v_3}{k_1^4 v_1}\right) Z_3^2 , \tag{33}$$

$$E_4 = Q\left[\begin{array}{l}\left\{-2\beta\sin^2\theta_4 - \cos 2\theta_4 - (\beta-1)\chi_4 \sin 2\theta_4 - \frac{\varsigma_4}{k_4^2} - \frac{\eta_4}{k_4^2}\right\}\{-\cos\theta_4 + \chi_4 \sin\theta_4\} \\ +(1-\beta)\{\chi_4 \cos 2\theta_4 + \sin 2\theta_4\}\{\sin\theta_4 + \chi_4 \cos\theta_4\}\end{array}\right]\left(\frac{k_4^4 v_4}{k_1^4 v_1}\right) Z_4^2 , \tag{34}$$

where,

$$Q = \left[\begin{array}{l}\left\{-2\beta\sin^2\theta_0 - \cos 2\theta_0 + (\beta-1)\chi_1 \sin 2\theta_0 - \frac{\varsigma_1}{k_1^2} - \frac{\eta_1}{k_1^2}\right\}\{\cos\theta_0 + \chi_1 \sin\theta_0\} \\ +(1-\beta)\{\chi_1 \cos 2\theta_0 - \sin 2\theta_0\}\{\sin\theta_0 - \chi_1 \cos\theta_0\}\end{array}\right]^{-1} ,$$

and each energy ratios $E_i (i = 1, 2, 3, 4)$ give the rate of energy transmission at the free plane surface for the respective reflected wave to the rate of energy transmission for an incident set of coupled P_1, P_2, P_3 and P_4 waves.

5. NUMERICAL RESULTS AND DISCUSSION

The absolute values of the reflection coefficients and the energy ratios are computed numerically for the following physical constants of the Aluminum metal at T_0=300 K [20]

$\lambda = 5.775\times10^{10}$ N m^{-2} , $\mu = 2.646\times10^{10}$ N m^{-2} ,
$\rho = 2700$ Kg m^{-3} ,
$C_E = 9.874\times10^{3}$ J Kg^{-1}K^{-1}, $K^* = 2.06\times10^{2}$ J s^{-1} m^{-1}K^{-1} ,
$\tau_0 = 0.05$ s , $\tau = 0.04$ s , $\alpha_t = 0.05$ K^{-1} ,
$\alpha_c = 0.06$ m^{3}K^{-1} , $\omega = 2$ s^{-1} , $a = 0.005$ m^{2}s^{-2}K^{-1} ,
$b = 0.05$ m^{5}s^{-2}K^{-1} , $D = 0.5$ Kgsm^{-3} .

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With the help of a Fortran program of Gauss elimination method, the reflection coefficients of various reflected waves are obtained numerically from the system of Eq. (29).

The absolute values of the reflection coefficients of reflected P_1, P_2, P_3 and P_4 waves are computed and shown graphically for the range $0° \le \theta_0 \le 90°$ of the angle of incidence of P_1 wave, when $\Omega/\omega = 5$, 10 and 15 in Fig. 2. Here, the solid and dashed curves correspond to variations for $\alpha^* = 0.005$/K and $\alpha^* = 0$, respectively. For $\Omega/\omega = 5$, the reflection coefficients of P_1 wave decrease in values due to the temperature dependence of material constants. For $\Omega/\omega = 10$ and 15, the absolute values of reflection coefficients of P_1 wave changes due to the temperature dependence of material constants. For $\Omega/\omega = 5$, 10 and 15, the reflection coefficients of P_2, P_3 and P_4 waves decrease due to the temperature dependence of material constants. For grazing incidence, there is no effect of temperature dependence on the reflection coefficients of the reflected waves. The comparison among solid curves in Fig. 2 shows the effect of rotation on the reflection coefficients of reflected P waves when $\alpha^* = 0.005$/K. However, this rotational effect on the reflected waves is different from that for $\alpha^* = 0$.

The energy ratios of reflected P_1, P_2, P_3 and P_4 waves given by Eqs. (31) to (34) are computed numerically and shown graphically for the range $0° \le \theta_0 \le 90°$ of the angle of incidence of P_1 wave, when $\Omega/\omega = 5$, 10 and 15. in Fig. 3. The solid and dashed curves in Fig. 3 correspond to the variations for $\alpha^* = 0.005$/K and $\alpha^* = 0$, respectively. For each value of Ω/ω, the sum of energy ratios remains one or less at each angle of incidence. The energy is not conserved at some of angles of incidence. This may be due to the application of Lord-Shulman theory of generalized thermoelasticity in this problem which allows the dissipation of heat and diffusion. The arbitrary choice of the diffusion and thermodiffusion constants may be another reason for obtaining such energy ratios. The variations of the energy ratios of reflected waves are similar to those for respective reflection coefficients. The energy ratios of all reflected waves are affected significantly by rotation parameter and the temperature dependence of material constants. It is also observed that the incident angle has not reached the critical angle (incident angle of an elastic wave beyond which a specified refracted wave or mode converted reflected wave no longer exists.) and hence no surface wave occur.

5. CONCLUSIONS

The plane harmonic solution of the governing equations leads to the existence of four coupled plane waves in a rotating temperature-dependent thermoelastic material with diffusion. The reflection of these plane waves from a thermally insulated free surface is studied. The relations between the reflection coefficients and the expressions for energy ratios of reflected waves are obtained for an obliquely incident wave. Numerical results show that the reflection coefficients and energy ratios of the reflected waves are affected significantly

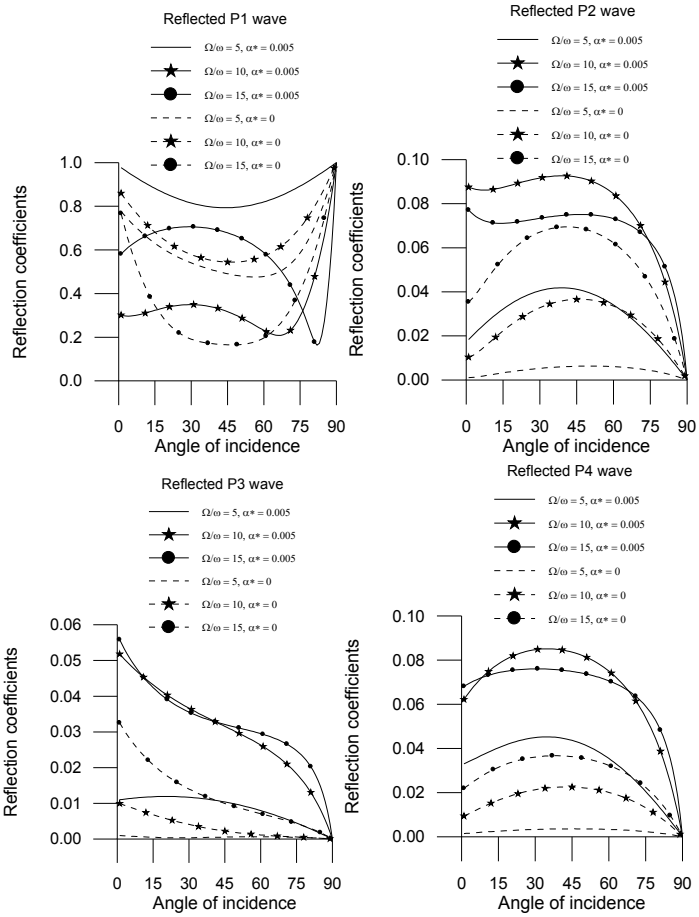


Fig. 2 Variations of the reflection coefficients of the reflected waves with the angle of incidence θ_0

by the rotation parameter and the temperature-dependence of material constants. The present theoretical results will provide interesting information for experimental seismologists and researchers working on problems in the area of thermoelasticity.

NOMENCLATURES

λ, μ	Lame's constants.
α_t	Coefficient of linear thermal expansion.
α_c	Coefficient of linear diffusion expansion.
T	Absolute temperature.
T_0	Reference temperature.
Θ	$T - T_0$, the change in temperature such that $\lvert\Theta/T_0\rvert << 1$.
σ_{ij}	Components of stress tensor.
e_{ij}	Components of strain tensor.
u_i	Components of displacement vector.
ρ	Density.
S	Entropy per unit mass.
P^*	Chemical potential per unit mass.
C	Mass concentration.
C_E	Specific heat at constant strain.
K	Coefficient of thermal conductivity.
D	Thermo-diffusion constant.
$E(T)$	Temperature-dependent modulus of elasticity.
$f(T)$	Non-dimensional function of temperature.
α^*	Empirical material constant [1/K].

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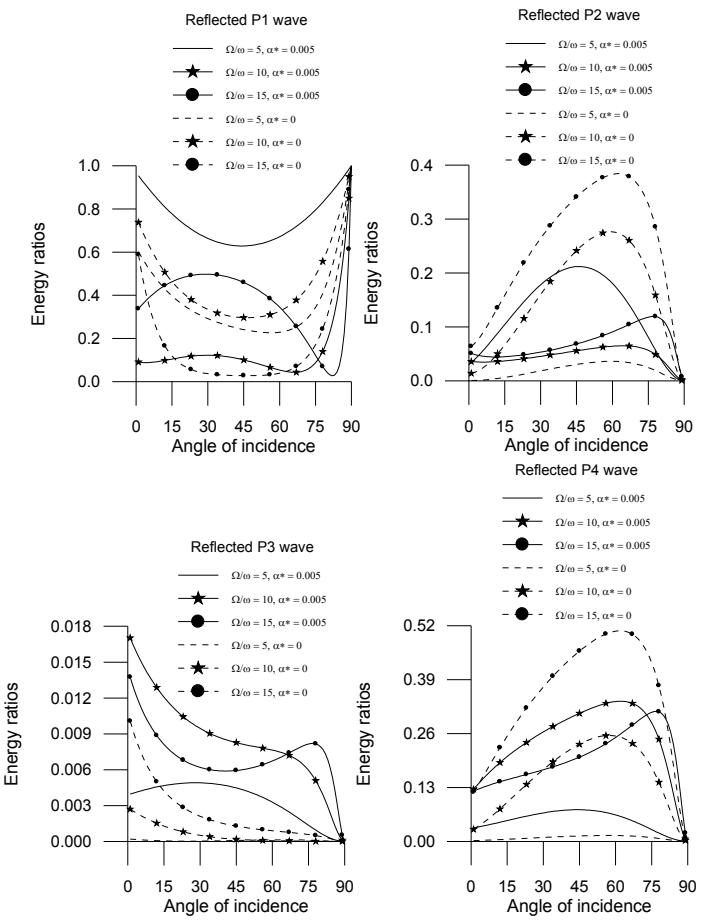


Fig. 3 Variations of the energy ratios of the reflected waves with the angle of incidence θ_0

E_0 Constant of modulus of elasticity at $\alpha^* = 0$.
ν Possion's ratio.
τ_0 Thermal relaxation time.
τ Diffusion relaxation time.
A Constant to measure the thermo-diffusion effects.
B Constant to measure the diffusive effects.
ϕ, ψ Scalar potentials
δ_{ij} Kronecker delta.
k Wave number.
V Phase speed.

The assumption $|\Theta/T_0| << 1$ in the nomenclature is made according to Lord and Shulman [1] theory and Sherief *et al.* [11] theory to obtain the significant effect of relaxation time τ, so that energy equation predicts a wave type phenomena.

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